Feasibility study of HQET to QCD matching of heavy-light axial and vector currents

Diploma thesis

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4th of July 2011

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Contents

Introduction

Standard Model of particle physics
Years of theoretical studies and experimental research have come to the conclusion that the universe is constructed of a specific number of elementary building blocks, which are governed by four fundamental forces. These perceptions provide a deep insight in the structure of the micro cosmos and help to achieve a better understanding of the construction and coherence of the matter. The Standard Model (SM) combines the fundamental particles and three of the forces to a theory, which explains most of the experimental results and predictions successfully so far.

The SM includes matter and interaction particles. The matter particles build two

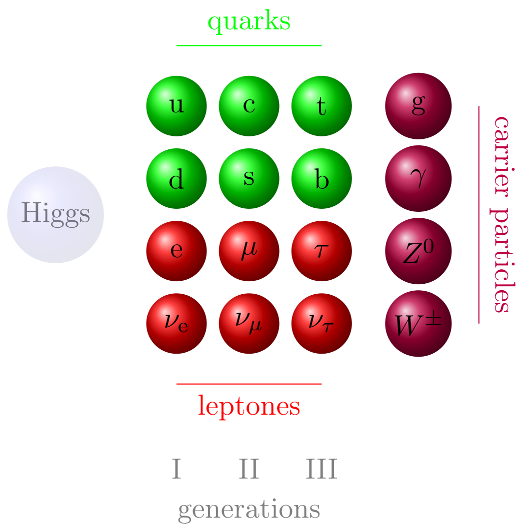


Figure 0.1: Particles in the Standard Model.

groups: 6 *quarks* and 6 *leptones*, which split up into 3 generations, see figure (0.1). The quarks form the particle family of the *hadrones*, which can be a combination of three or two quarks. Three quarks are the constituents of baryons, e.g. the neutron and proton, whereas mesons consist of a quark anti-quark pair. The leptones, i.e. neutrinos or the electron, with the quarks are assumed to be elementary and are the constituents of the matter that surrounds us. The elementary particles can be classified by the fundamental forces. There are four fundamental interactions, which bind the matter: gravitational force, electromagnetic force, strong and weak forces. They differ in range and strength as one can see in table (0.1).

force	range in m	strength	mediators (mass in GeV/c^2)	participants
strong	10^{-15}	$\alpha_S \approx 1$	8 gluons (0)	quarks and gluons
weak	10^{-18}	$\alpha_W \approx 10^{-5}$	$W^{\pm}, Z^0$ boson (80,91)	quarks and leptones
electrom.	∞	$\alpha \approx \frac{1}{137}$	photon (0)	el. charged particles
gravity	∞	$\approx 10^{-39}$	graviton (0)	all particles

Table 0.1: Four fundamental forces in physics.

To the forces belong associated carrier particles. They describe the interaction of the matter particles with each other by an exchange of mediators by which a discrete amount of energy is transferred. The interaction particles are gluons for the strong force, W and Z bosons for the weak force and photons for the electromagnetic force. For gravity a particle called graviton is assumed only theoretically. The first three interactions are explained by the SM whereas the gravitational force cannot be included in the theory successfully. The theory of relativity can explain the macro world but cannot be combined with the quantum theory yet. However in the low energy regime of particle physics the effects of gravity are so insignificant, that they can be ignored.

A further essential constituent of the SM is the *Higgs particle*. Although the Higgs particle remains illusive and is yet to be discovered, it is one possibility to provide mass to the other particles. This feature of the SM can be explained by the *Higgs mechanism*. It explains how the gauge bosons and matter particles in the SM obtain their masses. Therefore currently there is a global endeavor to proof the tangible existence of the Higgs particle. For example in the unified electroweak interaction the $W^{\pm}$ and Z bosons are related by the Higgs mechanism to a Higgs field. It interacts with itself and leads to a spontaneous symmetry breaking which finally gives three gauge bosons their masses.

A local gauge theory, here the Yang-Mills theory, furnish a unified specification of the three forces based on symmetries. The gauge symmetry group of the SM is the product of the gauge symmetry groups of each three forces

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_w \times \mathrm{U}(1)_Y \tag{0.1}$$

The gauge symmetry group constitutes the characteristics of the forces. The carrier particles are called gauge quanta and are bosons with spin 1.

Quantum Chromodynamics

The theory of the strong interaction is called Quantum Chromodynamics, or short QCD, and is a non-abelian gauge theory under the gauge group SU(3). The theory describes the interaction between the quarks governed by 8 gluons. In nature $N_f = 6$ quarks are known, which are distinguished by their quantum numbers and masses. The six quarks are up, down, strange, charm, bottom and top, f = u, d, s, c, b, t, and their characteristics are tabulated in (0.2). The particles affected by the strong coupling require an additional charge to preserve the Pauli principle in the QCD.

Therefore quarks and gluons have a colour charge, which occurs in red, green and blue. The colour charge is the origin of the name of the theory.

The QCD is determined by the strong coupling constant α_S and the quark masses.

quark		**flavour**	**charge Q (in e)**	**mass (in MeV)**
u	up	$I_z = \frac{1}{2}$	2/3	1.5 - 3.3
d	down	$I_z = -\frac{1}{2}$	$-1/3$	3.5 - 6.0
c	charm	$C = 1$	2/3	1270^{+70}_{-110}
s	strange	$S = -1$	$-1/3$	105^{+25}_{-35}
t	top	$T = 1$	2/3	≈ 17130
b	bottom	$B = -1$	$-1/3$	4200^{+170}_{-70}

Table 0.2: Properties of the six quark flavours. The given quantum numbers are: electric charge Q, isospin I, charmness C, strangeness S, topness T, bottomness B. The values are taken from the Particle Data Group [1].

A characteristic property of α_S is its behaviour in the low and high energy regime. Due to this it has acquired the name *running coupling*. For high energies, i.e. short distances, the running coupling decreases. The quarks behave like free particles, which is called *asymptotic freedom*. The reason for this is the self interaction of the gluons. In the low energy regime, which corresponds to the hadron physic, the coupling constant is large and the quarks are captured inside hadrons. This is called *Confinement*. The question arises how it is possible to combine the high energy physic of QCD with the observed properties of hadrons in the low energy regime, where perturbation theory is possible? A solution can be found in the non-perturbative method to transfer the QCD to a finite space-time lattice. With the discretization of the QCD the low energy regime can be explored by Monte Carlo simulations. The dimension of the lattice QCD is governed by the lattice spacing a and the lattice extent L. The finite lattice spacing a entails an automatically regularized continuum theory. In this case the lattice spacing complies with the condition of a regulator and prevents singularities to occur. The inverse of the lattice spacing imposes a cutoff proportional to $1/a$ to all momenta. Therefore the theory is automatically regularized and ultra violet divergences, which normally are encountered are avoided.

An advantage of the lattice discretization is that the QCD can be analysed by numerical simulations. The finer the lattice resolution a/L is selected the more accurate the approach to the continuum QCD in the limit $a \to 0$ will be. Certainly the evaluation of the calculations is restricted to the computer power. The mass of a considered particle on the lattice has the same proportionality as the cutoff. To simulate an acceptable propagation of a quark on the lattice, the cutoff has to be much larger than the quarks mass. With todays numerical capabilities it is not possible to simulate the b-quark on the lattice. The restriction $1 \gg am_{\mathrm{b}}$ would increase the resolution L/a that a correct propagation of the b-quark on the lattice is not possible. The discussion of a heavy quark demands an effective theory, which describes the theory of QCD correctly within its scope.

Heavy Quark Effective Theory
In a system with one heavy and one light quark, the light quark is regarded to be relativistic and the heavy quark to be nearly static. The strategy of the Heavy Quark Effective Theory (HQET) is that the observables of QCD are expanded in the inverse of the heavy quark mass $1/m_{\mathrm{b}}$. To obtain useful results for QCD from the effective theory, the two theories have to be connected. This step is called *matching*. The expansion coefficients are the *parameters of the effective theory* and can be obtained by a non-perturbative matching. In this work a full set of the parameters of HQET are determined at tree level. I will consider heavy-light axial and vector currents inside a finite volume, the Schrödinger Functional, and discuss the perturbative expansion of the appropriate correlation functions.

My study is a preliminary of the non-perturbative matching programme of QCD and HQET at order $1/m$. The parameters of HQET are essential for example when studying semileptonic decays as i.e. $\mathrm{B} \to \pi$ and $\mathrm{B} \to \rho$ at next-to-leading order in HQET. One has to introduce potentially suitable observables in order to perform the matching and from which the parameters can be extracted. I examine the observables to ascertain if they provide satisfactory information about the parameters and if they reproduce the classical values. The tree level behaviour is a prerequisite of the non-perturbative calculations. Thus if my complete set of tree level matching observables work out, they will also be well suited as observables in a fully non-perturbative matching calculation. From my results of the tree level coefficients it should be possible to infer the behaviour of the expansion coefficients in the heavy quark mass at $O(1/m_{\mathrm{b}})$. Furthermore indications of the corrections at order $1/m^2$ can be extracted.

In chapter 1 I will introduce the HQET first in the continuum and then transfer the theory to the lattice. I intend to include a finite volume scheme, the Schrödinger Functional scheme, to carry out my calculations. In chapter 2 I will add the $1/m$ corrections to the latter discussed static approximation. I will specify the parameters of the effective theory in chapter 3 and show the principle of a non-perturbative matching. Furthermore the observables needed to perform the matching will be illustrated. In chapter 4 I plan to reveal the perturbative expansion of the correlation functions and complete the tree level results. Finally in chapter 5 the results will be presented and discussed. I will finish my work with a conclusion and summary of the tree level results.

1 Heavy Quark Effective Theory

Heavy Quark Effective Theory has its main application in B-Physics. From decays of B-mesons, one can determine the parameters of the Standard Model, especially the CKM matrix elements, which have hadronic contributions one wants to understand with high precision. Latest experiments in B-Physics are BELLE, BarBar and LHCb.

HQET is an effective theory of QCD in the low energy regime to describe hadronic systems with one heavy quark. The theory describes an effective theory of QCD on the lattice with one heavy quark flavour ψ_{h}. The theory containing $N_{\mathrm{f}} - 1$ light quarks and one heavy quark can be formulated as an asymptotic expansion of the QCD-Lagrangian and observables in the inverse heavy quark mass m. This refers to the works of Eichten and Hill [2, 3, 4]. The scale of low energy QCD ranges from the mass of the pion $m_\pi \approx 140\mathrm{MeV}$ over $m_{\mathrm{D}} = 2\mathrm{GeV}$ to $m_{\mathrm{B}} = 5\mathrm{GeV}$. To obtain a good continuum limit $a \to 0$ of the lattice discretized observables the ultra violet cutoff $\Lambda_{\mathrm{UV}} = \frac{1}{a}$ has to be large compared to the physical energy scale. Furthermore the infrared cutoff $\Lambda_{\mathrm{IR}} = \frac{1}{L}$, arising by the limitation of the lattice extent, has to be much smaller than the QCD scale.

One than faces the scale problem of lattice QCD in the low energy regime to fulfill the restrictions

$$\Lambda_{\mathrm{IR}} = \frac{1}{L} \ll m_\pi, \ldots, m_{\mathrm{D}}, m_{\mathrm{B}} \ll \frac{1}{a} = \Lambda_{\mathrm{UV}}. \tag{1.1}$$

The challenge is now to simulate the b-quark on the lattice in such a way that Monte Carlo (MC) evaluations are meaningful. In an $O(a)$ improved theory one obtains with the mass of the c-quark an amount of lattice points about $L/a \approx 60 \ldots 120$. In particular the simualtion of the b-quark on the lattice would increase the lattice extent L/a by a factor 4.

The expansion of the Lagrangian and the observables in $1/m$ enables the cutoff a^{-1} to be much bigger than the mass of the bottom quark. This solves the problem of the scale hierarchy, because cutoff effects are much more feasible. The heavy quark can be considered as static in the limit $m \to \infty$. Thus the static quark field theory is a local renormalizable quantum field theory [3, 5].

The following chapter first gives the definition of the continuum HQET, with its action, heavy quark propagator and symmetries. Then I outline the HQET expansion of the discretized theory, first in the static approximation and afterwards in chapter 2, I include the $1/m$ corrections. Furthermore I consider a renormalization scheme in a finite volume, the Schrödinger Functional. According to my calculations it gives appropriate formulations of the QCD [6].

1.1 Effective theory in the continuum

The first deliberations of the effective theory of QCD was done by Eichten and Hill [3]. In order to measure matrix elements containing the B-meson decay constant the matrix element f_{b}[1], one has to extract the dependence of the heavy quark mass analytically. The heavy quark mass is considered to be static in the limit $m_{\mathrm{b}} = m \to \infty$, thus the b-quark is at rest in the rest-frame of the system and does not propagate in space.

In this work I consider B-mesons with one heavy and one light quark. The effective Lagrangian describes the dynamics of the light quark whereas the heavy quark just presents a colour source. In leading order in $1/m$ the field theory is called the static approximation but one can add $O((1/m)^n)$ corrections to the observables in a series expansion.

To distinguish between the two flavours I will label the light quark fields with an index l: $\psi_{\mathrm{l}}(x), \overline{\psi}_{\mathrm{l}}(x)$ and the heavy quark fields with a h in the static case: $\psi_{\mathrm{h}}(x), \overline{\psi}_{\mathrm{h}}(x)$ and a b in the relativistic case: $\psi_{\mathrm{b}}(x), \overline{\psi}_{\mathrm{b}}(x)$.

Lagrangian

One possible way to derive the classical effective Lagrangian is to decouple the components of the quark and anti-quark by the Fouldy Wouthuysen-Tani (FTW) transformations which is performed in [7]. The derivation is done order by order in $1/m$ to obtain an asymptotic expansion of QCD. The classical HQET Lagrangian up to next-to-leading order is

$$\mathscr{L} = \mathscr{L}_{\mathrm{h}}^{\mathrm{stat}} + \frac{1}{2m}\mathscr{L}_{\mathrm{h}}^{(1)} + \mathscr{L}_{\bar{\mathrm{h}}}^{\mathrm{stat}} + \frac{1}{2m}\mathscr{L}_{\bar{\mathrm{h}}}^{(1)} + O(\frac{1}{m^2}). \tag{1.2}$$

The static Lagrangians for the quark $\psi(x)$ and anti-quark fields $\psi_{\bar{\mathrm{h}}}(x)$ are

$$\mathscr{L}_{\mathrm{h}}^{\mathrm{stat}} = \overline{\psi}_{\mathrm{h}}(x)(D_0 + m)\psi_{\mathrm{h}}(x), \tag{1.3}$$

$$\mathscr{L}_{\bar{\mathrm{h}}}^{\mathrm{stat}} = \overline{\psi}_{\bar{\mathrm{h}}}(x)(-D_0 + m)\psi_{\bar{\mathrm{h}}}(x), \tag{1.4}$$

whereas the quark field propagates in forward time direction

$$P_+\psi_{\mathrm{h}}(x) = \psi_{\mathrm{h}}, \quad \overline{\psi}_{\mathrm{h}}P_+ = \overline{\psi}_{\mathrm{h}}, \tag{1.5}$$

and the anti-quark field propagates backwards in time

$$P_-\psi_{\bar{\mathrm{h}}}(x) = \psi_{\bar{\mathrm{h}}}(x), \quad \overline{\psi}_{\bar{\mathrm{h}}}(x)P_- = \overline{\psi}_{\bar{\mathrm{h}}}(x). \tag{1.6}$$

The projection operator is defined as $P_\pm = \frac{1}{2}(1 \pm \gamma_0)$. This property includes that the quark fields only have two degrees of freedom per space-time point. The other two of the spinor fields can be set to zero, because of the condition, that $P_\pm$ projects

[1] single hadronic parameter, parameterizing the bound state dynamics of this decay

onto orthogonal subspaces, $P_+P_- = 0$.

The $1/m$ Lagrangian is split into two parts

$$\mathscr{L}_{\mathrm{h}}^{(1)} = -(\mathcal{O}_{\mathrm{kin}} + \mathcal{O}_{\mathrm{spin}}), \tag{1.7}$$

$$\mathcal{O}_{\mathrm{kin}} = \overline{\psi}_{\mathrm{h}}(x)\mathbf{D}^2\psi_{\mathrm{h}}(x), \tag{1.8}$$

$$\mathcal{O}_{\mathrm{spin}} = \overline{\psi}_{\mathrm{h}}(x)\boldsymbol{\sigma}\mathbf{B}(x)\psi_{\mathrm{h}}(x), \tag{1.9}$$

$$\sigma_k = \frac{1}{2}\epsilon_{ijk}\sigma_{ijk}, \quad B_k = i\frac{1}{2}\epsilon_{ijk}F_{ij}. \tag{1.10}$$

The kinetic energy from the heavy quarks residual motion is described by the observable $\mathcal{O}_{\mathrm{kin}}$, where the chromomagnetic interaction with the gluon fields is described by $\mathcal{O}_{\mathrm{spin}}$. The field strength tensor $F_{\mu\nu}$ is defined by the vector potential

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \tag{1.11}$$

with the gauge fields $A_\mu(x) = A_\mu^a T^a$ and the the generator of the SU(3) colour group T^a. σ_k are the pauli matrices, which one can find in appendix A.

Propagator

In the static approximation the continuum propagator is discussed at zero velocity. The Green function $G_{\mathrm{h}}(x,y)$ of the static Dirac operator with the gauge field $A_\mu(x)$

$$(\partial_{x_0} + A_0(x) + m)G_{\mathrm{h}}(x,y) = \delta(x-y)P_+ \tag{1.12}$$

provides the continuum propagator

$$\begin{aligned} G_{\mathrm{h}}(x,y) = \theta(x_0-y_0)\delta(\mathbf{x}-\mathbf{y})\exp\{-m(x_0-y_0)\} \times \\ \mathcal{P}\exp\left\{-\int_{x_0}^{y_0} dz_0\, A_0(z_0,\mathbf{x})\right\} P_+. \end{aligned} \tag{1.13}$$

The anti-quark propagator is derived in the same way. The delta function $\delta(\mathbf{x}-\mathbf{y})$ denotes that the heavy quark is static in space and the heaviside function $\theta(x_0-y_0)$ together with the projection P_+ present forward time propagation. $\mathcal{P}$ is the path ordering product of the gauge fields A_μ. The mass of the bottom quark appears in the propagator in the explicit factor

$$\exp\{-m(x_0-y_0)\} \tag{1.14}$$

for every gauge field. It is possible to remove the mass from the effective Lagrangian by introducing an addend ϵ in the Lagrangian

$$\mathscr{L}_{\mathrm{h}}^{\mathrm{stat}} = \overline{\psi}_{\mathrm{h}}(x)(D_0+\epsilon)\psi_{\mathrm{h}}(x), \tag{1.15}$$

with the limit $\epsilon \to 0_+$, to reveal the appropriate propagation in time. Thus the heavy mass appears in the energies of the quarks

$$E_{\mathrm{h}/\bar{\mathrm{h}}}^{\mathrm{QCD}} = E_{\mathrm{h}/\bar{\mathrm{h}}}^{\mathrm{stat}} + m \tag{1.16}$$

with $m \geq 0$. This provides a shift of the energies to the sector of the Hilbert space containing only one heavy quark.

Symmetries

In the static approximation the Lagrangians (1.3) and (1.4) only contain local composite operators with mass dimension $d \leq 4$. The effective field theory is renormalizable by a finite number of counter terms. These counter terms are constrained by the symmetry conditions of the theory.

Flavour symmetry
The theory is invariant under flavour symmetry transformations [8]. If one assumes F to be the number of heavy quarks, one can add a flavour index to the fields

$$\psi_{\mathrm{h}} \to \psi_{\mathrm{h}} = (\psi_{h_1}, ..., \psi_{\mathrm{h}_F})^T, \quad \overline{\psi}_{\mathrm{h}} \to \overline{\psi}_{\mathrm{h}} = (\overline{\psi}_{\mathrm{h}_1}, ..., \overline{\psi}_{\mathrm{h}_F}). \tag{1.17}$$

Therefore one obtains the symmetry

$$\psi_{\mathrm{h}}(x) \to V\psi_{\mathrm{h}}(x), \quad \overline{\psi}_{\mathrm{h}}(x) \to \overline{\psi}_{\mathrm{h}}(x)V^\dagger, \quad V \in \mathrm{SU}(F). \tag{1.18}$$

In the large mass limit the symmetry emerges independent of how the limit is taken.

Spin symmetry
For each field exist two spin components, but $\mathscr{L}$ has no spin dependent interaction. The fermions are 4-component spinors, but have two zero entries. SU(2) rotations are expressed through spin matrices σ_k

$$\sigma_k = \frac{1}{2}\epsilon_{ijk}\sigma_{ij} \equiv \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}. \tag{1.19}$$

The spin rotation is transformed by

$$\psi_{\mathrm{h}}(x) \to e^{i\alpha_k\sigma_k}\psi_{\mathrm{h}}(x), \quad \overline{\psi}_{\mathrm{h}}(x) \to \overline{\psi}_{\mathrm{h}}(x)e^{-i\alpha_k\sigma_k}, \tag{1.20}$$

which act on each flavour component of the fields. The phases α_k are real parameters.

Local Flavour-number symmetry
The static Lagrangian does not contain spatial derivatives. This provides a local symmetry through

$$\psi_{\mathrm{h}}(x) \to e^{i\eta(\mathbf{x})}\psi_{\mathrm{h}}(x), \quad \overline{\psi}_{\mathrm{h}}(x) \to \overline{\psi}_{\mathrm{h}}(x)e^{-i\eta(\mathbf{x})}, \tag{1.21}$$

with a local phase $\eta(\mathbf{x})$.

Renormalization

For an effective field theory, in which the Lagrangian is made up from local fields, there is an unproven rule of renormalization. The effective theory is renormalizable if the mass dimension of the fields in the Lagrangian does not exceed the space-time dimension d [8]. Thus the ultra violet divergences can be absorbed by adding a complete set of local composite fields. These counter terms have to share the symmetry of the theory and the mass dimension has to be smaller than 5. With $[\psi_{\mathrm{h}}] = \frac{3}{2}$ only fermion fields with up to one derivative are possible. To preserve the local field invariance, no spatial derivatives are required. The static Lagrangian can be written as

$$\mathscr{L}_{\mathrm{h}}(x) = c_1 \mathcal{O}_1(x) + c_2 \mathcal{O}_2(x), \tag{1.22}$$

$$\mathcal{O}_1(x) = \overline{\psi}_{\mathrm{h}}(x)\psi_{\mathrm{h}}(x), \quad \mathcal{O}_2(x) = \overline{\psi}_{\mathrm{h}} D_0 \psi_{\mathrm{h}}. \tag{1.23}$$

The coefficient $c_2 = 1$ only fixes the unphysical field normalization and $c_1 = \delta m$ is the additive mass renormalization. The mass renormalization is possible because for a static quark it exists no chiral symmetry to forbid additive mass renormalization. All divergences can be absorbed in δm, which has the mass dimension $[\delta m] = 1$. Further QCD renormalization of coupling and light quark mass is necessary (see 1.2.2). One can write the energies of all states as

$$E^{\mathrm{QCD}}_{\mathrm{h},\bar{\mathrm{h}}} = E^{\mathrm{stat}}_{\mathrm{h},\bar{\mathrm{h}}}\Big|_{\delta m = 0} + m_{\mathrm{bare}} \tag{1.24}$$

$$m_{\mathrm{bare}} = \delta m + m, \tag{1.25}$$

with finite m, m_{bare} and δm, which is called the residual mass. They compensate the linear divergences of the static theory.

1.2 Lattice HQET in the static approximation

The standard formulation of the Lattice-QCD is given in [9] and [10]. The euclidean space-time $\mathbb{R}^4$ is replaced by a four dimensional hyper cube with lattice spacing a. The derivatives are defined as finite differences (A.8), whereas one has to distinguish between forward and backward derivatives. Furthermore the integrals are replaced by finite summations. The fermion fields $\psi(x)$ and $\overline{\psi}(x)$ are defined at the lattice points and carry Dirac-, colour- and flavour indices.

The gauge fields A^a_μ are replaced by the SU(3) matrices $U(x,\mu)$ for each lattice point and direction $\mu = 0, \ldots, 3$. Thereby $(x,\mu) = b$ is an ordered pair from one lattice point x to the next neighbour $x + a\hat{\mu}$. This pair is called *link* and represents the connection of two neighbouring lattice points. The *parallel transporter* $U(x,\mu) = U_\mu(x)$ is called *link variable.*

The parallel transporter of an arbitrary path on the lattice $\mathcal{C} = b_n \circ \ldots \circ b_1$ is defined by the the product of link variables

$$U(\mathcal{C}) = U(b_{\mathrm{n}}) \cdot \ldots \cdot U(b_1) \equiv \prod_{b \in \mathcal{C}} U(b). \tag{1.26}$$

The curve of the smallest closed oriented path on the lattice is called *plaquette* $p = (x;\mu,\nu)$, see figure (1.1). The amount of all link variables build the gauge lattice field.

The connection of the continuum gauge fields and the link variables is

$$U_\mu(x) = e^{igaT^a A^a_\mu} \tag{1.27}$$

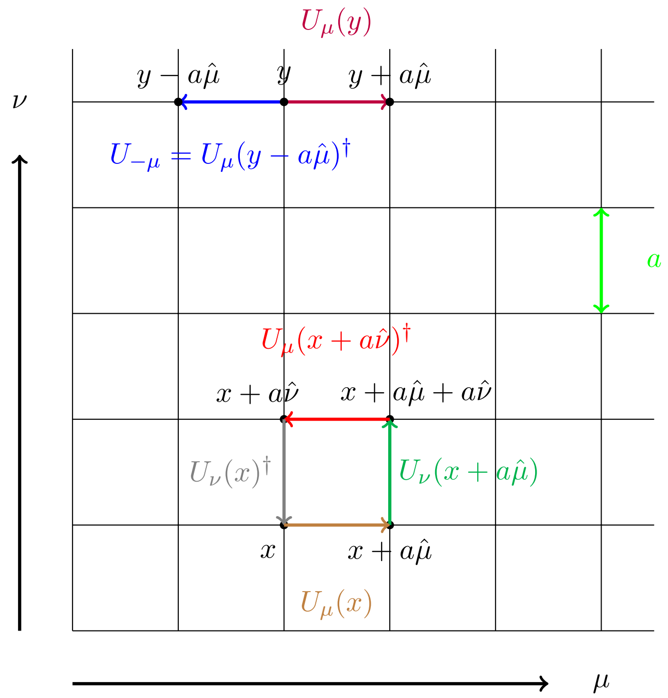


Figure 1.1: Illustration of the lattice definitions.

with the generators T^a of the Lie-algebra of the SU(N). In the case of the SU(3) the generators are the $N^2 - 1 = 8$ Gell-Mann matrices λ^a. The fermion and gauge fields have the following behaviour under local gauge transformations $\Lambda(x) \in \mathrm{SU}(N)$

$$\psi(x) \to \Lambda(x)\psi(x), \quad U_\mu(x) \to \Lambda(x)U_\mu(x)\Lambda(x + a\hat{\mu})^{-1}. \tag{1.28}$$

Lagrangian

To provide the discretized formulation of the HQET observables one has to transcribe the continuum observables to the lattice. The discretization of the quark propagator induces new poles, which can be seen as new quarks, named *doublers*. To avoid the doublers, Wilson [11] induced an additional term in the Dirac operator. The Wilson term causes that fermion modes with $p_\mu = 0$ vanish and modes with the lattice momenta $p_\mu = \frac{\pi}{a}$ provide a contribution of $\frac{2}{a}$. This contribution acts like an additional mass term. In the continuum limit the mass of the doublers become very heavy and they decouple from the theory. The unwanted poles vanish in the continuum limit and only the physical poles remain. Adopting the Dirac operator on the lattice takes the following form

$$D = \sum_{\mu=0}^{3} \gamma_\mu [\frac{1}{2}(\nabla_\mu^* + \nabla_\mu) \underbrace{- \frac{a}{2}\nabla_\mu^* \nabla_\mu}_{\text{Wilson term}}] \tag{1.29}$$

with the forward and backward gauge derivative on the lattice

$$\nabla_\mu \psi(x) = \frac{1}{a}[U_\mu(x)\psi(x+a\hat{\mu}) - \psi(x)], \tag{1.30}$$

$$\nabla^*_\mu \psi(x) = \frac{1}{a}[\psi(x-a\hat{\mu}) - U_\mu(x-a\hat{\mu})^{-1}\psi(x-a\hat{\mu})]. \tag{1.31}$$

To obtain the lattice Lagrangian one has to discretize (1.3), thus $D_0\psi_{\rm h}$. The time component of the Wilson Dirac operator is

$$D_0\gamma_0 \to \frac{1}{2}\left[(\nabla_0 + \nabla_0^*)\gamma_0 - a\nabla_0^*\nabla_0\right], \tag{1.32}$$

and the quark and anti-quark fields satisfy

$$P_+\psi_{\rm h}(x) = \psi_{\rm h}(x), \quad P_-\psi_{\bar{\rm h}} = \psi_{\bar{\rm h}}. \tag{1.33}$$

Therewith one obtains

$$\begin{aligned} D_0\psi_{\rm h}(x) = D_0 \underbrace{\gamma_0 P_+}_{P_+} \psi_{\rm h}(x) &\to \frac{1}{2}\left[(\nabla_0 + \nabla_0^*)\gamma_0 - a\nabla_0^*\nabla_0\right]\psi_{\rm h}(x) \\ &= \frac{1}{2a}\left\{\left[U_0(x)\psi_{\rm h}(x+a\hat{0}) - U_0(x-a\hat{0})^{-1}\psi_{\rm h}(x-a\hat{0})\right]\gamma_0 \right. \\ &\quad \left. - \left[U_0(x)\psi_{\rm h}(x+a\hat{0}) - 2\psi_{\rm h}(x) + U_0(x-a\hat{0})^{-1}\psi_{\rm h}(x-a\hat{0})\right]\right\} \\ &= \frac{1}{a}\left[\psi_{\rm h}(x) - U_0(x-a\hat{0})^{-1}\psi_{\rm h}(x-a\hat{0})\right] \\ &= \nabla_0^*\psi_{\rm h}(x) \end{aligned} \tag{1.34}$$

and in total analogy for the anti-quark

$$D_0\psi_{\bar{\rm h}}(x) = \nabla_0\psi_{\bar{\rm h}}. \tag{1.35}$$

With these replacements it is possible to write down the static Lagrangian for heavy quark and anti-quark fields, which was first introduced by Eichten and Hill [3]. For appropriate calculation a specific normalization factor is included

$$\mathscr{L}_{\rm h} = \frac{1}{1+a\delta m}\overline{\psi}_{\rm h}(x)\left(\nabla_0^* + \delta m\right)\psi_{\rm h}(x), \tag{1.36}$$

$$\mathscr{L}_{\bar{\rm h}} = \frac{1}{1+a\delta m}\overline{\psi}_{\bar{\rm h}}(x)\left(-\nabla_0 + \delta m\right)\psi_{\bar{\rm h}}(x). \tag{1.37}$$

The forward and backward propagation for the quark and anti-quark fields is selected in the forward and backward derivatives in the Lagrangian. The local mass counter term $\delta m \propto \frac{1}{a}$ appears due to the mixing of the kinetic and mass term in the static Lagrangian under renormalization. The lattice action reads in the static approximation

$$S_{\rm h} = a^4 \frac{1}{1+a\delta m}\sum_x \overline{\psi}_{\rm h}(x)\left(\nabla_0^* + \delta m\right)\psi_{\rm h}(x) \tag{1.38}$$

and it preserves all the continuum symmetries.

Quark Propagator

The static propagator can be obtained from the Green function

$$\frac{1}{1+a\delta m}(\nabla_0^* + \delta m)G_{\mathrm{h}}(x,y) = \delta(x-y)P_+ \equiv \frac{1}{a^4}\prod_\mu \delta_{x_\mu/a}\delta_{y_\mu/a}P_+. \tag{1.39}$$

In analogy to the continuum expression the propagator is proportional to $\delta(\mathbf{x}-\mathbf{y})$. One can choose the ansatz

$$G_{\mathrm{h}}(x,y) = g(n_0,k_0;\mathbf{x})\delta(\mathbf{x}-\mathbf{y})P_+ \tag{1.40}$$

with $x_0 = an_0$, $y_0 = ak_0$ and $g(n_0,k_0;\mathbf{x})$. This yields the recursion

$$g(n_0,k_0;\mathbf{x}) = \theta(n_0-k_0)\frac{1}{(1+a\delta m)^{-(n_0-k_0)}}\mathcal{P}(y,x;0)^\dagger. \tag{1.41}$$

$\mathcal{P}(y,x;0)$ is the parallel transporter in the fundamental representation from y to x along a time-like path. One has

$$\mathcal{P}(x,x;0) = 1 \quad \text{and} \quad \mathcal{P}(x,y+a\hat{0};0) = \mathcal{P}(x,y;0)U_0(y). \tag{1.42}$$

All in all the static propagator then reads

$$G_{\mathrm{h}}(x,y) = \theta(x_0-y_0)\delta(\mathbf{x}-\mathbf{y})\exp\left\{-\widehat{\delta m}(x_0-y_0)\right\}\mathcal{P}(y,x;0)^\dagger P_+ \tag{1.43}$$

with $\widehat{\delta m} = \frac{1}{a}\ln(1+a\delta m)$. The mass counter term δm introduces an energy shift of

$$E^{\mathrm{QCD}}_{\mathrm{h},\bar{\mathrm{h}}} = E^{\mathrm{stat}}_{\mathrm{h},\bar{\mathrm{h}}}\Big|_{\delta m=0} + m_{\mathrm{bare}} \tag{1.44}$$

$$m_{\mathrm{bare}} = \widehat{\delta m} + m \tag{1.45}$$

which applies to energies of systems with one heavy quark or anti-quark.

Symmetries

All the continuum symmetries of the static theory are preserved on the lattice. Especially the $U(2F)$ spin-flavour symmetry and the local flavour-number conservation. The transformations on the lattice in the static approximation can be obtained by replacing the continuum form with the lattice regularized fields. HQET symmetries are defined in terms of transformations of the heavy quark fields and the light quark fields do not change.

1.2.1 $O(a)$ improvement

Once one has computed observables from data of numerical simulations, they further depend on the lattice spacing a. To obtain physical quantities without a relation to the finite cutoff, the continuum limit has to be taken, yielding a well-defined and unique value after the approach of $a \to 0$. The behaviour of how the continuum limit is achieved is dictated by the *renormalization group equations* (see appendix A 4). They describe how the parameters of the theory act under a change of the scale, which is in our case the lattice spacing a.

If the considered observable $\mathcal{O}$ is a dimensionless quantity, the expectation value of the lattice regularized observable is

$$\langle \mathcal{O} \rangle^{\text{lattice}} = \langle \mathcal{O} \rangle^{\text{continuum}} + O(a^n). \tag{1.46}$$

The cutoff effects of $O(a^n)$, where n depends on the discretization of the QCD-Lagrangian. The quantity is determined for different lattice spacings a, to achieve the continuum limit by an extrapolation. The procedure to reduce the leading order cutoff effects in the operators and Lagrangian is called *improvement*. The idea of improvement is to reduce or even eliminate the $O(a^n)$ terms in the observables by adding appropriate combinations of operators with coefficients, which are chosen to reduce or eliminate the lattice effects. I want to refer to the $O(a)$ improvement of Symanzik in the following.

Symanziks programme of $O(a)$ improvement

The discretization of the Dirac operator requires the Wilson term (1.29) to avoid additional poles in the quark propagator. The disadvantage of the Wilson term is that it increases the lattice artifacts from $O(a^2)$ to $O(a)$. Therefore the $O(a)$ terms in the Lagrangian and observables have to be eliminated to obtain a meaningful continuum limit approach. A process was proposed by Symanzik [12, 13], the $O(a)$ *Symanzik improvement*. It is an effective continuum theory which describes the Wilson lattice action with finite spacing a and its approach to the continuum limit. The idea is that one can reduce the discretization errors by adding higher dimensional operators to the lattice action and composite fields. The expansion coefficient of the theory is the lattice spacing a. In the case of the static theory the effective action for the static quark is thus given by

$$S_{\text{eff}} = S_0 + aS_1 + a^2 S_2 + \ldots, \quad S_k = \int d^4x \, \mathscr{L}_k(x), \tag{1.47}$$

with the static continuum Lagrangian $\mathscr{L}_0(x) = \mathscr{L}_{\text{h}}^{stat}(x)$. $\mathscr{L}_k$ are local operators of dimension $k+4$, which are built from products of gluon and quark fields.

To achieve the $O(a)$ improvement one has to find all operators of dimension 5, which have the same symmetry as the lattice theory

$$\mathscr{L}_1(x) = \sum_{i=3}^{5} c_i \mathcal{O}_i(x). \tag{1.48}$$

$\mathscr{L}_1$ will contain additional derivatives or powers of the quark mass m_l, to have $d = 5$. Possible operators are

$$\mathcal{O}_3(x) = \overline{\psi}_\mathrm{h} D_0 D_0 \psi_\mathrm{h}, \tag{1.49}$$

$$\mathcal{O}_4(x) = m_\mathrm{l} \overline{\psi}_\mathrm{h} D_0 \psi_\mathrm{h}, \tag{1.50}$$

$$\mathcal{O}_5(x) = m_\mathrm{l}^2 \overline{\psi}_\mathrm{h} \psi_\mathrm{h}. \tag{1.51}$$

With the use of the equation of motion $D_0\psi_\mathrm{h} = 0$, the operators $\mathcal{O}_3$ and $\mathcal{O}_4$ can be canceled and the resulting operator in $O(a)$ is $\mathcal{O}_5$. The effective action in Symanziks theory is

$$S_\mathrm{eff} = \int d^4x \, \left(\mathscr{L}_\mathrm{h}^\mathrm{stat}(x) + a c_5 m_\mathrm{l}^2 \overline{\psi}_\mathrm{h} \psi_\mathrm{h} + \ldots \right) \tag{1.52}$$

and it induces a redefinition of the mass counter term δm, which depends therefore on m_l. I remark here that one has automatic on-shell $O(a)$ improvement for the static action, because almost in all applications the mass counter term δm can be canceled in the relation between the physical observables [8].

