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Algebraic Geometry
Exercise Sheet 5

To be hand in on 22.11.2018

Exercise 1:

Let X be a scheme.

(i) Show that X is a colimit of affine schemes.

(ii) Let A be a ring. Show that the natural map

$$\mathrm{Hom}_{\mathrm{Sch}}(X, \mathrm{Spec}\, A) = \mathrm{Hom}_{\mathrm{Rings}}(A, \mathcal{O}_X(X))$$

is a bijection.
(*Hint: Use Exercise 3 of the last exercise sheet.*)

Exercise 2:

(i) Let $f : X \to Y$ be a morphism of schemes, and suppose that Y can be covered by open subsets U_i, such that for each i, the induced map $f^{-1}(U_i) \to U_i$ is an isomorphism. Show that f is an isomorphism.

(ii) Show that a scheme X is affine if and only if there is a set of elements $f_1, \ldots, f_r \in \mathcal{O}_X(X)$, such that the open subsets $X_{f_i} = \{x \in X | f_i(x) \neq 0\}$ are affine, and $f_1, \ldots, f_r$ generate the unit ideal in $\mathcal{O}_X(X)$.
(*Hint: Use Exercise 4 of the last exercise sheet.*)

Exercise 3:

Let $f : X \to Y$ be a morphism of schemes. Show that the following are equivalent:

(a) For any affine open subscheme $V \subseteq Y$, $f^{-1}(V)$ is affine.

(b) There is an affine open covering U_i such that each $f^{-1}(U_i)$ is affine.

(*Hint: Use Exercise 2.*)

Exercise 4:

For a given scheme S, write $|S|$ for the underlying topological space. Let $f : Y \to X$ and $g : Z \to X$ be morphisms of schemes.

(i) Show that there is natural map:

$$t : |Y \times_X Z| \to |Y| \times_{|X|} |Z|$$

and t is surjective.

(ii) Give an example of fiber product such that t is not a homeomorphism.

Homepage: http://www.uni-muenster.de/Arithm/hellmann/veranstaltungen