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SoSe22

Geometry of Scalar Curvature

Sheet 5

For discussion on **20 May 2022**

Exercise 1: Minimal Surfaces and Positive Scalar Curvature

In lecture, you have seen that on a compact stably minimal hypersurface of dimension $n \geq 3$ in a Riemannian manifold of dimension $n+1$ with $\mathrm{Scal} > 0$, there is again a conformally equivalent metric with $\mathrm{Scal} > 0$. The argument does not work for $n = 2$ (i.e., a 2-dimensional minimal surface in a 3-manifold) because there is a term $u^{\frac{4}{n-2}}$.

Find a way to prove the assertion for $n = 2$.

Exercise 2: Small Deformation

Prove that for a 1-parameter family of Riemannian metrics g_t with $g = g_0$ and $h = \frac{\mathrm{d}}{\mathrm{d}t}\big|_{t=0} g_t$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\bigg|_{t=0} \mathrm{Scal}^{g_t} = -\Delta^g(\mathrm{tr}^g\, h) + \mathrm{div}^g(\mathrm{div}^g\, h) - g(\mathrm{Ric}^g, h).$$

You can start with a Taylor expansion around $t = 0$ and keep only terms linear in t.

If in struggle, you may consult Besse: *Einstein Manifolds*, Section 1.K.