1.2.2 Renormalization

The bare parameters of the theory receive a renormalization. That provides the following renormalized coupling and quark mass in the unimproved theory

$$\tilde{g}_\mathrm{R}^2 = g_0^2 Z_\mathrm{g}(g_0^2, a\mu), \tag{1.53}$$

$$\tilde{m}_\mathrm{R} = m_\mathrm{q} Z_\mathrm{m}(g_0^2, a\mu). \tag{1.54}$$

Thereby μ is the renormalization scheme. The Wilson term in the lattice fermion action breaks the chiral symmetry in the lattice regularized theory. Consequently in the $O(a)$ improved theory an additive renormalization constant for the coupling and quark mass is demanded [14]

$$g_\mathrm{R}^2 = \tilde{g}_0^2 Z_\mathrm{g}(\tilde{g}_0^2, a\mu), \tag{1.55}$$

$$m_\mathrm{R} = \tilde{m}_\mathrm{q} Z_\mathrm{m}(\tilde{g}_0^2, a\mu) \tag{1.56}$$

with the parameters

$$\tilde{g}_0^2 = g_0^2(1 + b_\mathrm{g}(g_0^2) a m_\mathrm{q}), \tag{1.57}$$

$$\tilde{m}_\mathrm{q} = m_\mathrm{q}(1 + b_\mathrm{m}(g_0^2) a m_\mathrm{q}). \tag{1.58}$$

The mass m_q denotes the substracted mass of the quarks $m_\mathrm{q} = m_0 - m_\mathrm{c}$, to restore the chiral symmetry on the lattice. The coefficients b_g and b_m depend on the coupling g_0^2 and have to be tuned to reduce the $O(a)$ lattice artifacts. The coefficient b_m is discussed in the quenched approximation in [15]

$$b_\mathrm{m}(g_0^2) = \left(-0.5 - 0.09623 g_0^2\right) \cdot \frac{1 - 0.6905 g_0^2 + 0.0584 g_0^4}{1 - 0.6905 g_0^2}. \tag{1.59}$$

and the perturbative expansion of the renormalization constant is given in [14]

$$Z_{\mathrm{g/m}}(\tilde{g}_0^2, a\mu) = 1 + Z^{(1)}_{\mathrm{g/m}}(a\mu)\tilde{g}_0^2 + Z^{(2)}_{\mathrm{g/m}}(a\mu)\tilde{g}_0^4 + \dots. \tag{1.60}$$

The perturbative expansion of the coupling constant and the quark mass can be determined in the *minimal substraction scheme*. The expansion coefficients are mass-independent and the renormalization constants at given order in the g_0^2 expansion are polynomials in $\ln(a\mu)$. The perturbative expansion leads to [16]

$$g_0^2 = g_{\mathrm{R}}^2 + O(g_{\mathrm{R}}^4), \tag{1.61}$$

$$m_0 = m_0^{(0)} + g_{\mathrm{R}}^2 m_0^{(1)} + O(g_{\mathrm{R}}^4). \tag{1.62}$$

At tree level the renormalized mass is given by

$$m_{\mathrm{R}}^{(0)} = m_0^{(0)}(1 - a\frac{m_0^{(0)}}{2}), \tag{1.63}$$

where I used the tree level values for $b_{\mathrm{m}} = -0.5$ and $Z_{\mathrm{m}} = 1$. This provides a tree level mass of

$$m_0^{(0)} = \frac{1}{a}\left(1 - \sqrt{1 - 2am_{\mathrm{R}}^{(0)}}\right). \tag{1.64}$$

Later I introduce the masses of the quarks through the quantity $z = \tilde{m}_{\mathrm{q}}L$. For the light quarks I employ $z = 0$ and for the heavy quarks I utilize different values for z to obtain a z dependence of HQET quantities.

1.3 HQET and the Schrödinger Functional

Numerical simulations require a discrete version of QCD in a finite volume. The easiest lattice formulation implicate periodic boundary conditions for the fields in space and time direction. If one considers periodic boundary conditions in three space dimensions and one fixed boundary value in the fourth dimension, finite volume boundary effects occur. A particular finite volume scheme is called the *Schrödinger Functional (SF)* [6].

In my thesis I use the advantage of the SF, that correlation functions, which are defined in this finite scheme, can be determined effectively. From these correlation functions new observables can be built, which are also relevant to B-Physics, e.g. to calculate the B-meson decay constant or mass.

1.3.1 Definition

The definition of the SF in the continuum Yang-Mills theory is given by specific boundary conditions and the euclidean partition function. The space-time manifold of the SF takes the shape of a finite cylinder with volume $L^3 \times T$. The extent in time direction is T and the extent in all three space directions is L.

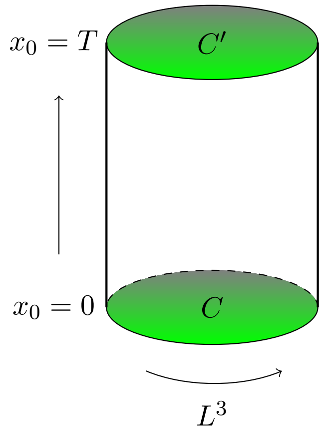


Figure 1.2: Illustration of the SF as a cylinder topology.

In time direction one has Dirichlet boundary conditions for the vector potential

$$A_k(x) = \begin{cases} C_k^\Lambda(\mathbf{x}) & \text{at } x_0 = 0, \\ C_k'(\mathbf{x}) & \text{at } x_0 = L, \end{cases} \tag{1.65}$$

where C, C' are classical gauge potentials. The gauge transformation obeys

$$C_k^\Lambda(\mathbf{x}) = \Lambda(\mathbf{x})C_k(\mathbf{x})\Lambda(\mathbf{x})^{-1} + \Lambda(\mathbf{x})\partial_k\Lambda(\mathbf{x})^{-1}, \quad \Lambda \in \mathrm{SU}(N). \tag{1.66}$$

Furthermore the SF has periodic boundary conditions in space for the gauge fields

$$A_k(x + L\hat{k}) = A_k(x), \quad \Lambda(\mathbf{x} + L\hat{k}) = \Lambda(\mathbf{x}). \tag{1.67}$$

An illustration of the SF manifold in 3 dimensions as a cylinder topology is shown in figure (1.2). These boundary conditions and the partition function define the SF

$$\mathcal{Z}[C', C] \equiv \int \mathrm{D}[\Lambda] \int \mathrm{D}[A]\, e^{-S_{\mathrm{G}}[A]}, \tag{1.68}$$

$$S_{\mathrm{G}}[A] = -\frac{1}{2g_0^2} \int \mathrm{d}^4x \ \mathrm{tr}\ \{F_{\mu\nu}F_{\mu\nu}\}, \tag{1.69}$$

$$\mathrm{D}[A] = \prod_{x,\mu,a} \mathrm{d}A_\mu^a(x), \quad \mathrm{D}[\Lambda] = \prod_{\mathbf{x}} \mathrm{d}\Lambda(\mathbf{x}) \tag{1.70}$$

where $\mathrm{d}\Lambda(\mathbf{x})$ is the Haar measure of $\mathrm{SU}(N)$. An important property of the SF is the gauge invariance of the partition function regarding to the boundary field C

$$\mathcal{Z}[C'^\Omega, C^\Omega] = \mathcal{Z}[C', C], \quad \Omega \in \mathrm{SU}(N). \tag{1.71}$$

Relevance of a renormalization scheme in a finite volume

Numerical calculations of a physical value, e.g. the running coupling, which depends on an energy scale μ, has to fulfill several conditions. To apply perturbation theory, calculations have to be possible in the high energy regime to connect it to other schemes. In lattice QCD the restrictions of the observables are given by the lattice spacing a and the finite lattice extent L. On the one hand the lattice cutoff has to be much larger than the energy scale. This implies, that the lattice has to be very fine, with a small a. On the other hand the lattice extent L has to be large compared to the confinement scale, to avoid finite size effects. All in all the energy scale μ has to satisfy the inequation

$$L \gg \frac{1}{\Lambda} \sim \frac{1}{0.4\mathrm{GeV}} \gg \frac{1}{\mu} \sim \frac{1}{10\mathrm{GeV}} \gg a. \tag{1.72}$$

Here the energy scales range the low and high energy regimes.

With present computer simulations it is not possible to fulfill the restrictions. Lattice resolutions are required, which cannot be achieved with todays computer capabilities. The clue to the puzzle is to consider the artifacts, which are produced by the finite volume, as observables [17]. The enregy scale and the lattice extent are connected that

$$\mu = \frac{1}{L}. \tag{1.73}$$

Thus calculations of the running coupling can be achieved in several steps, by increasing μ by 100% or reducing L by 50%. With the finite-size scaling technique it is possible to study the scale dependence of the observables [17]. Therefore in each step, no substantial differences of the energy occur and the continuum limit can be achieved. Hence the SF scheme is an intermediate renormalization scheme to relate the low and high energy regime of QCD. The SF renders a gap in the spectrum of the Dirac operator which enables the quark mass to be set to zero. Thus the SF is a mass independent renormalization scheme with simplified renormalization group invariants.

1.3.2 Discretization of the SF

An adoption of the lattice regularized theory within the SF scheme is possible, considering that the continuum limit $a \to 0$ has to be well defined and unique.

In the latter sections I discussed the SF pure Yang-Mills theory. To describe the full strong interaction I have to introduce fermionic degrees of freedom to the theory, which was first considered by Sint [18, 19]. In the following section light quark fields are discussed. The boundary fields of the pure gauge theory C, C' are joined by the boundary fields of the fermions and anti-fermions ρ, ρ', $\bar{\rho}$ and $\bar{\rho}'$. $\rho(\mathbf{x})$ and $\bar{\rho}(\mathbf{x})$ are the boundary quark fields in the SF at $x_0 = 0$, whereas $\rho'(\mathbf{x})$ and $\bar{\rho}'(\mathbf{x})$ are the boundary quark fields at $x_0 = T$.

Gauge action

The gauge invariance of the gauge action has to be preserved on the lattice. A gauge invariant quantity is constituted by the trace of a product of links along a closed path. Following Wilson [11] the gauge action in the SF is

$$S_{\mathrm{G}}[U] = \frac{1}{g_0^2} \sum_p \omega(p) \mathrm{tr} \{1 - U(p)\} . \tag{1.74}$$

The sum runs over all oriented plaquettes p on the lattice. The weight factors are given by [6]

$$\omega(p) = \begin{cases} 1 & \text{plaquette in the bulk,} \\ \frac{1}{2} & \text{plaquette at the boundary } x_0 = 0, T. \end{cases} \tag{1.75}$$

Fermion action

The fermion action on the lattice [20] is

$$S_{\mathrm{F}}[U, \overline{\psi}, \psi] = a^4 \sum_{0<x_\mu<L} \overline{\psi}(x)(D + m_0)\psi(x) \tag{1.76}$$

with the bare quark mass m_0. D is given by the Wilson Dirac operator (1.29). The disadvantage of the Wilson term is that it breaks the chiral symmetry and increases discretization errors from $O(a^2)$ to $O(a)$. Concerning the restoration of the chiral symmetry on the lattice an additive and multiplicative renormalization constant of the quark mass is needed. This was discussed in 1.2.2.

$O(a)$ improvement

Adding counter terms proportional to a to the lattice action reduces the $O(a)$ discretization effects. This is possible because they vanish in the continuum limit. In the usual lattice regularized QCD with periodic boundary conditions in spatial and time direction only one counter term is needed to improve the fermionic action. It was defined by Sheikholeslami and Wohlert [21]

$$S_{\mathrm{SW}}[U, \overline{\psi}, \psi] = a^5 \sum_x c_{\mathrm{SW}} \overline{\psi}(x) \frac{i}{4} \sigma_{\mu\nu} \widehat{F}_{\mu\nu} \psi(x). \tag{1.77}$$

The field strength tensor on the lattice is a sum of plaquettes

$$\widehat{F}_{\mu\nu} = \frac{1}{8a^2} \left(Q_{\mu\nu}(x) - Q_{\nu\mu}(x) \right), \tag{1.78}$$

$$\begin{aligned} Q_{\mu\nu} = {} & U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu(x + a\hat{\nu})^{-1} U_\nu(x)^{-1} \\ & + U_\nu(x) U_\mu(x - a\hat{\mu} + a\hat{\nu})^{-1} U_\nu(x - a\hat{\mu})^{-1} U_\mu(x - a\hat{\mu}) \\ & + U_\mu(x - a\hat{\mu})^{-1} U_\nu(x - a\hat{\mu} - a\hat{\nu})^{-1} U_\mu(x - a\hat{\mu} - a\hat{\nu}) U_\nu(x - a\hat{\nu}) \\ & + U_\nu(x - a\hat{\nu})^{-1} U_\mu(x - a\hat{\nu}) U_\nu(x + a\hat{\mu} - a\hat{\nu}) U_\mu(x)^{-1}. \end{aligned} \tag{1.79}$$

$\widehat{F}_{\mu\nu}$ is also called the clover leaf representation in correspondence to the look of the plaquettes, see figure (1.3).

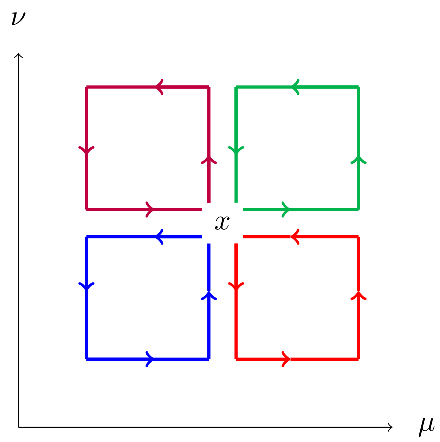


Figure 1.3: Illustration of the field strength tensor as a clover leaf. The different coloured plaquettes correspond to the 4 terms in (1.79).

In [22] the improvement coefficient was discussed non-perturbatively in the quenched approximation

$$c_{\mathrm{SW}} = \frac{1 - 0.656 g_0^2 - 0.152 g_0^4 - 0.054 g_0^6}{1 - 0.922 g_0^2}, \quad 0 \leq g_0 \leq 1. \tag{1.80}$$

In the SF scheme the boundary fields in time direction have to respect Dirichlet boundary conditions, thus to improve the gauge action one can alter the weight factor (1.75) to

$$\omega(p) = \begin{cases} c_{\mathrm{t}}(g_0) & \text{time plaquette related to the boundary} \\ \frac{1}{2} c_{\mathrm{s}}(g_0) & \text{spatial plaquette at the boundary } x_0 = 0, T \\ 1 & \text{else} \end{cases}. \tag{1.81}$$

The coefficients in space c_{s} and time c_{t} have to be chosen correctly in order to compensate the boundary effects. To improve the fermion action Lüscher et. al. [14] considered the counter terms

$$S_{\mathrm{F,b}} = a^4 \sum_{\mathbf{x}} \{ (\tilde{c}_{\mathrm{s}} - 1)[\mathcal{O}_{\mathrm{s}}(\mathbf{x}) - \mathcal{O}'_{\mathrm{s}}(\mathbf{x})] + (\tilde{c}_{\mathrm{t}} - 1)[\mathcal{O}_{\mathrm{t}}(\mathbf{x}) - \mathcal{O}'_{\mathrm{t}}(\mathbf{x})] \}, \tag{1.82}$$

with $\tilde{c}_{\mathrm{t}} = \tilde{c}_{\mathrm{t}}(g_0)$, $\tilde{c}_{\mathrm{s}} = \tilde{c}_{\mathrm{s}}(g_0)$ and

$$\mathcal{O}_{\mathrm{s}}(\mathbf{x}) = \frac{1}{2} \overline{\rho}(\mathbf{x}) \gamma_k (\nabla_k^* + \nabla_k) \rho(\mathbf{x}), \tag{1.83}$$

$$\mathcal{O}'_{\mathrm{s}}(\mathbf{x}) = \frac{1}{2} \overline{\rho}'(\mathbf{x}) \gamma_k (\nabla_k^* + \nabla_k) \rho'(\mathbf{x}), \tag{1.84}$$

$$\mathcal{O}_{\mathrm{t}}(\mathbf{x}) = \frac{1}{2} \left[\overline{\psi}(x) (P_- \nabla_0 + P_+ \overleftarrow{\nabla}_0^*) \psi(x) \right]_{x_0 = a}, \tag{1.85}$$

$$\mathcal{O}'_{\mathrm{t}}(\mathbf{x}) = \frac{1}{2} \left[\overline{\psi}(x) (P_+ \nabla_0 + P_- \overleftarrow{\nabla}_0^*) \psi(x) \right]_{x_0 = T - a}. \tag{1.86}$$

1.3.3 Classical quark propagation

The lattice Dirac operator was given in (1.29). Due to the appearance of $O(a)$ discretization effects one has to add a counter term to the Dirac operator of $O(a)$

$$\delta D = \delta D_{\mathrm{V}} + \delta D_{\mathrm{b}} \tag{1.87}$$

which is a sum of a volume and a boundary term

$$\delta D_{\mathrm{V}} \psi(x) = c_{\mathrm{SW}} a \frac{i}{4} \sigma_{\mu\nu} \widehat{F}_{\mu\nu} \psi(x), \tag{1.88}$$

$$\begin{aligned} \delta D_{\mathrm{b}} \psi(x) = (\tilde{c}_{\mathrm{t}} - 1) \frac{1}{a} \big\{ & \delta_{x_0,a} [\psi(x) - U_0(x - a\hat{0})^{-1} P_+ \psi(x - a\hat{0})] \\ & + \delta_{x_0,T-a} [\psi(x) - U_0(x)^{-1} P_- \psi(x + a\hat{0})] \big\} . \end{aligned} \tag{1.89}$$

In the bulk of the SF the quark propagator is defined through the Green function

$$(D + \delta D + m_0)\, S(x,y) = \frac{1}{a^4} \delta_{x,y}, \qquad x_0 \in (0,T), \tag{1.90}$$

whereas at the boundaries it has to fulfill

$$P_+ S(x,y)|_{x_0=0} = P_- S(x,y)|_{x_0=T} = 0. \tag{1.91}$$

Furthermore the quark propagator satisfies the hermiticity property

$$\gamma_5 S(y,x) \gamma_5 = S(x,y)^\dagger . \tag{1.92}$$

The classical solution of the lattice Dirac operator in the SF with $0 < x_0 < T$ can be obtained from

$$(D + \delta D + m_0)\, \psi_{\mathrm{cl}}(x) = 0. \tag{1.93}$$

At the boundaries $x_0 = 0$ and $x_0 = T$, the classical quark field obeys

$$P_+ \psi_{\mathrm{cl}}(x)|_{x_0=0} = \rho(\mathbf{x}), \quad P_- \psi_{\mathrm{cl}}(x)|_{x_0=T} = \rho'(\mathbf{x}). \tag{1.94}$$

The solution of (1.93) is therefore

$$\begin{aligned} \psi_{\mathrm{cl}}(x) = a^3 \sum_{\mathbf{y}} \tilde{c}_{\mathrm{t}} \Big(& S(x,y) U(y - a\hat{0}, 0)^{-1} \rho(\mathbf{y}) \big|_{y_0=a} \\ & + S(x,y) U(y,0) \rho'(\mathbf{y}) \big|_{y_0=T-a} \Big) . \end{aligned} \tag{1.95}$$

One finds the expression for the anti-quark in total analogy by using the adjoint expression.

1.3.4 Heavy and light quarks on the SF

In the HQET the static and relativistic quark fields have to be considered separately in the SF scheme. Accepting that static fermions do not propagate in space, they differ from the light quark fields in their boundary behaviour and their corresponding action.

Light quarks

The relativistic quarks are periodic up to a phase

$$\psi_{\rm l}(x+L\hat{k}) = e^{i\theta_k}\psi_{\rm l}(x), \quad \overline{\psi}_{\rm l}(x+L\hat{k}) = e^{-i\theta_k}\overline{\psi}_{\rm l}(x), \tag{1.96}$$

with the phase angles θ_k, where $k = 1, \ldots 3$, which are real parameters. The phase is introduced due to practical reasons in numerical calculations. With the additional phase, small eigenvalues of the fermion matrix are prevented, thus they provide an easy inversion of the fermion matrix [23]. The connection of the angles and the momentum of the fermions on the lattice is

$$p_k = \frac{2\pi l_k}{L} + \frac{\theta_k}{L}, \quad l_k \in \mathbb{Z}. \tag{1.97}$$

In total equivalence it is possible to include the phase into the spatial components of the covariant derivative

$$D_\mu = \delta_\mu + A_\mu + i\frac{\theta_\mu}{L} \tag{1.98}$$

with $\theta_0 = 0$. This provides periodic fermion fields. I use for the integers $l_k = 0$, therefore the lattice momenta in space dimensions are linked with the periodicity angles via $\mathbf{p} = \frac{1}{L}(\theta_x, \theta_y, \theta_z)$. If one uses, in a system with one heavy and one light quark, e.g. for the axial current $A_0(x) = \overline{\psi}_{\rm l}(x)\gamma_0\gamma_5\psi_{\rm b}(x)$, same periodicity angles $\vec{\theta}_{\rm l} = \vec{\theta}_{\rm b}$, the momentum of the quark is compensated by the opposite momentum of the anti-quark. Therefore considering a bilinear system with $\vec{\theta}_{\rm l} = \vec{\theta}_{\rm b}$ entails zero momentum of the system.

The Dirac operator is an operator of first order, thus the equation of motion derived from the light quark action is a differential equation of first order. Therefore only half of the fermionic field components can be fixed at the boundaries $x_0 = 0, T$, to obtain a unique solution of the Dirac equation. Boundary conditions for the light quarks provide explicit boundary functions

$$P_+\psi_{\rm l}(x)|_{x_0=0} = \rho_{\rm l}(\mathbf{x}), \quad P_-\psi_{\rm l}(x)|_{x_0=L} = \rho_{\rm l}'(\mathbf{x}), \tag{1.99}$$

$$\overline{\psi}_{\rm l}(x)P_-|_{x_0=0} = \overline{\rho}_{\rm l}(\mathbf{x}), \quad \overline{\psi}_{\rm l}(x)P_+|_{x_0=L} = \overline{\rho}_{\rm l}'(\mathbf{x}). \tag{1.100}$$

Finally the light quark action in the SF is [18]

$$S_{\rm l}[U, \overline{\psi}_{\rm l}, \psi_{\rm l}] = a^4\sum_x \overline{\psi}_{\rm l}(x)(D + m_0)\psi_{\rm l}(x) \tag{1.101}$$

with the Wilson Dirac operator D and the boundary conditions

$$\psi_{\rm l}(x) = 0 \quad \text{if} \quad x_0 < 0 \quad \text{or} \quad x_0 > T \tag{1.102}$$

and

$$P_-\psi_{\rm l}(x)|_{x_0=0} = P_+\psi_{\rm l}(x)|_{x_0=T} = 0. \tag{1.103}$$

Static quarks

With the condition

$$P_+\psi_{\mathrm{h}}(x) = \psi_{\mathrm{h}}(x), \quad \overline{\psi}_{\mathrm{h}}(x)P_- = \overline{\psi}_{\mathrm{h}}(x) \tag{1.104}$$

and the fact that static quarks do not propagate in space, no spatial boundary conditions are needed. In time direction the boundary fields are defined as

$$\psi_{\mathrm{h}}(x)\big|_{x_0=0} = \rho_{\mathrm{h}}(\mathbf{x}), \quad \overline{\psi}_{\mathrm{h}}(x)\big|_{x_0=T} = \overline{\rho}'_{\mathrm{h}}(\mathbf{x}). \tag{1.105}$$

Finally the static quark action can be written as [20]

$$S_{\mathrm{h}}[U,\overline{\psi}_{\mathrm{h}},\psi_{\mathrm{h}}] = a^4 \sum_x \overline{\psi}_{\mathrm{h}}(x)\nabla_0^*\psi_{\mathrm{h}}(x) \tag{1.106}$$

with the boundary conditions

$$\psi_{\mathrm{h}}(x) = 0 \quad \text{if} \quad x_0 < 0 \quad \text{or} \quad x_0 \geq T. \tag{1.107}$$

Schrödinger Functional action in HQET

The complete action with one heavy and one light quark in the static approximation is

$$S[U,\overline{\psi}_{\mathrm{l}},\psi_{\mathrm{l}},\overline{\psi}_{\mathrm{h}},\psi_{\mathrm{h}}] = S_G[U] + S_{\mathrm{l}}[U,\overline{\psi}_{\mathrm{l}},\psi_{\mathrm{l}}] + S_{\mathrm{h}}[U,\overline{\psi}_{\mathrm{h}},\psi_{\mathrm{h}}]. \tag{1.108}$$

The SF in the continuum is defined through the partition function

$$\begin{aligned}&\mathcal{Z}[C',\overline{\rho}'_{\mathrm{l}},\rho'_{\mathrm{l}},\overline{\rho}'_{\mathrm{h}};C,\overline{\rho}_{\mathrm{l}},\rho_{\mathrm{l}},\rho_{\mathrm{h}}]\\ &= \int \mathrm{D}[A]\mathrm{D}[\psi_{\mathrm{l}}]\mathrm{D}[\overline{\psi}_{\mathrm{l}}]\mathrm{D}[\psi_{\mathrm{h}}]\mathrm{D}[\overline{\psi}_{\mathrm{h}}]\, e^{S[A,\psi_{\mathrm{l}},\psi_{\mathrm{l}},\overline{\psi}_{\mathrm{h}},\psi_{\mathrm{h}}]}.\end{aligned} \tag{1.109}$$

The expectation value of an operator $\mathcal{O}$ is determined for vanishing boundary fields to obtain simple renormalization properties

$$\langle\mathcal{O}\rangle = \left\{\frac{1}{\mathcal{Z}}\int \mathrm{D}[A]\mathrm{D}[\psi_{\mathrm{l}}]\mathrm{D}[\overline{\psi}_{\mathrm{l}}]\mathrm{D}[\psi_{\mathrm{h}}]\mathrm{D}[\overline{\psi}_{\mathrm{h}}]\mathcal{O}e^{S[A,\psi_{\mathrm{l}},\psi_{\mathrm{l}},\overline{\psi}_{\mathrm{h}},\psi_{\mathrm{h}}]}\right\}_{\substack{\overline{\rho}'_{\mathrm{l}}=\rho'_{\mathrm{l}}=\\ \overline{\rho}_{\mathrm{l}}=\rho_{\mathrm{l}}=\\ \overline{\rho}'_{\mathrm{h}}=\rho_{\mathrm{h}}=0}} \tag{1.110}$$

The operator can contain light and heavy quark fields through the sources

$$\zeta_{\mathrm{l}}(\mathbf{x}) = \frac{\delta}{\delta\overline{\rho}_{\mathrm{l}}(\mathbf{x})}, \qquad \bar{\zeta}_{\mathrm{l}}(\mathbf{x}) = -\frac{\delta}{\delta\rho_{\mathrm{l}}(\mathbf{x})}, \tag{1.111}$$

$$\zeta'_{\mathrm{l}}(\mathbf{x}) = \frac{\delta}{\delta\bar{\rho}'_{\mathrm{l}}(\mathbf{x})}, \qquad \bar{\zeta}'_{\mathrm{l}}(\mathbf{x}) = -\frac{\delta}{\delta\rho'_{\mathrm{l}}(\mathbf{x})}, \tag{1.112}$$

$$\zeta'_{\mathrm{h}}(\mathbf{x}) = \frac{\delta}{\delta\bar{\rho}'_{\mathrm{h}}(\mathbf{x})}, \qquad \bar{\zeta}_{\mathrm{h}}(\mathbf{x}) = -\frac{\delta}{\delta\rho_{\mathrm{h}}(\mathbf{x})}. \tag{1.113}$$

1.3.5 Correlation functions and their renormalization

Correlation functions in the SF are expectation values of products of observables. They are ideal to be considered in this context, because they are proportional to the probability amplitude of quark propagation in the SF. Two-point SF correlation functions in the static approximation from boundary-to-bulk in the pseudo-scalar and vector channel are [24]

$$f_{\mathrm{A}}^{\mathrm{stat}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle A_0^{\mathrm{stat}}(x)\;\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{1.114}$$

$$k_{\mathrm{V}}^{\mathrm{stat}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = -\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle V_k^{\mathrm{stat}}(x)\;\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_k\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{1.115}$$

$$k_{\mathrm{V}_{11}}^{\mathrm{stat}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle V_1^{\mathrm{stat}}(x)\;\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_1\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{1.116}$$

with the static axial and vector current

$$A_\mu^{\mathrm{stat}}(x) = \overline{\psi}_{\mathrm{l}}(x)\gamma_\mu\gamma_5\psi_{\mathrm{h}}(x), \tag{1.117}$$

$$V_\mu^{\mathrm{stat}}(x) = \overline{\psi}_{\mathrm{l}}(x)\gamma_\mu\psi_{\mathrm{h}}(x). \tag{1.118}$$

$k_{\mathrm{V}}^{\mathrm{stat}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}})$ is a correlator between the space component of the vector current in the bulk of the SF and a vector boundary source $\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_k\zeta_{\mathrm{l}}(\mathbf{z})$ at the surface at $x_0 = 0$. It describes the propagation of a quark anti-quark pair from the surface $x_0 = 0$ to the space-time point x, where they annihilate each other.

Including the time component of the vector current, I introduce the correlation function

$$k_{\mathrm{V}_0}^{\mathrm{stat}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle V_0^{\mathrm{stat}}(x)\;\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_k\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle. \tag{1.119}$$

To consider the space components of the axial current I write down the correlators

$$f_{\mathrm{A}_k}^{\mathrm{stat}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle A_k^{\mathrm{stat}}(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{1.120}$$

$$f_{\mathrm{AV}_{21}}^{\mathrm{stat}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = i\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle A_2^{\mathrm{stat}}(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_1\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle. \tag{1.121}$$

Furthermore I consider the SF correlation functions from boundary-to-boundary

$$k_1^{\mathrm{stat}}(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = -\frac{a^{12}}{6L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z},k}\left\langle\bar{\zeta}'_{\mathrm{l}}(\mathbf{u})\gamma_k\zeta'_{\mathrm{h}}(\mathbf{v})\;\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_k\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{1.122}$$

$$f_1^{\mathrm{stat}}(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}}\left\langle\bar{\zeta}'_{\mathrm{l}}(\mathbf{u})\gamma_5\zeta'_{\mathrm{h}}(\mathbf{v})\;\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle. \tag{1.123}$$

with two boundary sources at $x_0 = 0$ and $x_0 = T$.

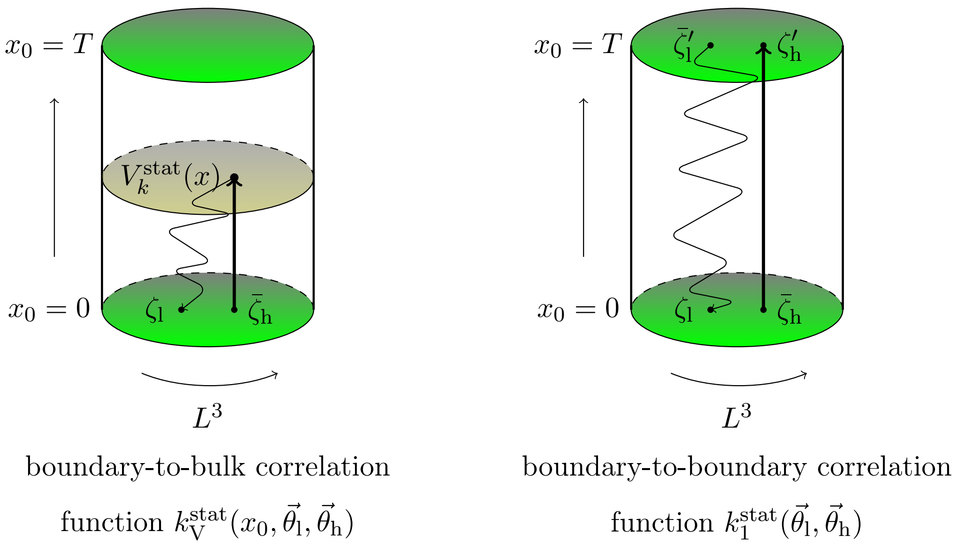


Figure 1.4: Schrödinger Functional correlation functions in the vector channel. The straight lines symbolize the static quark and the wavy lines denote the light quarks. The left correlation function has the space component of the vector current $V_k^{\text{stat}}(x)$ as an insertion. The quark anti-quark pair is created at point $x_0 = 0$ and annihilate each other at the insertion point of the vector current. I also consider the first space component V_1. The right correlation function contains vector boundary fields at the surfaces $x_0 = 0$ and $x_0 = T$.

In the relativistic case one has to replace the index for the heavy quark by a b: $\text{h} \to \text{b}$ and the axial and vector currents have the $O(a)$ improved form

$$(A_{\text{I}})_\mu = \overline{\psi}_{\text{l}}\gamma_\mu\gamma_5\psi_{\text{b}} + ac_{\text{A}}\tilde{\partial}_\mu\overline{\psi}_{\text{l}}\gamma_5\psi_{\text{b}}, \tag{1.124}$$

$$(V_{\text{I}})_\mu = \overline{\psi}_{\text{l}}\gamma_\mu\psi_{\text{b}} + ac_{\text{V}}\tilde{\partial}_\mu\overline{\psi}_{\text{l}}i\sigma_{\mu\nu}\psi_{\text{b}}. \tag{1.125}$$

The improvement coefficients c_{A} was determined non-perturbatively by Lüscher et al. in [22] in the quenched approximation and is given by

$$c_{\text{A}} = -0.00756g_0^2 \cdot \frac{1 - 0.748g_0^2}{1 - 0.977g_0^2}, \quad 0 \leq g_0 \leq 1. \tag{1.126}$$

The improvement coefficient of the vector current c_{V} is discussed perturbatively in [16]. Non-perturbative results are shown in the work of Guagnelli and Sommer in [25].

The renormalization of the correlation functions is straightforward, i.e. for vanishing light quark mass [16]. One needs a multiplicative renormalization constant for the current and boundary quark fields and an additional improvement coefficient for the heavy quark mass

$$[f_{\mathrm{A}}]_{\mathrm{R}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = Z_{\mathrm{A}}(1+\frac{1}{2}b_{\mathrm{A}}am_{\mathrm{q,b}})Z_\zeta^2(1+b_\zeta am_{\mathrm{q,b}})f_{\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}), \qquad (1.127)$$

$$[k_{\mathrm{V}}]_{\mathrm{R}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = Z_{\mathrm{V}}(1+\frac{1}{2}b_{\mathrm{V}}am_{\mathrm{q,b}})Z_\zeta^2(1+b_\zeta am_{\mathrm{q,b}})k_{\mathrm{V}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}), \qquad (1.128)$$

$$[f_1]_{\mathrm{R}}(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = Z_\zeta^4(1+b_\zeta am_{\mathrm{q,b}})^2 f_1(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}), \qquad (1.129)$$

$$[k_1]_{\mathrm{R}}(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = Z_\zeta^4(1+b_\zeta am_{\mathrm{q,b}})^2 k_1(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}). \qquad (1.130)$$

The renormalization coefficients are given by [26]

$$Z_{\mathrm{A}}(g_0) = \frac{1-0.8496g_0^2+0.0610g_0^4}{1-0.7332g_0^2}, \qquad (1.131)$$

$$Z_{\mathrm{V}}(g_0) = \frac{1-0.7663g_0^2+0.0488g_0^4}{1-0.6369g_0^2} \qquad (1.132)$$

and the values for the improvement coefficients are known from [16] and [26]

$$b_{\mathrm{A}}(g_0) = 1+0.1522g_0^2, \qquad (1.133)$$

$$b_{\mathrm{V}}(g_0) = \frac{1-0.6518g_0^2-0.1226g_0^4}{1-0.8467g_0^2}. \qquad (1.134)$$

All coefficients are defined in the quenched approximation for values of the bare gauge coupling $0 \le g_0 \le 1$. Besides the $O(a)$ improved axial and vector currents can be renormalized by those means

$$(A_{\mathrm{R}})_\mu = Z_{\mathrm{A}}(1+b_{\mathrm{A}}am_{\mathrm{q,b}})\,(A_{\mathrm{I}})_\mu\,, \qquad (1.135)$$

$$(V_{\mathrm{R}})_\mu = Z_{\mathrm{V}}(1+b_{\mathrm{V}}am_{\mathrm{q,b}})\,(V_{\mathrm{I}})_\mu\,. \qquad (1.136)$$

2 Including $1/m$ corrections

In the latter chapter I discussed the effective theory in the static approximation. To perform the matching of HQET parameters at order $1/m$ the corrections of $O(1/m)$ have to be included in the theory.

In this chapter I directly work in the lattice regularized theory. To find the descretized expressions one can transcribe the continuum observables (see section 1.1) to the lattice.

2.1 Lagrangian and expectation values

Additionally to the leading order expression of (1.36) one has to add the $1/m$ corrections. This provides the lattice action in HQET beyond the static approximation

$$S_{\mathrm{HQET}} = a^4 \sum_x \left(\mathscr{L}_{\mathrm{h}}^{\mathrm{stat}}(x) + \sum_{k=1}^{n} \mathscr{L}^{(k)}(x) \right), \tag{2.1}$$

$$\mathscr{L}^{(k)}(x) = \sum_i \omega_i^{(k)} \mathscr{L}_i^{(k)}(x). \tag{2.2}$$

The HQET Lagrangians $\mathscr{L}^{(k)}$ are of order $1/m^k$ and the static Lagrangian reads $\mathscr{L}_{\mathrm{h}}^{\mathrm{stat}} = \overline{\psi}_{\mathrm{h}}(x)(\nabla_0^* + \delta m)\psi_{\mathrm{h}}(x)$. The coefficients δm, $\omega_i(g_0, m)$ have to be determined from the matching condition. They are the bare parameters of the effective theory. In next-to-leading order in $1/m$ the HQET Lagrangian reads

$$\mathscr{L}_{\mathrm{h}}^{(1)} = -\left(\omega_{\mathrm{kin}}\mathcal{O}_{\mathrm{kin}} + \omega_{\mathrm{spin}}\mathcal{O}_{\mathrm{spin}}\right) \tag{2.3}$$

with the $O(1/m)$ operators $\mathcal{O}_{\mathrm{kin}}, \mathcal{O}_{\mathrm{spin}}$ as in (1.8),(1.9) only with discretized transcriptions

$$D_k D_k \to \nabla_k^* \nabla_k, \quad F_{kl} \to \widehat{F}_{kl}, \tag{2.4}$$

where $\widehat{F}_{kl}$ is the clover leaf representation defined in (1.78). The normalization is chosen with the result that the classical values of the coefficients are $\omega_{\mathrm{kin}} = \omega_{\mathrm{spin}} = \frac{1}{2m_{\mathrm{b}}}$. The problem of the effective theory is that the path integral with the weight

$$P_{\mathrm{NRQCD}} \propto \exp\left\{ -a^4 \sum_x \left(\mathscr{L}_{\mathrm{light}}(x) + \mathscr{L}_{\mathrm{h}}^{\mathrm{stat}}(x) + \mathscr{L}_{\mathrm{h}}^{(1)} \right) \right\} \tag{2.5}$$

is not renormalizable since the terms consist of composite fields of dimension 5. In perturbation theory new divergences will occur in each order therefore an infinite number of counter terms have to be added to the observables and composite fields.

