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„Distance estimates in the spin setting and the positive mass theorem.“

Abstract:

The positive mass theorem states that a complete asymptotically Euclidean manifold of nonnegative scalar curvature has nonnegative ADM mass. It relates quantities that are defined using geometric information localized in the Euclidean ends (the ADM mass) with global geometric information on the ambient manifold (the nonnegativity of the scalar curvature). It is natural to ask whether the positive mass theorem can be ``localized'', that is, whether the nonnegativity of the ADM mass of a single asymptotically Euclidean end can be deduced by the nonnegativity of the scalar curvature in a suitable neighborhood of E.

I will present the following localized version of the positive mass theorem in the spin setting. Let E be an asymptotically Euclidean end in a connected Riemannian spin manifold (M,g). If E has negative ADM-mass, then there exists a constant R > 0, depending only on the geometry of E, such that M must either become incomplete or have a point of negative scalar curvature in the R-neighborhood around E in M. This gives a quantitative answer, for spin manifolds, to Schoen and Yau's question on the positive mass theorem with arbitrary ends. Similar results have recently been obtained by Lesourd, Unger, and Yau without the spin condition in dimensions <8 assuming Schwarzschild asymptotics on the end E. I will also present explicit quantitative distance estimates in case the scalar curvature is uniformly positive in some region of the chosen end E. The bounds obtained are reminiscent of Gromov's metric inequalities with scalar curvature. This is joint work with Rudolf Zeidler.