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Estimation of Detection Thresholds for Redirected Walking Techniques

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Abstract—In immersive virtual environments (IVEs) users can control their virtual viewpoint by moving their tracked head and by walking through the real world. Usually, movements in the real world are mapped one-to-one to virtual camera motions. With redirection techniques, the virtual camera is manipulated by applying gains to user motion so that the virtual world moves differently than the real world. Thus, users can walk through large-scale IVEs while physically remaining in a reasonably small workspace.

In psychophysical experiments with a two-alternative-forced-choice tasks we have quantified how much humans can unknowingly be redirected on virtual paths which are different from the physical paths that they actually walk. We tested 12 subjects in three different experiments: (E1) discrimination between virtual and physical rotation, (E2) discrimination between virtual and physical translation, and (E3) discrimination of path curvature. In experiment E1 subjects performed rotations with different gains, and then had to choose whether the visually perceived rotation was smaller or greater than the physical rotation. In experiment E2, subjects chose whether the physical walk was shorter or longer than the visually perceived scaled travel distance. In experiment E3 subjects estimate the path curvature when walking a curved path in the real world while the visual display shows a straight path in the virtual world.

Our results show that users can be turned physically about 49% more or 20% less than the perceived virtual rotation, distances can be downscaled by 14% and up-scaled by 26%, and users can be redirected on a circular arc with a radius greater than 22m while they believe they are walking straight.

Index Terms—Virtual reality, virtual locomotion, redirected walking.

1 Introduction

In the real world we navigate with ease by walking, running, driving etc., but in immersive virtual environments (IVEs) realistic simulation of these locomotion techniques is difficult to achieve. While moving in the real world, sensory information such as vestibular, proprioceptive, and efferent copy signals as well as visual information create consistent multi-sensory cues that indicate one's own motion, i.e., acceleration, speed and direction of travel. In this context walking is the most basic and intuitive way of moving within the real world. Keeping such an active and dynamic ability to navigate through large-scale immersive virtual environments is of great interest for many 3D applications demanding locomotion, such as in urban planning, tourism, or 3D entertainment.

Locomotion in Virtual Environments

Often, IVEs are characterized by head-mounted displays (HMDs) and a tracking system for measuring position and orientation data [8]. Immersive virtual environments

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were initially restricted to visual displays, combined with interaction devices, e.g. joystick or mouse, for providing (often unnatural) inputs to generate self-motion. More and more research groups are investigating natural, multimodal methods of generating self-motion. Traveling through immersive virtual environments by means of *real walking* is an important activity to increase naturalness of virtual reality (VR)-based interaction.

Many domains are inherently three-dimensional and advanced visual simulations often provide a good sense of locomotion, but exclusive visual stimuli alone cannot sufficiently address the vestibular-proprioceptive system. Furthermore, as a matter of fact real walking is a more presence-enhancing locomotion technique than other navigation metaphors [35]. However, real walking in IVEs is often not possible [39].

Indeed, an obvious approach is to transfer the user's tracked head movements to changes of the camera in the virtual world by means of a one-to-one mapping. Then, a one meter movement in the real world is mapped to a one meter movement of the virtual camera in the corresponding direction in the VE. This technique has the drawback that the users' movements are restricted by a limited range of the tracking sensors and a rather small workspace in the real world. The size of the virtual world often differs from the size of the tracked laboratory space so that a straightforward implementation of omni-directional and unlimited walking is not possible. Thus, concepts for virtual locomotion methods are needed that enable walking over large distances in

the virtual world while remaining within a relatively small space in the real world. Various prototypes of interface devices have been developed to prevent a displacement in the real world. These devices include torus-shaped omni-directional treadmills [5], [6], motion foot pads, robot tiles [18], [19] and motion carpets [30]. Although these hardware systems represent enormous technological achievements, they are still very expensive and will not be generally accessible in the foreseeable future.

Hence there is a tremendous demand for more applicable approaches. As a solution to this challenge, traveling by exploiting walk-like gestures has been proposed in many different variants, giving the user the impression of walking. For example, the walking-in-place approach exploits walk-like gestures to travel through an IVE, while the user remains physically at nearly the same position [12], [34], [40]. However, real walking has been shown to be a more presence-enhancing locomotion technique than other navigation metaphors [35].

Redirected Walking

Cognition and perception research suggests that costefficient as well as natural alternatives exist. It is known from perceptive psychology that vision often dominates proprioception and vestibular sensation when they disagree [2], [10]. In perceptual experiments, where human participants can use only vision to judge their motion through a virtual scene they can successfully estimate their momentary direction of self-motion but are much less good in perceiving their paths of travel [4], [23]. Therefore, since users tend to unwittingly compensate for small inconsistencies during walking it is possible to guide them along paths in the real world which differ from the path perceived in the virtual world. This redirected walking enables users to explore a virtual world that is considerably larger than the tracked working space [28].

As illustrated in Figure 1 a path that a user walks in the physical world can be scaled and bended, and real-world rotations of users can be in- or decreased when the motions are applied to the virtual camera. However, until now, not much effort has been spend in order to identify thresholds which show how much users can be manipulated while waking.