In this case the continuum limit of the lattice QCD does not exist.

One possibility to avoid this problem is to expand the weight in $1/m$ and counting the terms proportional to $\omega_{\rm kin} = \omega_{\rm spin} = O(1/m)$. This is possible if the HQET expansion only reproduces the next-to-leading order expansion of the observables. One obtains for the path integral weight

$$P_{\rm HQET} \equiv \exp\left\{-a^4 \sum_x \left(\mathscr{L}_{\rm light}(x) + \mathscr{L}_{\rm h}^{\rm stat}(x)\right)\right\}\left\{1 - a^4 \sum_x \mathscr{L}_{\rm h}^{(1/m)}(x)\right\}, \quad (2.6)$$

which is now renormalizable.

In order to compute SF correlation functions in the effective theory I need expectation values in HQET. Including $O(1/m)$ the expectation value of an observable $\mathcal{O}$ is defined as

$$\begin{aligned}\langle \mathcal{O}\rangle &= \langle \mathcal{O}\rangle_{\rm stat} + \omega_{\rm kin} a^4 \sum_x \langle \mathcal{O}\mathcal{O}_{\rm kin}(x)\rangle_{\rm stat} + \omega_{\rm spin} a^4 \sum_x \langle \mathcal{O}\mathcal{O}_{\rm spin}(x)\rangle_{\rm stat} \\ &\equiv \langle \mathcal{O}\rangle_{\rm stat} + \omega_{\rm kin}\langle \mathcal{O}\rangle_{\rm kin} + \omega_{\rm spin}\langle \mathcal{O}\rangle_{\rm spin} \end{aligned} \quad (2.7)$$

with the path integral average

$$\langle \mathcal{O}\rangle_{\rm stat} = \frac{1}{\mathcal{Z}} \int_{\rm fields} \mathcal{O} \exp\left\{-a^4 \sum_x \left(\mathscr{L}_{\rm light}(x) + \mathscr{L}_{\rm h}^{\rm stat}(x)\right)\right\}. \quad (2.8)$$

2.2 Renormalized HQET heavy-light currents

In order to consider matrix elements and correlation functions composite fields are required, for example the axial and vector current. In order to eliminate the $O(a)$ effects of the static heavy-light currents, correction terms have to be added. The Symanzik $O(a)$ improvement can be applied to the currents in a similar way as to the action as it was done in section 1.2.1. For a detailed discussion I want to refer to the work of M. Kurth and R. Sommer in [20]. The $O(a)$ improved currents in the static approximation are

$$\left(A_{\rm I}^{\rm stat}\right)_0 = A_0^{\rm stat} + a c_{\rm A}^{\rm stat} \overline{\psi}_{\rm l}\gamma_i\gamma_5 \frac{1}{2}(\overleftarrow{\nabla}_i + \overleftarrow{\nabla}_i^*)\psi_{\rm h}, \quad (2.9)$$

$$\left(V_{\rm I}^{\rm stat}\right)_0 = V_0^{\rm stat} + a c_{\rm V}^{\rm stat} \overline{\psi}_{\rm l}\gamma_i \frac{1}{2}(\overleftarrow{\nabla}_i + \overleftarrow{\nabla}_i^*)\psi_{\rm h}, \quad (2.10)$$

whereas the coefficients $c_{\rm A}^{\rm stat}$ and $c_{\rm V}^{\rm stat}$ depend on the coupling g_0 and are independent of the relativistic quark mass $m_{\rm q}$. The renormalization of the static axial and vector current demand a multiplicative constant $Z_X^{\rm stat}$, X = A,V, which depend on the renormalization scale and the bare coupling. The improved and renormalized version of the heavy-light currents therefore is

$$\left(A_{\rm R}^{\rm stat}\right)_0 = Z_{\rm A}^{\rm stat}(g_0, a\mu)(1 + b_{\rm A}^{\rm stat} a m_{\rm q})\left(A_{\rm I}^{\rm stat}\right)_0, \quad (2.11)$$

$$\left(V_{\rm R}^{\rm stat}\right)_0 = Z_{\rm V}^{\rm stat}(g_0, a\mu)(1 + b_{\rm V}^{\rm stat} a m_{\rm q})\left(V_{\rm I}^{\rm stat}\right)_0. \quad (2.12)$$

The space components of the axial and vector current are not discussed here, because they can be related to the time component by spin symmetry.

To find the $1/m$ corrections in the HQET expansion one has to find all dimension four operators with the right flavor structure and the right transformation under spatial lattice rotations and parity. The time component of the axial current in the HQET expansion is

$$A_0^{\mathrm{HQET}}(x) = Z_{\mathrm{A}}^{\mathrm{HQET}}[A_0^{\mathrm{stat}}(x) + \sum_{i=1}^{2} c_{\mathrm{A}}^{(i)} A_0^{(i)}(x)], \tag{2.13}$$

$$A_0^{(1)}(x) = \overline{\psi}_{\mathrm{l}}(x)\frac{1}{2}\gamma_5\gamma_i(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\psi_{\mathrm{h}}(x), \tag{2.14}$$

$$A_0^{(2)}(x) = -\frac{1}{2}\tilde{\partial}_i A_i^{\mathrm{stat}}(x) \tag{2.15}$$

with the symmetric derivatives

$$\tilde{\partial}_i = \frac{1}{2}\left(\partial_i + \partial_i^*\right), \quad \overleftarrow{\nabla}_i^{\mathrm{S}} = \frac{1}{2}\left(\overleftarrow{\nabla}_i + \overleftarrow{\nabla}_i^*\right), \quad \nabla_i^{\mathrm{S}} = \frac{1}{2}\left(\nabla_i + \nabla_i^*\right). \tag{2.16}$$

For the spatial components one obtains

$$A_k^{\mathrm{HQET}}(x) = Z_{\mathrm{A}}^{\mathrm{HQET}}[A_k^{\mathrm{stat}}(x) + \sum_{i=3}^{6} c_{\mathrm{A}}^{(i)} A_k^{(i)}(x)], \tag{2.17}$$

$$A_k^{(3)}(x) = \overline{\psi}_{\mathrm{l}}(x)\frac{1}{2}(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\gamma_5\gamma_k\psi_{\mathrm{h}}(x), \tag{2.18}$$

$$A_k^{(4)}(x) = \overline{\psi}_{\mathrm{l}}(x)\frac{1}{2}(\nabla_k^{\mathrm{S}} - \overleftarrow{\nabla}_k^{\mathrm{S}})\gamma_5\psi_{\mathrm{h}}(x), \tag{2.19}$$

$$A_k^{(5)}(x) = \frac{1}{2}\tilde{\partial}_i(\overline{\psi}_{\mathrm{l}}(x)\gamma_i\gamma_5\gamma_k\psi_{\mathrm{h}}(x)), \tag{2.20}$$

$$A_k^{(6)}(x) = \frac{1}{2}\tilde{\partial}_k A_0^{\mathrm{stat}}(x). \tag{2.21}$$

The vector current can be obtained by the corresponding expansion only dropping the γ_5 and replacing $c_{\mathrm{A}}^{(i)}$ with $c_{\mathrm{V}}^{(i)}$ and $Z_{\mathrm{A}}^{\mathrm{HQET}}$ with $Z_{\mathrm{V}}^{\mathrm{HQET}}$

$$V_0^{\mathrm{HQET}}(x) = Z_{\mathrm{V}}^{\mathrm{HQET}}[V_0^{\mathrm{stat}}(x) + \sum_{i=1}^{2} c_{\mathrm{V}}^{(i)} V_0^{(i)}(x)], \tag{2.22}$$

$$V_k^{\mathrm{HQET}}(x) = Z_{\mathrm{V}}^{\mathrm{HQET}}[V_k^{\mathrm{stat}}(x) + \sum_{i=3}^{6} c_{\mathrm{V}}^{(i)} V_k^{(i)}(x)], \tag{2.23}$$

$$V_0^{(1)}(x) = \overline{\psi}_{\mathrm{l}}(x)\frac{1}{2}\gamma_i(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\psi_{\mathrm{h}}(x), \tag{2.24}$$

$$V_0^{(2)}(x) = -\frac{1}{2}\tilde{\partial}_i V_i^{\mathrm{stat}}(x), \tag{2.25}$$

$$V_k^{(3)}(x) = \overline{\psi}_{\mathrm{l}}(x)\frac{1}{2}(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\gamma_k\psi_{\mathrm{h}}(x), \tag{2.26}$$

$$V_k^{(4)}(x) = \overline{\psi}_{\mathrm{l}}(x)\frac{1}{2}(\nabla_k^{\mathrm{S}} - \overleftarrow{\nabla}_k^{\mathrm{S}})\psi_{\mathrm{h}}(x), \tag{2.27}$$

$$V_k^{(5)}(x) = \frac{1}{2}\tilde{\partial}_i(\overline{\psi}_{\mathrm{l}}(x)\gamma_i\gamma_k\psi_{\mathrm{h}}(x)), \tag{2.28}$$

$$V_k^{(6)}(x) = \frac{1}{2}\tilde{\partial}_k V_0^{\mathrm{stat}}(x). \tag{2.29}$$

Transformation to a new basis

The HQET expansion of the time component of the heavy-light axial current was given in (2.13). For zero space-time momentum i.e. $\vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{h}}$, the contribution $A_0^{(2)}$ vanishes. Following [27] I transform to a new basis to consider all $1/m$ corrections. In this basis I can express $A_0^{(1)}$ and $A_0^{(2)}$ as terms with derivatives only acting on the heavy or light quark fields

$$\begin{aligned} A_0^{(1)} &= \frac{1}{2}(\underbrace{\overline{\psi}_{\mathrm{l}}(x)\gamma_5\gamma_i\nabla_i^{\mathrm{S}}\psi_{\mathrm{h}}(x)}_{=\delta_{\mathrm{h}}A_0} - \underbrace{\overline{\psi}_{\mathrm{l}}(x)\gamma_5\gamma_i\overleftarrow{\nabla}_i^{\mathrm{S}}\psi_{\mathrm{h}}(x)}_{=\delta A_0}) \\ &= \frac{1}{2}(\delta_{\mathrm{h}}A_0 - \delta A_0), \end{aligned} \tag{2.30}$$

$$\begin{aligned} A_0^{(2)} &= \frac{1}{2}(\overline{\psi}_{\mathrm{l}}(x)\gamma_5\gamma_i\nabla_i^{\mathrm{S}}\psi_{\mathrm{h}}(x) + \overline{\psi}_{\mathrm{l}}(x)\gamma_5\gamma_i\overleftarrow{\nabla}_i^{\mathrm{S}}\psi_{\mathrm{h}}(x)) \\ &= \frac{1}{2}(\delta_{\mathrm{h}}A_0 + \delta A_0). \end{aligned} \tag{2.31}$$

This transformation leads to new coefficients, with the following correspondence at tree level

$$c_{\delta\mathrm{A}} = \frac{1}{2}c_{\mathrm{A}}^{(2)} - \frac{1}{2}c_{\mathrm{A}}^{(1)}, \tag{2.32}$$

$$c_{\delta_{\mathrm{h}}\mathrm{A}} = \frac{1}{2}c_{\mathrm{A}}^{(1)} + \frac{1}{2}c_{\mathrm{A}}^{(2)}. \tag{2.33}$$

In a similar way I transform the space components of the axial currents (2.20) and (2.21) to

$$A_k^{(5)}(x) = \frac{1}{2}\left[\overline{\psi}_{\mathrm{l}}(x)(\nabla_i^{\mathrm{S}} + \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\gamma_5\gamma_k\psi_{\mathrm{h}}(x)\right], \tag{2.34}$$

$$A_k^{(6)}(x) = -\frac{1}{2}\left[\overline{\psi}_{\mathrm{l}}(x)(\nabla_k^{\mathrm{S}} + \overleftarrow{\nabla}_k^{\mathrm{S}})\gamma_5\psi_{\mathrm{h}}(x)\right]. \tag{2.35}$$

The expressions of the vector currents (2.25), (2.28) and (2.29) with the new basis are

$$V_0^{(2)}(x) = \frac{1}{2}\left[\overline{\psi}_{\mathrm{l}}(x)(\nabla_i^{\mathrm{S}} + \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\psi_{\mathrm{h}}(x)\right], \tag{2.36}$$

$$V_k^{(5)}(x) = \frac{1}{2}\left[\overline{\psi}_{\mathrm{l}}(x)(\nabla_i^{\mathrm{S}} + \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\gamma_k\psi_{\mathrm{h}}(x)\right], \tag{2.37}$$

$$V_k^{(6)}(x) = \frac{1}{2}\left[\overline{\psi}_{\mathrm{l}}(x)(\nabla_k^{\mathrm{S}} + \overleftarrow{\nabla}_k^{\mathrm{S}})\psi_{\mathrm{h}}(x)\right]. \tag{2.38}$$

I must bear in mind that I only get a $\vec{\theta}_{\mathrm{h}}$ dependence if a spatial derivative is acting on a heavy quark field. Furthermore the static correlation functions only depend on $\vec{\theta}_{\mathrm{l}}$, because the heavy quark propagator is replaced in the static approximation by the projection P_+ (see section 4.4.1).

2.3 Heavy-light correlation functions and their renormalization in next-to-leading order

The HQET expansion in $1/m$ of the correlation functions is straightforward

$$[f_{\rm A}]_R = Z_{\rm A}^{\rm HQET} Z_{\zeta_{\rm h}} Z_\zeta e^{-m_{\rm bare}x_0} \times \left\{ f_{\rm A}^{\rm stat} + \sum_{i=1}^{2} c_{\rm A}^{(i)} f_{{\rm A}^{(i)}}^{\rm stat} + \omega_{\rm kin} f_{\rm A}^{\rm kin} + \omega_{\rm spin} f_{\rm A}^{\rm spin} \right\}, \tag{2.39}$$

$$[f_{{\rm A}_k}]_R = Z_{{\rm A}_k}^{\rm HQET} Z_{\zeta_{\rm h}} Z_\zeta e^{-m_{\rm bare}x_0} \times \left\{ f_{{\rm A}_k}^{\rm stat} + \sum_{i=3}^{6} c_{\rm A}^{(i)} f_{{\rm A}_k^{(i)}}^{\rm stat} + \omega_{\rm kin} f_{{\rm A}_k}^{\rm kin} + \omega_{\rm spin} f_{{\rm A}_k}^{\rm spin} \right\}, \tag{2.40}$$

$$[f_{{\rm AV}_{21}}]_R = Z_{{\rm A}_k}^{\rm HQET} Z_{\zeta_{\rm h}} Z_\zeta e^{-m_{\rm bare}x_0} \times \left\{ f_{{\rm AV}_{21}}^{\rm stat} + \sum_{i=3}^{6} c_{\rm A}^{(i)} f_{{\rm AV}_{21}^{(i)}}^{\rm stat} + \omega_{\rm kin} f_{{\rm AV}_{21}}^{\rm kin} + \omega_{\rm spin} f_{{\rm AV}_{21}}^{\rm spin} \right\}, \tag{2.41}$$

$$[k_{\rm V}]_R = Z_{\rm V}^{\rm HQET} Z_{\zeta_{\rm h}} Z_\zeta e^{-m_{\rm bare}x_0} \times \left\{ k_{\rm V}^{\rm stat} + \sum_{i=3}^{6} c_{\rm V}^{(i)} k_{{\rm V}^{(i)}}^{\rm stat} + \omega_{\rm kin} k_{\rm V}^{\rm kin} + \omega_{\rm spin} k_{\rm V}^{\rm spin} \right\}, \tag{2.42}$$

$$[k_{{\rm V}_{11}}]_R = Z_{\rm V}^{\rm HQET} Z_{\zeta_{\rm h}} Z_\zeta e^{-m_{\rm bare}x_0} \times \left\{ k_{{\rm V}_{11}}^{\rm stat} + \sum_{i=3}^{6} c_{\rm V}^{(i)} k_{{\rm V}_{11}^{(i)}}^{\rm stat} + \omega_{\rm kin} k_{{\rm V}_{11}}^{\rm kin} + \omega_{\rm spin} k_{{\rm V}_{11}}^{\rm spin} \right\}, \tag{2.43}$$

$$[k_{{\rm V}_0}]_R = Z_{{\rm V}_0}^{\rm HQET} Z_{\zeta_{\rm h}} Z_\zeta e^{-m_{\rm bare}x_0} \times \left\{ k_{{\rm V}_0}^{\rm stat} + \sum_{i=1}^{2} c_{\rm V}^{(i)} k_{{\rm V}_0^{(i)}}^{\rm stat} + \omega_{\rm kin} k_{{\rm V}_0}^{\rm kin} + \omega_{\rm spin} k_{{\rm V}_0}^{\rm spin} \right\}, \tag{2.44}$$

$$[f_1]_R = Z_{\zeta_{\rm h}}^2 Z_\zeta^2 e^{-m_{\rm bare}T} \left\{ f_1^{\rm stat} + \omega_{\rm kin} f_1^{\rm kin} + \omega_{\rm spin} f_1^{\rm spin} \right\}, \tag{2.45}$$

$$[k_1]_R = Z_{\zeta_{\rm h}}^2 Z_\zeta^2 e^{-m_{\rm bare}T} \left\{ k_1^{\rm stat} + \omega_{\rm kin} k_1^{\rm kin} + \omega_{\rm spin} k_1^{\rm spin} \right\}. \tag{2.46}$$

The factor $e^{-m_{\rm bare}x_0}$ alternatively $e^{-m_{\rm bare}T}$ corresponds to the energy shift between QCD and HQET. The correlation functions, containing the kinetic and spin term, have the form

$$f_{\rm A}^{\rm kin}(x_0, \vec\theta_{\rm l}, \vec\theta_{\rm h}) = -\frac{a^{10}}{2} \sum_{\mathbf{y},\mathbf{z},u} \left\langle A_0^{\rm stat}(x)\, \bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z})\, \mathcal{O}_{\rm kin}(u) \right\rangle, \tag{2.47}$$

$$f_{\rm A}^{\rm spin}(x_0, \vec\theta_{\rm l}, \vec\theta_{\rm h}) = -\frac{a^{10}}{2} \sum_{\mathbf{y},\mathbf{z},u} \left\langle A_0^{\rm stat}(x)\, \bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z})\, \mathcal{O}_{\rm spin}(u) \right\rangle. \tag{2.48}$$

The corresponding ones for the vector correlation functions are similar. Due to the fact that I also have a $1/m$ expansion of the currents I obtain additional terms in $f_{\rm A}$,$f_{{\rm A}_k}$,$f_{{\rm AV}_{21}}$, $k_{\rm V}$ and $k_{{\rm V}_{11}}$, which contain the $1/m$ corrections of the currents. For the axial correlation functions they have the form

$$f_{\mathrm{A}^{(i)}}^{\mathrm{stat}}(x_0,\vec{\theta}_\mathrm{l},\vec{\theta}_\mathrm{h}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle A_0^{(i)}(x)\,\bar{\zeta}_\mathrm{h}(\mathbf{y})\gamma_5\zeta_\mathrm{l}(\mathbf{z})\right\rangle \tag{2.49}$$

and for the other correlation functions, they can be obtained in a similar way. At tree level I can exploit some useful relations between the correlation functions [28], due to the spin symmetry of the lattice action. The relations valid for isotropic angles are

$$f_\mathrm{A}^\mathrm{kin} = 6\frac{x_0}{a^2}\left(\cos(a\frac{\theta_\mathrm{h}}{L})-1\right) f_\mathrm{A}^\mathrm{stat} \tag{2.50}$$

$$k_\mathrm{V}^\mathrm{kin} = 6\frac{x_0}{a^2}\left(\cos(a\frac{\theta_\mathrm{h}}{L})-1\right) k_\mathrm{V}^\mathrm{stat}, \tag{2.51}$$

and

$$f_1^\mathrm{kin} = 6\frac{T-a}{a^2}\left(\cos(a\frac{\theta_\mathrm{h}}{L})-1\right) f_1^\mathrm{stat}, \tag{2.52}$$

$$k_1^\mathrm{kin} = 6\frac{T-a}{a^2}\left(\cos(a\frac{\theta_\mathrm{h}}{L})-1\right) k_1^\mathrm{stat}. \tag{2.53}$$

The further relations are valid for any θ combinations

$$f_1 = k_1, \tag{2.54}$$

$$f_1^\mathrm{stat} = k_1^\mathrm{stat}, \tag{2.55}$$

$$k_\mathrm{V}^\mathrm{stat} = -f_\mathrm{A}^\mathrm{stat} = k_{\mathrm{V}_{11}}^\mathrm{stat}, \tag{2.56}$$

$$k_{\mathrm{V}^{(i)}}^\mathrm{stat} = -f_{\mathrm{A}^{(i-1)/2}}^\mathrm{stat} = k_{\mathrm{V}_{11}^{(i)}}^\mathrm{stat}, \quad i=3,5 \tag{2.57}$$

$$k_{\mathrm{V}^{(i)}}^\mathrm{stat} = -\frac{1}{3}f_{\mathrm{A}^{(i-1)/2}}^\mathrm{stat}, \quad i=4,6, \tag{2.58}$$

$$k_\mathrm{V}^\mathrm{kin} = -f_\mathrm{A}^\mathrm{kin} = k_{\mathrm{V}_{11}}^\mathrm{kin}, \tag{2.59}$$

$$k_\mathrm{V}^\mathrm{spin} = \frac{1}{3}f_\mathrm{A}^\mathrm{spin}, \tag{2.60}$$

$$k_1^\mathrm{kin} = f_1^\mathrm{kin}, \tag{2.61}$$

$$f_1^\mathrm{spin} = -3k_1^\mathrm{spin}, \tag{2.62}$$

$$f_\mathrm{A}^\mathrm{spin} = 3k_\mathrm{V}^\mathrm{spin}, \tag{2.63}$$

$$f_\mathrm{A}^\mathrm{spin} = f_1^\mathrm{spin} = 0 \tag{2.64}$$

the last equation is valid for vanishing background fields only. From now on all correlation functions have the renormalized and improved form if not otherwise specified. I will drop the subscription I and R for a better readability.

3 Parameters of HQET

In the introduction I mentioned that the HQET and the QCD need to be linked to successfully achieve conclusions of the effective theory. A non-perturbative strategy to determine the HQET parameters is the *matching*, which was first considered by Heitger and Sommer [29, 30].

3.1 Matching of QCD and HQET

To obtain the matching condition between the HQET and the QCD there is on the one hand the observables, e.g. dimensionless renormalized correlation functions or energies, of HQET Φ^{HQET} and on the other the QCD observables Φ^{QCD}. The N_{HQET} unknown parameters are determined from the equation

$$\Phi_k^{\mathrm{HQET}}(m,a) = \Phi_k^{\mathrm{QCD}}(m), \quad k = 1,\ldots,N_{\mathrm{HQET}}. \tag{3.1}$$

On the QCD side one assumes the continuum limit has already been taken, whereas on the HQET side one have a dependence in a. The bare parameters of the theory are defined at the lattice spacing a. Further the HQET observables have an explicit mass dependence in the parameters. The determination of Φ_k^{QCD} is difficult, because the cutoff effects of $O((am_{\mathrm{b}})^2)$ are large, which appear by simulating the b-quark on the lattice. The remedy for this problem is to apply a finite volume with accessible small lattice spacings. In this way one uses the SF boundary conditions. A definition of the QCD observables on a finite volume provides much smaller lattice spacings, thus a well-defined and unique continuum limit can be achieved. With a lattice extent of $L = L_1 \approx 0.4\mathrm{fm}$ it is known from tests of HQET [24] that it provides $\frac{1}{z} = \frac{1}{LM_{\mathrm{b}}} \approx \frac{1}{10}$ and an accurate expansion in $1/m$. M_{b} is the renormalization group invariant (RGI) quarkmass, which is defined in appendix A. The matching condition yields

$$\Phi_k^{\mathrm{HQET}}(L_1,M_{\mathrm{b}},a) = \Phi_k^{\mathrm{QCD}}(L_1,M_{\mathrm{b}}), \quad k = 1,\ldots,N_{\mathrm{HQET}}. \tag{3.2}$$

The HQET observables require a fine resolution of the lattice, otherwise the $O(a)$ effects have a strong influence on the calculations. Therefore in the effective theory larger physical volumes are required.

Step-Scaling

With the matching condition the parameters are defined for any value of the lattice spacing. With an extent of $L_1 \approx 0.4\mathrm{fm}$ rather small lattice spacings are achieved from $a \approx 0.02\text{fm}$ to $a \approx 0.05\text{fm}$. Although one needs larger volumes to determine the physical mass spectrum or matrix elements. To bridge the gap between lower and higher volumes one makes use of a well-defined procedure [17] by establishing the step-scaling functions

$$\Phi_k^{\text{HQET}}(sL, M, a) = \sigma_k \left\{ \left[\Phi_j^{\text{HQET}}(L, M, a), j = 1, \ldots, N_{\text{HQET}} \right] \right\}, \tag{3.3}$$

with $k = 1, \ldots, N_{\text{HQET}}$. Usually one uses the scaling $s = 2 \Rightarrow L \to 2L$. The step-scaling functions σ_k are dimensionless and relate observables in two different volumes with L and sL. Furthermore the step-scaling functions have a dependence on the

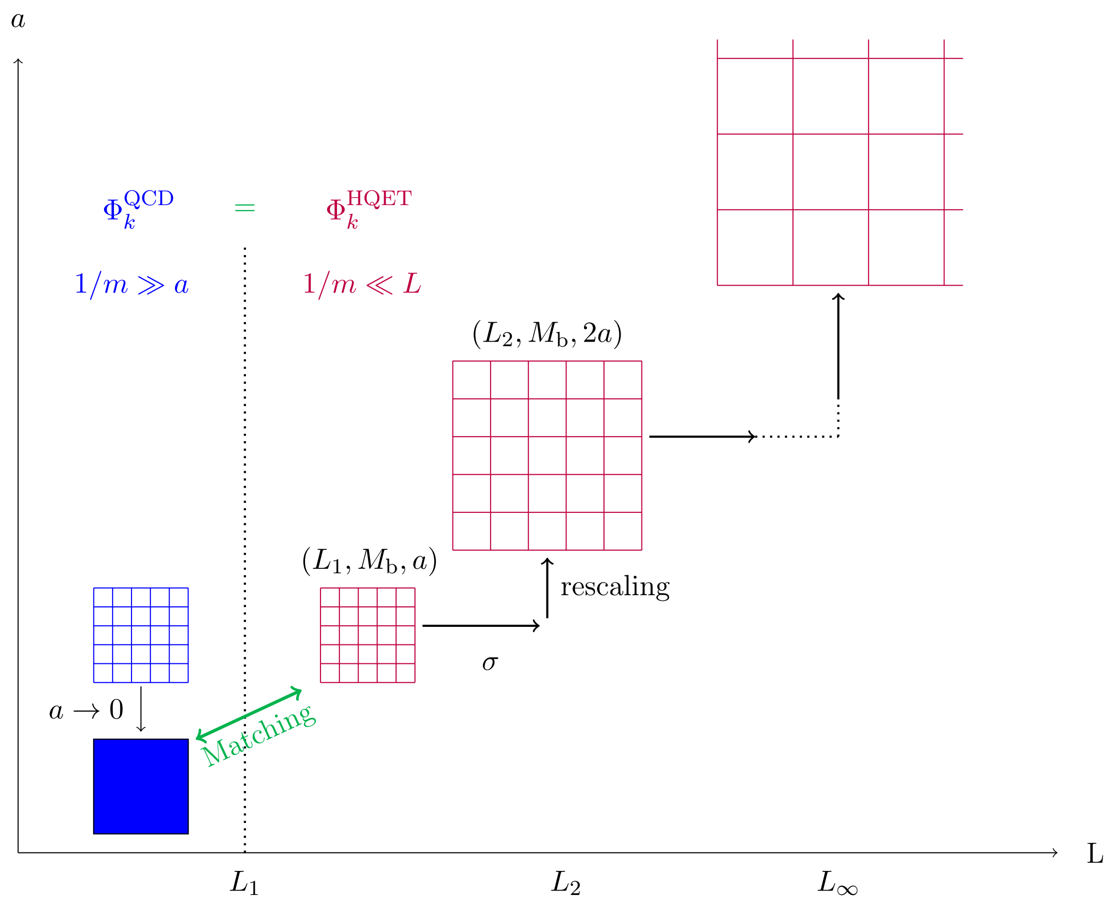


Figure 3.1: The matching procedure and the step-scaling method to connect the once determined HQET parameters to physically large volumes.

lattice quantities a and L. The dependence is described by the lattice step-scaling function $\Sigma(u, \frac{a}{L})$. To obtain the continuum observable $\sigma(u)$ one has to determine the lattice step-scaling functions for several values of a/L and extrapolate the data to the continuum limit

$$\sigma(s, u) = \lim_{a \to 0} \Sigma(s, u, \frac{a}{L})\Big|_{\bar{g}^2(L)=u}. \tag{3.4}$$

Similar step-scaling functions are applied on the HQET side of the matching equation frequently until the extent of the lattice reaches the infinte-volume region, which is about $L_\infty \approx 2$fm. The continuum limit of the HQET observables is extrapolated in each step. The strategy is shown in figure (3.1).

I have to note here, that my calculations are at first order perturbation theory, thus I obtain tree level results. I will perform the matching in the continuum limit. On this account I do not apply the step-scaling method.

3.2 HQET parameters

In [28] a non-perturbative determination of the parameters of the HQET-Lagrangian and of the time component of the heavy-light axial-vector current was performed. The computation of the parameters was accomplished in the quenched approximation and the results show a qualitative agreement with perturbation theory.

A first step to extend the work is to determine the parameters of the lattice HQET heavy-light currents at tree level, including the space components of the axial and vector current to obtain further $1/m$-coefficients. To obtain the full set of $1/m$-coefficients the currents are inserted in the SF correlation functions with zero momentum, e.g. $\vec{\theta}_\mathrm{h} = \vec{\theta}_\mathrm{l}$ and with non-zero momentum, e.g. $\vec{\theta}_\mathrm{h} \neq \vec{\theta}_\mathrm{l}$. Furthermore I use unisotropic angles to obtain sensitivity to individual insertions of the currents.

The bare parameters of the effective theory $\omega_\mathrm{kin}, \omega_\mathrm{spin}, Z^\mathrm{HQET}_{\mathrm{A},V}, c^\mathrm{HQET}_{\mathrm{A},V}$, together with the energy shift m_bare are sufficient to absorb all divergences in the effective theory up to $O(1/m)$. A complete set of all HQET parameters up to next-to-leading order are tabulated in (3.1). An ideal determination of the HQET coefficients would be in the non-perturbative way, but tree level results give a first impression of the behaviour of the parameters in $1/z$. Furthermore they are an indicator of the magnitude of the corrections $1/z^2$. My aim is to study the tree level coefficients as functions in $1/z$, where $z = L\tilde{m}_\mathrm{q}$. This is achieved by performing the matching in the continuum limit.

The classical values of the coefficients should agree with the tree level results. The classical values are summarized in table (3.1)

classical values of HQET parameters							
ω_kin	ω_spin	$c_\mathrm{A}^{(1)}$	$c_\mathrm{A}^{(2)}$	$c_\mathrm{V}^{(1)}$	$c_\mathrm{V}^{(2)}$	$Z^\mathrm{HQET}_{\mathrm{A}_{(k)}}$	$Z^\mathrm{HQET}_{\mathrm{V}_{(0)}}$
$\frac{1}{2m_\mathrm{b}}$	$\frac{1}{2m_\mathrm{b}}$	$-\frac{1}{2m_\mathrm{b}}$	$-\frac{1}{2m_\mathrm{b}}$	$\frac{1}{2m_\mathrm{b}}$	$\frac{1}{2m_\mathrm{b}}$	1	1
$c_\mathrm{A}^{(3)}$	$c_\mathrm{A}^{(4)}$	$c_\mathrm{A}^{(5)}$	$c_\mathrm{A}^{(6)}$	$c_\mathrm{V}^{(3)}$	$c_\mathrm{V}^{(4)}$	$c_\mathrm{V}^{(5)}$	$c_\mathrm{V}^{(6)}$
$\frac{1}{2m_\mathrm{b}}$	$\frac{1}{m_\mathrm{b}}$	$\frac{1}{2m_\mathrm{b}}$	$-\frac{1}{m_\mathrm{b}}$	$\frac{1}{2m_\mathrm{b}}$	$-\frac{1}{m_\mathrm{b}}$	$\frac{1}{2m_\mathrm{b}}$	$-\frac{1}{m_\mathrm{b}}$

Table 3.1

3.3 Matching observables

In this section observables are required for the determination of the HQET parameters. They are built from SF correlation functions with suitable renormalized combinations. The quantities depend on the lattice resolution a/L and in particular they are universal, e.g. the continuum limit does exist. Furthermore the observables depend on the periodicity angles of the light and the heavy quark $\vec{\theta}_\mathrm{l}$ and $\vec{\theta}_\mathrm{h}$. In previous calculations [28] isotropic and equal angles for the light and the heavy quark $\vec{\theta}_\mathrm{l} = \vec{\theta}_\mathrm{h}$ were chosen. Besides the parameters of the action and of the time component of the axial current were discussed in the non-perturbative studies. I extend the non-perturbative calculations and include a further current, the vector current. Furthermore to obtain sensitivity to further HQET parameters I require $\vec{\theta}_\mathrm{l} \neq \vec{\theta}_\mathrm{h}$ for some observables. By my choice of observables I orientated myself on the quantities, which were used in the non-perturbative studies. I expect that, with the use of the observables at tree level the classical values of the parameters can be reproduced.

I introduce the observables concerning the axial correlation functions. Using isotropic periodicity angles for the light and heavy quark with $\theta_\mathrm{l} = \theta_\mathrm{h} = \theta$ the following QCD observables were introduced in [28]. However the quantities are defined by the periodicity angles of the light and the heavy quark and thus depend on $\vec{\theta}_\mathrm{l}$ and $\vec{\theta}_\mathrm{h}$. For a better readability I write for the observables depending on θ, which is to be interpreted as depending on $\vec{\theta}_\mathrm{l}$ and $\vec{\theta}_\mathrm{h}$.

$$R_1 = \frac{1}{4}(\ln(f_1(\theta_1)k_1(\theta_1)^3) - \ln(f_1(\theta_2)k_1(\theta_2)^3))\Big|_{(T=\frac{L}{2})} \tag{3.5}$$

$$R_\mathrm{A} = \ln\left(\frac{f_\mathrm{A}(x_0,\theta_1)}{f_\mathrm{A}(x_0,\theta_2)}\right)\Big|_{(x_0=\frac{T}{2},T=L)} \tag{3.6}$$

$$\zeta_\mathrm{A} = \ln\left(\frac{-f_\mathrm{A}(x_0,\theta)}{\sqrt{f_1(\theta)}}\right)\Big|_{(x_0=\frac{T}{2},T=L)} \tag{3.7}$$

The corresponding observables in the $1/m$ expansion are deduced from the HQET expansion directly. The static quantities are

$$R_1^\mathrm{stat} = \ln\left(\frac{f_1^\mathrm{stat}(\theta_1)}{f_1^\mathrm{stat}(\theta_2)}\right)\Big|_{(T=\frac{L}{2})}, \tag{3.8}$$

$$R_\mathrm{A}^\mathrm{stat} = \ln\left(\frac{f_\mathrm{A}^\mathrm{stat}(x_0,\theta_1)}{f_\mathrm{A}^\mathrm{stat}(x_0,\theta_2)}\right)\Big|_{(x_0=\frac{T}{2},T=L)}, \tag{3.9}$$

$$\zeta_\mathrm{A}^\mathrm{stat} = \ln\left(\frac{-f_\mathrm{A}^\mathrm{stat}(x_0,\theta)}{\sqrt{f_1^\mathrm{stat}(\theta)}}\right)\Big|_{(x_0=\frac{T}{2},T=L)}, \tag{3.10}$$

and the observables in $O(1/m)$ can be written as

$$R_1^{\mathrm{kin}} = \left(\frac{f_1^{\mathrm{kin}}(\theta_1)}{f_1^{\mathrm{stat}}(\theta_1)} - \frac{f_1^{\mathrm{kin}}(\theta_2)}{f_1^{\mathrm{stat}}(\theta_2)} \right) \Bigg|_{(T=\frac{L}{2})}, \tag{3.11}$$

$$R_{\mathrm{A}}^{\mathrm{kin}} = \left(\frac{f_{\mathrm{A}}^{\mathrm{kin}}(x_0,\theta_1)}{f_{\mathrm{A}}^{\mathrm{stat}}(x_0,\theta_1)} - \frac{f_{\mathrm{A}}^{\mathrm{kin}}(x_0,\theta_2)}{f_{\mathrm{A}}^{\mathrm{stat}}(x_0,\theta_2)} \right) \Bigg|_{(x_0=\frac{T}{2}, T=L)}, \tag{3.12}$$

$$R_{\mathrm{A}^{(i)}}^{\mathrm{stat}} = \left(\frac{f_{\mathrm{A}^{(i)}}^{\mathrm{stat}}(x_0,\theta_1)}{f_{\mathrm{A}}^{\mathrm{stat}}(x_0,\theta_1)} - \frac{f_{\mathrm{A}^{(i)}}^{\mathrm{stat}}(x_0,\theta_2)}{f_{\mathrm{A}}^{\mathrm{stat}}(x_0,\theta_2)} \right) \Bigg|_{(x_0=\frac{T}{2}, T=L)}, \tag{3.13}$$

$$\Psi_{\mathrm{A}}^{\mathrm{kin}} = \left(\frac{f_{\mathrm{A}}^{\mathrm{kin}}(x_0,\theta)}{f_{\mathrm{A}}^{\mathrm{stat}}(x_0,\theta)} - \frac{1}{2} \frac{f_1^{\mathrm{kin}}(\theta)}{f_1^{\mathrm{stat}}(\theta)} \right) \Bigg|_{(x_0=\frac{T}{2}, T=L)}, \tag{3.14}$$

$$\rho_{\mathrm{A}^{(i)}}^{\mathrm{stat}} = \left(\frac{f_{\mathrm{A}^{(i)}}^{\mathrm{stat}}(x_0,\theta)}{f_{\mathrm{A}}^{\mathrm{stat}}(x_0,\theta)} \right) \Bigg|_{(x_0=\frac{T}{2}, T=L)}. \tag{3.15}$$

The observables for the space components of the axial current and the vector matching are defined analogous. One just has to replace the index A with a A_k, V or V_0 and use the corresponding correlation function and their normalization.

