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„Periodicity and stability in mapping class groups and other E_2 algebras."

Abstract:
Let $\Gamma_{g,1}$ denote the mapping class group of a genus g surface with one parametrized boundary component, and $B\Gamma_{g,1}$ its classifying space. Boundary connected sum can be used to define a product on the chains $R = \oplus_g C_*(B\Gamma_{g,1}; k)$, making it into a bigraded DGA. Two famous theorems may be phrased in terms of this bigraded DGA and an element $\sigma \in C_0(B\Gamma_{1,1}; k)$ representing the point class: Harer's homological stability theorem asserts that the quotient R/σ is acyclic in a range of bidegrees, while Madsen–Weiss' theorem calculates the localization $R[\sigma^{-1}]$. I will argue that a similar point of view may be taken on R/σ and a graded self-map $\kappa : R/\sigma \to R/\sigma$ of bidegree (3,2), replacing R and σ. We prove that the quotient $R/(\sigma, \kappa)$ is acyclic in a larger range of bidegrees, and make preliminary calculations concerning the localization $(R/\sigma)[\kappa^{-1}]$. Time permitting, I will report on a similar approach to general linear groups of fields and local rings. All is joint work with Alexander Kupers and Oscar Randal-Williams.