Since redirected walking techniques are based on imperfections in human visual path perception one has to study human perception of self-motion to identify thresholds for tolerable amounts of deviation between virtual and real movement. When visual, vestibular, and proprioceptive sensory signals that normally support perception of self-motion are in conflict, such as when visual motion stimuli are presented to stationary human subjects, vision can dominate vestibular and proprioceptive information. For example, in the illusion of linear vection [3] observers feel themselves moving although they are physically stationary simply because they are

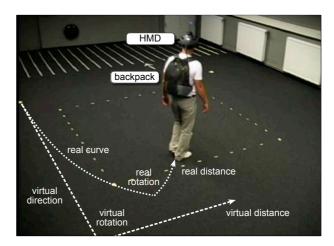


Fig. 1. Redirected walking scenario: a user walks in the real environment on a different path with a different length in comparison to the perceptual path in the virtual world.

presented with large field visual motion that resembles the motion pattern normally experienced during real self-motion. This visual motion pattern is called *optical flow*, and much research has shown that humans can in principle extract self-motion information from optical flow (cf. [23], [37]).

In this article we present a series of experiments in which we have quantified how much humans can be redirected without observing inconsistencies between real and virtual motions. We performed three psychophysical studies in which subjects had to discriminate between real and virtual motions, in particular rotations, translations and walking directions.

The remainder of this article is structured as follows. Section 2 summarizes previous work related to locomotion and perception in virtual reality environments. Section 3 presents a taxonomy of redirected walking techniques as used in the experiments that are described in Section 4. Section 5 summarizes the results and discusses implications for the design of virtual locomotion user interfaces. The last section also gives an overview about future work.

2 RELATED WORK

Currently locomotion and perception in IVEs are the focus of many research groups analyzing perception in both the real world and virtual worlds. For example, researchers have described that distances in virtual worlds are underestimated in comparison to the real world [15], [16], [25], that visual speed during walking is underestimated in VEs [1] and that the distance one has traveled is also underestimated [13]. Sometimes, users have general difficulties in orienting themselves in virtual worlds [29].

From an egocentric perspective the real world appears stationary as we move around or rotate our head and eyes. Both visual and extraretinal cues that come from other parts of the mind and body help us to perceive the world as stable [7], [36], [38]. Extraretinal cues come from the vestibular system, proprioception, our cognitive model of the world, or from an efference copy of the motor commands that move the respective body parts. When one or more of these cues conflicts with other cues, as is often the case for IVEs (e.g., due to tracking errors or latency) the virtual world may appear to be spatially unstable. Experiments demonstrate that the user tolerates a certain amount of inconsistency between visual and proprioceptive sensation in IVEs [9], [21], [22], [27], [28], [33], [20]. In this context redirected walking provides a promising solution to the problem of limited tracking space and the challenge of providing users with the ability to explore a virtual world by walking [28]. With this approach the user is redirected via manipulations applied to the displayed scene, causing users to unknowingly compensate scene motion by repositioning and/or reorienting themselves.

Different approaches to redirect a user in an IVE have been proposed. An obvious approach is to scale translational movements, for example, to cover a virtual distance that is larger than the distance walked in the physical space. Interrante et al. suggest to apply the scaling exclusively to the main walking direction in order to prevent unintended lateral shifts [17]. With most reorientation techniques, the virtual world is imperceptibly rotated around the center of a stationary user until she is oriented in such a way that no physical obstacles are in front of him/her [22], [27], [28]. Then, the user can continue to walk in the desired virtual direction. Alternatively, reorientation can also be applied while the user walks [14], [28], [33]. For instance, if the user wants to walk straight ahead for a long distance in the virtual world, small rotations of the camera redirect him/her to walk unconsciously on an arc in the opposite direction in the real world. When redirecting a user, the visual sensation is consistent with motion in the IVE, but proprioceptive sensation reflects motion in the physical world. However, if the induced manipulations are small enough, the user has the impression of being able to walk in the virtual world in any direction without restrictions. In the scope of this article we address the question how much manipulation applied to the virtual camera is unnoticeable for humans.

Redirection techniques have been applied particularly in robotics for controlling a remote robot by walking [14]. For such scenarios much effort has been undertaken to prevent collisions—sophisticated path prediction is therefore essential [14], [26]. These techniques guide users on physical paths for which lengths as well as turning angles of the visually perceived paths are maintained, but the user observes the discrepancy between both worlds.

Until recently, little research has been undertaken in order to identify thresholds which indicate the tolerable amount of deviation between vision and proprioception while the user is moving. Preliminary studies have

shown that in general redirected walking works [33], [27], [28]. In these experiments users had to remark after they walked a manipulated path, if they noticed a manipulation or not. Quantified analyzes of thresholds were not taken in these experiments. Some work has been done in order to identify thresholds for detecting scene motion during head rotation [21], [36], [20], but walking was not considered in these experiments. Steinicke et al. [31] performed psychophysical studies to identify detection thresholds for redirected walking gains. Similar to the experiments described in this article, subjects had to discriminate between virtual and real motions. Afterwards, they decided in a yes/no-judgment whether a physical movement was greater than the virtual counterpart or not. This yes/no-judgment has the drawback that it potentially induces a bias, since a subject that is uncertain about the true answer might favor the "or not" unless the movement is clearly greater.

In summary, substantial efforts have been made to allow a user to walk through a large-scale VE, but much research is needed to improve the sense of natural walking.