Time component of the axial current

In [28, 29] the quantities for the determination of ω_{kin}, $c_{\mathrm{A}}^{(1)}$ and $\ln Z_{\mathrm{A}}^{\mathrm{HQET}}$ were introduced for isotropic angles. Different combinations of angles are denoted by θ_1 and θ_2 in this case. To determine $c_{\mathrm{A}}^{(2)}$ a new quantity $\Phi_{4,\mathrm{A}}$ is introduced with $\vec{\theta}_{\mathrm{l}} \neq \vec{\theta}_{\mathrm{h}}$. For the determination of the coefficients one uses

$$\begin{aligned} \Phi_{1,\mathrm{A}} &= R_1 - R_1^{\mathrm{stat}} \\ &= \omega_{\mathrm{kin}}\, R_1^{\mathrm{kin}}, \end{aligned} \tag{3.16}$$

$$\begin{aligned} \Phi_{2,\mathrm{A}} &= R_{\mathrm{A}} - R_{\mathrm{A}}^{\mathrm{stat}} \\ &= c_{\mathrm{A}}^{(1)}\, R_{\mathrm{A}^{(1)}}^{\mathrm{stat}} + \omega_{\mathrm{kin}}\, R_{\mathrm{A}}^{\mathrm{kin}}, \end{aligned} \tag{3.17}$$

$$\begin{aligned} \Phi_{3,\mathrm{A}} &= \zeta_{\mathrm{A}} \\ &= \ln Z_{\mathrm{A}}^{\mathrm{HQET}} + \zeta_{\mathrm{A}}^{\mathrm{stat}} + c_{\mathrm{A}}^{(1)}\, \rho_{\mathrm{A}^{(1)}}^{\mathrm{stat}} + \omega_{\mathrm{kin}} \Psi_{\mathrm{A}}^{\mathrm{kin}}, \end{aligned} \tag{3.18}$$

$$\begin{aligned} \Phi_{4,\mathrm{A}} &= R_{\mathrm{A}}(T/2, \vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{l}}', \vec{\theta}_{\mathrm{h}}, \vec{\theta}_{\mathrm{h}}') \\ &= \omega_{\mathrm{kin}}\, R_{\mathrm{A}}^{\mathrm{kin}}(T/2, \vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{l}}', \vec{\theta}_{\mathrm{h}}, \vec{\theta}_{\mathrm{h}}') \\ &\quad + c_{\mathrm{A}}^{(1)}\, R_{\mathrm{A}^{(1)}}^{\mathrm{stat}}(T/2, \vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{l}}', \vec{\theta}_{\mathrm{h}}, \vec{\theta}_{\mathrm{h}}') + c_{\mathrm{A}}^{(2)}\, R_{\mathrm{A}^{(2)}}^{\mathrm{stat}}(T/2, \vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{l}}', \vec{\theta}_{\mathrm{h}}, \vec{\theta}_{\mathrm{h}}'). \end{aligned} \tag{3.19}$$

In the setup with vanishing background field there is no sensitivity to ω_{spin}, because the corresponding correlation functions $f_{\mathrm{A}}^{\mathrm{spin}}$ and f_1^{spin} are zero, see eq. (2.64).

Time component of the vector current

Analogous to the observables of the time component of the axial current I use the observables (3.17), (3.18) and (3.19) for the matching of the time component of the vector current. I can extract the HQET coefficients from the matching equations

$$\begin{aligned}
\Phi_{1,\mathrm{V}_0} &= R_{\mathrm{V}_0} - R^{\mathrm{stat}}_{\mathrm{V}_0} \\
&= c^{(1)}_{\mathrm{V}}\, R^{\mathrm{stat}}_{\mathrm{V}_0{}^{(1)}} + \omega_{\mathrm{kin}}\, R^{\mathrm{kin}}_{\mathrm{V}_0}, \qquad (3.20)\\
\Phi_{2,\mathrm{V}_0} &= \zeta_{\mathrm{V}_0} \\
&= \ln Z^{\mathrm{HQET}}_{\mathrm{V}_0} + \zeta^{\mathrm{stat}}_{\mathrm{V}_0} + c^{(1)}_{\mathrm{V}}\, \rho^{\mathrm{stat}}_{\mathrm{V}^{(1)}_0} + \omega_{\mathrm{kin}} \Psi^{\mathrm{kin}}_{\mathrm{V}_0}, \qquad (3.21)\\
\Phi_{3,\mathrm{V}_0} &= R_{\mathrm{V}_0}(T/2, \vec\theta_{\mathrm{l}} = \vec\theta'_{\mathrm{l}}, \vec\theta_{\mathrm{h}}, \vec\theta'_{\mathrm{h}}) \\
&= \omega_{\mathrm{kin}}\, R^{\mathrm{kin}}_{\mathrm{V}_0}(T/2, \vec\theta_{\mathrm{l}} = \vec\theta'_{\mathrm{l}}, \vec\theta_{\mathrm{h}}, \vec\theta'_{\mathrm{h}}) \\
&+ c^{(1)}_{\mathrm{V}}\, R^{\mathrm{stat}}_{\mathrm{V}^{(1)}_0}(T/2, \vec\theta_{\mathrm{l}} = \vec\theta'_{\mathrm{l}}, \vec\theta_{\mathrm{h}}, \vec\theta'_{\mathrm{h}}) + c^{(2)}_{\mathrm{V}}\, R^{\mathrm{stat}}_{\mathrm{V}^{(2)}_0}(T/2, \vec\theta_{\mathrm{l}} = \vec\theta'_{\mathrm{l}}, \vec\theta_{\mathrm{h}}, \vec\theta'_{\mathrm{h}}). \qquad (3.22)
\end{aligned}$$

The observables (3.20) and (3.21) are applied with equal and isotropic θ angles for the light and the heavy quark respectively. The matching equation (3.22) has sensitivity to the coefficients ω_{kin}, $c^{(1)}_{\mathrm{V}}$ and $c^{(2)}_{\mathrm{V}}$ by using $\vec\theta_{\mathrm{l}} \neq \vec\theta_{\mathrm{h}}$, recall that only then the observable $R^{\mathrm{stat}}_{\mathrm{V}^{(2)}_0}$ does not vanish.

Space components of the vector current

For the matching of the space components of the vector current I introduce new observables depending in k_{V} and k_1. With $\vec\theta_{\mathrm{l}} = \vec\theta_{\mathrm{h}}$ I choose the observables

$$\begin{aligned}
\Phi_{1,\mathrm{V}} = \zeta_{\mathrm{V}} &= \ln\left(\frac{k_{\mathrm{V}}(T/2, \vec\theta_{\mathrm{l}} = \vec\theta_{\mathrm{h}} = \vec\theta)}{\sqrt{k_1(\vec\theta_{\mathrm{l}} = \vec\theta_{\mathrm{h}} = \vec\theta)}}\right) \\
&= \ln Z^{\mathrm{HQET}}_{\mathrm{V}} + \zeta^{\mathrm{stat}}_{\mathrm{V}} + \omega_{\mathrm{kin}}\, \Psi^{\mathrm{kin}}_{\mathrm{V}} + \sum_{i=3}^{4} c^{(i)}_{\mathrm{V}}\, \rho^{(i)}_{\mathrm{V}}, \qquad (3.23)\\
\Phi_{2,\mathrm{V}} = R_{\mathrm{V}} &= \ln\left(\frac{k_{\mathrm{V}}(T/2, \vec\theta_{\mathrm{l}} = \vec\theta_{\mathrm{h}} = \vec\theta)}{k_{\mathrm{V}}(T\ 2, \vec\theta'_{\mathrm{l}} = \vec\theta'_{\mathrm{h}} = \vec\theta')}\right) \\
&= R^{\mathrm{stat}}_{\mathrm{V}} + \omega_{\mathrm{kin}}\, R^{\mathrm{kin}}_{\mathrm{V}} + \sum_{i=3}^{4} c^{(i)}_{\mathrm{V}}\, R^{\mathrm{stat}}_{\mathrm{V}^{(i)}}. \qquad (3.24)
\end{aligned}$$

The two quantities are sensitive to the parameters ω_{kin}, $c^{(3)}_{\mathrm{V}}$ and $c^{(4)}_{\mathrm{V}}$, whereas (3.23) also has sensitivity to the renormalization constant $Z^{\mathrm{HQET}}_{\mathrm{V}}$. The coefficients $c^{(5)}_{\mathrm{V}}$ and $c^{(6)}_{\mathrm{V}}$ do not contribute because at zero-momentum the insertions $V^{(5)}_k$ and $V^{(6)}_k$ vanish.

Further observables thus will be determined with different angles for the light and the heavy quark. A natural choice which provides further sensitivity to $c_{\mathrm{V}}^{(5)}$ and $c_{\mathrm{V}}^{(6)}$ would be

$$\begin{aligned}\Phi_{3,\mathrm{V}} &= \ln\left(\frac{k_{\mathrm{V}}(T/2,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}})}{k_{\mathrm{V}}(T/2,\vec{\theta}'_{\mathrm{l}},\vec{\theta}'_{\mathrm{h}})}\right)\\ &= R_{\mathrm{V}}^{\mathrm{stat}}(T/2,\vec{\theta}_{\mathrm{l}},\vec{\theta}'_{\mathrm{l}},\vec{\theta}_{\mathrm{h}},\vec{\theta}'_{\mathrm{h}}) + \omega_{\mathrm{kin}}\, R_{\mathrm{V}}^{\mathrm{kin}}(T/2,\vec{\theta}_{\mathrm{l}},\vec{\theta}'_{\mathrm{l}},\vec{\theta}_{\mathrm{h}},\vec{\theta}'_{\mathrm{h}})\\ &\quad + \sum_{i=3}^{6} c_{\mathrm{V}}^{(i)}\, R_{\mathrm{V}^{(i)}}^{\mathrm{stat}}(T/2,\vec{\theta}_{\mathrm{l}},\vec{\theta}'_{\mathrm{l}},\vec{\theta}_{\mathrm{h}},\vec{\theta}'_{\mathrm{h}}). \end{aligned} \tag{3.25}$$

To obtain all the five parameters two more observables are needed. Let me remark at this point that to handle on all $1/m$-coefficients I introduce a further correlation function $k_{\mathrm{V}_{11}}$. The contribution from $V_k^{(4)}$ is proportional to $\theta_{\mathrm{l},x}^2$ at tree level for large L/a [31]. If one uses $\theta_{\mathrm{l},x} = 0$ the correlator $k_{\mathrm{V}_{11}}$ has no sensitivity to $c_{\mathrm{V}}^{(4)}$. At tree level possible choices would be

$$\Phi_{4,\mathrm{V}} = \ln\left(\frac{k_{\mathrm{V}_{11}}(T/2,\vec{\theta}_{\mathrm{l}}=\vec{\theta}_{\mathrm{h}},\theta_{\mathrm{l},x}=0)}{k_{\mathrm{V}_{11}}(T/2,\vec{\theta}'_{\mathrm{l}}=\vec{\theta}'_{\mathrm{h}},\theta'_{\mathrm{l},x}=0)}\right), \tag{3.26}$$

$$\Phi_{5,\mathrm{V}} = \ln\left(\frac{k_{\mathrm{V}_{11}}(T/2,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}},\theta_{\mathrm{l},x}=\theta_{\mathrm{h},x}=0)}{k_{\mathrm{V}_{11}}(T/2,\vec{\theta}'_{\mathrm{l}},\vec{\theta}'_{\mathrm{h}},\theta_{\mathrm{l},x}=\theta'_{\mathrm{h},x}=0)}\right). \tag{3.27}$$

where as $\Phi_{4,\mathrm{V}}$ is sensitive to ω_{kin}, $c_{\mathrm{V}}^{(3)}$ at tree level and $\Phi_{5,\mathrm{V}}$ has sensitivity to ω_{kin}, $c_{\mathrm{V}}^{(3)}$ and $c_{\mathrm{V}}^{(5)}$.
The HQET expansions of the two observables are analogous to (3.25).

Space components of the axial current

The observables for the matching of the space components of the axial current should be imposed in the same way as for the space components of the vector current. With $\vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{h}}$ I choose the observables

$$\begin{aligned}\Phi_{1,\mathrm{A}_k} = \zeta_{\mathrm{A}_k} &= \ln\left(\frac{f_{\mathrm{A}_k}(T/2,\vec{\theta}_{\mathrm{l}}=\vec{\theta}_{\mathrm{h}}=\vec{\theta})}{\sqrt{f_1(\vec{\theta}_{\mathrm{l}}=\vec{\theta}_{\mathrm{h}}=\vec{\theta})}}\right)\\ &= \ln Z_{\mathrm{A}_k}^{\mathrm{HQET}} + \zeta_{\mathrm{A}_k}^{\mathrm{stat}} + \omega_{\mathrm{kin}}\,\Psi_{\mathrm{A}_k}^{\mathrm{kin}} + \sum_{i=3}^{4} c_{\mathrm{A}}^{(i)}\,\rho_{\mathrm{A}_k}^{(i)}, \end{aligned}\tag{3.28}$$

$$\begin{aligned}\Phi_{2,\mathrm{A}_k} = R_{\mathrm{A}_k} &= \ln\left(\frac{f_{\mathrm{A}_k}(T/2,\vec{\theta}_{\mathrm{l}}=\vec{\theta}_{\mathrm{h}}=\vec{\theta})}{f_{\mathrm{A}_k}(T\ 2,\vec{\theta}'_{\mathrm{l}}=\vec{\theta}'_{\mathrm{h}}=\vec{\theta}')}\right)\\ &= R_{\mathrm{A}_k}^{\mathrm{stat}} + \omega_{\mathrm{kin}}\,R_{\mathrm{A}_k}^{\mathrm{kin}} + \sum_{i=3}^{4} c_{\mathrm{A}}^{(i)}\,R_{\mathrm{A}_k^{(i)}}^{\mathrm{stat}}. \end{aligned}\tag{3.29}$$

and with $\vec{\theta}_\mathrm{l} \neq \vec{\theta}_\mathrm{h}$ I use

$$\begin{aligned}\Phi_{3,\mathrm{A}_k} &= \ln\left(\frac{f_{\mathrm{A}_k}(T/2,\vec{\theta}_\mathrm{l},\vec{\theta}_\mathrm{h})}{f_{\mathrm{A}_k}(T/2,\vec{\theta}'_\mathrm{l},\vec{\theta}'_\mathrm{h})}\right)\\ &= R^\mathrm{stat}_{\mathrm{A}_k}(T/2,\vec{\theta}_\mathrm{l},\vec{\theta}'_\mathrm{l},\vec{\theta}_\mathrm{h},\vec{\theta}'_\mathrm{h}) + \omega_\mathrm{kin}\, R^\mathrm{kin}_{\mathrm{A}_k}(T/2,\vec{\theta}_\mathrm{l},\vec{\theta}'_\mathrm{l},\vec{\theta}_\mathrm{h},\vec{\theta}'_\mathrm{h})\\ &\quad + \sum_{i=3}^{6} c_\mathrm{A}^{(i)}\, R^\mathrm{stat}_{\mathrm{A}_k^{(i)}}(T/2,\vec{\theta}_\mathrm{l},\vec{\theta}'_\mathrm{l},\vec{\theta}_\mathrm{h},\vec{\theta}'_\mathrm{h}).\end{aligned} \tag{3.30}$$

To obtain all the five parameters two more observables are needed and I introduce the quantity $f_{\mathrm{AV}_{21}}$ from (1.121). The correlation function has no sensitivity to the insertions $A_k^{(4)}$ and $A_k^{(6)}$ at tree level. Therefore possible choices are

$$\Phi_{4,\mathrm{A}_k} = \ln\left(\frac{f_{\mathrm{AV}_{21}}(T/2,\vec{\theta}_\mathrm{l}=\vec{\theta}_\mathrm{h})}{f_{\mathrm{AV}_{21}}(T/2,\vec{\theta}'_\mathrm{l}=\vec{\theta}'_\mathrm{h})}\right), \tag{3.31}$$

$$\Phi_{5,\mathrm{A}_k} = \ln\left(\frac{f_{\mathrm{AV}_{21}}(T/2,\vec{\theta}_\mathrm{l},\vec{\theta}_\mathrm{h})}{f_{\mathrm{AV}_{21}}(T/2,\vec{\theta}'_\mathrm{l},\vec{\theta}'_\mathrm{h})}\right). \tag{3.32}$$

These observables are sensitive to ω_kin, $c_\mathrm{A}^{(3)}$ and $c_\mathrm{A}^{(5)}$, whereas (3.31) has no sensitivity to $c_\mathrm{A}^{(5)}$ when $\vec{\theta}_\mathrm{l} = \vec{\theta}_\mathrm{h}$.

4 Perturbative expansion of the heavy-light correlation functions

The perturbative expansion of the correlation functions follows [32] and [33]. At first I will present the integration over the quark and anti-quark fields. Furthermore some definitions are introduced to accomplish the perturbative expansion of the SF correlation functions. Next in 4.3 the perturbative expansion of the relativistic correlation functions is exposed in detail and tree level results are shown. Finally the analogous expansion of the static correlation functions and the $1/m$-corrections are presented.

4.1 Quark functional intergal

Expectation values of observables in the quark functional framework are of interest. One can write them as

$$\langle \mathcal{O} \rangle = \langle [\mathcal{O}]_{\mathrm{F}} \rangle_{\mathrm{G}} \tag{4.1}$$

with $[\ldots]_{\mathrm{F}}$ the expectation value generated with the fermionic action[1]

$$S_{\mathrm{F}}[U, \overline{\psi}_{\mathrm{b}}, \psi_{\mathrm{b}}, \overline{\psi}_{\mathrm{l}}, \psi_{\mathrm{l}}] = S_{\mathrm{l}}[U, \overline{\psi}_{\mathrm{l}}, \psi_{\mathrm{l}}] + S_{\mathrm{b}}[U, \overline{\psi}_{\mathrm{b}}, \psi_{\mathrm{b}}] \tag{4.2}$$

and the expectation value $\langle \ldots \rangle_{\mathrm{G}}$ calculated with the effective gauge field action

$$S_{\mathrm{eff}}[U] = S_{\mathrm{G}}[U] - \mathrm{tr}\left\{ \ln(D + \delta D + m_0) \right\}. \tag{4.3}$$

One can write the generating functional by using the source fields $\eta_{\mathrm{l}}, \bar{\eta}_{\mathrm{l}}, \eta_{\mathrm{b}}$, and $\bar{\eta}_{\mathrm{b}}$ for the quark fields

$$\begin{aligned}
&\mathcal{Z}_{\mathrm{F}}[\bar{\rho}_{\mathrm{l}}', \rho_{\mathrm{l}}'; \bar{\rho}_{\mathrm{l}}, \rho_{\mathrm{l}}; \bar{\rho}_{\mathrm{b}}', \rho_{\mathrm{b}}; U] = \\
&\quad \int D[\psi_{\mathrm{l}}] D[\overline{\psi}_{\mathrm{l}}] D[\psi_{\mathrm{b}}] D[\overline{\psi}_{\mathrm{b}}] \exp\big\{ -S_{\mathrm{F}}[U, \overline{\psi}_{\mathrm{b}}, \psi_{\mathrm{b}}, \overline{\psi}_{\mathrm{l}}, \psi_{\mathrm{l}}] \\
&\quad + a^4 \sum_x \left[\overline{\psi}_{\mathrm{l}}(x) \eta_{\mathrm{l}}(x) + \bar{\eta}_{\mathrm{l}}(x) \psi_{\mathrm{l}}(x) \right] \\
&\quad + a^4 \sum_x \left[\overline{\psi}_{\mathrm{b}}(x) \eta_{\mathrm{b}}(x) + \bar{\eta}_{\mathrm{b}}(x) \psi_{\mathrm{b}}(x) \right] \big\}.
\end{aligned} \tag{4.4}$$

After substituting the quark fields with derivatives of the sources

$$\psi_{\mathrm{l}}(x) \to \frac{\delta}{\delta \bar{\eta}_{\mathrm{l}}(x)}, \quad \overline{\psi}_{\mathrm{l}}(x) \to -\frac{\delta}{\delta \eta_{\mathrm{l}}(x)}, \tag{4.5}$$

$$\psi_{\mathrm{b}}(x) \to \frac{\delta}{\delta \bar{\eta}_{\mathrm{b}}(x)}, \quad \overline{\psi}_{\mathrm{b}}(x) \to -\frac{\delta}{\delta \eta_{\mathrm{b}}(x)} \tag{4.6}$$

[1] The gauge field U has to be fixed

I can write the fermionic expectation value as

$$[\mathcal{O}]_{\mathrm{F}} = \left\{ \frac{1}{\mathcal{Z}_{\mathrm{F}}} \mathcal{O} \mathcal{Z}_{\mathrm{F}} \right\}_{\bar{\rho}'_{\mathrm{l}}=\ldots=\eta_{\mathrm{b}}=0} . \tag{4.7}$$

I can write the quark fields as a sum of the classical values and of fluctuation fields,

$$\psi_{\mathrm{l}}(x) = \psi_{\mathrm{l,cl}}(x) + \chi_{\mathrm{l}}(x), \quad \overline{\psi}_{\mathrm{l}}(x) = \overline{\psi}_{\mathrm{l,cl}}(x) + \bar{\chi}_{\mathrm{l}}(x), \tag{4.8}$$

$$\psi_{\mathrm{b}}(x) = \psi_{\mathrm{b,cl}}(x) + \chi_{\mathrm{b}}(x), \ \overline{\psi}_{\mathrm{b}}(x) = \overline{\psi}_{\mathrm{b,cl}}(x) + \bar{\chi}_{\mathrm{b}}(x), \tag{4.9}$$

which have to be zero at $x_0 = 0$ and $x_0 = T$, to fulfill the SF boundary conditions. The fermion action can be split into a sum of an action depending on the classic values for the quark fields and an action depending on the fluctuation fields. I use the specific boundary condition of the SF and the equation of motion in the bulk for the light and heavy quark fields,

$$S_{\mathrm{F}}[U, \overline{\psi}_{\mathrm{l}}, \psi_{\mathrm{l}}, \overline{\psi}_{\mathrm{b}}, \psi_{\mathrm{b}}] = S_{\mathrm{F}}[U, \overline{\psi}_{\mathrm{l,cl}}, \psi_{\mathrm{l,cl}}, \overline{\psi}_{\mathrm{b,cl}}, \psi_{\mathrm{b,cl}}] + S_{\mathrm{F}}[U, \bar{\chi}_{\mathrm{l}}, \chi_{\mathrm{l}}, \bar{\chi}_{\mathrm{b}}, \chi_{\mathrm{b}}]. \tag{4.10}$$

Thereby the generating functional provides, including a change of the integration variable from ψ to χ

$$\begin{aligned} \ln \mathcal{Z}_{\mathrm{F}} &= \ln \mathcal{Z}_{\mathrm{F}}|_{\bar{\rho}'_{\mathrm{l}}=\ldots=\eta_{\mathrm{b}}=0} - S_{\mathrm{F}}[U, \overline{\psi}_{\mathrm{l,cl}}, \psi_{\mathrm{l,cl}}, \overline{\psi}_{\mathrm{b,cl}}, \psi_{\mathrm{b,cl}}] \\ &+ a^8 \sum_{x,y} \bar{\eta}_{\mathrm{l}}(x) S_{\mathrm{l}}(x,y) \eta_{\mathrm{l}}(y) + a^4 \sum_{x} \left[\bar{\eta}_{\mathrm{l}}(x) \psi_{\mathrm{l,cl}}(x) + \overline{\psi}_{\mathrm{l,cl}}(x) \eta_{\mathrm{l}}(x) \right] \\ &+ a^8 \sum_{x,y} \bar{\eta}_{\mathrm{b}}(x) S_{\mathrm{b}}(x,y) \eta_{\mathrm{b}}(y) + a^4 \sum_{x} \left[\bar{\eta}_{\mathrm{b}}(x) \psi_{\mathrm{b,cl}}(x) + \overline{\psi}_{\mathrm{b,cl}}(x) \eta_{\mathrm{b}}(x) \right], \end{aligned} \tag{4.11}$$

where I distinguish between S_{l}, the light quark propagator and S_{b}, the heavy quark propagator. From [32] one knows that the improved action receives boundary contributions

$$\begin{aligned} & S_{\mathrm{F,impr}}[U, \overline{\psi}_{\mathrm{cl}}, \psi_{\mathrm{cl}}] = \\ & a^3 \sum_{\mathbf{x}} \left\{ \frac{1}{2} a \tilde{c}_{\mathrm{s}} \left[\bar{\rho}(\mathbf{x}) \gamma_k (\nabla_k^* + \nabla_k) \rho(\mathbf{x}) + \bar{\rho}'(\mathbf{x}) \gamma_k (\nabla_k^* + \nabla_k) \rho'(\mathbf{x}) \right] \right. \\ & \left. - \tilde{c}_{\mathrm{t}} \left[\bar{\rho}(\mathbf{x}) U(x - a\hat{0}, 0) \psi_{\mathrm{cl}}(x) \big|_{x_0=a} + \bar{\rho}'(\mathbf{x}) U(x,0)^{-1} \psi_{\mathrm{cl}}(x) \big|_{x_0=T-a} \right] \right\}, \end{aligned} \tag{4.12}$$

which is valid for light and heavy quarks. The action of a bilinear system consisting of a light and a heavy quark would be the sum of the light quark action and the heavy quark action, as one can see in (4.2). The two-point functions can be determined by differentiating the generating functional (4.11) with respect to the source fields.

This results in [32]

$$\begin{aligned}
\left[\psi(x)\overline{\psi}(y)\right]_{\mathrm{F}} &= S(x,y), && (4.13)\\
\left[\psi(x)\bar{\zeta}(\mathbf{y})\right]_{\mathrm{F}} &= \tilde{c}_{\mathrm{t}} S(x,y) P_+ U(y-a\hat{0},0)^{-1} P_+\big|_{x_0=a}\,, && (4.14)\\
\left[\psi(x)\bar{\zeta}'(\mathbf{y})\right]_{\mathrm{F}} &= \tilde{c}_{\mathrm{t}} S(x,y) P_+ U(y,0)^{-1} P_-\big|_{x_0=T-a}\,, && (4.15)\\
\left[\zeta(\mathbf{x})\overline{\psi}(y)\right]_{\mathrm{F}} &= \tilde{c}_{\mathrm{t}} P_+ U(x-a\hat{0},0) S(x,y)\big|_{x_0=a}\,, && (4.16)\\
\left[\zeta'(\mathbf{x})\overline{\psi}(y)\right]_{\mathrm{F}} &= \tilde{c}_{\mathrm{t}} P_+ U(x,0)^{-1} S(x,y)\big|_{x_0=T-a}\,, && (4.17)\\
\left[\zeta(\mathbf{x})\bar{\zeta}(\mathbf{y})\right]_{\mathrm{F}} &= \tilde{c}_{\mathrm{t}} P_- U(x-a\hat{0},0) S(x,y) U(y-a\hat{0},0)^{-1} P_+\big|_{x_0=y_0=a} \\
&\quad - \frac{1}{2}\tilde{c}_{\mathrm{s}} P_- \gamma_k (\nabla_k^* + \nabla_k) a^{-2}\delta_{\mathbf{xy}}, && (4.18)\\
\left[\zeta(\mathbf{x})\bar{\zeta}'(\mathbf{y})\right]_{\mathrm{F}} &= \tilde{c}_{\mathrm{t}}^2 P_+ U(x-a\hat{0},0)\ S(x,y)\ U(y,0)^{-1} P_-\big|_{x_0=a,y_0=T-a}\,, && (4.19)\\
\left[\zeta'(\mathbf{x})\bar{\zeta}(\mathbf{y})\right]_{\mathrm{F}} &= \tilde{c}_{\mathrm{t}}^2 P_+ U(x,0)^{-1}\ S(x,y)\ U(y-a\hat{0},0)^{-1} P_-\big|_{x_0=T-a,y_0=a}\,, && (4.20)\\
\left[\zeta'(\mathbf{x})\bar{\zeta}'(\mathbf{y})\right]_{\mathrm{F}} &= \tilde{c}_{\mathrm{t}}^2 P_+ U(x,0)^{-1}\ S(x,y)\ U(y,0)^{-1} P_-\big|_{x_0=a,y_0=T-a} \\
&\quad - \frac{1}{2}\tilde{c}_{\mathrm{s}} P_+ \gamma_k (\nabla_k^* + \nabla_k) a^{-2}\delta_{\mathbf{xy}}. && (4.21)
\end{aligned}$$

The Wick contractions are valid for heavy and light fermion fields.

4.2 Tree level quark propagator

For simplicity some definitions are needed to evaluate SF tree level correlation functions. The periodicity angles lead to a shift of the lattice momenta

$$p_k = \frac{2\pi n_k}{L}, \qquad -\frac{\pi}{a} < p_k < \frac{\pi}{a} \tag{4.22}$$

by

$$p_\mu^+ = p_\mu + \frac{\theta_\mu}{L}. \tag{4.23}$$

I use the abbreviation for any momentum q_μ

$$\mathring{q}_\mu = \frac{1}{a}\sin(aq_\mu), \tag{4.24}$$

$$\hat{q}_\mu = \frac{2}{a}\sin(aq_\mu/2). \tag{4.25}$$

One now can define an effective mass for the light quark field

$$M(q) = m_0 + \frac{1}{2}a\hat{q}^2. \tag{4.26}$$

The tree level solution of the free lattice Dirac equation can be written as

$$\psi_{\mathrm{l,cl}}^{(0)} = u_1 e^{ipx} \tag{4.27}$$

and only considering states with positive energy, i.e. $\operatorname{Im} p_0 \geq 0$, provides

$$p_0 = p_0^+ = i\omega(\mathbf{p}^+) \bmod 2\pi/a \tag{4.28}$$

with

$$\sinh\left(\frac{a}{2}\omega(\mathbf{q})\right) = \frac{a}{2}\left[\frac{\mathring{\mathbf{q}}^2 + (m_0 + \frac{1}{2}a\hat{\mathbf{q}}^2)^2}{1 + a(m_0 + \frac{1}{2}a\hat{\mathbf{q}}^2)}\right]^{1/2}. \tag{4.29}$$

The tree level propagator for the light quark can be obtained from

$$(D + m_0)S_{\mathrm{l}}^{(0)}(x,y)\Big|_{g_0=0} = \delta(x-y) \tag{4.30}$$

with the boundary conditions

$$\begin{aligned} S_{\mathrm{l}}^{(0)}(x,y)\Big|_{x_0=0} &= S_{\mathrm{l}}^{(0)}(x,y)\Big|_{y_0=0} = \\ S_{\mathrm{l}}^{(0)}(x,y)\Big|_{x_0=T} &= S_{\mathrm{l}}^{(0)}(x,y)\Big|_{y_0=T} = 0. \end{aligned} \tag{4.31}$$

The Fourier transform is defined as

$$\tilde{S}_{\mathrm{l}}^{(0)}(x_0,y_0;\mathbf{p}) = a^3\sum_{\mathbf{x}} e^{-i\mathbf{p}(\mathbf{x}-\mathbf{y})}S_{\mathrm{l}}^{(0)}(x,y). \tag{4.32}$$

For $m_0 = 0$ and $\mathbf{p}^+ = 0$ the tree level propagator has the form

$$\tilde{S}_{\mathrm{l}}^{(0)}(x_0,y_0;\mathbf{p}) = \begin{cases} P_+ & \text{if } x_0 > y_0, \\ P_- & \text{if } x_0 < y_0, \\ 1 & \text{if } x_0 = y_0. \end{cases} \tag{4.33}$$

For all other cases the tree level expression is written in appendix B.

The classical solution for the free Wilson Dirac equation for $a \le x_0 \le T$ can be written as

$$\psi_{\mathrm{l,cl}}^{(0)} = a^3\sum_{\mathbf{y}}\left(S_{\mathrm{l}}^{(0)}(x,y)\rho_{\mathrm{l}}(\mathbf{y})\Big|_{y_0=a} + S_{\mathrm{l}}^{(0)}(x,y)\rho'_{\mathrm{l}}(\mathbf{y})\Big|_{y_0=T-a}\right). \tag{4.34}$$

4.3 Perturbative expansion of the relativistic correlation functions

In this section I consider the perturbative expansion of the QCD correlation functions f_{A}, f_1 and k_{V} in detail. The expansion of k_1, k_{V_0}, $k_{\mathrm{V}_{11}}$ and f_{A_k}, $f_{\mathrm{AV}_{21}}$ is done analogously.

