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Geometry of Scalar Curvature

Sheet 2

For discussion on **29 April 2022**

Exercise 1: Whitney Trick

The Whitney trick can be stated approximately as follows: Given a simply connected oriented manifold M^n and two compact embedded submanifolds R^k, S^{n-k} intersecting transversely with intersection number $[R^k : S^{n-k}]$, under certain conditions, R^k can be smoothly isotoped such that it intersects S^{n-k} in exactly $[R^k : S^{n-k}]$ points.

Consult the literature* to find precise conditions when this works and sketch the idea of the proof.

Exercise 2: A Chain Complex

Let

$$0 \longleftarrow \mathbb{Z}^{d_1} \xleftarrow{\partial_2} \mathbb{Z}^{d_2} \xleftarrow{\partial_3} \cdots$$

be an exact chain complex of free abelian groups, i.e., the homology groups $H_i = \frac{\mathrm{Ker}(\partial_i)}{\mathrm{Im}(\partial_{i+1})}$ vanish.

We denote the canonical basis for $\mathbb{Z}^{d_i}$ by $(e_j^{(i)})$. Show that if for an element $e_j^{(1)}$ there is only one $e_k^{(2)}$ such that the coefficient $n_{k,j}$ in $\partial_2(e_k^{(2)}) = \sum_\ell n_{k,\ell} e_\ell^{(1)}$ is non-vanishing, then $n_{k,j} = \pm 1$. Where do we use this in the proof of the h-cobordism theorem?

Exercise 3: CW Structure for Handlebodies

If we attach handles to an $(n+1)$-dimensional handlebody in the order of nondecreasing degree, this inductively gives a CW complex of the same homotopy type as our handlebody as follows: We reduce every k-handle to its core D^k and then attach its boundary S^{k-1} to the $(k-1)$-skeleton of the previously existing CW complex.

Convince yourself that the intersection number of the attachment sphere S_{a}^{k-1} of a k-handle with the belt sphere S_{b}^{n-k} of a $(k-1)$-handle agrees with the mapping degree of the corresponding CW attachment map. (You can find more details in Chapter 6 and Appendix A of Rourke, Sanderson: *Introduction to Piecewise-Linear Topology*, Springer 1972.)

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*E.g., J. Milnor: *Lectures on the h-Cobordism Theorem*, Princeton University Press 1965.