3 TAXONOMY OF REDIRECTED WALKING TECHNIQUES

A fundamental task of an IVE is to synchronize images presented on the display surface with the user's head movements in such a way that the elements of the virtual scene appear stable in world space. Redirected walking and reorientation techniques take advantage of the imperfections of the human visual-vestibular system by intentionally injecting imperceivable motions of the scene. When a user navigates through an IVE by means of real walking, motions are composed of translational and rotational movements. Translational movements are used to get from one position to another, rotational movements are used to reorient in the IVE. By combining both types of movements users can navigate on curvelike trajectories. We classify redirection techniques with respect to these types of locomotion.

Redirected walking can be implemented using gains which define how tracked real-world motions are mapped to the VE. These gains are specified with respect to a coordinate system. For example, they can be defined by uniform scaling factors that are applied to the virtual world registered with the tracking coordinate system such that all motions are scaled. However, when *all* motions are scaled simultaneously, lateral and vertical motions are also affect, which complicates intuitive and natural movements [15].

3.1 Human Locomotion Triple

In [32] we introduced the *human locomotion triple (HLT)* (s,u,w) by three normalized vectors, i. e., strafe vector s, up vector u and direction of walk w. The user's direction of walk can be determined by the actual tracked walking

direction or using the users pose, for example, defined by the orientation of the limbs or the view direction. In our experiments we define w by the actual walking direction tracked and filtered by the tracking system. The strafe vector, a.k.a. right vector, is orthogonal to the direction of walk and parallel to the walk plane. Whereas the direction of walk and the strafe vector are orthogonal to each other, the up vector u is not constrained to the crossproduct of s and w. Hence, if a user walks up a slope the direction of walk is defined according to the walk plane's orientation, whereas the up vector is not orthogonal to this tilted plane. When walking on slopes humans tend to lean forward, so the up vector is invers to the direction of gravity. As long as the direction of walk holds $w \neq (0, 1, 0)$, the HLT composes a coordinate system. In the following sections we describe how gains can be applied to this locomotion triple. We define u by the up vector of the tracked head orientation. In our experiments we considered only planar grounds.

3.2 Translation Gains

Assume that the tracking and virtual world coordinate systems are calibrated and registered. When the tracking system detects a change of the user's real world position defined by the vector $T_{\rm real} = P_{\rm cur} - P_{\rm pre}$, where $P_{\rm cur}$ is the current position and $P_{\rm pre}$ is the previous position, $T_{\rm real}$ is mapped one-to-one to the virtual camera with respect to the registration between virtual scene and tracking coordinates system. Then, the virtual camera is moved by $|T_{\rm real}|$ units in the corresponding direction in the virtual world coordinate system. The tracking system updates the change of position several times per second as long as the user remains within the range of the tracking system.

A translation gain $g_T \in \mathbb{R}^3$ is defined for each component of the HLT (see Section 3.1) by the quotient of the mapped virtual world translation T_{virtual} and the tracked real world translation T_{real} , i.e., $g_T := \frac{T_{\text{virtual}}}{T_{\text{real}}}$.

When a translation gain g_T is applied to a translational movement $T_{\rm real}$ the virtual camera is moved by the vector $g_T \cdot T_{\rm real}$ in the corresponding direction. This is particularly useful if the user wants to explore IVEs whose size differs significantly from the size of the tracked space. For instance, if a user wants to explore molecular structures, movements in the real world must be scaled down when they are mapped to virtual movements, e. g., $g_T \approx 0$. In contrast, the exploration of a football field by means of real walking in a working space requires a translation gain $g_T \approx 20$.

Such uniform gains allow exploration of IVEs whose sizes differ from the size of the working space, but often restrict natural movements. Besides scaling movements in the direction of walk, lateral and vertical movements are affected by uniform gains. In most VR-based scenarios users benefit from the ability to explore close objects via head movements which may be hindered by scaling also vertical or lateral movements, and therefore uniform gains are often inadequate. Non-uniform

translation gains are used to distinguish between movements in the main walking direction, lateral movements and vertical movements [15]. Generic gains for translational movements can be expressed by $(g_{T[s]},g_{T[u]},g_{T[w]})$, where each component is applied to the corresponding vector s,u and w respectively composing the translation. In our experiments we have focused on sensitivity to translation gains $g_{T[w]}$, and have filtered both lateral and vertical movements.

3.3 Rotation Gains

Real-world head rotations can be specified by a vector consisting of three angles, i.e., $R_{\text{real}} :=$ $(pitch_{real}, yaw_{real}, roll_{real})$. The tracked orientation change is applied to the virtual camera. Analogous to Section 3.2, rotation gains are defined for each component (pitch/yaw/roll) of the rotation and are applied to the axes of the locomotion triple. A rotation gain g_R is defined by the quotient of the considered component of a virtual world rotation R_{virtual} and the real world rotation R_{real} , i.e., $g_R := \frac{R_{\text{virtual}}}{R_{\text{real}}}$. When a rotation gain g_R is applied to a real world rotation α , the virtual camera is rotated by $\alpha \cdot g_R$ instead of α . This means that if $g_R = 1$ the virtual scene remains stable considering the head's orientation change. In the case $g_R > 1$ the virtual scene appears to move against the direction of the head turn, whereas a gain $g_R < 1$ causes the scene to rotate in the direction of the head turn. For instance, if the user rotates her head by 90° degree, a gain $g_R = 1$ maps this motion one-to-one to a 90° degree rotation of the virtual camera in the VE. The appliance of a gain $g_R = 0.5$ means that the user has to rotate the head by 180° physically in order to achieve a 90° virtual rotation; a gain $g_R = 2$ means that the user has to rotate the head by only 45° physically in order to achieve a 90° virtual rotation.

