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„eta-periodic motivic stable homotopy theory.”

Abstract:

The classical Hopf map can be described as the quotient map C^2-0 -> CP^1. Stably, this defines the well-known "first Hopf element" S^1 -> S^0 in the first stable stem. It is, of course, nilpotent.

In the world of algebraic varieties, there is a geometric incarnation of the Hopf map, just given by the quotient map A^2-0 -> P^1, which defines the (first) motivic stable Hopf element eta: Gm -> S^0. Perhaps surprisingly, this is *not* nilpotent: Morel has shown that the endomorphism ring of the eta-periodic sphere over the field k is given by the Witt ring W(k) of symmetric bilinear forms over k.

The eta-periodic sphere has since been studied by various authors. I will report on work with Mike Hopkins, which relates (over fields of characteristic different from 2) the eta-periodic sphere to the Hermitian K-theory spectrum. This is curiously similar to the classical relationship between the K(1)-local sphere and the complex K-theory spectrum. As applications we can determine all homotopy groups of the eta-periodic sphere (settling a conjecture of Ormsby-Röndigs), as well as the eta-periodized algebraic special linear and symplectic cobordism groups.