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Thomas Schick (Universität Göttingen)

„Rigidity of scalar curvature."

Abstract:

A famous and old problem at the interface between topology and geometry (differentiable versions) is:
how does a given topological type constrain the possible geometric shapes.

A prototypical result in this direction is the Gauss-Bonnet theorem: it implies that the only closed orientable surface which carries a metric with everywhere positive (scalar) curvature is the sphere.

The next level of question then asks: how round can we make the sphere. Here, a fundamental result of Llarull says:
if we pick a smooth Riemannian metric g on the n-dimensional sphere such that no tangent vector is in this metric shorter than in the standard round metric, and if the scalar curvature is everywhere >= the one for the standard metric (=n(n-1)) then the metric g is already the standard metric.
In other words: the standard metric is extremal/rigid when it comes to positive scalar content.

In the talk, we will discuss generalizations and related results. In particular, we will advertise the methods (spectral and index theory of Dirac opeators), and we will learn about low regularity versions (the latter achieved in joint work with Cecchini and Hanke).