Again, gains are defined for each component of the rotation, i.e., yaw, pitch, and roll, and are applied to the axes of the locomotion triple. Rotation gains can be expressed by $(g_{R[s]}, g_{R[u]}, g_{R[w]})$, where the gain $g_{R[s]}$ specified for pitch is applied to s, the gain $g_{R[u]}$ specified for yaw is applied to u, and $g_{R[w]}$ specified for roll is applied to w. In our experiments we have focused on rotation gains for yaw rotation $g_{R[u]}$. Yaw is the most often manipulated rotation for redirected walking [9], [21], [22], [27], [28], [33].

3.4 Curvature Gains

Instead of multiplying gains with translations or rotations, offsets can be added to real world movements. Thereby, camera manipulations are enforced if only one kind of motion is tracked, for example, user turns the head, but stands still, or the user moves straight without head rotations. If the injected manipulations are reasonably small, the user will unknowingly compensate for these offsets resulting in walking a curve. The gains can be applied in order to inject rotations, while users virtually walk straight, or gains can be applied in order

to inject translations, while users only rotate their heads. The curvature gain g_C denotes the resulting bend of a real path. For example, when the user moves straight ahead, a curvature gain that causes reasonably small iterative camera rotations to one side enforces the user to walk along a curve in the opposite direction in order to stay on a straight path in the virtual world. The curve is determined by a circular arc with radius r, and we define $g_C := \frac{1}{r}$. In case no curvature is applied it is $r = \infty \Rightarrow g_C = 0$, whereas if the curvature causes the user to rotate by 90° clockwise after $\frac{\pi}{2}$ m the user has covered a quarter circle with radius $r = 1 \Rightarrow g_C = 1$.

Alternatively, gains can be applied as translation offsets while the user turns the head and no translational movements are intended. While the user turns, such a gain causes the camera to shift to one direction such that the user will unknowingly move to the opposite direction in order to compensate an unintended displacement in the virtual world. Potentially, such gains can be applied to each axes of the HLT. However, in our experiments we focused on the common procedure which enforce users to walk on an arc parallel to the walk plane by means of curvature gains $g_{C[w]}$. Furthermore, gains can be applied time-depandently, but this approach is not in the scope of this article.

4 EXPERIMENTS

In this section we present three experiments in which we have quantified how much humans can unknowingly be redirected. We have analyzed the appliance of translation $g_{T[w]}$, rotation $g_{R[u]}$, and curvature gains $g_{C[w]}$.

4.1 Experimental Design

Since the main objective of our experiments is to allow users to walk unlimitedly in 3D city environments, the visual stimulus consisted of virtual scenes of the city of Münster (see Figure 2). Before each trial a random place and a horizontal gaze direction were chosen. The only restriction for this starting scene was that no vertical objects were within 10m of the starting position in order to prevent collisions in the VE.

Hardware Setup

We performed all experiments in a $10m \times 7m$ darkened laboratory room. The subjects wore an HMD (3DVisor Z800, 800x600@60Hz, 40° diagonal field of view (FoV)) for the stimulus presentation. On top of the HMD an infrared LED was fixed. We tracked the position of this LED within the room with an active optical tracking system (Precise Position Tracking of World Viz), which provides sub-millimeter precision and sub-centimeter accuracy. The update rate was 60Hz providing real-time positional data of the active markers. For three degrees of freedom (DoF) orientation tracking we used an InertiaCube 2 (InterSense) with an update rate of 180Hz. The InertiaCube was also fixed on top of the

HMD. In the experiments we used an Intel computer for visual display, system control and logging purposes with dual-core processors, 4GB of main memory and an *n*Vidia GeForce 8800.

The virtual scene was rendered using OpenGL and our own software with which the system maintained a frame rate of 30 frames per second. During the experiment the room was entirely darkened in order to reduce the user's perception of the real world. The subjects received instructions on slides presented in the HMD. A Nintendo WII remote controller served as an input device via which the subjects judged their motions.

We connected the HMD display with a 12m VGA cable, which ensured that no assistant had to walk beside the user during the entire experiment to keep an eye on the wires. In order to focus subjects on the tasks no communication between experimenter and subject was performed during the experiment. All instructions were displayed in the VE, and subjects responded via the WII device. Acoustic feedback was used for ambient city noise in the experiment such that an orientation by means of auditory feedback in the real world was not possible.

Participants

9 male and 5 female (age 19-50, \varnothing : 25.54) subjects participated in the study. Most subjects were students or members of the departments (computer science, mathematics, psychology, geoinformatics, and physics). All had normal or corrected to normal vision; 8 wear glasses or contact lenses. 2 had no game experience, 6 had some, and 6 had much game experience. 3 of the subjects had experience with walking in VR environments using an HMD setup. 12 subjects were right-handed, 2 were lefthanded. Two of the authors served as subjects; all other subjects were naïve to the experimental conditions. Some subjects obtained class credit for their participation. The total time per subject including pre-questionnaire, instructions, training, experiment, breaks, and debriefing took 3 hours. Subjects were allowed to take breaks at any time.

For all experiments we used the method of constant stimuli in a *two-alternative forced-choice* (2AFC) task. In the method of constant stimuli, the applied gains are not related from one trial to the next, but presented randomly and uniformly distributed. The subject chooses between one of two possible responses, e.g. "Was the virtual movement *smaller* or *greater* than the physical movement?"; responds like "I can't tell." were not allowed. In this version, when the subject cannot detect the signal, she must guess, and will be correct on average in 50% of the trials. The question "Was this greater or not?" that was used in [31], might have introduced a bias to respond "no" in the case of uncertainty. The two alternative-forced choice question "greater or smaller" that was used in the present study avoids this problem.

