Surreal Pontryagin classes

Manuel Krannich

January 24, 2024

Pontryagin classes were originally considered as invariants of real vector bundles, but thanks to work of Novikov, Sullivan, and Kirby–Siebenmann it was realised in the 60s that these invariants in fact do not depend on the linear structure on the fibres and can be defined more generally for Euclidean fibre bundles, that is, fibre bundles whose fibres are homeomorphic to Euclidean space. This led to the question whether the well-known fact that the kth Pontryagin class of a d-dimensional vector bundle vanishes for k > d/2 continues to hold in the setting of Euclidean fibre bundles. To the surprise of many, Michael Weiss proved in 2015 that it does not. I will speak about this result of Michaels and about some of the subsequent developments in high-dimensional geometric topology that were inspired by his proof.