

SELECTED TOPICS IN DIFFERENTIAL GEOMETRY

Sheet 7

Due on **22 June 2023**

Exercise 1: Sum of Negative Scalar Curvature Metrics

Find a Riemannian manifold M and two negative scalar curvature metrics g, g' on M , such that the sum $g + g'$ does not have negative scalar curvature everywhere.

Exercise 2: Gluing Totally Geodesic Boundaries

Given two Riemannian manifolds M and N with totally geodesic boundaries, we want to glue them along an isometry $\varphi : \partial M \rightarrow \partial N$.

- (a) Realize that there is a canonical way to obtain a smooth structure on the resulting space $M \cup_{\varphi} N$.
- (b) Show that the induced metric on $M \cup_{\varphi} N$ is C^2 . You can e.g. use Fermi coordinates.
- (c) Find an example where the induced metric is not C^3 .
- (d) Argue that the induced metric can be approximated by a C^{∞} metric agreeing with the original metric outside a small neighborhood of the gluing area and still satisfying open curvature bounds that are true on M and N , such as $\text{Scal} > 0$ or $\text{Ric} < 0$.

Exercise 3: Fréchet Manifolds

Show that the space $S^-(M)$ of negative scalar curvature metrics on a closed manifold M , endowed with the C^{∞} topology, is a Fréchet manifold.

Hint: The main part is to figure out what a Fréchet manifold is.