With the heavy-light axial and vector current the SF correlation functions read

$$f_{\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle\overline{\psi}_{\mathrm{l}}(x)\gamma_0\gamma_5\psi_{\mathrm{b}}(x)\,\bar{\zeta}_{\mathrm{b}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{4.35}$$

$$k_{\mathrm{V}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = -\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z}.k}\left\langle\overline{\psi}_{\mathrm{l}}(x)\gamma_k\psi_{\mathrm{b}}(x)\,\bar{\zeta}_{\mathrm{b}}(\mathbf{y})\gamma_k\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{4.36}$$

$$f_1(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}}\left\langle\bar{\zeta}'_{\mathrm{l}}(\mathbf{u})\gamma_5\zeta'_{\mathrm{b}}(\mathbf{v})\,\bar{\zeta}_{\mathrm{b}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle. \tag{4.37}$$

Applying Wick's theorem [34] provides the following formulations for f_{A}, k_{V} and f_1

$$f_{\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = \frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle \mathrm{tr}\left\{[\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)]_{\mathrm{F}}\gamma_0\gamma_5[\psi_{\mathrm{b}}(x)\bar{\zeta}_{\mathrm{b}}(\mathbf{y})]_{\mathrm{F}}\gamma_5\right\}\right\rangle_{\mathrm{G}} \tag{4.38}$$

$$k_{\mathrm{V}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = \frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k} \left\langle \mathrm{tr}\left\{[\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)]_{\mathrm{F}}\gamma_k[\psi_{\mathrm{b}}(x)\bar{\zeta}_{\mathrm{b}}(\mathbf{y})]_{\mathrm{F}}\gamma_k\right\}\right\rangle_{\mathrm{G}}, \tag{4.39}$$

$$f_1(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = \frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \left\langle \mathrm{tr}\left\{[\zeta_{\mathrm{l}}(\mathbf{z})\bar{\zeta}'_{\mathrm{l}}(\mathbf{u})]_{\mathrm{F}}\gamma_5[\zeta'_{\mathrm{b}}(\mathbf{v})\bar{\zeta}_{\mathrm{b}}(\mathbf{y})]\gamma_5\right\}\right\rangle_{\mathrm{G}}. \tag{4.40}$$

The trace runs over Dirac and colour indices. The hermiticity property of the quark propagator (1.92) can be utilized for the contractions in the following way

$$\gamma_5[\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)]_{\mathrm{F}}\gamma_5 = [\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}, \tag{4.41}$$

$$\gamma_5[\zeta_{\mathrm{l}}(\mathbf{z})\bar{\zeta}'_{\mathrm{l}}(\mathbf{u})]_{\mathrm{F}}\gamma_5 = [\zeta'_{\mathrm{l}}(\mathbf{u})\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}. \tag{4.42}$$

Furthermore I use the relation of the Dirac matrices in the chiral representation (A.5) and I obtain

$$f_{\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle \mathrm{tr}\left\{[\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}\gamma_0[\psi_{\mathrm{b}}(x)\bar{\zeta}_{\mathrm{b}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}}, \tag{4.43}$$

$$k_{\mathrm{V}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = \frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k} \left\langle \mathrm{tr}\left\{\gamma_5[\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}\gamma_5\gamma_k[\psi_{\mathrm{b}}(x)\bar{\zeta}_{\mathrm{b}}(\mathbf{y})]_{\mathrm{F}}\gamma_k\right\}\right\rangle_{\mathrm{G}}, \tag{4.44}$$

$$f_1(\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = \frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \left\langle \mathrm{tr}\left\{[\zeta'_{\mathrm{l}}(\mathbf{u})\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}[\zeta'_{\mathrm{b}}(\mathbf{v})\bar{\zeta}_{\mathrm{b}}(\mathbf{y})]\right\}\right\rangle_{\mathrm{G}}. \tag{4.45}$$

I can write the contractions as derivatives of the classical fields with respect to the source fields

$$\left[\psi(x)\bar{\zeta}(\mathbf{y})\right]_{\mathrm{F}} = \frac{\delta\psi_{\mathrm{cl}}(x)}{\delta\rho(\mathbf{y})}. \tag{4.46}$$

This is valid for heavy and light quarks. It provides the expressions for the SF heavy-light correlation functions

$$f_{\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle \mathrm{tr}\left\{\left(\frac{\delta\psi_{\mathrm{l,cl}}(x)}{\delta\rho_{\mathrm{l}}(\mathbf{z})}\right)^{\dagger}\gamma_0\left(\frac{\delta\psi_{\mathrm{b,cl}}(x)}{\delta\rho_{\mathrm{b}}(\mathbf{y})}\right)\right\}\right\rangle_{\mathrm{G}}, \tag{4.47}$$

$$k_{\mathrm{V}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{b}}) = \frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z}} \left\langle \mathrm{tr}\left\{\gamma_5\left(\frac{\delta\psi_{\mathrm{l,cl}}(x)}{\delta\rho_{\mathrm{l}}(\mathbf{z})}\right)^{\dagger}\gamma_5\gamma_k\left(\frac{\delta\psi_{\mathrm{b,cl}}(x)}{\delta\rho_{\mathrm{b}}(\mathbf{y})}\right)\gamma_k\right\}\right\rangle_{\mathrm{G}}. \tag{4.48}$$

It is appropriate to define matrices for the heavy and the light quarks and as correspondingly for the boundary fields as in [22]. The matrix $H(x)$ describes the quark propagator from boundary at time 0 to the point x in the interior of the space-time volume,

$$H(x) = a^3\sum_{\mathbf{y}}\frac{\delta\psi_{\mathrm{cl}}(x)}{\delta\rho(\mathbf{y})}. \tag{4.49}$$

Furthermore one finds in [20] the matrices for the relativistic heavy and light quark, which present the quark propagator from the boundary at time 0 to the boundary at time T

$$K = \tilde{c}_{\mathrm{t}} \frac{a^3}{L^3} \sum_{\mathbf{x}} P_+ U(x,0)^{-1} H(x) \,|_{x_0=T-a} \,. \tag{4.50}$$

With the use of the expressions for the quark fields I can write the heavy-light correlation functions as

$$f_{\mathrm{A}}(x_0, \vec{\theta}_{\mathrm{l}}, \vec{\theta}_{\mathrm{b}}) = -\frac{1}{2} \left\langle \mathrm{tr}\left\{ H_{\mathrm{l}}(x)^{\dagger} \gamma_0 H_{\mathrm{b}}(x) \right\} \right\rangle_{\mathrm{G}}, \tag{4.51}$$

$$k_{\mathrm{V}}(x_0, \vec{\theta}_{\mathrm{l}}, \vec{\theta}_{\mathrm{b}}) = \frac{1}{6} \left\langle \mathrm{tr}\left\{ \gamma_5 H_{\mathrm{l}}(x)^{\dagger} \gamma_5 \gamma_k H_{\mathrm{b}}(x) \gamma_k \right\} \right\rangle_{\mathrm{G}}, \tag{4.52}$$

$$f_1(\vec{\theta}_{\mathrm{l}}, \vec{\theta}_{\mathrm{b}}) = \frac{1}{2} \left\langle \mathrm{tr}\left\{ K_{\mathrm{l}}^{\dagger} K_{\mathrm{b}} \right\} \right\rangle_{\mathrm{G}}. \tag{4.53}$$

Finally the perturbative expansion can be deduced by expanding the correlation functions and the matrices in the coupling g_0

$$f_{\mathrm{A}}(x_0) = f_{\mathrm{A}}^{(0)}(x_0) + g_0^2 f_{\mathrm{A}}^{(1)}(x_0) + O(g_0^4), \tag{4.54}$$

$$k_{\mathrm{V}}(x_0) = k_{\mathrm{V}}^{(0)}(x_0) + g_0^2 k_{\mathrm{V}}^{(1)}(x_0) + O(g_0^4), \tag{4.55}$$

$$f_1 = f_1^{(0)} + g_0^2 f_1^{(1)} + O(g_0^4), \tag{4.56}$$

with the matrix expansions

$$H_{\mathrm{l}}(x) = H_{\mathrm{l}}^{(0)}(x) + g_0 H_{\mathrm{l}}^{(1)}(x) + g_0^2 H_{\mathrm{l}}^{(2)}(x) + O(g_0^3), \tag{4.57}$$

$$H_{\mathrm{b}}(x) = H_{\mathrm{b}}^{(0)}(x) + g_0 H_{\mathrm{b}}^{(1)}(x) + g_0^2 H_{\mathrm{b}}^{(2)}(x) + O(g_0^3), \tag{4.58}$$

$$K_{\mathrm{l}}(x) = K_{\mathrm{l}}^{(0)} + g_0 K_{\mathrm{l}}^{(1)} + g_0^2 K_{\mathrm{l}}^{(2)} + O(g_0^3), \tag{4.59}$$

$$K_{\mathrm{b}}(x) = K_{\mathrm{b}}^{(0)} + g_0 K_{\mathrm{b}}^{(1)} + g_0^2 K_{\mathrm{b}}^{(2)} + O(g_0^3). \tag{4.60}$$

4.3.1 Tree level expressions

I have to consider the matrix $H(x)$ at tree level. The matrix is defined as the derivative of the quark fields with respect to the corresponding boundary source

$$H^{(0)}(x) = a^3 \sum_{\mathbf{y}} \frac{\delta \psi_{\mathrm{cl}}^{(0)}(x)}{\delta \rho(\mathbf{y})}. \tag{4.61}$$

With (B.3) to (B.5) one obtains after some calculations for the tree level propagator for $\mathbf{p} = 0$

$$\begin{aligned} H^{(0)}(x) = \frac{1}{R(p^+)} \Big\{ & (M(p^+) - i\mathring{p}_0^+ - i\gamma_k \mathring{p}_k^+) e^{-\omega(\mathbf{p}^+)x_0} \\ & - (M(p^+) + i\mathring{p}_0^+ - i\gamma_k \mathring{p}_k^+) e^{-\omega(\mathbf{p}^+)(2T-x_0)} \Big\} P_+. \end{aligned} \tag{4.62}$$

The tree level propagator does not depend on the space components of a point but on the time component. Therefore I can introduce the function

$$\chi(x_0) = H^{(0)}(x). \tag{4.63}$$

Moreover the tree level results for the K matrices can be written as

$$K^{(0)} = P_+\chi(T-a). \tag{4.64}$$

In QCD I have the correlation functions with two relativistic quarks labeled with 1 and 2, for the light and the heavy quark. They have the bare masses $m_{0,1}$ and $m_{0,2}$ and zero momentum $\mathbf{p} = 0$.

All in all I get the QCD two-point correlation functions at tree level

$$f_{\mathrm{A}}^{(0)}(x_0, \vec{\theta}_{\mathrm{l}}, \vec{\theta}_{\mathrm{b}}) = -\frac{1}{2}\left\langle \mathrm{tr}\left\{\chi_1(x_0)^\dagger\gamma_0\chi_2(x_0)\right\}\right\rangle_{\mathrm{G}}, \tag{4.65}$$

$$k_{\mathrm{V}}^{(0)}(x_0, \vec{\theta}_{\mathrm{l}}, \vec{\theta}_{\mathrm{b}}) = \frac{1}{6}\left\langle \mathrm{tr}\left\{\gamma_5\chi_1(x_0)^\dagger\gamma_5\gamma_k\chi_2(x_0)\gamma_k\right\}\right\rangle_{\mathrm{G}}, \tag{4.66}$$

$$f_1^{(0)}(\vec{\theta}_{\mathrm{l}}, \vec{\theta}_{\mathrm{b}}) = \frac{1}{2}\left\langle \mathrm{tr}\left\{\chi_1(T-a)^\dagger P_+\chi_2(T-a)\right\}\right\rangle_{\mathrm{G}}. \tag{4.67}$$

4.4 Perturbative expansion of the the correlation functions in the static approximation

The boundary-to-bulk correlation functions in the static approximation for the pseudo-scalar and vector channel are

$$f_{\mathrm{A}}^{\mathrm{stat}}(x_0, \vec{\theta}_{\mathrm{l}}, \vec{\theta}_{\mathrm{h}}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle A_0^{\mathrm{stat}}(x)\ \bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{4.68}$$

$$k_{\mathrm{V}}^{\mathrm{stat}}(x_0, \vec{\theta}_{\mathrm{l}}, \vec{\theta}_{\mathrm{h}}) = -\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle V_k^{\mathrm{stat}}(x)\ \bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_k\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle. \tag{4.69}$$

The computation is associated with the one in the relativistic case. I apply Wick's theorem and make use of the hermiticity property and the representation through the boundary fields

$$\begin{aligned}\gamma_5[\zeta(\mathbf{z})\overline{\psi}(x)]_{\mathrm{F}}\gamma_5 &= \left[\psi(x)\bar{\zeta}(\mathbf{z})\right]_{\mathrm{F}}^\dagger \\ &= \left(\frac{\delta\psi_{\mathrm{cl}}(x)}{\delta\rho(\mathbf{z})}\right)^\dagger.\end{aligned} \tag{4.70}$$

It is convenient for the calculations to introduce the heavy and light quark matrices as for the relativistic quarks (4.49). Thus one defines

$$H_{\mathrm{l}}(x) = a^3\sum_{\mathbf{y}}\frac{\delta\psi_{\mathrm{l,cl}}(x)}{\delta\rho_{\mathrm{l}}(\mathbf{y})}, \tag{4.71}$$

$$H_{\mathrm{h}}(x) = a^3\sum_{\mathbf{y}}\frac{\delta\psi_{\mathrm{h,cl}}(x)}{\delta\rho_{\mathrm{h}}(\mathbf{y})}. \tag{4.72}$$

Furthermore the matrices for the heavy and light quarks, which present the quark propagator from the boundary at time 0 to the boundary at time T are [33]

$$K_{\rm l} = \tilde{c}_t \frac{a^3}{L^3} \sum_{\mathbf{x}} P_+ U(x,0)^{-1} H_{\rm l}(x) \,|_{x_0=T-a} \,, \tag{4.73}$$

$$K_{\rm h} = \frac{a^3}{L^3} \sum_{\mathbf{x}} P_+ U(x,0)^{-1} H_{\rm h}(x) \,|_{x_0=T-a} \,. \tag{4.74}$$

The correlation functions $f_{\rm A}^{\rm stat}$ and $k_{\rm V}^{\rm stat}$ can be written with the use of the matrices (4.71) and (4.72)

$$f_{\rm A}^{\rm stat}(x_0, \vec{\theta}_{\rm l}, \vec{\theta}_{\rm h}) = -\frac{1}{2} \left\langle {\rm tr} \left\{ H_{\rm l}(x)^\dagger \gamma_0 H_{\rm h}(x) \right\} \right\rangle_{\rm G} , \tag{4.75}$$

$$k_{\rm V}^{\rm stat}(x_0, \vec{\theta}_{\rm l}, \vec{\theta}_{\rm h}) = \frac{1}{6} \left\langle {\rm tr} \left\{ \gamma_k \gamma_5 \, H_{\rm l}(x)^\dagger \gamma_5 \gamma_k \, H_{\rm h}(x) \right\} \right\rangle_{\rm G} . \tag{4.76}$$

In analogy to $f_{\rm A}^{\rm stat}(x_0)$ I get for the boundary-to-boundary correlation function

$$f_1^{\rm stat}(\theta) = \frac{1}{2} \left\langle {\rm tr} \left\{ K_{\rm l}(x)^\dagger K_{\rm h}(x) \right\} \right\rangle_{\rm G} . \tag{4.77}$$

The perturbative expansion of the correlations functions is straightforward

$$f_{\rm A}^{\rm stat}(x_0) = f_{\rm A}^{\rm stat(0)}(x_0) + g_0^2 f_{\rm A}^{\rm stat(1)}(x_0) + O(g_0^4), \tag{4.78}$$

$$k_{\rm V}^{\rm stat}(x_0) = k_{\rm V}^{\rm stat(0)}(x_0) + g_0^2 k_{\rm V}^{\rm stat(1)}(x_0) + O(g_0^4), \tag{4.79}$$

$$f_1^{\rm stat} = f_1^{\rm stat(0)} + g_0^2 f_1^{\rm stat(1)} + O(g_0^4) \tag{4.80}$$

4.4.1 Tree level expressions

In the static case the tree level expression for the matrix (4.72), setting $\delta m = 0$ is

$$\chi_{\rm h}(x_0) = H_{\rm h}^{(0)}(x) = P_+ \tag{4.81}$$

and for the light quark mass I just add an index l to (4.63). The tree level results for the K matrices in the static case are

$$K_{\rm l}^{(0)} = P_+ \chi_{\rm l}(T-a), \quad K_{\rm h}^{(0)} = P_+. \tag{4.82}$$

This leads to the tree level formulations for the static correlation functions

$$f_{\rm A}^{\rm stat\,(0)}(x_0, \vec{\theta}_{\rm l}) = -\frac{1}{2} \left\langle {\rm tr} \left\{ \chi_{\rm l}(x_0)^\dagger P_+ \right\} \right\rangle_{\rm G} , \tag{4.83}$$

$$k_{\rm V}^{\rm stat\,(0)}(x_0, \vec{\theta}_{\rm l}) = \frac{1}{6} \left\langle {\rm tr} \left\{ \gamma_k \gamma_5 \, \chi_{\rm l}(x_0)^\dagger \gamma_5 \gamma_k \, P_+ \right\} \right\rangle_{\rm G} \tag{4.84}$$

and

$$f_1^{\rm stat\,(0)}(\vec{\theta}_{\rm l}) = \frac{1}{2} \left\langle {\rm tr} \left\{ \chi_{\rm l}(T-a)^\dagger P_+ \right\} \right\rangle_{\rm G} , \tag{4.85}$$

$$k_1^{\rm stat\,(0)}(\vec{\theta}_{\rm l}) = \frac{1}{6} \left\langle {\rm tr} \left\{ \gamma_k \gamma_5 \chi_{\rm l}(T-a)^\dagger P_+ \gamma_5 \gamma_k P_+ \right\} \right\rangle_{\rm G} . \tag{4.86}$$

4.5 Next-to-leading order in $1/m$

To determine the coefficients of $O(1/m)$ I have to compute the $1/m$ corrections of the heavy light correlation functions. I will present the perturbative expansion and tree level results of the $1/m$ corrections of the axial correlation functions. For the vector correlation functions I refer to to appendix B.

In section 2.2 the currents $f^{\mathrm{stat}}_{\delta_{\mathrm{h}}\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}})$ and $f^{\mathrm{stat}}_{\delta\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}})$ were introduced to obtain sensitivity to all $O(1/m)$ terms in the HQET expansion. With the relation at tree level

$$f^{\mathrm{stat}}_{\mathrm{A}^{(1)}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = \frac{1}{2}\left(f^{\mathrm{stat}}_{\delta_{\mathrm{h}}\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) - f^{\mathrm{stat}}_{\delta\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}})\right), \tag{4.87}$$

$$f^{\mathrm{stat}}_{\mathrm{A}^{(2)}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = \frac{1}{2}\left(f^{\mathrm{stat}}_{\delta_{\mathrm{h}}\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) + f^{\mathrm{stat}}_{\delta\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}})\right) \tag{4.88}$$

the correlation functions $f^{\mathrm{stat}}_{\mathrm{A}^{(1)}}$ and $f^{\mathrm{stat}}_{\mathrm{A}^{(2)}}$, which are used for the observables, i.e. in (3.19), can be obtained.

Perturbative expansion of $f^{\mathrm{stat}}_{\delta_{\mathrm{h}}\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}})$

The correlation function is given by

$$f^{\mathrm{stat}}_{\delta_{\mathrm{h}}\mathrm{A}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle \delta_{\mathrm{h}}\mathrm{A}_0(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{4.89}$$

with the insertion

$$\delta_{\mathrm{h}}\mathrm{A}_0(x) = \overline{\psi}_{\mathrm{l}}(x)\gamma_5\gamma_i\nabla^S_i\psi_{\mathrm{h}}(x). \tag{4.90}$$

Making use of Wick's theorem yields

$$\begin{aligned} f^{\mathrm{stat}}_{\delta_{\mathrm{h}}\mathrm{A}} &= \frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle \mathrm{tr}\left\{\gamma_5[\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)]_F\gamma_5\gamma_i[\nabla^S_i\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_F\right\}\right\rangle_G \\ &= \frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle \mathrm{tr}\left\{[\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]^\dagger_F\gamma_i[\nabla^S_i\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_F\right\}\right\rangle_G \\ &= \frac{a^3}{2}\sum_{\mathbf{y}}\left\langle \mathrm{tr}\left\{H_{\mathrm{l}}(x)^\dagger\gamma_i[\nabla^S_i\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_F\right\}\right\rangle_G. \end{aligned} \tag{4.91}$$

Furthermore I applied the previous steps as the hermiticity condition (1.92) and the introduction of the quark matrix (4.71). It remains to evaluate the second Wick contraction

$$[\nabla^S_i\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_F. \tag{4.92}$$

At first I calculate the covariant derivative acting on the heavy quark field

$$\nabla^S_i\psi_{\mathrm{h}}(x) = \frac{1}{2a}\left[\lambda_{\mathrm{h},i}U_i(x)\psi_{\mathrm{h}}(x+a\hat{i}) - \lambda^{-1}_{\mathrm{h},i}U_i(x-a\hat{i})^{-1}\psi_{\mathrm{h}}(x-a\hat{i})\right]. \tag{4.93}$$

The phases $\lambda_{\mathrm{h},i} = \exp\{ia\frac{\theta_{\mathrm{h},i}}{L}\}$ depend on the heavy quark angles. Thus I can deduce for the contraction

$$\begin{aligned}
&[\nabla_i^S \psi_\mathrm{h}(x)\bar\zeta_\mathrm{h}(\mathbf{y})]_F \\
&= \frac{1}{2a}\left\{\lambda_{\mathrm{h},i}U_i(x)[\psi_\mathrm{h}(x+a\hat{i})\zeta_\mathrm{h}(\mathbf{y})]_F - \lambda_{\mathrm{h},i}^{-1}U_i(x-a\hat{i})^{-1}[\psi_\mathrm{h}(x-a\hat{i})\zeta_\mathrm{h}(\mathbf{y})]_F\right\} \\
&= \frac{1}{2a}\left\{\lambda_{\mathrm{h},i}U_i(x)\left(\frac{\delta\psi_{\mathrm{h,cl}}(x+a\hat{i})}{\delta\rho_\mathrm{h}(\mathbf{y})}\right) - \right. \\
&\qquad\qquad \left. \lambda_{\mathrm{h},i}^{-1}U_i(x-a\hat{i})^{-1}\left(\frac{\psi_{\mathrm{h,cl}}(x-a\hat{i})}{\delta\rho_\mathrm{h}(\mathbf{y})}\right)\right\}.
\end{aligned} \tag{4.94}$$

Hence the correlation function at tree level can be written as

$$\begin{aligned}
&f_{\delta_\mathrm{h}\mathrm{A}}^{\mathrm{stat}\,(0)} \\
&= \frac{1}{2}\left\langle \mathrm{tr}\left\{H_\mathrm{l}^{(0)}(x)^\dagger\gamma_i\frac{1}{2a}\left[\lambda_{\mathrm{h},i}H_\mathrm{h}^{(0)}(x+a\hat{i}) - \lambda_{\mathrm{h},i}^{-1}H_\mathrm{h}^{(0)}(x-a\hat{i})\right]\right\}\right\rangle_G
\end{aligned} \tag{4.95}$$

where I used the definition of the quark matrices (4.72). With the tree level heavy quark matrices (4.81) I obtain

$$\begin{aligned}
f_{\delta_\mathrm{h}\mathrm{A}}^{\mathrm{stat}\,(0)} &= \frac{1}{2}\left\langle \mathrm{tr}\left\{\chi_\mathrm{l}(x_0)^\dagger\gamma_i\frac{1}{2a}P_+\left[\lambda_{\mathrm{h},i} - \lambda_{\mathrm{h},i}^{-1}\right]\right\}\right\rangle_G \\
&= \frac{i}{2a}\sum_i \sin(a\frac{\theta_{\mathrm{h},i}}{L})\left\langle \mathrm{tr}\left\{\chi_\mathrm{l}(x_0)^\dagger\gamma_i P_+\right\}\right\rangle_G
\end{aligned} \tag{4.96}$$

Perturbative expansion of $f_{\delta\mathrm{A}}^{\mathrm{stat}}(x_0, \vec\theta_\mathrm{l})$

The correlation function is given by

$$f_{\delta\mathrm{A}}^{\mathrm{stat}}(x_0,\vec\theta_\mathrm{l}) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle \delta\mathrm{A}_0(x)\,\bar\zeta_\mathrm{h}(\mathbf{y})\gamma_5\zeta_\mathrm{l}(\mathbf{z})\right\rangle, \tag{4.97}$$

and has only a $\vec\theta_\mathrm{l}$ dependence, because the operator

$$\delta\mathrm{A}_0(x) = \overline{\psi}_\mathrm{l}(x)\gamma_5\gamma_i\overleftarrow{\nabla}_i^S\psi_\mathrm{h}(x) \tag{4.98}$$

contains derivatives acting on the light quark fields. Making use of Wick's theorem yields

$$\begin{aligned}
f_{\delta\mathrm{A}}^{\mathrm{stat}} &= \frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\left\langle \mathrm{tr}\left\{\gamma_5[\zeta_\mathrm{l}(\mathbf{z})\overline{\psi}_\mathrm{l}(x)\overleftarrow{\nabla}_i^S]_F\gamma_5\gamma_i[\psi_\mathrm{h}(x)\bar\zeta_\mathrm{h}(\mathbf{y})]_F\right\}\right\rangle_G \\
&= \frac{a^3}{2}\sum_{\mathbf{z}}\left\langle \mathrm{tr}\left\{[\nabla_i^S\psi_\mathrm{l}(x)\bar\zeta_\mathrm{l}(\mathbf{z})]_F^\dagger\gamma_i H_\mathrm{h}(x)\right\}\right\rangle_G.
\end{aligned} \tag{4.99}$$

I calculate the symmetric derivative acting on the light quark field at first and afterwards I replace the boundary fields by the source fields

$$
\begin{aligned}
&[\nabla_i^S \psi_\mathrm{l}(x)\bar{\zeta}_\mathrm{l}(\mathbf{z})]_F \\
&= \left[\frac{1}{2a}\left(\lambda_{\mathrm{l},i} U_i(x)\psi_\mathrm{l}(x+a\hat{i}) - \lambda_{\mathrm{l},i}^{-1} U_i(x-a\hat{i})^{-1}\psi_\mathrm{l}(x-a\hat{i})\right)\bar{\zeta}_\mathrm{l}(\mathbf{z})\right]_F .
\end{aligned} \tag{4.100}
$$

At tree level this expression reduces to

$$
\begin{aligned}
&\frac{\lambda_{\mathrm{l},i}}{2a}\left[\psi_\mathrm{l}^{(0)}(x+a\hat{i})\bar{\zeta}_\mathrm{l}(\mathbf{z})\right]_F - \frac{\lambda_{\mathrm{l},i}^{-1}}{2a}\left[\psi_\mathrm{l}^{(0)}(x-a\hat{i})\bar{\zeta}_\mathrm{l}(\mathbf{z})\right]_F \\
=&\frac{\lambda_{\mathrm{l},i}}{2a}\left(\frac{\delta\psi_{\mathrm{l,cl}}^{(0)}(x+a\hat{i})}{\delta\rho_\mathrm{l}(\mathbf{z})}\right) - \frac{{\lambda_{\mathrm{l},i}}^{-1}}{2a}\left(\frac{\delta\psi_{\mathrm{l,cl}}^{(0)}(x-a\hat{i})}{\delta\rho_\mathrm{l}(\mathbf{z})}\right).
\end{aligned} \tag{4.101}
$$

It emerges for the tree level correlation function

$$
\begin{aligned}
f_{\delta\mathrm{A}}^{\mathrm{stat}\,(0)} &= \frac{1}{2}\left\langle \mathrm{tr}\left\{\left[\frac{\lambda_{\mathrm{l},i}^{-1}}{2a}H_\mathrm{l}^{(0)}(x+a\hat{i})^\dagger - \frac{\lambda_{\mathrm{l},i}}{2a}H_\mathrm{l}^{(0)}(x-a\hat{i})^\dagger\right]\gamma_i H_\mathrm{h}^{(0)}(x)\right\}\right\rangle_G \\
&= \frac{1}{2}\left\langle \mathrm{tr}\left\{\left[\frac{\lambda_{\mathrm{l},i}^{-1}}{2a}\chi_\mathrm{l}(x_0)^\dagger - \frac{\lambda_{\mathrm{l},i}}{2a}\chi_\mathrm{l}(x_0)^\dagger\right]\gamma_i H_\mathrm{h}^{(0)}(x)\right\}\right\rangle_G
\end{aligned} \tag{4.102}
$$

and I make use of the light quark matrices (4.71) at tree level

$$
\begin{aligned}
f_{\delta\mathrm{A}}^{\mathrm{stat}\,(0)} &= \frac{1}{4a}\left\langle \mathrm{tr}\left\{\chi_\mathrm{l}(x_0)^\dagger\left[\lambda_{\mathrm{l},i}^{-1} - \lambda_{\mathrm{l},i}\right]\gamma_i H_\mathrm{h}^{(0)}(x)\right\}\right\rangle_G \\
&= -\frac{i}{2a}\sum_i \sin(a\frac{\theta_{\mathrm{l},i}}{L})\left\langle \mathrm{tr}\left\{\chi_\mathrm{l}(x_0)^\dagger\gamma_i P_+\right\}\right\rangle_G .
\end{aligned} \tag{4.103}
$$

If I assume equal periodicity angles in all space directions $\theta_{\mathrm{h},i} = \theta_\mathrm{h}$ and $\theta_{\mathrm{l},i} = \theta_\mathrm{l}$, I obtain the following tree level formulation

$$
f_{\delta_\mathrm{h}\mathrm{A}}^{\mathrm{stat}\,(0)} = -\frac{\sin(a\frac{\theta_\mathrm{h}}{L})}{\sin(a\frac{\theta_\mathrm{l}}{L})} f_{\delta\mathrm{A}}^{\mathrm{stat}\,(0)} . \tag{4.104}
$$

Finally the tree level results for the $1/m$ correlation functions in the axial channel are

$$
\begin{aligned}
&f_{\mathrm{A}^{(1)}}^{\mathrm{stat}\,(0)}(x_0,\vec{\theta}_\mathrm{l},\vec{\theta}_\mathrm{h}) \\
&= \frac{i}{4a}\sum_i\left(\sin(a\frac{\theta_{\mathrm{h},i}}{L}) + \sin(a\frac{\theta_{\mathrm{l},i}}{L})\right)\left\langle \mathrm{tr}\left\{\chi_\mathrm{l}(x_0)^\dagger\gamma_i P_+\right\}\right\rangle_G ,
\end{aligned} \tag{4.105}
$$

and

$$
\begin{aligned}
&f_{\mathrm{A}^{(2)}}^{\mathrm{stat}\,(0)}(x_0,\vec{\theta}_\mathrm{l},\vec{\theta}_\mathrm{h}) \\
&= \frac{i}{4a}\sum_i\left(\sin(a\frac{\theta_{\mathrm{h},i}}{L}) - \sin(a\frac{\theta_{\mathrm{l},i}}{L})\right)\left\langle \mathrm{tr}\left\{\chi_\mathrm{l}(x_0)^\dagger\gamma_i P_+\right\}\right\rangle_G .
\end{aligned} \tag{4.106}
$$

5 Tree level matching

The matching of the QCD and HQET observables constructed from the axial and vector correlation functions is discussed at tree level. I show the behaviour of the parameters of the effective theory for the time and space components of the currents in $1/z$. The results indicate a trend of the $1/z^2$ corrections of the coefficients. In the case of a non-perturbative matching one defines the quantity $z = LM$, where M is the RGI quark mass and L is the extent of the matching volume. With this definition the scale of the bottom and charm quark mass can be specified. To set my values in correspondence to the simulations with the ALPHA collaboration[1] I choose for the matching volume $L_1 \approx 0.5$fm and the quenched RGI quark mass for the charm quark $M_{\mathrm{c}} = 1.60(2)$MeV [35] and for the bottom quark $M_{\mathrm{b}} = 6.758(86)$MeV [36]. That implies a bottom scale of $1/z_{\mathrm{b}} = 0.058(5)$ and a charm scale of $1/z_{\mathrm{c}} = 0.247(126)$.

To study the coefficients as a function of $1/z$ I determine equations for $z = \tilde{m}_{\mathrm{q}}L = 4, 8, 12, 16, 20, 24, 28, 32, 64$ and different θ combinations. In the following chapter I illustrate my calculations for a full set of HQET parameters at tree level. The charm and bottom scale are presented by grey dashed lines. In the discussion of the results in 5.4, a linear and quadratic fit of the determined tree level values are shown, to ascertain the behaviour of the coefficients in $O(1/z^2)$ and higher orders. The quadratic fit is performed with all values for $z = 4, \ldots, 64$. The linear fit of the results is performed with the values for $z = 16, 20, 24, 28, 32$ and 64. This region overlaps with the scale of the b-quark mass. Since the non-perturbative matching is performed in the region of the b-quark mass, a linear behaviour of the tree level results would indicate that higher order corrections in $1/z$ can be neglected.

5.1 Continuums extrapolation

To obtain the desired continuum limit of the observables I have to apply an extrapolation of the data to $a/L \to 0$. The continuum extrapolation is governed by a fit function, which produces the best developing of the numerical data. From [37] I infer the fit routine for my problem to be the *Linear Least Squares Function.* An optimal set of parameters $\{a_k\}$ is imposed and governs the fit function. The parameters are constituted by the *Merit function*

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - \sum_{k=1}^{M} a_k X_k(x_i)}{\sigma_i} \right)^2 . \tag{5.1}$$

[1] http://www-zeuthen.desy.de/alpha/

This function describes the difference of the data point y_i from the value of $f(x_i) = \sum_{k=1}^{M} a_k X_k(x_i)$, which are weighted with measurement errors σ_i. To obtain the optimal fitroutine in this case can be considered by minimizing (5.1). It is essential to evaluate the matrix equation

$$\sum_{j=1}^{M} \alpha_{kj} a_j = \beta_k \tag{5.2}$$

with

$$\alpha_{kj} = \sum_{i=1}^{N} \frac{X_j(x_i) X_k(x_i)}{\sigma_i^2}, \tag{5.3}$$

$$\beta_k = \sum_{i=1}^{N} \frac{y_i X_k(x_i)}{\sigma_i}. \tag{5.4}$$

For the continuum extrapolation of the observables I choose the fit function

$$f(a/L) = a_1(a/L)^4 + a_2(a/L)^3 + a_3(a/L)^2 + a_4 \tag{5.5}$$

containing no linear term due to $O(a)$ improvement of the observables.

The continuum limit of the observables can be computed by extrapolating data from $L/a = 84, \ldots, 256$, although for $z = 64$ only the finest resolutions are considered. Discretization effects occur with $z = 64$ and consequently a meaningful continuum limit extrapolation is not feasible. Thus one has to extrapolate the data from lattices so that $\chi^2/\text{dof} \approx 0.5$[2]. This restriction provides an acceptable continuum limit. If not otherwise specified I use the last 50 lattice points. The errors of the observables are determined by error propagation. I refer to appendix C for more details.

5.2 Axial Matching

5.2.1 ω_{kin}

I can extract ω_{kin} from the relation (3.16) up to $O(\frac{1}{m^2})$. I use the matching at $T = L/2$ and for isotropic and equal angles $\theta_{\text{h}} = \theta_{\text{l}} = \theta$ I obtain

$$\frac{R_1 - R_1^{\text{stat}}}{R_1^{\text{kin}}} = \omega_{\text{kin}} + O(\frac{1}{m^2}). \tag{5.6}$$

The observables are given as ratios of correlation functions. For a detailed definition of the observables I want to refer to 3.3. With the tree level relations (2.52) and (2.54) the observables yield

$$R_1 = \ln\left(\frac{f_1(\theta_1)}{f_1(\theta_2)}\right), \tag{5.7}$$

$$R_1^{\text{kin}} = \frac{6}{a^2}(T - a)\left(\cos(a\frac{\theta_1}{L}) - \cos(a\frac{\theta_2}{L})\right). \tag{5.8}$$

[2]dof means degrees of freedom

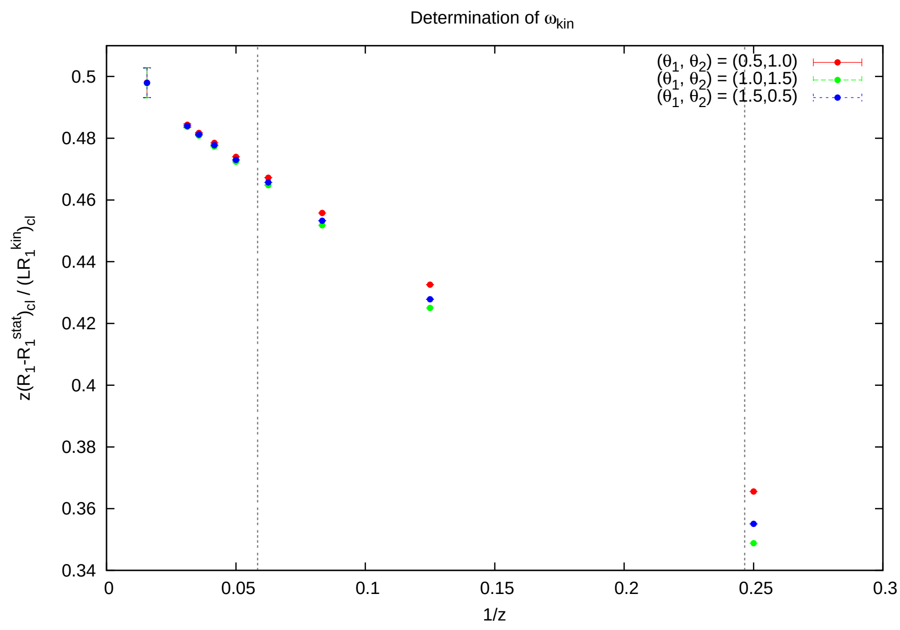


Figure 5.1: Tree level results for the determination of $\omega_{\rm kin}$ for different θ combinations.

In this case only R_1 depends on the heavy quark mass. The other observables are determined in the static approximation.
Regarding that the classical value for $\omega_{\rm kin}$ is $\frac{1}{2m_{\rm b}}$, I consider $\omega_{\rm kin} \cdot \tilde{m}_{\rm q}$, with $\tilde{m}_{\rm q} = z/L$

$$\frac{z(R_1 - R_1^{\rm stat})_{\rm cl}}{(LR_1^{\rm kin})_{\rm cl}} = \frac{1}{2} + O(\frac{1}{z}). \tag{5.9}$$

From this equation one expects that $\omega_{\rm kin} \cdot z/L$ is a linear function in $\frac{1}{z}$ with the approach to $\frac{1}{2}$ for $\frac{1}{z} \to 0$.
The continuum limit of $L \cdot R_1^{\rm kin}$ can be extracted analytically

$$\begin{aligned} L \cdot R_1^{\rm kin} &= L \cdot \frac{6}{a^2}\left(\frac{L}{2} - a\right)\left(\cos(a\frac{\theta_1}{L}) - \cos(a\frac{\theta_2}{L})\right) \\ &= \left(3\frac{L^2}{a^2} - 6\frac{L}{a}\right)\left(1 - \frac{1}{2}\frac{a^2\theta_1^2}{L^2} - 1 + \frac{1}{2}\frac{a^2\theta_2^2}{L^2}\right) \\ &= \left(\frac{3}{2} - 3\frac{a}{L}\right)\left(\theta_2^2 - \theta_1^2\right) \\ &\xrightarrow{a/L \to 0} \frac{3}{2}\left(\theta_2^2 - \theta_1^2\right). \end{aligned} \tag{5.10}$$

In figure (5.1) the results for $(\theta_1, \theta_2) = (0.5, 1.0), (1.0, 1.5)$ and $(1.5, 0.5)$ are shown.

5.2.2 $c_{\mathbf{A}}^{(1)}$

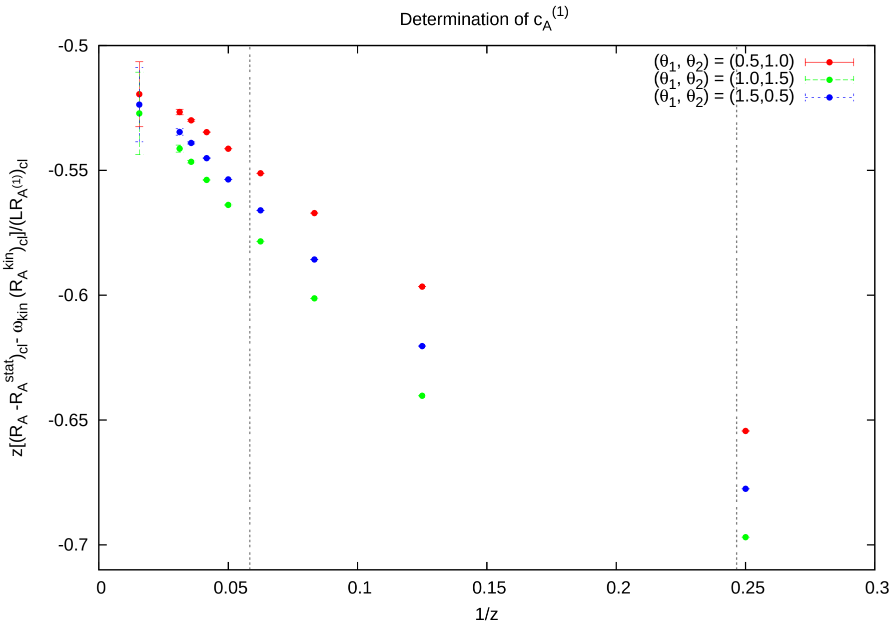


Figure 5.2: Tree level results for the determination of $c_{\mathrm{A}}^{(1)}$ for different θ combinations. I used the results from ω_{kin} for the associated θ combination.

I can extract $c_{\mathrm{A}}^{(1)}$ from the relation (3.17) up to $O(\frac{1}{m^2})$. If I demand zero momentum, I do not get sensitivity to $c_{\mathrm{A}}^{(2)}$, because the correlation function $f_{\mathrm{A}^{(2)}}^{\mathrm{stat}}$ is zero for $\vec{\theta}_{\mathrm{h}} = \vec{\theta}_{\mathrm{l}}$. I use the matching equation at $T = L$, $x_0 = T/2$ and isotropic angles. $c_{\mathrm{A}}^{(1)}$ can be determined from

$$\frac{R_{\mathrm{A}} - R_{\mathrm{A}}^{\mathrm{stat}} - \omega_{\mathrm{kin}} R_{\mathrm{A}}^{\mathrm{kin}}}{R_{\mathrm{A}^{(1)}}^{\mathrm{stat}}} = c_{\mathrm{A}}^{(1)}. \tag{5.11}$$

At tree level I obtain for the kinetic observable

$$R_{\mathrm{A}}^{\mathrm{kin}} = \frac{6}{a^2} x_0 \left(\cos(a\frac{\theta_1}{L}) - \cos(a\frac{\theta_2}{L}) \right). \tag{5.12}$$

The classical value of $c_{\mathrm{A}}^{(1)}$ is $-\frac{1}{2m_{\mathrm{b}}}$. To show the z dependence of $c_{\mathrm{A}}^{(1)}$ it is a possibility to plot $c_{\mathrm{A}}^{(1)} \cdot \tilde{m}_{\mathrm{q}}$. Multiplying (5.11) with $\tilde{m}_{\mathrm{q}} = z/L$ provides

$$\frac{z\left[\left(R_{\mathrm{A}} - R_{\mathrm{A}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}}\left(R_{\mathrm{A}}^{\mathrm{kin}}\right)_{\mathrm{cl}}\right]}{\left(LR_{\mathrm{A}^{(1)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = -\frac{1}{2} + O(1/z). \tag{5.13}$$

The continuum limit of $LR_{\mathrm{A}}^{\mathrm{kin}}$ can be again extracted analytically to $L \cdot R_{\mathrm{A}}^{\mathrm{kin}} \overset{a/L\to 0}{\longrightarrow} \frac{3}{2}\left(\theta_{2\mathrm{h}}^2 - \theta_{1\mathrm{h}}^2\right)$. The previously determined values of $\omega_{\mathrm{kin}} \cdot \tilde{m}_{\mathrm{q}}$ are included in the determination. In figure (5.2) the coefficient is plotted for the combinations $(\theta_1, \theta_2) = (0.5, 1.0), (1.0, 1.5)$ and $(1.5, 0.5)$.

