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„Fredholm operators as classifying spaces for K-theory."

Abstract:
A classical theorem of Atiyah and Jänich says that the space Fred(H) of Fredholm operators on a separable infinite dimensional Hilbert space H is a "classifying spaces" for K-theory. Concretely, there is a canonical bijection between homotopy classes of maps [X,Fred(H)] and K^0(X), the Grothendieck group of isomorphism classes of vector bundles. Here, X has to be a reasonable space (CW complex works well).
An the Fredholm operators are an open subset of the space of bounded operators B(H) with the usual norm topology.

Segal extended this result to equivariant K-theory for actions of a compact group G.

In the talk, we will:
*oresebt the main ideas of a proof of the Atiyah-Jänich-Segal theorem (using operator K-theory) *discuss the role of the norm topology, it turns out that other topologies naturally come up *discuss improvements: how to make the model for the classifying space even nicer
*discuss the extension to only locally compact (second countable Hausdorff) groups, eg Lie groups like SL_n(R) or p-adic gropus like SL_n(Q_p) *indicate applications of a nice model for the classifying space of equivariant K-theory in this situation

(the novel parts are joint work with Paul Baum and Anne Prepeneit)