Equivariant $\mathcal{D}$-modules on rigid analytic spaces

Programm Oberseminar SS 2018

Talk 1: An overview of Beilinson-Bernstein localization

See [A-Tor] sections 1 and 2 for a summary. Discuss [A] Section 5.1 (up to and including Lemma 5.1.5): it explains precisely the meaning of the "infinitesimal action" over general commutative rings. Then explain [AW] section 4. At the end verify rigorously the calculations on pages 3 and 4 of [A-Tor].

Talks 2, 3, 4: Separating the Lie algebra from the group inside the locally analytic distribution algebra

GOAL: Understand [A] Theorem 6.5.1. It may be useful to see the particular presentation of $D(N, K)$ as a Frechet-Stein algebra as it serves as a leitmotif for the main constructions of Section 3.

2: Sections 2.1 and 2.2 on trivializations of skew-group rings. This language is fundamental to the whole paper. 6.5.11 on Frommer's/Schmidt's identification of $D_{r_n}(N, K)$ as a crossed product written in Ardakov's language of trivializations.

3: 6.5.3 on the Lazard isomorphism. 6.2.1 - 6.2.6 on trivializations of affinoid enveloping algebras.

4: 6.5.8 on the Lazard isomorphism for $D_{r_n}(N, K)$. 6.2.7 (read the start of section 3.3 in parallel), 6.5.9, and 6.5.12.

The 3 speakers should decide whether they can save time and include a discussion of the extension from uniform groups N to arbitrary p-adic Lie groups G: Section 6.1 leading up to Example 6.1.17 (can be skipped if willing to take associativity and well-definition of $\widehat{U}(\mathfrak{g}, G)$ on trust). 6.2.11, 6.2.12, and proof of 6.5.1.

Talks 5, 6, 7: Coadmissible equivariant $\mathcal{D}$-modules

GOAL: Understand [A] Definition 3.6.7. Mainly run through 3.1 until 3.6.7. Some things to highlight:

5: Section 3.1: 3.1.5 (maybe skip proof), 3.1.8, and 3.1.12. Section 3.2: 3.2.5, 3.2.11, 3.2.12 (very important), and 3.2.15.

6: 3.3.1 (central definition), 3.3.4, 3.3.12, 3.4.4, 3.4.8 (Frechet-Stein) (postpone proof), 3.4.9 (formalism required for defining the localization functor), and 3.4.10.

7: Section 2.3: Grothendieck's language of equivariant modules. 3.5.1, 3.5.3, 3.5.5, 3.5.8, 3.5.12, 3.5.11 (Tate) (postpone proof), all of 3.6.1 - 3.6.7, and 3.7.8.

Talks 8, 9, 10: Main properties and proofs

GOAL: State and prove 4 main technical theorems:

8: [A] Theorem 3.4.8 (Frechet-Stein) with proof in section 4.1.

9,10: [A] Theorems 3.5.11 (Tate), 3.6.11 (Kiehl), and 3.7.1 (flatness) with proofs in sections 4.2 - 4.4. (**The speakers have to divide it up suitably.**)

Talks 11, 12: Localization for $D(G, K)$

GOAL: Understand the proof of [A] Theorem C/Theorem 6.4.8.

11: Section 5.2 (in particular, 5.2.5 and 5.2.10). Then 6.4.2 and 6.4.4 (needs all of section 6.3 which may be skipped).

12: Sections 5.3 and 5.4, and proof of 6.4.8.

References

[A] Ardakov K.: *Equivariant $\mathcal{D}$-modules on rigid analytic spaces*. Preprint 2017

[A-Tor] Ardakov K.: *Notes of talk at Toronto.*

[AW] Ardakov K., Wadsley S.: *On irreducible representations of compact p-adic analytic groups*. Ann. Math. 178, 453 - 557 (2013)