- **Schwarz, Michael: Courant's Nodal Domain Theorem for Regular Dirichlet Forms (204)**
  The classical nodal domain theorem of Courant states that an eigenfunction f for the n-th eigenvalue of the Dirichlet Laplacian has at most n nodal domains, i.e., \emph{topological} connected components of $\{f>0\}$ and $\{f<0\}$. Later, in 2001, Davies, Gladwell, Leydold, and Stadler proved a discrete version of Courant's theorem. Their theorem states that the upper bound of the number of nodal domains of an eigenfunction f for the n-th eigenvalue of the Laplacian on a finite graph is given by $n+k-1$, where k denotes the multiplicity of the eigenvalue and a nodal domain is a \emph{graph} connected component of $\{f>0\}$ or $\{f<0\}$.
  The stronger bound in the classical theorem is the result of a unique continuation principle, which holds for the Laplacian on $\mathbb{R}^N$ but not for the Laplacian on a graph. We are discussing Courant's nodal domain theorem in the setting of regular Dirichlet forms, which includes both examples mentioned above. To do this, we first introduce a notion of nodal domains which does only depend on the Dirichlet form and not the topology of the space. We show that in this setting, the number of nodal domains of an eigenfunction for the n-th eigenvalue is bounded from above by $n+k-1$, where k denotes the multiplicity of the eigenvalue. Furthermore, if the form is local and satisfies a unique continuation principle, we show that an eigenfunction for the n-th eigenvalue has at most n nodal domains.