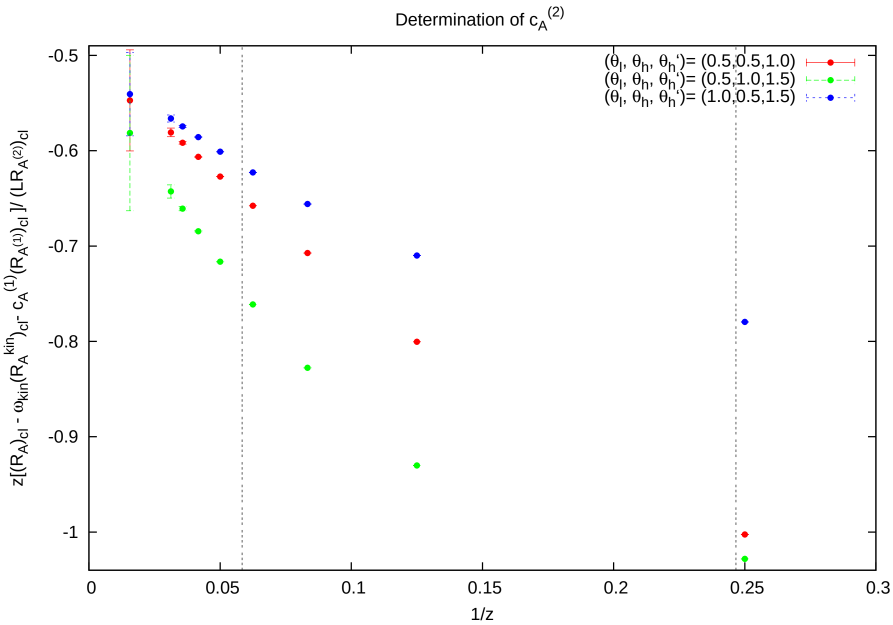


Figure 5.3: Tree level results for the determination of $c_{\mathrm{A}}^{(2)}$ for different θ combinations. I used the results for the coefficients ω_{kin} and $c_{\mathrm{A}}^{(1)}$ for the combination $(\theta_1, \theta_2) = (0.5, 1.0)$.

5.2.3 $c_{\mathrm{A}}^{(2)}$

The coefficient $c_{\mathrm{A}}^{(2)}$ can be extracted from eq. (3.19) with $x_0 = T/2$ and $T = L$. If one uses different periodicity angles for the light and the heavy quark one obtains sensitivity to $c_{\mathrm{A}}^{(2)}$

$$\frac{R_{\mathrm{A}} - \omega_{\mathrm{kin}} R_{\mathrm{A}}^{\mathrm{kin}} - c_{\mathrm{A}}^{(1)} R_{\mathrm{A}^{(1)}}^{\mathrm{stat}}}{R_{\mathrm{A}^{(2)}}^{\mathrm{stat}}} = c_{\mathrm{A}}^{(2)}. \tag{5.14}$$

I employ the observables for isotropic θ combinations $(\theta_1, \theta_{1\mathrm{h}}, \theta_{2\mathrm{h}}) = (0.5, 0.5, 1.0)$, $(0.5, 1.0, 1.5)$ and $(1.0, 0.5, 1.5)$, with the use of $\theta_{1\mathrm{l}} = \theta_{2\mathrm{l}} = \theta_{\mathrm{l}}$. With the previously determined values of $c_{\mathrm{A}}^{(1)}$ and ω_{kin} for $(\theta_1, \theta_2) = (0.5, 1.0)$ I can extract $c_{\mathrm{A}}^{(2)} \cdot \tilde{m}_{\mathrm{q}}$ by

$$\frac{z\left[(R_{\mathrm{A}})_{\mathrm{cl}} - \omega_{\mathrm{kin}}\left(R_{\mathrm{A}}^{\mathrm{kin}}\right)_{\mathrm{cl}} - c_{\mathrm{A}}^{(1)}\left(R_{\mathrm{A}^{(1)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}\right]}{\left(LR_{\mathrm{A}^{(2)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = -\frac{1}{2} + O(1/z). \tag{5.15}$$

In figure (5.3) the $1/z$ dependence of $c_{\mathrm{A}}^{(2)}$ is shown.

5.2.4 $\ln Z_{\mathrm{A}}^{\mathrm{HQET}}$ and $\ln Z_{\mathrm{A}}^{\mathrm{stat}}$

The renormalization coefficient $Z_{\mathrm{A}}^{\mathrm{HQET}}$ can be extracted from eq. (3.18). With $x_0 = T/2$ and $T = L$ and isotropic angles for $\vec{\theta}_{\mathrm{h}} = \vec{\theta}_{\mathrm{l}} = \vec{\theta}$ I obtain

$$\left(\zeta_{\mathrm{A}} - \zeta_{\mathrm{A}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - c_{\mathrm{A}}^{(1)} \left(\rho_{\mathrm{A}}^{(1)}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(\Psi_{\mathrm{A}}^{\mathrm{kin}}\right)_{\mathrm{cl}} = \ln Z_{\mathrm{A}}^{\mathrm{HQET}}. \tag{5.16}$$

At tree level the continuum limit of $L\Psi_{\mathrm{A}}^{\mathrm{kin}}$ is 0. With equal periodicity angles for the light and the heavy quark (5.16) has only sensitivity to the $1/m$ -coefficient $c_{\mathrm{A}}^{(1)}$. In figure (5.4) the coefficient $\ln Z_{\mathrm{A}}^{\mathrm{HQET}}$ is presented for $\theta = 0.5, 1.0$ and 1.5. The static quantity $\ln Z_{\mathrm{A}}^{\mathrm{stat}}$ can be obtained by dropping the term proportional to $c_{\mathrm{A}}^{(1)}$ in (5.16). The dependence of $1/z$ is shown in figure (5.4).

5.2.5 $c_{\mathrm{A}}^{(3)}$

The coefficient can be extracted from equation (3.31) by

$$\frac{\left(R_{\mathrm{AV}_{21}} - R_{\mathrm{AV}_{21}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(R_{\mathrm{AV}_{21}}^{\mathrm{kin}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{AV}_{21}^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = c_{\mathrm{A}}^{(3)}. \tag{5.17}$$

I use equal periodicity angles for the light and the heavy quark, $\vec{\theta}_{1\mathrm{l}} = \vec{\theta}_{1\mathrm{h}} = \vec{\theta}_1$ and $\vec{\theta}_{2\mathrm{l}} = \vec{\theta}_{2\mathrm{h}} = \vec{\theta}_2$. At tree level the axial correlation function $f_{\mathrm{AV}_{21}}$ has no sensitivity to $c_{\mathrm{A}}^{(4)}$. The continuum limit of $LR_{\mathrm{AV}_{21}}^{\mathrm{kin}}$ can be determined analytically to

$$LR_{\mathrm{AV}_{21}}^{\mathrm{kin}} \overset{a/L \to 0}{\longrightarrow} \frac{3}{2} \left(\theta_2^2 - \theta_1^2\right). \tag{5.18}$$

I employ the results of ω_{kin} for $(\theta_1, \theta_2) = (0.5, 1.0)$ and determine the values of the coefficient $c_{\mathrm{A}}^{(3)} \cdot \tilde{m}_{\mathrm{q}}$ for $(\theta_1, \theta_2) = (0.5, 1.0), (1.0, 1.5)$ and $(1.5, 0.5)$. They are shown in figure (5.5).

5.2.6 $c_{\mathrm{A}}^{(4)}$

The coefficient can be extracted from equation (3.29) by

$$\frac{\left(R_{\mathrm{A}_k} - R_{\mathrm{A}_k}^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(R_{\mathrm{A}_k}^{\mathrm{kin}}\right)_{\mathrm{cl}} - c_{\mathrm{A}}^{(3)} \left(R_{\mathrm{A}_k^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{A}_k^{(4)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = c_{\mathrm{A}}^{(4)}. \tag{5.19}$$

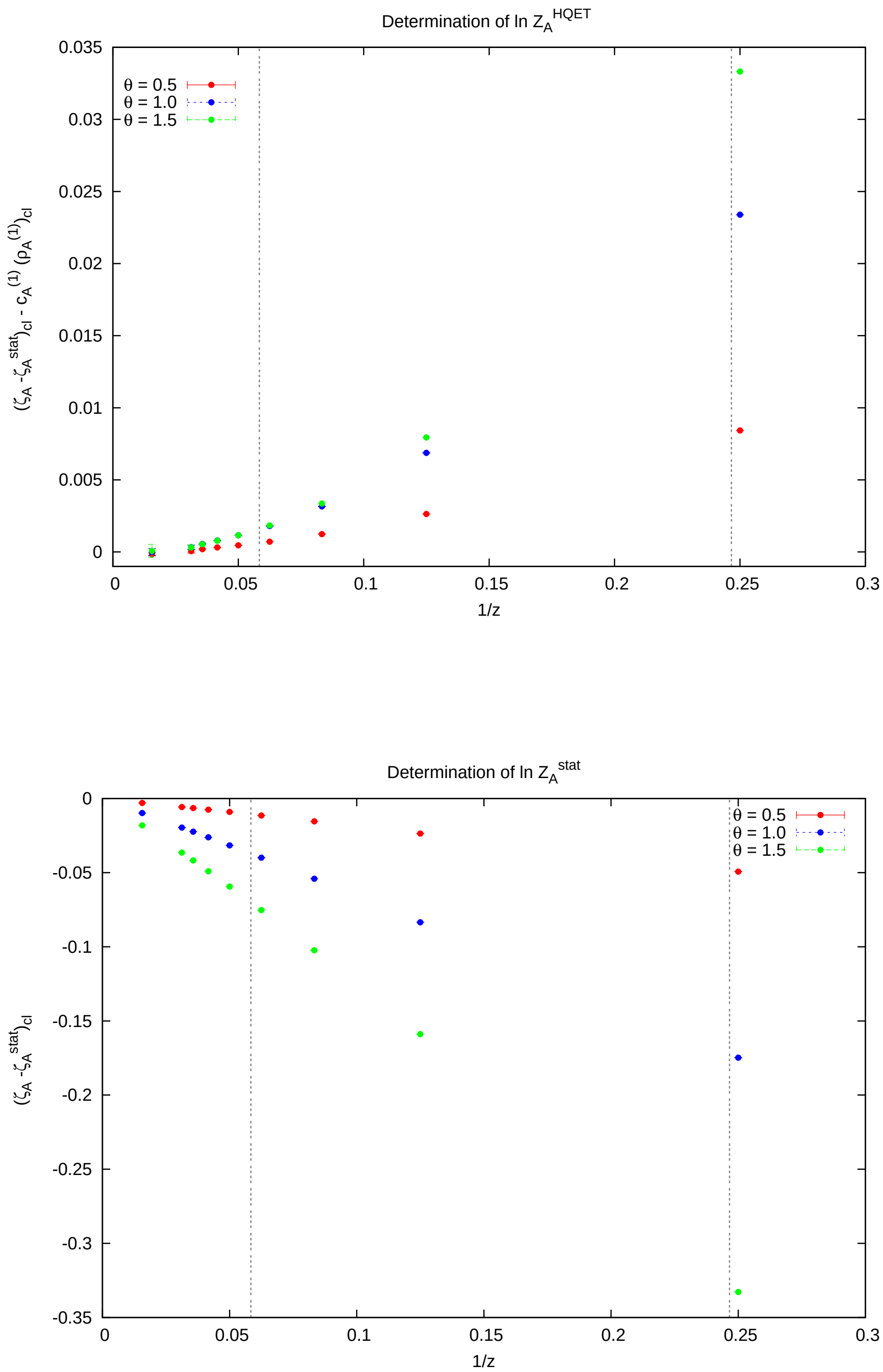


Figure 5.4: Tree level results of the HQET (top) and static (bottom) renormalization constant for different θ combinations. I used the results of the coefficient $c_{\mathrm{A}}^{(1)}$ for the combination $(\theta_1, \theta_2) = (0.5, 1.0)$.

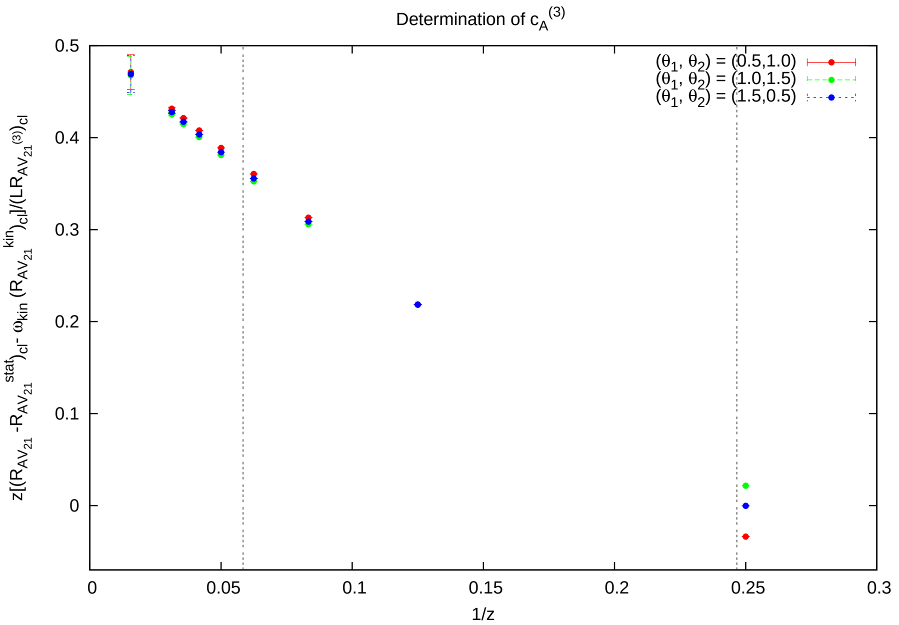


Figure 5.5: Tree level results for the determination of $c_{\mathrm{A}}^{(3)}$ for different θ combinations.

I employ equal periodicity angles for the light and the heavy quark, $\vec{\theta}_{1\mathrm{l}} = \vec{\theta}_{1\mathrm{h}} = \vec{\theta}_1$ and $\vec{\theta}_{2\mathrm{l}} = \vec{\theta}_{2\mathrm{h}} = \vec{\theta}_2$ and use isotropic angles. The continuum limit of $LR^{\mathrm{kin}}_{\mathrm{A}_k}$ at tree level is

$$LR^{\mathrm{kin}}_{\mathrm{A}_k} \overset{a/L\to 0}{\longrightarrow} \frac{3}{2}\left(\theta_2^2 - \theta_1^2\right). \tag{5.20}$$

I use the results of ω_{kin} and $c_{\mathrm{A}}^{(3)}$ for $(\theta_1, \theta_2) = (0.5, 1.0)$ and determine the values of the coefficient $c_{\mathrm{A}}^{(4)} \cdot \tilde{m}_{\mathrm{q}}$ for $(\theta_1, \theta_2) = (0.5, 1.0), (1.0, 1.5)$ and $(1.5, 0.5)$. In figure (5.6) the behaviour in $1/z$ is presented.

5.2.7 $c_{\mathrm{A}}^{(5)}$

The coefficient can be extracted from equation (3.31) by

$$\frac{(R_{\mathrm{AV}_{21}})_{\mathrm{cl}} - \omega_{\mathrm{kin}}\left(R^{\mathrm{kin}}_{\mathrm{AV}_{21}}\right)_{\mathrm{cl}} - c_{\mathrm{A}}^{(3)}\left(R^{\mathrm{stat}}_{\mathrm{AV}^{(3)}_{21}}\right)_{\mathrm{cl}}}{\left(R^{\mathrm{stat}}_{\mathrm{AV}^{(5)}_{21}}\right)_{\mathrm{cl}}} = c_{\mathrm{A}}^{(5)} \tag{5.21}$$

by using unequal periodicity angles for the light and the heavy quark, $\vec{\theta}_{1\mathrm{l}} \neq \vec{\theta}_{1\mathrm{h}}$ and $\vec{\theta}_{2\mathrm{l}} \neq \vec{\theta}_{2\mathrm{h}}$. Furthermore with the use of $\vec{\theta}_{1\mathrm{l}} = \vec{\theta}_{2\mathrm{l}} = \vec{\theta}_{\mathrm{l}}$ the static observable $R^{\mathrm{stat}}_{\mathrm{A_{V21}}}$ is zero. At tree level the correlation function $f_{\mathrm{AV}_{21}}$ has no sensitivity to the $O(1/m)$

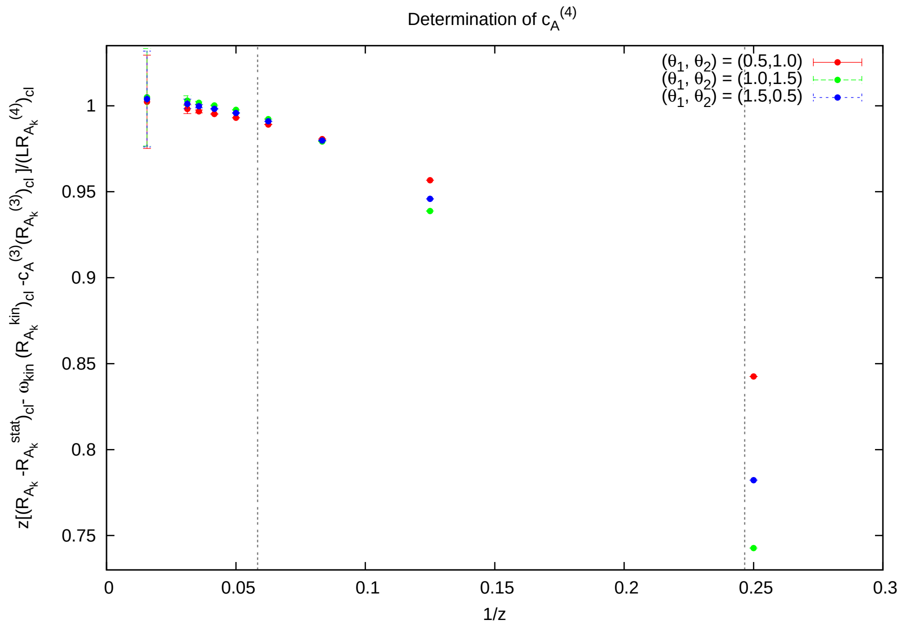


Figure 5.6: Tree level results for the determination of $c_{\mathrm{A}}^{(4)}$ for different θ combinations.

coefficients $c_{\mathrm{A}}^{(4)}$ and $c_{\mathrm{A}}^{(6)}$, thus I only obtain a dependence on $c_{\mathrm{A}}^{(3)}$ and $c_{\mathrm{A}}^{(5)}$. I determine the coefficient $c_{\mathrm{A}}^{(5)}$ for the combinations $(\theta_{\mathrm{l}}, \theta_{1\mathrm{h}}, \theta_{2\mathrm{h}}) = (0.5, 1.0, 1.5), (1.0, 0.5, 1.5)$ and $(1.5, 0.5, 1.0)$ by including the values of ω_{kin} and $c_{\mathrm{A}}^{(3)}$ for $(\theta_1, \theta_2) = (0.5, 1.0)$. In figure (5.7) the behaviour in $1/z$ is shown.

5.2.8 $c_{\mathbf{A}}^{(6)}$

The coefficient can be extracted from equation (3.30) by

$$c_{\mathrm{A}}^{(6)} = \frac{(R_{\mathrm{A}_k})_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(R_{\mathrm{A}_k}^{\mathrm{kin}}\right)_{\mathrm{cl}} - c_{\mathrm{A}}^{(3)} \left(R_{\mathrm{A}_k^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{A}_k^{(6)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} - \frac{c_{\mathrm{A}}^{(4)} \left(R_{\mathrm{A}_k^{(4)}}^{\mathrm{stat}}\right)_{\mathrm{cl}} + c_{\mathrm{A}}^{(5)} \left(R_{\mathrm{A}_k^{(5)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{A}_k^{(6)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} \tag{5.22}$$

by using unequal periodicity angles for the light and the heavy quark, $\vec{\theta}_{1\mathrm{l}} \neq \vec{\theta}_{1\mathrm{h}}$ and $\vec{\theta}_{2\mathrm{l}} \neq \vec{\theta}_{2\mathrm{h}}$. Furthermore with the use of $\vec{\theta}_{1\mathrm{l}} = \vec{\theta}_{2\mathrm{l}} = \vec{\theta}_{\mathrm{l}}$ the static observable

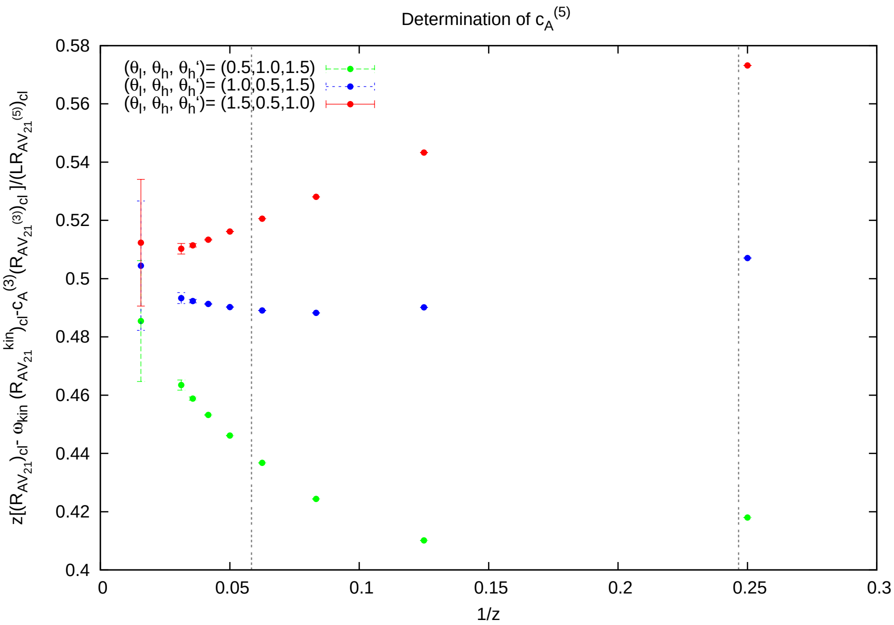


Figure 5.7: Tree level results for the determination of $c_{\mathrm{A}}^{(5)}$ for different θ combinations.

is again zero. I determine the coefficient $c_{\mathrm{A}}^{(6)}$ for the combinations $(\theta_{\mathrm{l}}, \theta_{1\mathrm{h}}, \theta_{2\mathrm{h}}) = (0.5, 1.0, 1.5), (1.0, 0.5, 1.5)$ and $(1.5, 0.5, 1.0)$. I employ the results of ω_{kin}, $c_{\mathrm{A}}^{(3)}$ and $c_{\mathrm{A}}^{(4)}$ for $(\theta_1, \theta_2) = (0.5, 1.0)$ and the results of $c_{\mathrm{A}}^{(5)}$ for $(\theta_{\mathrm{l}}, \theta_{1\mathrm{h}}, \theta_{2\mathrm{h}}) = (0.5, 1.0, 1.5)$. The $1/z$ dependence of $c_{\mathrm{A}}^{(6)}$ is shown in figure (5.8).

5.2.9 $\ln Z_{\mathbf{A}_k}^{\mathbf{HQET}}$ and $\ln Z_{\mathbf{A}_k}^{\mathbf{stat}}$

The renormalization coefficients can be extracted with $T = L$ and $x_0 = T/2$ from equation (3.28) by

$$\left(\zeta_{\mathrm{A}_k} - \zeta_{\mathrm{A}_k}^{\mathrm{stat}}\right)_{\mathrm{cl}} - c_{\mathrm{A}}^{(3)} \left(\rho_{\mathrm{A}_k^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - c_{\mathrm{A}}^{(4)} \left(\rho_{\mathrm{A}_k^{(4)}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(\Psi_{\mathrm{A}_k}^{\mathrm{kin}}\right)_{\mathrm{cl}} = \ln Z_{\mathrm{A}_k}^{\mathrm{HQET}} \qquad (5.23)$$

with the use of isotropic angles and $\vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{h}} = \vec{\theta}$. In the continuum limit the tree level result of $L\Psi_{\mathrm{A}}^{\mathrm{kin}}$ is zero, thus there is only sensitivity to the $O(1/m)$ coefficients $c_{\mathrm{A}}^{(3)}$ and $c_{\mathrm{A}}^{(4)}$. I employ the values of $c_{\mathrm{A}}^{(3)}$ and $c_{\mathrm{A}}^{(4)}$ from $(\theta_1, \theta_2) = (0.5, 1.0)$. The renormalization constants in HQET and in the static approximation are determined for the angles $\theta = 0.5, 1.0$ and 1.5 and shown in figure (5.9).

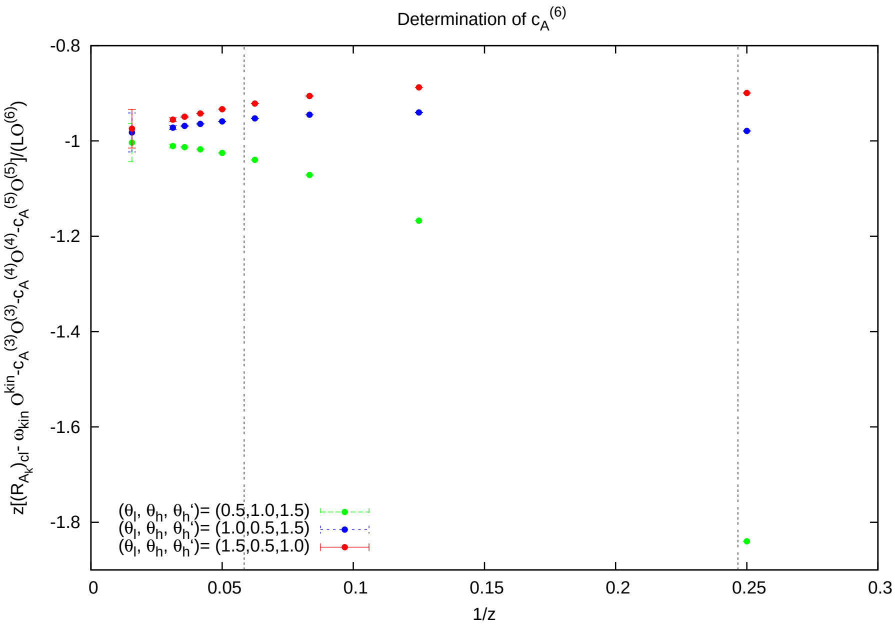


Figure 5.8: Tree level results for the determination of $c_A^{(6)}$ for different θ combinations. For a better readability are the observables from (5.22) replaced with an O.

5.3 Vector Matching

The determination of the coefficients obtained by the vector matching is analogous to the determination in the axial-vector channel.

5.3.1 $c_V^{(1)}$

The coefficient can be extracted from equation (3.20) by

$$\frac{\left(R_{\mathrm{V}_0} - R_{\mathrm{V}_0}^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(R_{\mathrm{V}_0}^{\mathrm{kin}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{V}_0^{(1)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = c_{\mathrm{V}}^{(1)} \tag{5.24}$$

using isotropic angles for the light and the heavy quark, $(\theta_1, \theta_2) = (0.5, 1.0)$, $(1.0, 1.5)$ and $(1.5, 0.5)$. The tree level continuum limit of $LR_{\mathrm{V}_0}^{\mathrm{kin}}$ can be obtained analytically and I employ the tree level results of ω_{kin} for $(\theta_1, \theta_2) = (0.5, 1.0)$. The $1/z$ dependence of $c_{\mathrm{V}}^{(1)}$ is shown in figure (5.10).

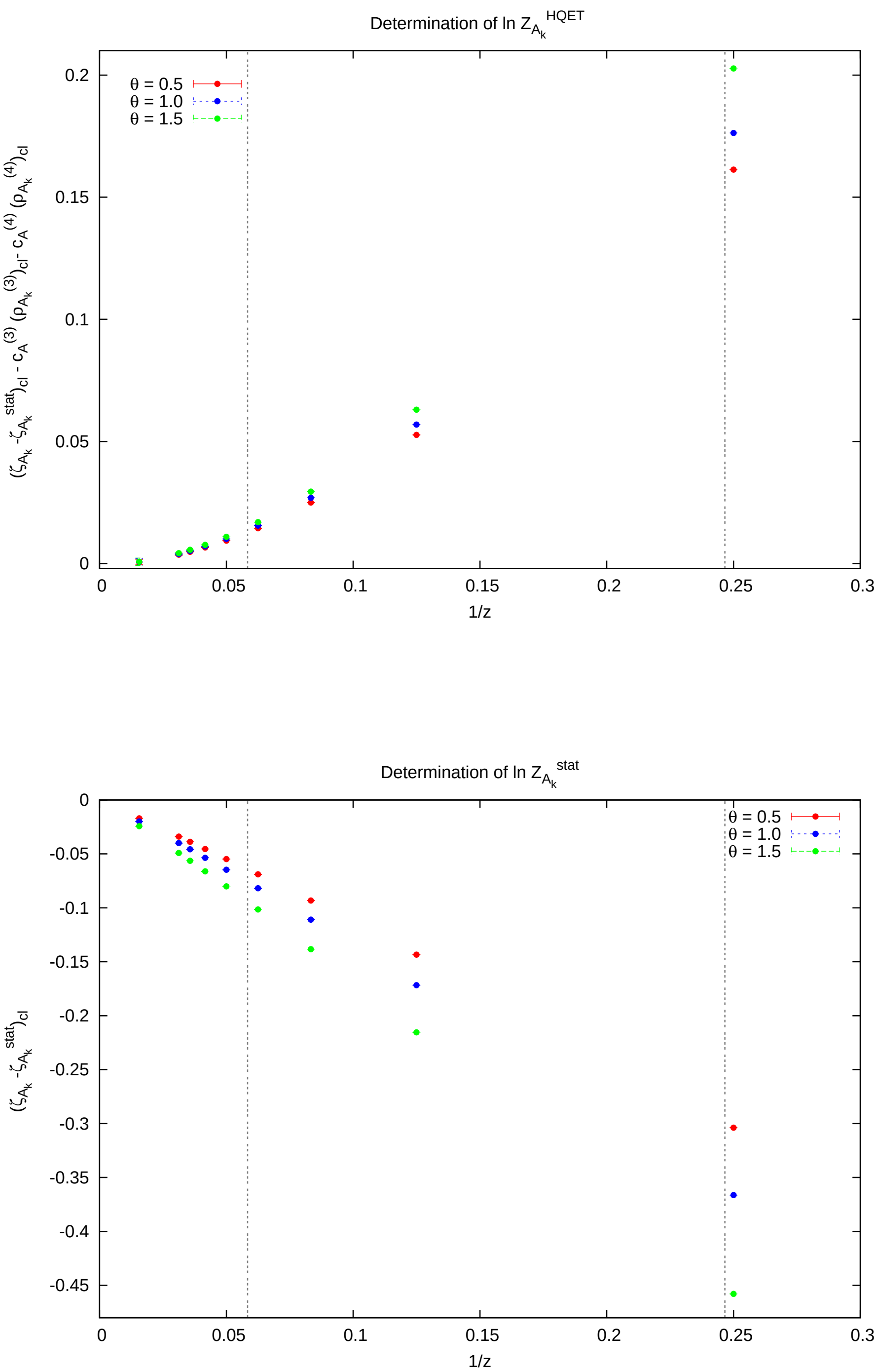


Figure 5.9: Tree level results of the HQET (top) and static (bottom) renormalization constant for different θ combinations.

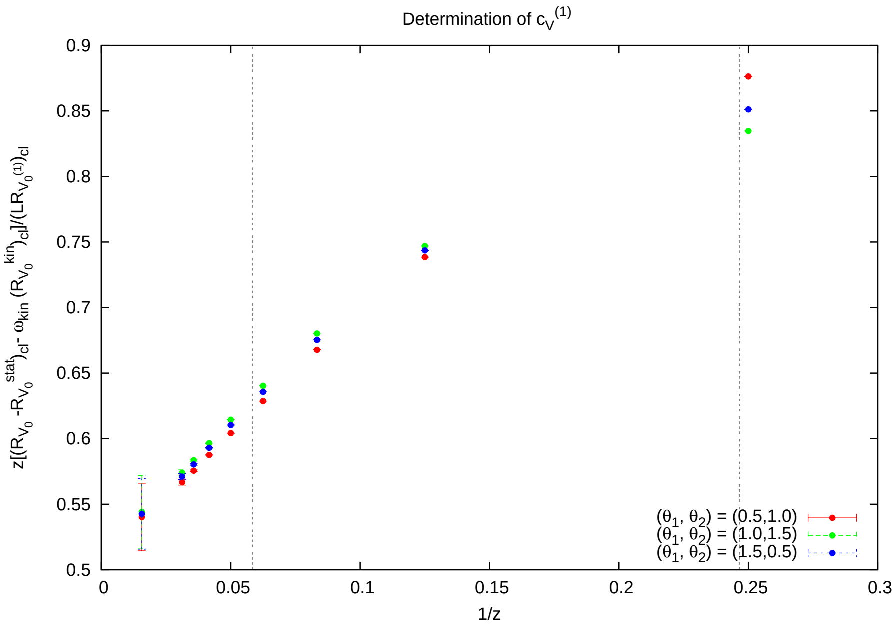


Figure 5.10: Tree level results for the determination of $c_{\mathrm{V}}^{(1)}$ for different θ combinations.

5.3.2 $c_{\mathbf{V}}^{(2)}$

The coefficient can be extracted from equation (3.22) by

$$\frac{(R_{\mathrm{V}_0})_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(R_{\mathrm{V}_0}^{\mathrm{kin}}\right)_{\mathrm{cl}} - c_{\mathrm{V}_0}^{(1)} \left(R_{\mathrm{V}_0^{(1)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{V}_0^{(2)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = c_{\mathrm{V}}^{(2)} \tag{5.25}$$

using isotropic but different angles for the light and the heavy quarks $(\theta_{\mathrm{l}}, \theta_{1\mathrm{h}}, \theta_{2\mathrm{h}}) = (0.5, 0.5, 1.0), (0.5, 1.0, 1.5)$ and $(1.0, 0.5, 1.5)$. Furthermore I demand $\theta_{1\mathrm{l}} = \theta_{2\mathrm{l}} = \theta_{\mathrm{l}}$ and obtain $R_{\mathrm{V}_0}^{\mathrm{stat}} = 0$. I employ the tree level results of ω_{kin} and $c_{\mathrm{V}}^{(1)}$ for $(\theta_1, \theta_2) = (0.5, 1.0)$. The $1/z$ dependence of $c_{\mathrm{V}}^{(2)}$ is shown in figure (5.11).

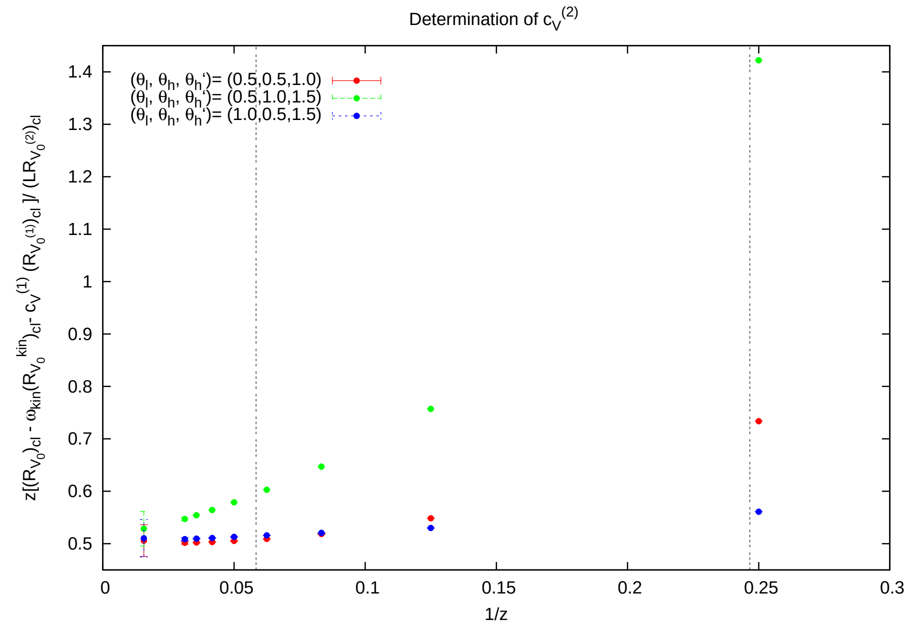


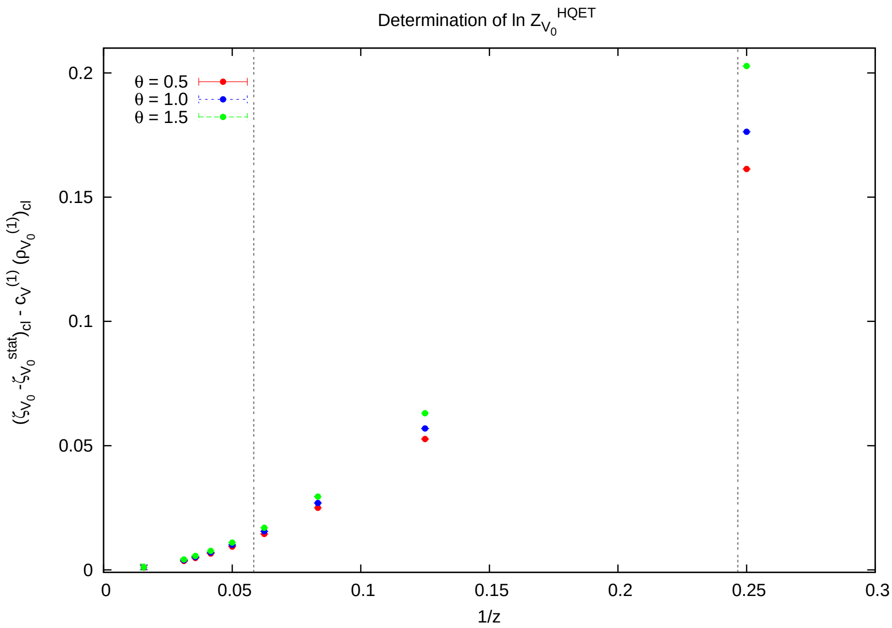
Figure 5.11: Tree-level results for the determination of $c_{\mathrm{V}}^{(2)}$ for different θ combinations.