The gain at which the subject responds "smaller" in half of the trials is taken as the *point of subjective equality*



Fig. 2. Example scene from Virtual Münster as used for the experiments E1 and E2. Subjects had to walk until the green dot turned red. No obstacles are within a 10m distance from the user.

(PSE), at which the subject perceives the physical and the virtual movement as identical. As the gain decreases or increases from this value the ability of the subject to detect the difference between physical and virtual movement increases, resulting in a psychometric curve for the discrimination performance. A threshold is the point of intensity at which subjects can just detect a discrepancy between physical and virtual motion. However, stimuli at values close to thresholds will often be detectable. Therefore, thresholds are considered to be the gains at which the manipulation is detected only some proportion of the time. In psychophysical experiments, usually the point at which the curve reaches the middle between the chance level and 100% is usually taken as threshold. Therefore, we define the detection threshold (DTs) for gains smaller than the PSE to be the value of the gain at which the subject has 75% probability of choosing the "smaller" response correctly and the detection threshold for gains greater than the PSE to be the value of the gain at which the subject chooses the "smaller" response in only 25% of the trials (since the correct response "greater" was then chosen in 75% of the trails).

In this article we focus on the range of gains over which the subject cannot reliably detect the difference as well as the gain at which subjects perceive physical and virtual movement as identical. The 25% to 75% range of gains will give us an interval of possible manipulations which can be used for redirected walking. The PSEs give indications about how to map user movements to the virtual camera such that virtual motions appear naturally to users.

4.2 Experiment 1 (E1): Discrimination between Virtual and Physical Rotation

In this experiment we investigated subject's ability to discriminate whether a physical rotation was smaller or greater than the simulated virtual rotation (see Sec-

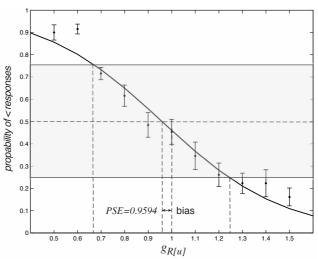


Fig. 3. Pooled results of the discrimination between virtual and physical rotation. The x-axis shows the applied rotation gain $g_{R[u]}$, the y-axis shows the probability of estimating a virtual rotation smaller than the physical counterpart.

tion 3.3). Therefore, we instructed the subjects to rotate on a physical spot and we mapped this rotation to a corresponding virtual rotation to which different gains were applied.

4.2.1 Material and Methods for E1

At the beginning of each trial the virtual scene was presented on the HMD together with the written instruction to physically turn right or left until a red dot drawn at eye height was directly in front of the subject's gaze direction. The subjects indicated the end of the turn with a button press on the WII controller. Afterwards the subjects had to decide whether the simulated virtual rotation was smaller (down button) or greater (up button) than the physical rotation. Before the next trial started, subjects turned to a new orientation. We indicated the reorientation process in the IVE setup by a white screen and two orientation markers (current orientation and target orientation). We implemented this random reorientation to prevent that subjects get wrapped by the wires. The virtual rotation was always 90° either to the right or left of the starting orientation. We varied the gain $g_{R[u]}$ between the physical and virtual rotation randomly in the range between 0.5 (180° physical rotation resulted in a 90° virtual rotation) and 1.5 (60° physical rotation resulted in a 90° virtual rotation) in steps of 0.1. We tested each gain 10 times in randomized order. 14 subjects participated in this experiment.

4.2.2 Results of E1

Figure 3 shows the mean detection thresholds together with the standard error over all subjects for the tested gains. The x-axis shows the applied rotation gain $g_{R[u]}$,

the y-axis shows the probability for estimating a physical rotation greater than the mapped virtual rotation. The solid line shows the fitted psychometric function of the form $f(x) = \frac{1}{1+e^{a\cdot x}+b}$ with real numbers a and b. We found no difference between rotations to the left and rotations to the right and therefore pooled the two conditions. We had to dismiss the data set of two subjects from further analyses, because these subjects either mixed up the answer buttons or misunderstood the task.

From the psychometric function we determined a bias for the point of subjective equality at PSE=0.96. As illustrated in Table 1 for individual subjects, we found the PSE to vary between 0.83 and 1.34 (6 subjects with PSE greater than 1.0, 7 subjects less than 1.0). Detection thresholds of 75% were reached at gains of 0.67 for greater responses and at 1.24 for smaller responses. Gain differences within this range cannot be reliably estimated, i. e., subjects had problems to discriminate between a 90° virtual from real rotations ranging between 72.6° and 134.3° .

4.2.3 Discussion of E1

According to previous results [31], [21] we assumed an asymmetric characteristic of the psychometric function that could be reproduced in our experiment. The asymmetry is shown in Figure 3, where the 75% DT is further away from the PSE than the 25% DT. The results show that subjects can be turned physically about 49% more or 20% less than the perceived virtual rotation. This result is similar to the result found in [31], where the detection thresholds indicated that subjects could be turned physically about 68% more or 10% less than the perceived virtual rotation. The deviation between both experiments might be caused by the small number of participants and/or the bias inherent in the previous experiment.