5.3.3 $\ln Z_{\mathbf{V}_0}^{\mathbf{HQET}}$ and $\ln Z_{\mathbf{V}_0}^{\mathbf{stat}}$

The renormalization coefficients can be extracted with $T = L$ and $x_0 = T/2$ from equation (3.21) by

$$\left(\zeta_{\mathrm{V}_0} - \zeta_{\mathrm{V}_0}^{\mathrm{stat}}\right)_{\mathrm{cl}} - c_{\mathrm{V}_0}^{(1)} \left(\rho_{\mathrm{V}_0^{(1)}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(\Psi_{\mathrm{V}_0}^{\mathrm{kin}}\right)_{\mathrm{cl}} = \ln Z_{\mathrm{V}_0}^{\mathrm{HQET}} \tag{5.26}$$

with the use of isotropic angles and $\vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{h}}$. In the continuum limit the tree level result of $L\Psi_{\mathrm{V}_0}^{\mathrm{kin}}$ is zero, thus there is only sensitivity to the $O(1/m)$ coefficient $c_{\mathrm{V}}^{(1)}$. I employ the values of $c_{\mathrm{V}}^{(1)}$ from $(\theta_1, \theta_2) = (0.5, 1.0)$. The renormalization constants are determined for the angles $\theta = 0.5, 1.0$ and 1.5 and their behaviour in $1/z$ is shown in figure (5.12).



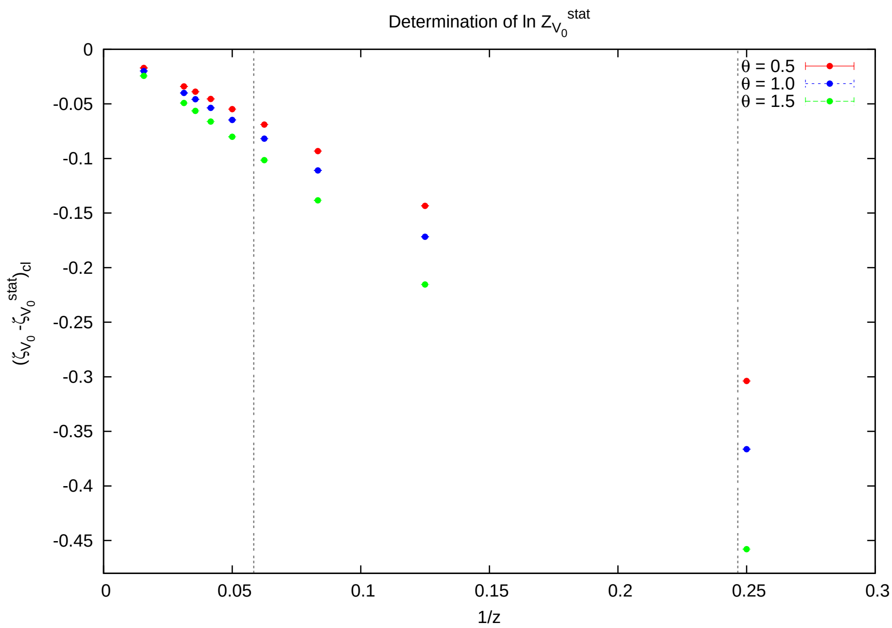


Figure 5.12: Tree level results of the HQET and static renormalization constant for different θ combinations. I used the results for the coefficient $c_A^{(1)}$ for the combination $(\theta_1, \theta_2) = (0.5, 1.0)$.

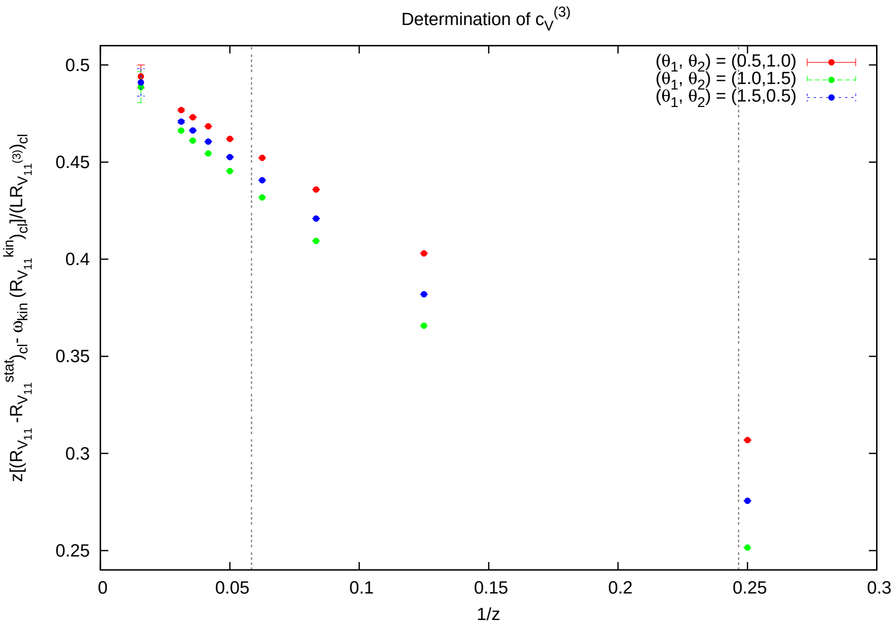


Figure 5.13: Tree level results for the determination of $c_{\mathrm{V}}^{(3)}$ for different θ combinations.

5.3.4 $c_{\mathbf{V}}^{(3)}$

The coefficient can be extracted from equation (3.26) by

$$\frac{\left(R_{\mathrm{V}_{11}} - R_{\mathrm{V}_{11}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}}\left(R_{\mathrm{V}_{11}}^{\mathrm{kin}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{V}_{11}^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = c_{\mathrm{V}}^{(3)}. \tag{5.27}$$

I use equal periodicity angles for the light and the heavy quark, $\vec{\theta}_{1\mathrm{l}} = \vec{\theta}_{1\mathrm{h}} = \vec{\theta}_1$ and $\vec{\theta}_{2\mathrm{l}} = \vec{\theta}_{2\mathrm{h}} = \vec{\theta}_2$, however to obatin no sensitivity to $c_{\mathrm{V}}^{(4)}$ I demand $\theta_{1,x} = \theta_{2,x} = 0$. The continuum limit of $LR_{\mathrm{V}_{11}}^{\mathrm{kin}}$ can be determined analytically to

$$LR_{\mathrm{V}_{11}}^{\mathrm{kin}} \overset{a/L\to 0}{\longrightarrow} \frac{1}{2}\left(\theta_{2,y}^2 + \theta_{2,z}^2 - \theta_{1,y}^2 - \theta_{1,z}^2\right). \tag{5.28}$$

I employ the results of ω_{kin} for $(\theta_1, \theta_2) = (0.5, 1.0)$ and determine the values of the coefficient $c_{\mathrm{V}}^{(3)} \cdot \tilde{m}_{\mathrm{q}}$ for $(\theta_1, \theta_2) = (0.5, 1.0), (1.0, 1.5)$ and $(1.5, 0.5)$. In this case θ_i, for $i = 1, 2$, only label the y and z component of the angles, with $\theta_{i,y} = \theta_{i,z} = \theta_i$. The results are shown in figure (5.13).

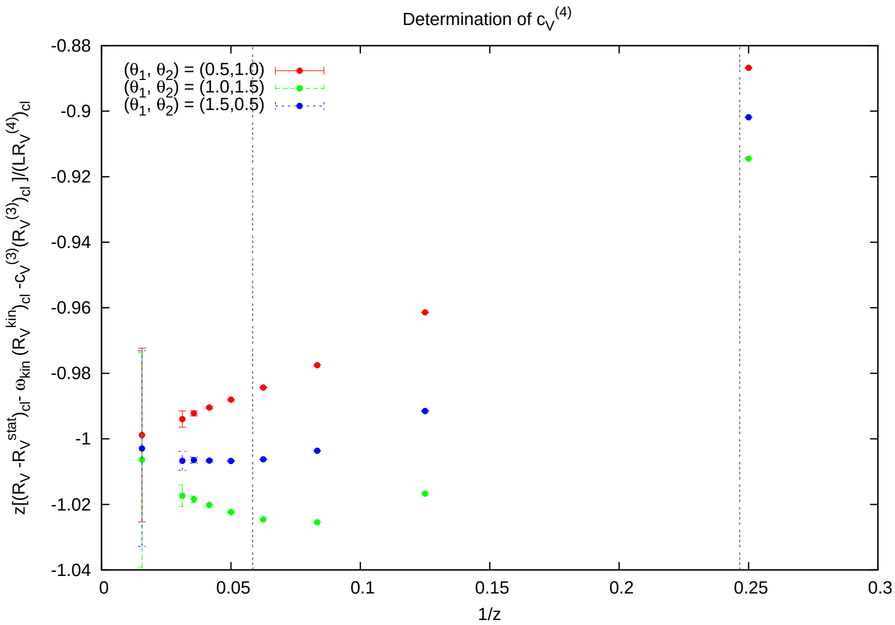


Figure 5.14: Tree level results for the determination of $c_V^{(4)}$ for different θ combinations.

5.3.5 $c_V^{(4)}$

The coefficient can be extracted from equation (3.24) by

$$\frac{\left(R_V - R_V^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}}\left(R_V^{\mathrm{kin}}\right)_{\mathrm{cl}} - c_V^{(3)}\left(R_{V^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}}{\left(R_{V^{(4)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = c_V^{(4)}. \tag{5.29}$$

I use equal periodicity angles for the light and the heavy quark, $\vec{\theta}_{1\mathrm{l}} = \vec{\theta}_{1\mathrm{h}} = \vec{\theta}_1$ and $\vec{\theta}_{2\mathrm{l}} = \vec{\theta}_{2\mathrm{h}} = \vec{\theta}_2$. In comparison to the determination of $c_V^{(3)}$ I demand $\theta_{i,x} \neq 0$ and I use isotropic angles. The continuum limit of LR_V^{kin} at tree level is $LR_V^{\mathrm{kin}} \stackrel{a/L\to 0}{\longrightarrow} \frac{3}{2}\left(\theta_2^2 - \theta_1^2\right)$. I use the results of ω_{kin} and $c_V^{(3)}$ for $(\theta_1, \theta_2) = (0.5, 1.0)$ and determine the values of the coefficients $c_V^{(4)} \cdot \tilde{m}_{\mathrm{q}}$ for $(\theta_1, \theta_2) = (0.5, 1.0), (1.0, 1.5)$ and $(1.5, 0.5)$. In figure (5.14) the tree level results for $c_V^{(4)}$ are shown for different θ combinations.

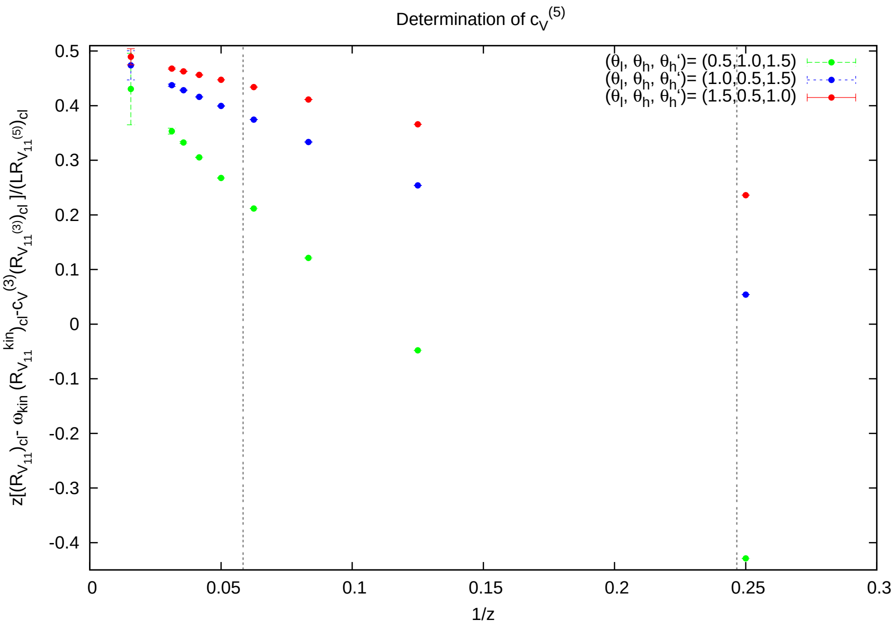


Figure 5.15: Tree level results for the determination of $c_{\mathrm{V}}^{(5)}$ for different θ combinations.

5.3.6 $c_{\mathrm{V}}^{(5)}$

The coefficient can be extracted from equation (3.27) by

$$\frac{(R_{\mathrm{V}_{11}})_{\mathrm{cl}} - \omega_{\mathrm{kin}}\left(R_{\mathrm{V}_{11}}^{\mathrm{kin}}\right)_{\mathrm{cl}} - c_{\mathrm{V}}^{(3)}\left(R_{\mathrm{V}_{11}^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{V}_{11}^{(5)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} = c_{\mathrm{V}}^{(5)} \tag{5.30}$$

by using unequal periodicity angles for the light and the heavy quark, $\vec{\theta}_{1\mathrm{l}} \neq \vec{\theta}_{1\mathrm{h}}$ and $\vec{\theta}_{2\mathrm{l}} \neq \vec{\theta}_{2\mathrm{h}}$. Furthermore with the use of $\vec{\theta}_{1\mathrm{l}} = \vec{\theta}_{2\mathrm{l}} = \vec{\theta}_{\mathrm{l}}$ the static observable $R_{\mathrm{V}_{11}}^{\mathrm{stat}}$ is zero. Regarding to the determination of $c_{\mathrm{V}}^{(3)}$, I demand the x components of the angles to be zero, $\theta_{\mathrm{l},x} = \theta_{1\mathrm{h},x} = \theta_{2\mathrm{h},x} = 0$, to cancel the terms proportional to $c_{\mathrm{V}}^{(4)}$. I determine the coefficient $c_{\mathrm{V}}^{(5)}$ for the combinations $(\theta_{\mathrm{l}}, \theta_{1\mathrm{h}}, \theta_{2\mathrm{h}}) = (0.5, 1.0, 1.5), (1.0, 0.5, 1.5)$ and $(1.5, 0.5, 1.0)$. In this notation the y and z component of the appropriate angles are equal. The tree level expression of $\left(LR_{\mathrm{V}_{11}}^{\mathrm{kin}}\right)_{\mathrm{cl}}$ is $\frac{1}{2}\left(\theta_{2\mathrm{h},y}^2 + \theta_{2\mathrm{h},z}^2 - \theta_{1\mathrm{h},y}^2 - \theta_{1\mathrm{h},z}^2\right)$. I employ the results of ω_{kin} and $c_{\mathrm{V}}^{(3)}$ for $(\theta_1, \theta_2) = (0.5, 1.0)$. The behaviour of $c_{\mathrm{V}}^{(5)}$ in $1/z$ is shown in figure (5.15).

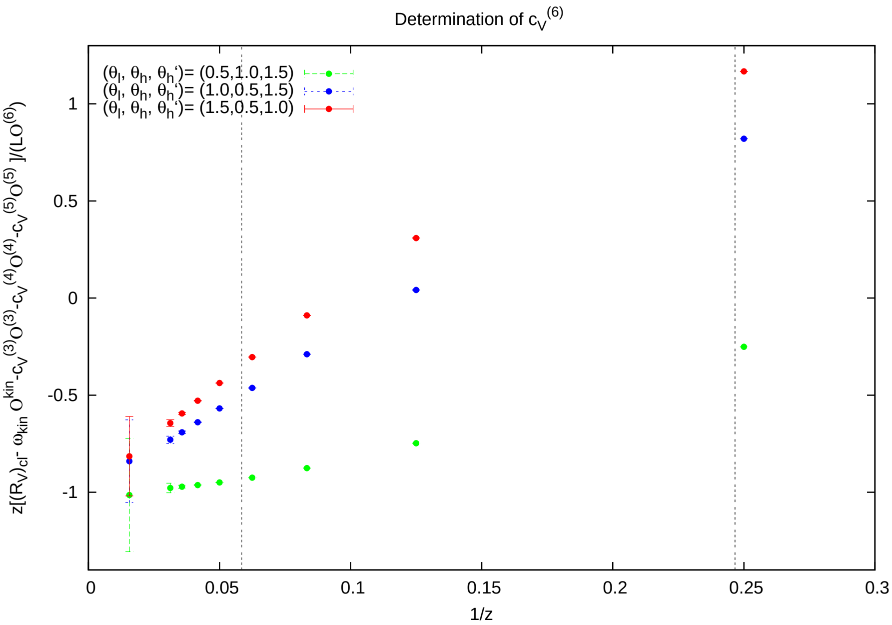


Figure 5.16: Tree level results for the determination of $c_{\mathrm{V}}^{(6)}$ for different θ combinations.

5.3.7 $c_{\mathbf{V}}^{(6)}$

The coefficient can be extracted from equation (3.25) by

$$c_{\mathrm{V}}^{(6)} = \frac{(R_{\mathrm{V}})_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(R_{\mathrm{V}}^{\mathrm{kin}}\right)_{\mathrm{cl}} - c_{\mathrm{V}}^{(3)} \left(R_{\mathrm{V}^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{V}^{(6)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} - \frac{c_{\mathrm{V}}^{(4)} \left(R_{\mathrm{V}^{(4)}}^{\mathrm{stat}}\right)_{\mathrm{cl}} + c_{\mathrm{V}}^{(5)} \left(R_{\mathrm{V}^{(5)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}}{\left(R_{\mathrm{V}^{(6)}}^{\mathrm{stat}}\right)_{\mathrm{cl}}} \tag{5.31}$$

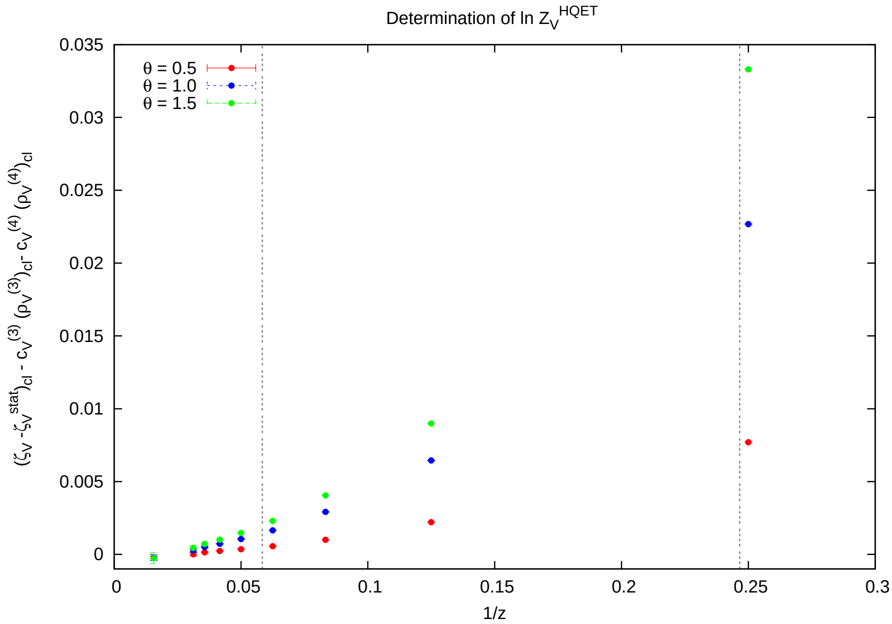
by using unequal periodicity angles for the light and the heavy quark, $\vec{\theta}_{1\mathrm{l}} \neq \vec{\theta}_{1\mathrm{h}}$ and $\vec{\theta}_{2\mathrm{l}} \neq \vec{\theta}_{2\mathrm{h}}$. Furthermore with the use of $\vec{\theta}_{1\mathrm{l}} = \vec{\theta}_{2\mathrm{l}} = \vec{\theta}_{\mathrm{l}}$ the static observable $R_{\mathrm{V}}^{\mathrm{stat}}$ is zero. I determine the coefficient $c_{\mathrm{V}}^{(6)}$ for the combinations $(\theta_{\mathrm{l}}, \theta_{1\mathrm{h}}, \theta_{2\mathrm{h}}) = (0.5, 1.0, 1.5), (1.0, 0.5, 1.5)$ and $(1.5, 0.5, 1.0)$. The tree level expression of $\left(LR_{\mathrm{V}}^{\mathrm{kin}}\right)_{\mathrm{cl}}$ is $\frac{3}{2}\left(\theta_{2\mathrm{h}}^2 - \theta_{1\mathrm{h}}^2\right)$. I employ the results of ω_{kin}, $c_{\mathrm{V}}^{(3)}$ and $c_{\mathrm{V}}^{(4)}$ for $(\theta_1, \theta_2) = (0.5, 1.0)$ and the results of $c_{\mathrm{V}}^{(5)}$ for $(\theta_{\mathrm{l}}, \theta_{1\mathrm{h}}, \theta_{2\mathrm{h}}) = (0.5, 1.0, 1.5)$. In figure (5.16) the $1/z$ dependence of $c_{\mathrm{V}}^{(6)}$ is shown.

5.3.8 $\ln Z_{\mathrm{V}}^{\mathrm{HQET}}$ and $\ln Z_{\mathrm{V}}^{\mathrm{stat}}$

The renormalization coefficients can be extracted with $T = L$ and $x_0 = T/2$ from equation (3.23) by

$$\left(\zeta_{\mathrm{V}} - \zeta_{\mathrm{V}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - c_{\mathrm{V}}^{(3)} \left(\rho_{\mathrm{V}^{(3)}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - c_{\mathrm{V}}^{(4)} \left(\rho_{\mathrm{V}^{(4)}}^{\mathrm{stat}}\right)_{\mathrm{cl}} - \omega_{\mathrm{kin}} \left(\Psi_{\mathrm{V}}^{\mathrm{kin}}\right)_{\mathrm{cl}} = \ln Z_{\mathrm{V}}^{\mathrm{HQET}} \tag{5.32}$$

with the use of isotropic angles and $\vec{\theta}_{\mathrm{l}} = \vec{\theta}_{\mathrm{h}}$. In the continuum limit the tree level result of $L\Psi_{\mathrm{V}}^{\mathrm{kin}}$ is zero, thus there is only sensitivity to the $1/m$-coefficients $c_{\mathrm{V}}^{(3)}$ and $c_{\mathrm{V}}^{(4)}$. I employ the values of $c_{\mathrm{V}}^{(3)}$ and $c_{\mathrm{V}}^{(4)}$ from $(\theta_1, \theta_2) = (0.5, 1.0)$. The renormalization constants in HQET and in the static approximation are determined for the angles $\theta = 0.5, 1.0$ and 1.5. and shown in figure (5.17).



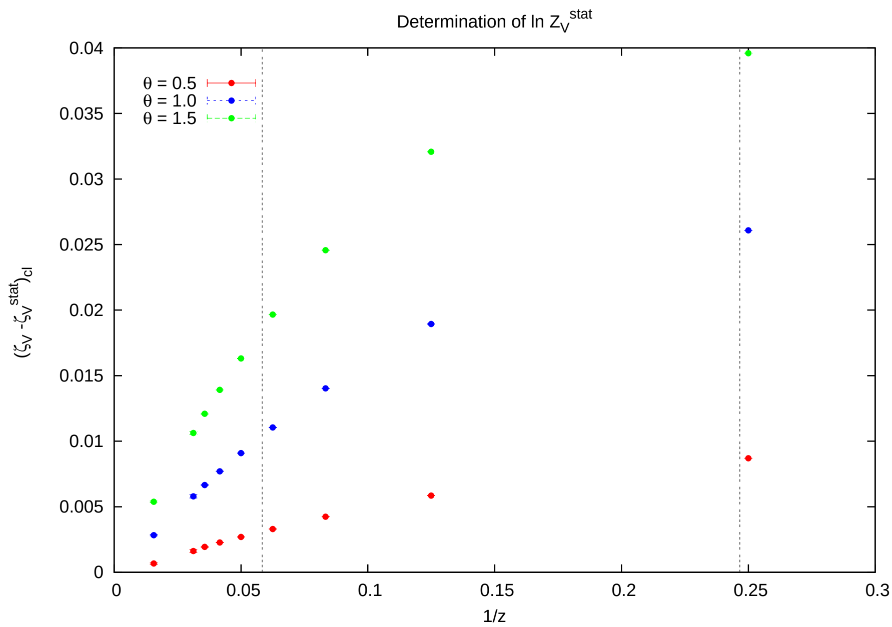


Figure 5.17: Tree level results of the HQET, $\ln Z_{\mathrm{V}}^{\mathrm{HQET}}$, and static renormalization constant, $\ln Z_{\mathrm{V}}^{\mathrm{stat}}$, for different θ combinations.

5.4 Discussion of the tree level HQET parameters

One can see from figures (5.1) to (5.17), that the coefficients arrive at the classical continuum values for $1/z \to 0$. The classical values are displayed in table (3.1).

For a discussion of the behaviour of the coefficients in the region of the heavy quark mass, I include a linear and quadratic fit of the coefficients in the plots, see figures (5.18) to (5.30). For large z the quark mass lattice corrections increase and the discretization effects implicate large errorbars, i.e. for $z = 64$. The tree level result tables are presented in appendix C.

For a discussion of the effective higher order coefficients I include plots, which show the behaviour of $z \times (\text{coeff.} - \text{classical value})$ in $1/z$. To obtain a feasible linear fit through the points in the region of small $1/z$, I exclude the points for $z = 4$ and $z = 8$. For small $1/z$ the higher order corrections are expected to be small and there the fits must be asymptotically more and more sensitive to the leading order correction. The effective coefficients are shown in figures (5.18) to (5.30).

At first sight the coefficients show a linear behaviour in the region of the b-quark mass. This is also confirmed by the linear fits through the data, which show that a good linear approximation can be achieved. Although for $c_{\mathrm{A}}^{(4)}$, $c_{\mathrm{A}}^{(5)}$ and $c_{\mathrm{V}}^{(4)}$ in figures (5.22), (5.23) and (5.28) small deviations from the linear fits to the data can be seen, especially for the green values, which correspond to the θ combination concerning the largest heavy quark angles. The quadratic fits show a good approximation of the data in the whole region for all coefficients. Besides in (5.24) and (5.26) the quadratic fits of the coefficients $c_{\mathrm{A}}^{(6)}$ and $c_{\mathrm{V}}^{(2)}$ corresponding to the combination $(0.5, 1.0, 1.5)$ show deviations to the data in the region of the b-quark mass. Here the linear fits show a more accurate behaviour of the points.

The slopes of the fit functions show a influence of the heavy quark angles. The values of the slopes increases with increasing θ_{h}. In figures (5.18), (5.19), (5.21) and (5.25), (5.27) the slopes increase from $(0.5, 1.0)$ over $(1.5, 0.5)$ to $(1.0, 1.5)$. In the case of $c_{\mathrm{A}}^{(4)}$ the slopes of the linear fits of the different θ combinations do not show the same behaviour. Here the data points with the most sensitivity to the angles are from $(0.5, 1.0)$. Concerning the coefficient $c_{\mathrm{A}}^{(2)}$ in figure (5.20) the value of the slopes increase with $(1.0, 0.5, 1.5)$ over $(0.5, 0.5, 1.0)$ to $(0.5, 1.0, 1.5)$, where the last two numbers give the components of two different heavy quark angles. In this case the conclusion, that the slope for the largest heavy quark angles shows the most sensitivity, is verified, as in the cases of $c_{\mathrm{A}}^{(5)}$, $c_{\mathrm{V}}^{(3)}$ and $c_{\mathrm{V}}^{(5)}$. This is not the case for the HQET parameter $c_{\mathrm{V}}^{(6)}$ in (5.30), where the θ combination $(1.5, 0.5, 1.0)$ has the largest slope. In figure (5.23) one can see a change in the signs of the slopes for $c_{\mathrm{A}}^{(5)}$. For the red points $(1.5, 0.5, 1.0)$ one has a positive slope and the behaviour of the data is nearly linear in $1/z$, where as the quadratic or higher order terms occur with higher heavy quark angles. The slopes of the fit functions for the combinations $(1.0, 0.5, 1.5)$ and $(0.5, 1.0, 1.5)$ are negative and the largest slope is the one with the largest heavy quark angles. A similar behaviour shows the coefficient $c_{\mathrm{V}}^{(4)}$ in figure (5.28).

In the figures (5.18) to (5.30) the effective $O(1/z^2)$ coefficients can be estimated from the y-intercept of the linear fits in the lower graphs. With an accurate linear fit of the data points for large z one obtains an indication of the leading order corrections. In all figures a feasible fit through the data points for small $1/z$ is possible, whereas the points for $z = 64$ do not contribute very much due of the large errorsbars. To get an idea of the magnitude of the leading order corrections, the values of the y-intercept are tabulated in (5.1) and (5.2).

The behaviour of the axial and vector renormalization constants at tree level is illustrated in (5.31) to (5.34). The HQET quantities, which are shown at the top of the figures show a quadratic behaviour in $1/z$. In the range of $[-0.0003(4), 0.0333164(1)]$ the behaviours of the values of $\ln Z_{\mathrm{A}}^{\mathrm{HQET}}$ and $\ln Z_{\mathrm{V}}^{\mathrm{HQET}}$ are very similar. They have a higher sensitivity to the angles, where as the quantities $\ln Z_{\mathrm{A}_k}^{\mathrm{HQET}}$ and $\ln Z_{\mathrm{V}_0}^{\mathrm{HQET}}$ show more sensitivity to $1/z$. The behaviour of $\ln Z_{\mathrm{A}_k}^{\mathrm{HQET}}$ and $\ln Z_{\mathrm{V}_0}^{\mathrm{HQET}}$ are almost identically and in the range of $[0.001(1), 0.2027507(4)]$ they are of the same order. For the static quantities $\ln Z_{\mathrm{A}}^{\mathrm{stat}}$, $\ln Z_{\mathrm{A}_k}^{\mathrm{stat}}$ and $\ln Z_{\mathrm{V}_0}^{\mathrm{stat}}$ the quadratic fits coincide with the values. The linear fits in the region of the bottom quark mass describes the behaviour of the data accurately. This is not the case in figure (5.34) for $\ln Z_{\mathrm{V}}^{\mathrm{stat}}$. The $1/z^2$ effects have a great impact on the behaviour, especially for $\theta = 1.5$, however a linear approximation in the region of the bottom quark mass is possible. Just as concluded in the latter discussion of the HQET coefficients, the renormalization constants show an increasing sensitivity to $1/z$ when the angles are increased. When comparing the HQET coefficients to HQET renormalization constants one can see, that the coefficients are nearly linear in $1/z$ in the b-quark mass region, whereas the constants show a rather quadratically behaviour. The $1/z$ corrections of the HQET coefficients are small compared to the $1/z$ corrections of the constants. This can be explained by the fact that the coefficients are of $O(1/z)$ at leading order themselves, so that their corrections are of absolute order $O(1/z^2)$ only, whereas the HQET renormalization constants receive $O(1/z)$ corrections to their leading order behaviour.

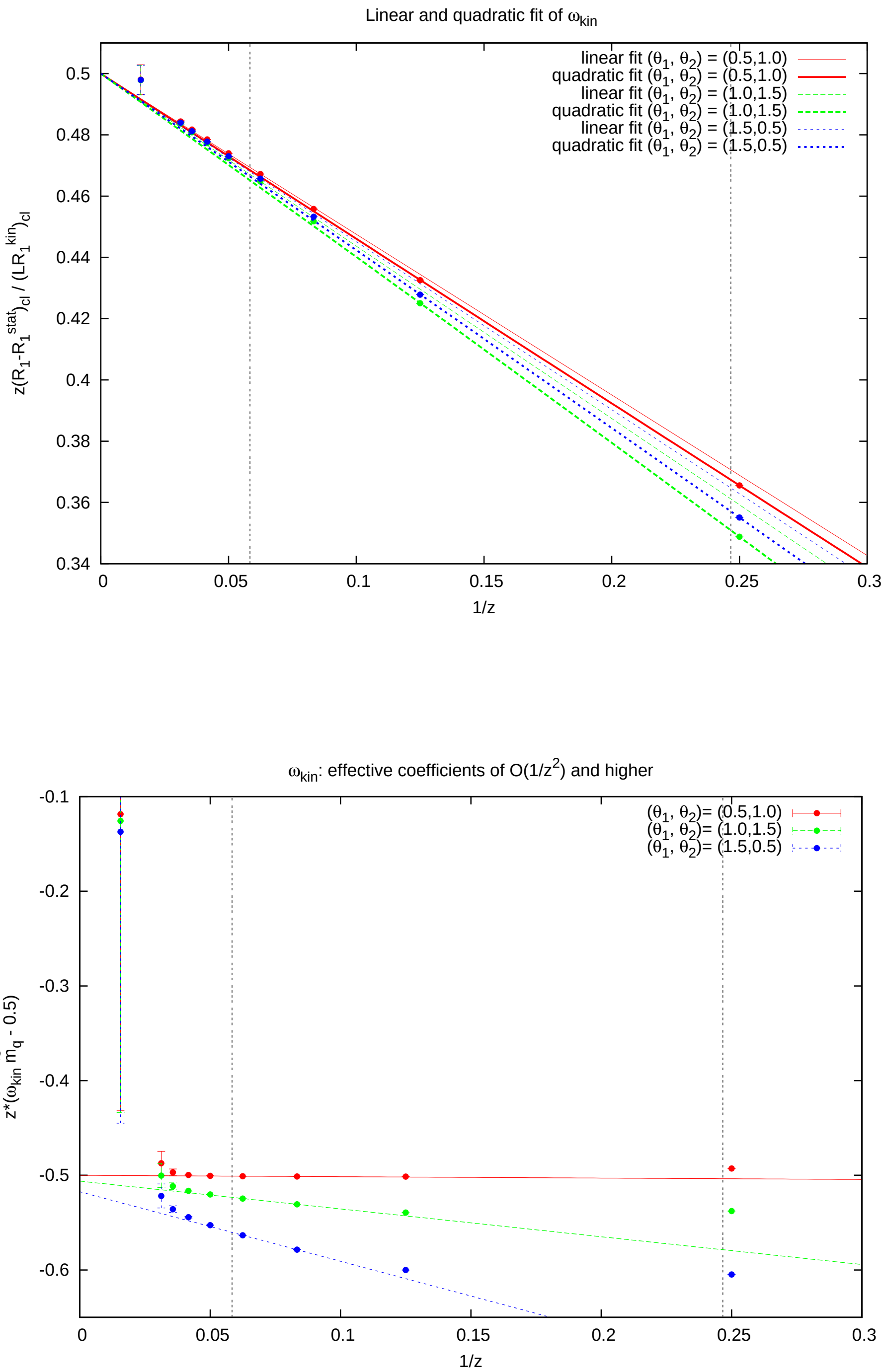
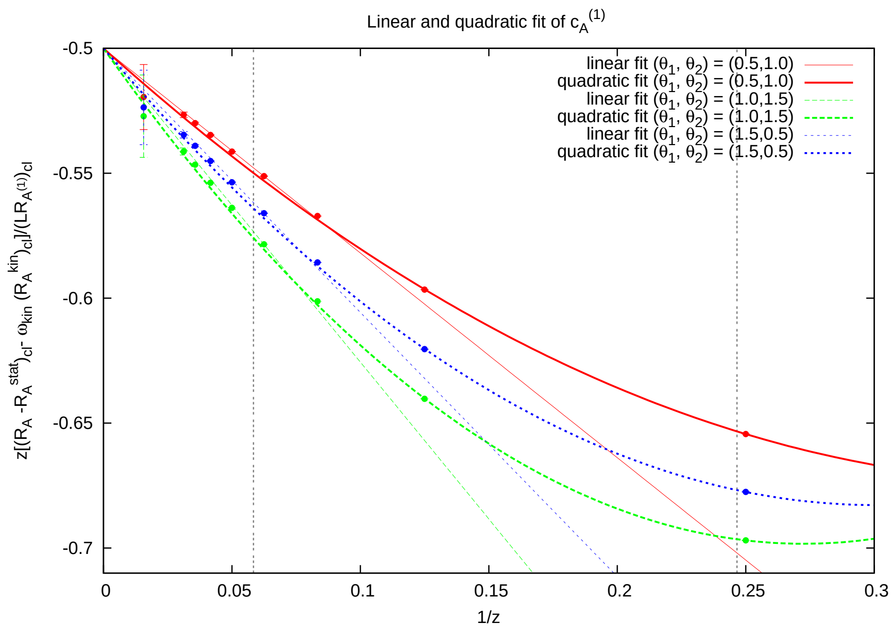


Figure 5.18: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.