The asymmetry of the detection thresholds implies that a gain $g_{R[u]} < 1$ downscaling a physical rotation is less noticeable for the subjects. In this case the scene seems to move slightly with the head rotation as shown in previous research [21]. Figure 3 shows that the mean PSE was at $g_{R[u]} = 0.96$, indicating that subjects estimated a virtual 90° rotation scaled with a gain $g_{R[u]} = 0.96$ identical to the physical 90° rotation. With such a gain users have to rotate by approximately 95° in order to achieve a 90° virtual rotation, i. e., subjects underestimate this rotation by approximately 5%. In previous experiments [31] Steinicke et al. found a larger bias (PSE = 0.8403), which could be caused by the estimation based on a yes/no-judgment (cf. Section 2).

Considering also results of other researchers [11], [21], [31], it seems that subjects tend to underestimate virtual rotations; although some researchers have found the opposite results (overestimation of rotations) [20]. Underestimation of movement distance have also previously been reported for translations [13], [16], [24]. The observed underestimation of rotations might be related to that reported for translations, but this has to be verified

in further analyses. In summary the experiment shows that subjects could not discriminate physical from virtual rotations over the reported range of gains. Consequently, reorientating users via rotation gains is a valid technique to redirect users without them noticing.

4.3 Experiment 2 (E2): Discrimination between Virtual and Physical Straightforward Movement

In this experiment we analyzed the ability to discriminate between virtual and physical straightforward movements. The virtual movement in the walk direction was scaled with a corresponding translation gain $g_{T[w]}$ (see Section 3.2).

4.3.1 Material and Methods for E2

In the IVE subjects always had to walk a virtual distance of 5m. The walking direction was indicated by a green dot in front of the subjects (see Figure 2). When the subjects travelled 5m in the virtual scene, the dot turned red to indicate the end of the distance. The dot was constant in size and positioned on the subject's eye level above the ground. The physical distance subjects had to walk varied between 3m and 7m, i.e., gain $g_{T[w]}$ was between 0.6 and 1.4 in steps of 0.1. We presented the gains each 8 times in a randomized order. The task was to judge whether the virtual travel distance was smaller or larger than the physical walking distance. After each trial the subject had to walk back to the starting position, guided by two reference markers on an otherwise white screen. One marker showed the actual position of the subject relative to the second fixed marker, which represented the starting position. 15 subjects participated in this experiment.

4.3.2 Results of E2

Figure 4 shows mean (over all subjects) probability that a subject estimates that the virtual distance is smaller than the physical perceived distance against the tested gains. The error bars show standard errors for each tested gain. A translation gain $g_{T[w]}$ which satisfies $g_{T[w]} < 1$ results in a larger physical walking distance relative to the virtual distance. A gain $g_{T[w]} > 1$ results in a smaller physical walking distance relative to the virtual distance. We fitted the data with the same sigmoidal function as in experiment E1. We dismissed the data set of two subjects from further analysis. One subject always indicated that the virtual walking distance was shorter than the physical distance. The second subject either mixed up the answer buttons or misunderstood the task. The PSE for the pooled data of the remaining 12 subjects is 1.07. This means that subjects estimate that they have walked the 5m distance after waking only 4.69. The PSEs for individual subjects are shown in Table 1. The calculated PSE for the single subjects varied between 0.93 and 1.22 (5 subjects with PSE above or equal, 8 less than 1.07). DTs for estimation of straightforward movements are given at gains smaller than 0.86 or greater than

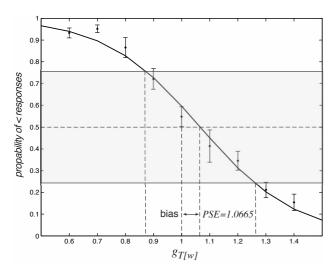


Fig. 4. Pooled results of the discrimination between virtual and physical straightforward movement. The x-axis shows the applied translation gain $g_{T[w]}$, the y-axis shows the probability that subjects estimate the virtual straightforward movement smaller than the physical motion.

1.26. The DTs at gains $g_{T[w]}=0.86$ or greater than $g_{T[w]}=1.26$ mean that subjects could not discriminate reliably between 4.3m and 6.3m physical distance while they walked 5m in the virtual world.

4.3.3 Discussion of E2

Figure 4 shows that subjects can be manipulated physically by about 14% more or 26% less than the perceived virtual translation. The PSE is at $g_{T[w]}=1.07$. In the results of the experiments performed in [31], we found similar detection thresholds $g_{T[w]}=0.78$ and $g_{T[w]}=1.22$, but no asymmetry in the range of detection thresholds could be verified. Again, this may be caused due an estimation which was based on the yes/no-judgment instead of the 2AFCT.

A PSE greater than one is consistent with earlier findings that subjects tend to underestimate travel distances in the virtual world [13], [15], [16], [25]. A gain $g_{T[w]}=1.07$ appears natural to subjects, which need to walk only 4.69m in the real world in order to walk 5m virtually. This corresponds to a 7% overestimation of the physical walked distance, which in other words underlines the underestimations of virtual distances.

One might argue, on the other hand, that 7% percent underestimation is not much, considering the difficulty of the task in VE. From this viewpoint, the results indicate that human can discriminate between virtual and real translational movements quite accurately when actually walking a distance in a familiar environment such as realistic 3D city model. Since subjects knew the VE from the real world, they were able to exploit distance cues such as the height of trees, street sizes etc. As stated in [17] such cues rather support subjects when



Fig. 5. Example scene from Virtual Münster as used for the experiment E3. The pavement that supports subjects during walking was added to the scene. No obstacles are within a 10m distance from the user.

estimating distances in comparison to evaluate features in artificial environments.

4.4 Experiment 3 (E3): Discrimination of Path Curvature

In this experiment we analyze sensitivity to curvature gains which enforce the user to walk on a curve in order to stay on a straight path (see Section 3.4). Subjects were instructed to walk along a straight line in the VE, but because the path was manipulated they physically had to walk along a curved path in order for the virtual path to stay straight. We asked whether subjects were able to discriminate the direction of bending of the physical path, and, if so, at which threshold they start to do so reliably.

A problem in such experiments is that subjects are typically uncertain during the first steps [31], and have difficulty staying on track during the first steps. For instance, after two steps, subjects in an earlier study left the pavement and had to reorient themselves to the target and continue the walk. Consequently, they tend to walk in a triangle rather than on an arc. To avoid this problem, subjects started with a 2m walk without scene manipulation, before manipulations to the virtual camera were applied by means of curvature gain $g_{C[w]}$.

4.4.1 Material and Methods for E3

To support users to virtually walk on a straight path we introduced a 1m wide pavement (see Figure 5). In level with the subject's eye height we added a green dot in the scene, which turned red when the subjects had walked 2m + 5m towards it. While the subjects walked along the pavement, we rotated the scene to either side with a velocity linked to the subject's movement velocity.

The scene rotated by 5, 10, 15, 20 and 30 degrees after 5m walking distance. This corresponds to a curvature radius of approximately 57.3, 28.65, 19.10, 14.32 and 9.55m respectively. Hence, the curvature gains were given by $g_{C[w]} = \left\{ \pm \frac{\pi}{30}, \pm \frac{\pi}{45}, \pm \frac{\pi}{60}, \pm \frac{\pi}{90}, \pm \frac{\pi}{180} \right\}$.

The rotation of the virtual camera started after subjects had walked the 2m start-up phase. After subjects walked a total distance of 7m in the virtual world, the screen turned white and the question of the discrimination task appeared. The subject's task was to decide whether the physical path was curved to the left or to the right by pressing the corresponding "left" or "right" button on the WII controller. The subject then walked back to the starting position guided by the markers (one indicted the current and one the target position/orientation) on an otherwise white screen. 12 subjects participated in this experiment.

4.4.2 Results of E3

In Figure 6 we plotted the mean probability for the response that that the physical path was curved to the left against the curvature gains $g_{C[w]}$. Error bars correspond to the standard error. The PSE for the pooled data is $\frac{\pi}{1423} = 0.002$. At this PSE the subjects have in fact walked on a circular arc with a radius of 453.14m, and rotated by less than one degree after 5m. The PSEs for individual subjects are shown in Table 1. They varied between $\frac{\pi}{62.51} = -0.019$ and $\frac{\pi}{60.90} = 0.052$ (10 subjects with PSE above or equal, 2 less than 0.0022). The detection thresholds are given by the stimulus intensity at which subjects correctly detect the bending of the path 75% of the time. Detection thresholds were $g_{C[w]} = \pm 0.045$, i.e., $g_{C[w]} = -rac{\pi}{69.23}$ for leftward bended paths and $g_{C[w]} =$ $+\frac{\pi}{69.23}$ for rightward bended paths. At these threshold values, subjects walked physically on a circular arc with a radius of approximately 22.03m. Within this range of detection thresholds subjects cannot estimate reliably if they walk straight or on a curve.

4.4.3 Discussion of E3

The results show that subjects can be reoriented by 13° to the left or to the right after walking a 5m distance, which corresponds to walking along a circular arc with a radius of approximately 22m. Hence, if the laboratory space covers an area of approximately $40m \times 40m$, it gets possible to guide the user on a circular arc in the physical world, whereas the user can walk straight in the VE unlimitedly.

5 CONCLUSION AND DISCUSSION

In this article, we analyzed the users' ability to recognize redirected walking manipulations in three different experiments. We introduced generic concepts for redirection techniques and tested the corresponding gains in a practical useful range for their perceptibility. The findings include detection thresholds, which have essential implications for the design of future locomotion user interfaces, which are based on redirected walking.

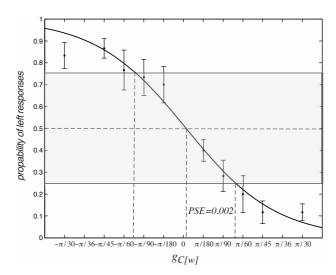


Fig. 6. Pooled results of the discrimination of path curvature. The x-axis shows the applied curvature gain which bends the walked path either to the left $(g_{C[w]} < 0)$ or the right $(g_{C[w]} > 0)$, the y-axis shows the proportion of subjects' left responses.

5.1 Summary of the Results

Our results show that users can be turned physically about 49% more or 20% less than the perceived virtual rotation without noticing the difference. We determined a bias for the point of subjective equality resulting in a PSE=0.95 for which virtual rotations appear most natural to users. Our results agree with previous findings [21], [31] that users are more sensitive to scene motion if the scene moves against head rotation than if the scene moves with head rotation. The observed underestimation of rotations might be related to that reported for translations, but this has to be verified in further analyses.

Walked distances in the real world can be down-scaled by 14% and up-scaled by 26%, when they are mapped to virtual motions. This asymmetry coincides with previous findings that users tend to underestimate distances [13], [15], [16], [25]. The PSE for the pooled data of the subjects is 1.07. This means that subjects estimate that they have walked 5m distance after waking only 4.69. Further experimentation and analysis could be performed to examine if this underestimation coincides with the gait length which is usually smaller for subjects wearing an HMD [39].

When applying curvature gains users can be redirected such that they unknowingly walk on a circular arc when the radius is greater or equal to 22m.

In comparison to the study presented in [31], the design of the experiment based on a 2AFCT probably has diminished most of the bias caused by questions based on yes/no-judgements.

Certainly, redirected walking is a subjective matter, but the results have potential to serve as thresholds for the development of future locomotion interfaces. The detection thresholds derived from our experiments are conservative estimates, since a subject's task was to detect discrepancies between vestibular, proprioceptive, as well as efferent copy signals perceived in the real world and visual feedback perceived in the virtual environment. In actual VR-based applications based on redirected walking users will not be confronted with such discrepancies in an obvious way, instead users will focus on other tasks such as selection or manipulation of objects in space. We have experienced that subjects tolerate substantially greater gains when they are not aware of the manipulation, in particular if they are engaged in their primary tasks. For example, in [33] we found that curvature gains up to $g_C = 0.64$ are noticeable, but still not overly distracting. In this case users walk on a circular arc with radius of approximately 3.3m which is much more practical for most VR-based setups. Hence, the thresholds proposed in this article provide lower and upper bounds for human's sensitivity to redirected walking, but in most scenarios much greater gains can be applied without user's noticing that they are manipulated.

Post-Questionaires

After the experiments we have performed further questionnaires in order to identify potential drawbacks of the experimental design. The subjects estimated the difficulty of the tasks with 1.57 in average on a 4-point Likert-scale (0 corresponds to very easy, 4 corresponds to very difficulty). Furthermore, we have asked subjects about their fear of colliding with physical objects. The subjects revealed their level of fear on a four point Liker-scale (0 corresponds to no fear, 4 corresponds to a high level of fear). On average the evaluation approximates 1.36 which shows that the subjects felt quite safe even though they were wearing an HMD and knew that they were being manipulated. Further post-questionnaires based on a comparable Likert-scale show that the subjects only had marginal positional and orientational indications due to environmental audio (0.5), visible (0.14) or haptic (1.21) cues.

We measured simulator sickness by means of Kennedy's Simulator Sickness Questionnaire (SSQ). The Pre-SSQ score averages for all subjects to 8.55 and the Post-SSQ score to 24.04. We conducted a follow-up test on another day for subjects with high Post-SSQ scores in order to examine whether the sickness was caused by the applied redirected walking manipulations or not. However, we could not identify any evidence that the described redirected walking techniques contribute to or subtract from simulator sickness symptoms.

5.2 Future Work

In the future we will consider other redirection approaches presented in the taxonomy of redirected walking techniques [32], which have not been analyzed in

Subject	Exp. 1	Exp. 2	Exp.3
ab	0.8624	1.1432	0.0169
bb	1.3369	1.0613	l —
df	1.0072	1.2161	0.0186
dw	0.9849	0.9958	-0.0194
fs	0.9241	1.0320	0.0013
gb	0.8592	1.0107	-0.0009
jp	0.9067	1.0416	-0.0164
kk	1.1404	1.0841	_
tb	0.8341	1.1968	0.0526
ee	_	_	0.0100
fz	0.8108	1.0552	-0.0100
sw	1.1871	0.9340	-0.0047
ms	0.8720	0.9938	0.0150
cs	1.1661	1.1623	0.0084
mh	_	_	
Ø	0.9594	1.0665	0.0022

TABLE 1
Individual PSE values of the subjects participated in experiments E1, E2 and E3. We dismissed some data sets, due to the reasons mentioned in Section 4.

the scope of this article. We plan to extend these concepts also to backward movements. Moreover, further conditions have to be taken into account and tested for their impact on redirected walking, for example, distances of scene objects, level of detail, contrast etc. Informal tests have motivated that manipulations can be intensified in some cases, e.g., when less objects are close to the camera, which could provide further motion cues while the user walks. Furthermore, we plan to examine the influence of adaptation. From our experiences we believe that gains can be increased gradually over time without user's noticing. Hence, it may be possible to provide greater gain ranges where a scene manipulation is unnoticeable for users.

The presented redirected walking approach has some limitations. For instance, it may happen that users are guided to points where they face directly into walls; the physical movements are constrained in such a situation. Then, it may get impossible to redirected users in such a way that they cannot observe the manipulation; sometimes it is not possible to redirect users by visual stimuli at all, for instance, if a user walks blindfolded. For such extreme situations, larger manipulations have to be taken into account in order to support sufficient manipulation. Furthermore, certain security mechanisms have to be implemented. For example, we fade out the visualization on the HMD and display an acoustic warning signal to the user when she gets close to a physical wall.

It has been shown that certain factors may have an impact on the range of gains where scene manipulations are unnoticeable for users. For example, Peck et al. [27] use virtual objects (e. g., a butterfly) in front of the user to distract the user from reorientation allowing much larger manipulations. In this article we have considered the situation where subjects are focused on detecting the discrepancy between virtual and real motion. Therefore,

we are confident that the detection thresholds presented in this article have great potential to hold across different conditions and can be applied during the design process of other locomotion user interfaces based on redirected walking.

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