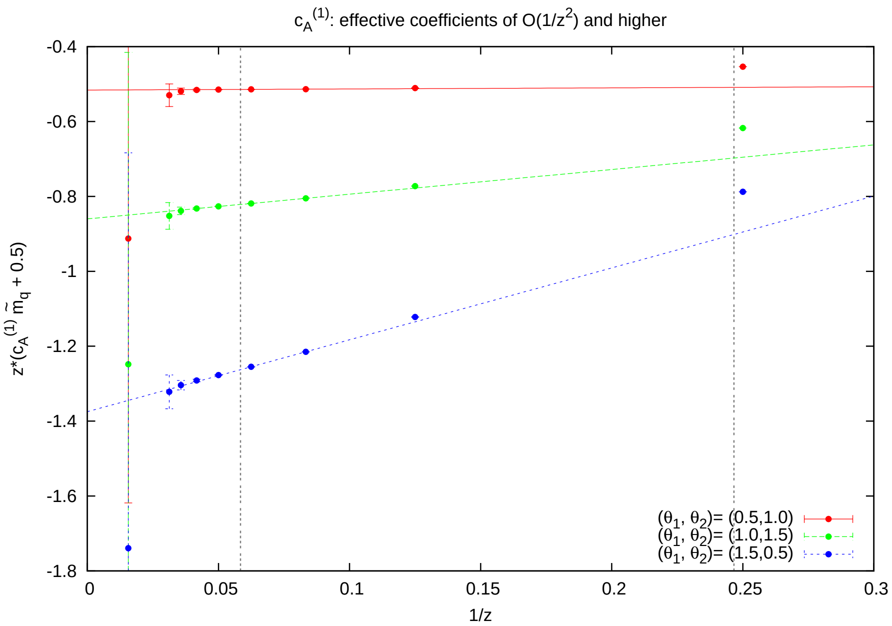


Figure 5.19: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.

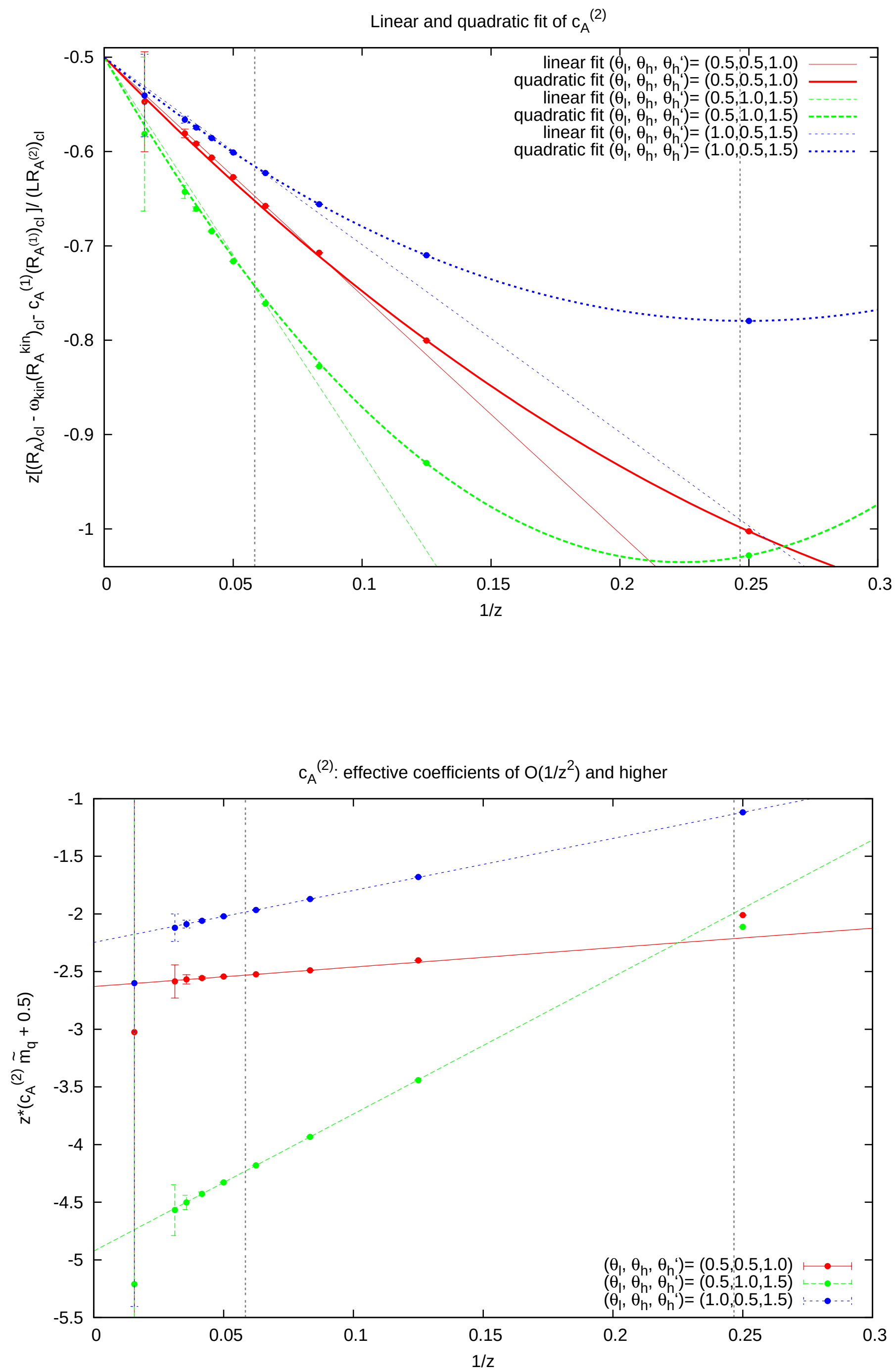


Figure 5.20: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.

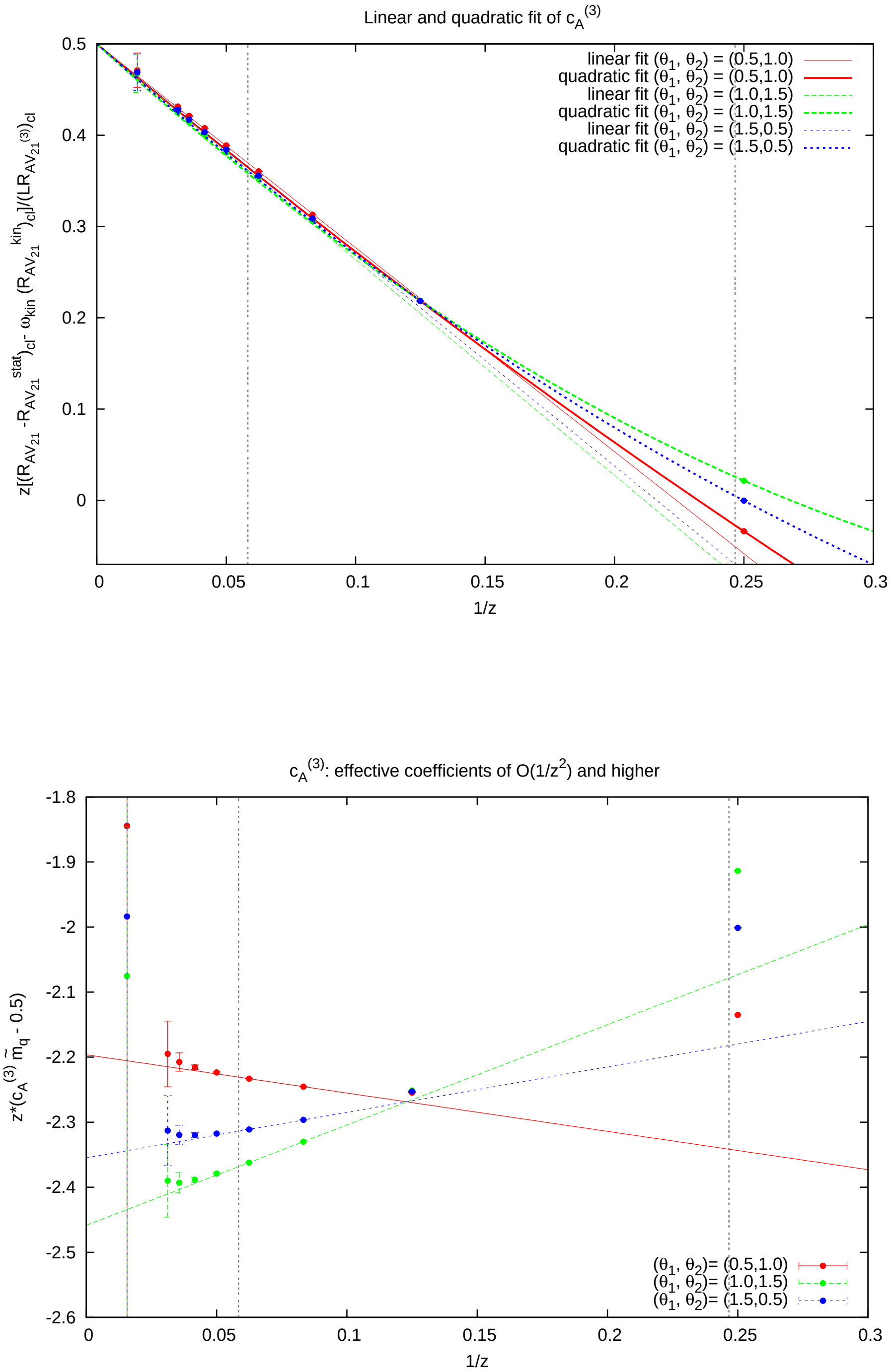
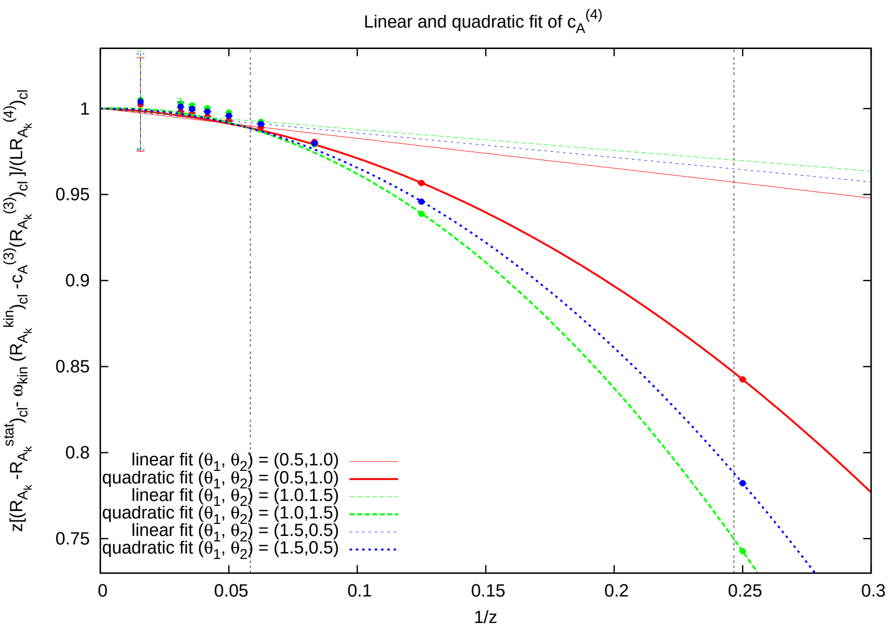


Figure 5.21: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.



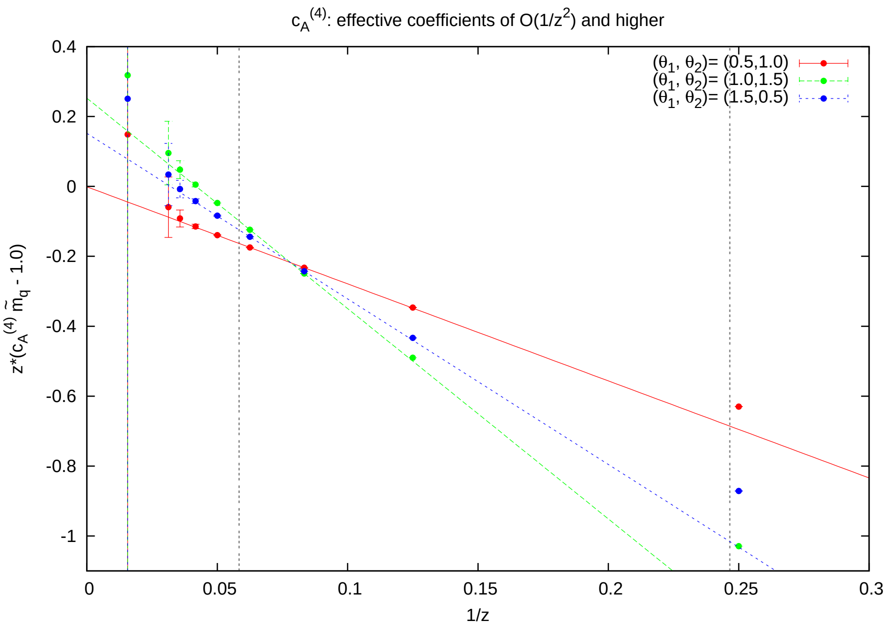


Figure 5.22: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.

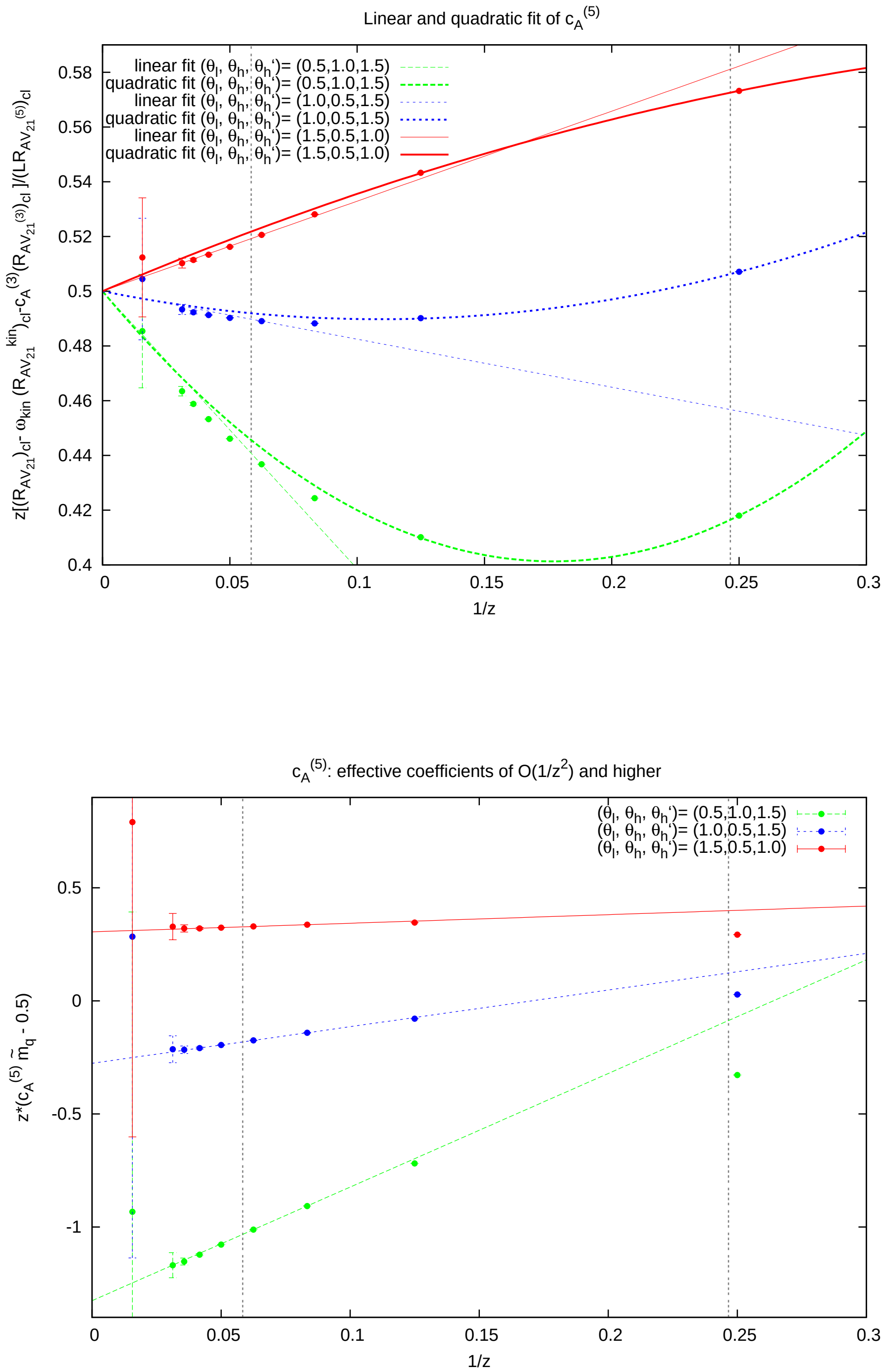
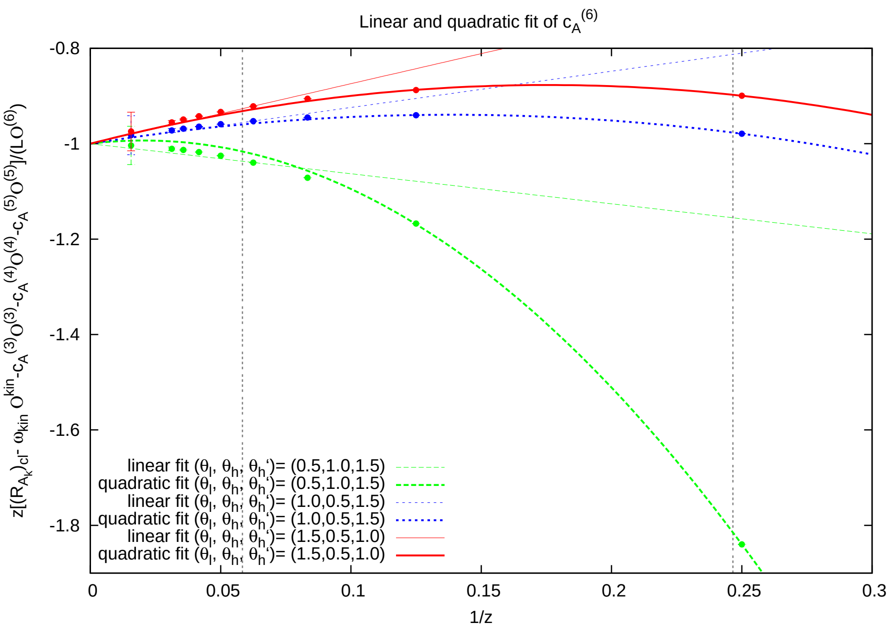


Figure 5.23: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.



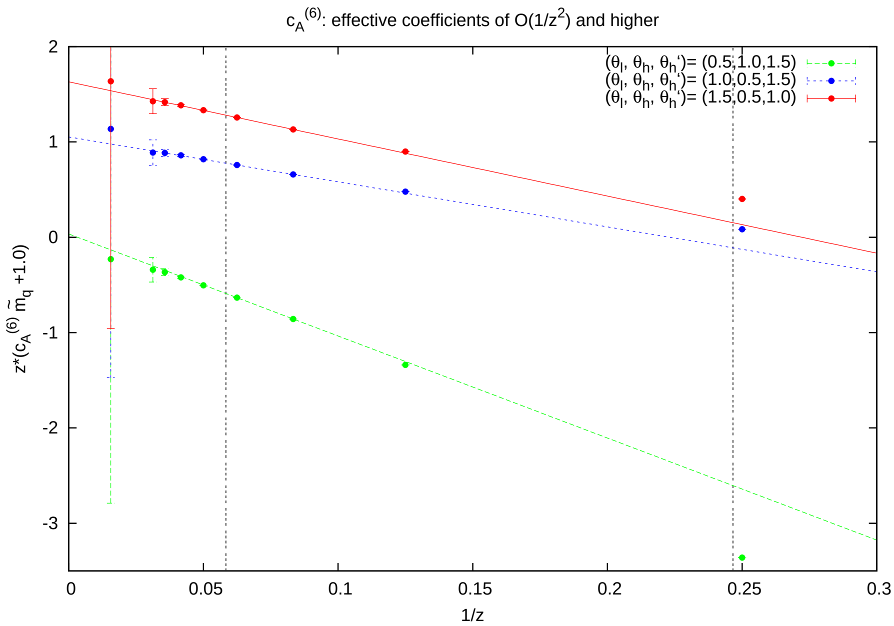


Figure 5.24: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.

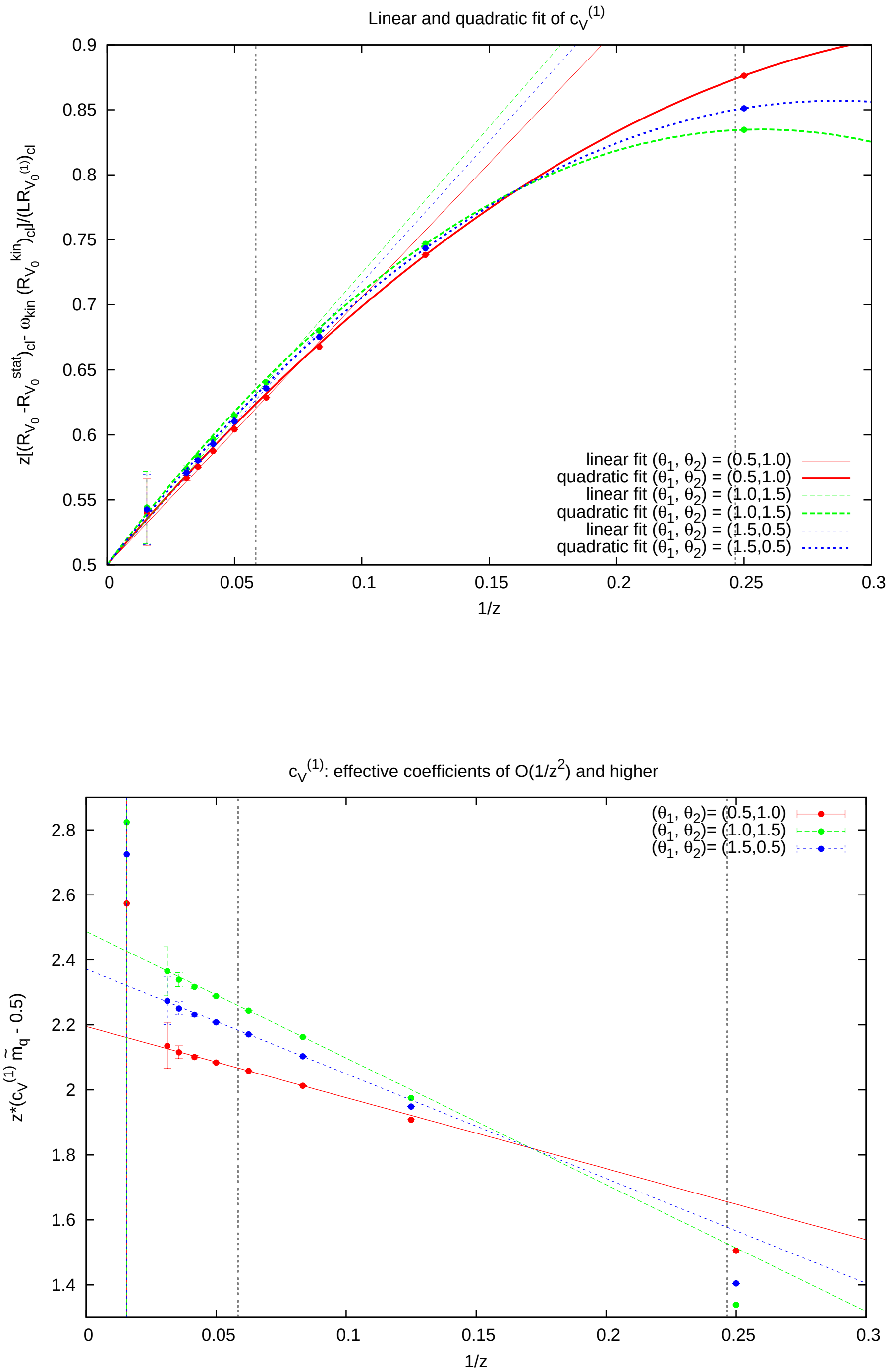
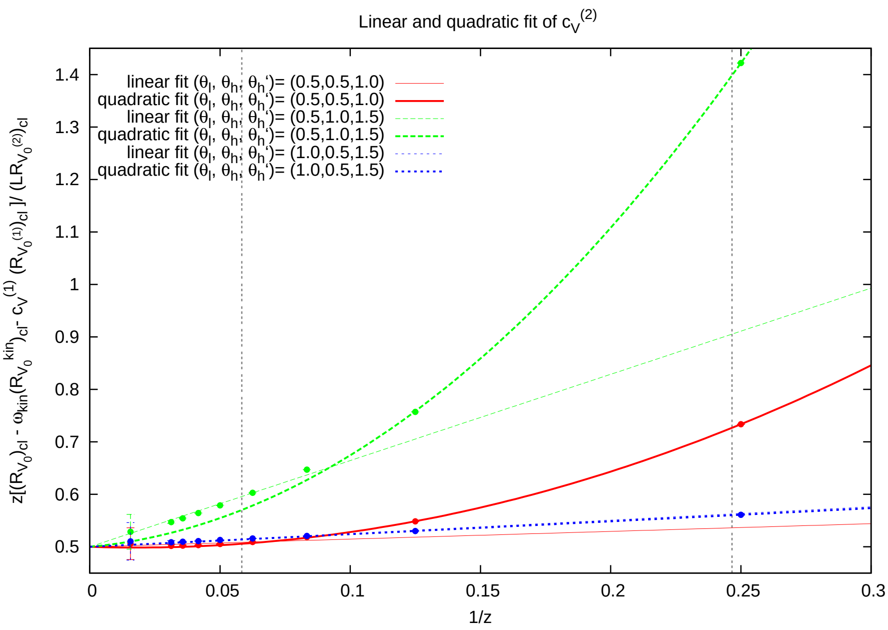


Figure 5.25: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.



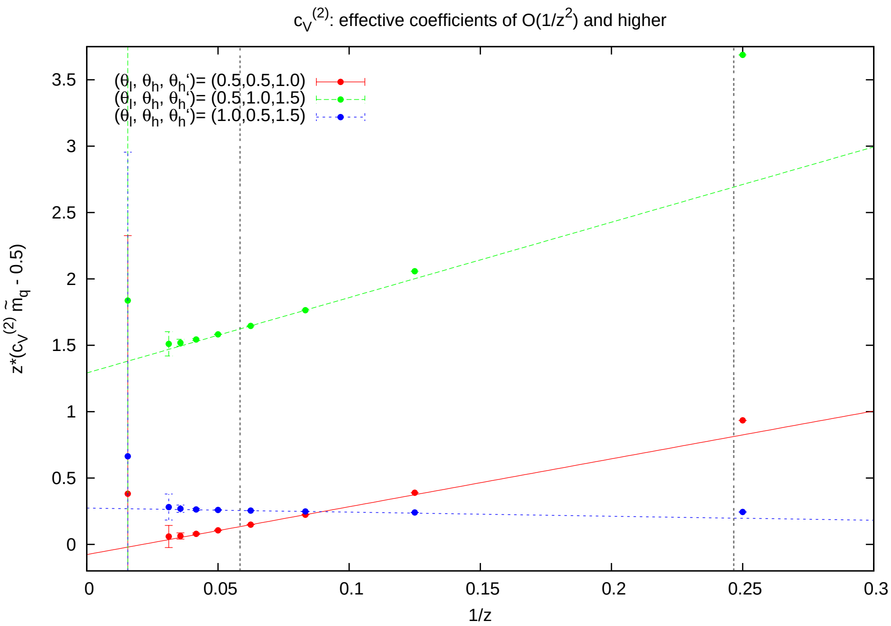


Figure 5.26: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.

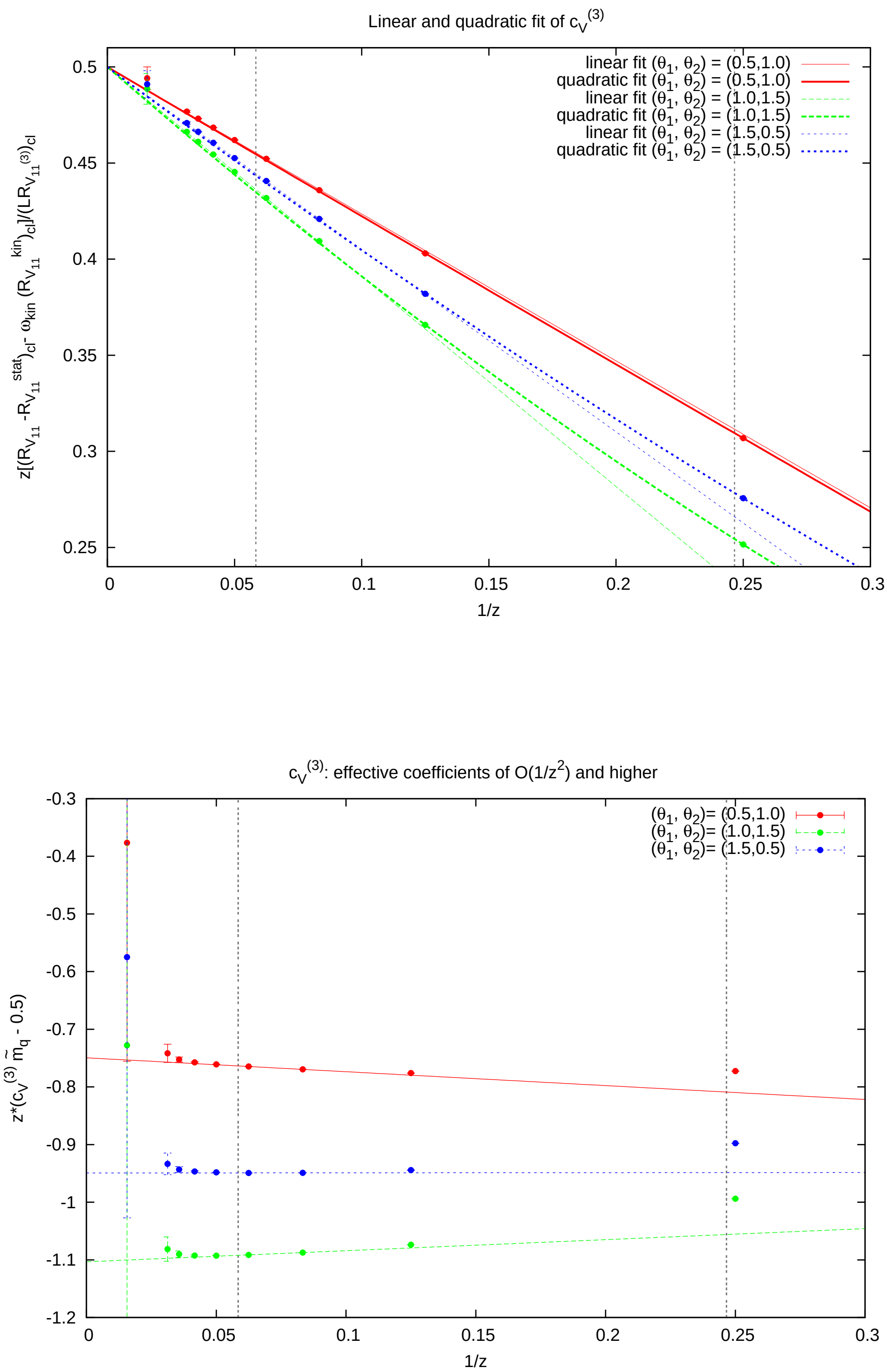


Figure 5.27: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.

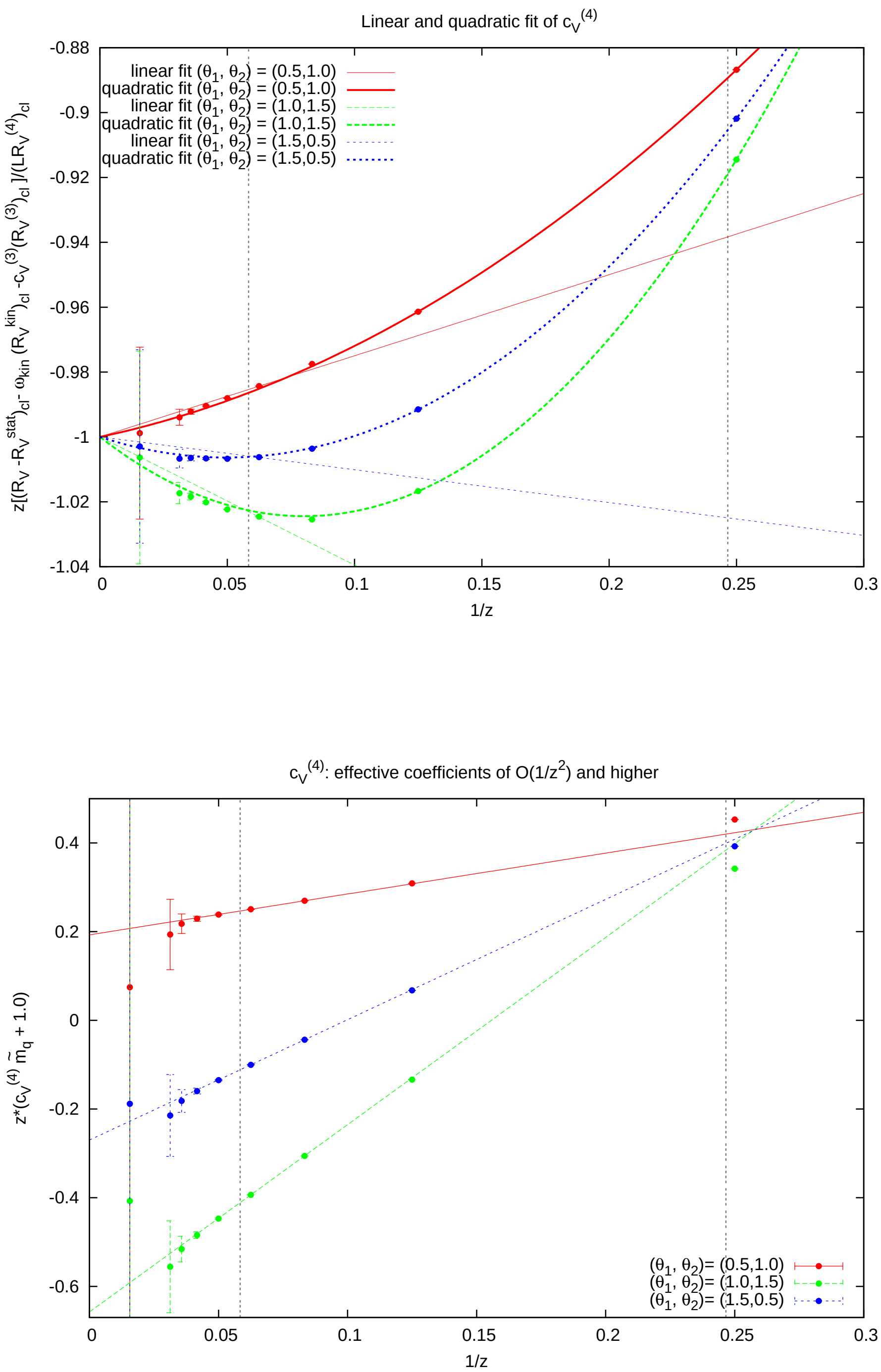
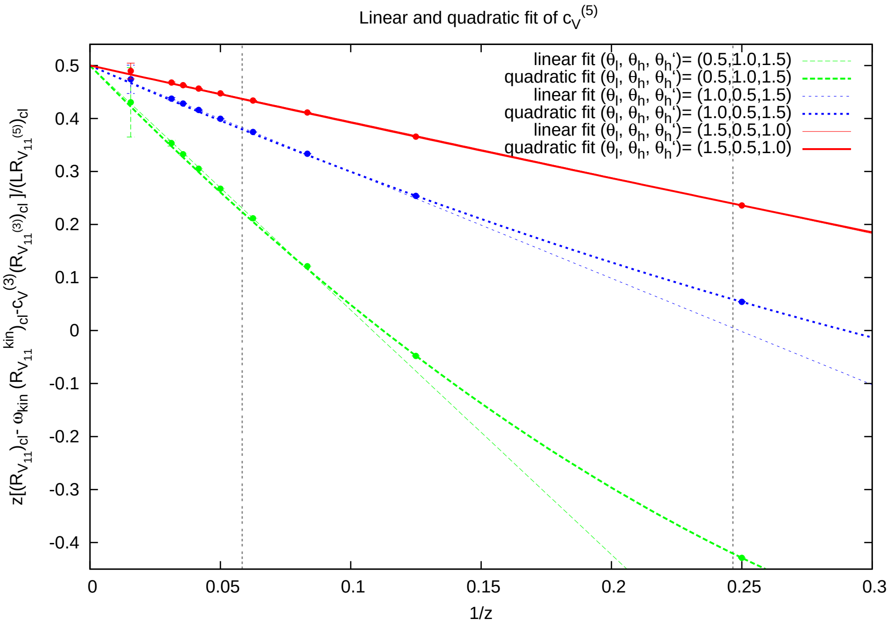


Figure 5.28: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.



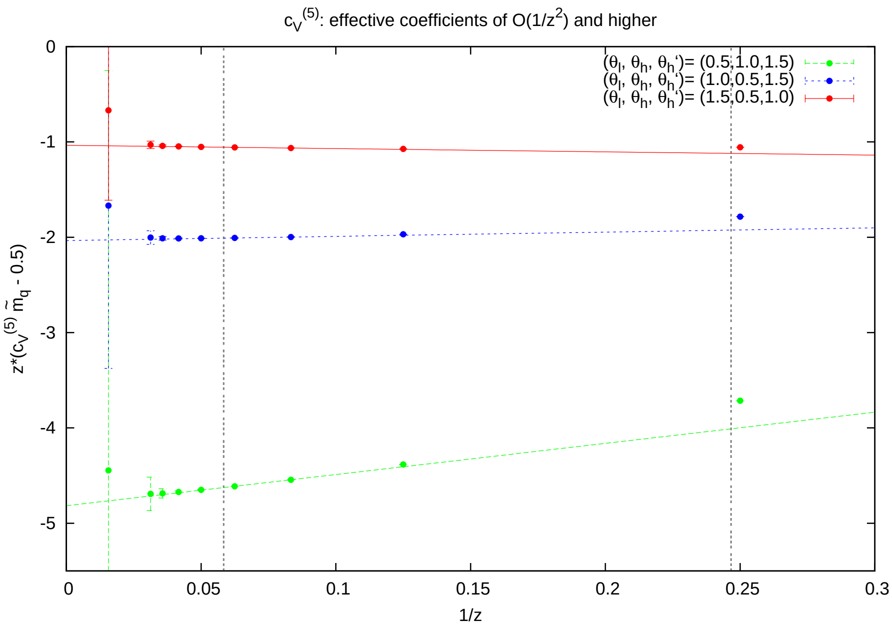
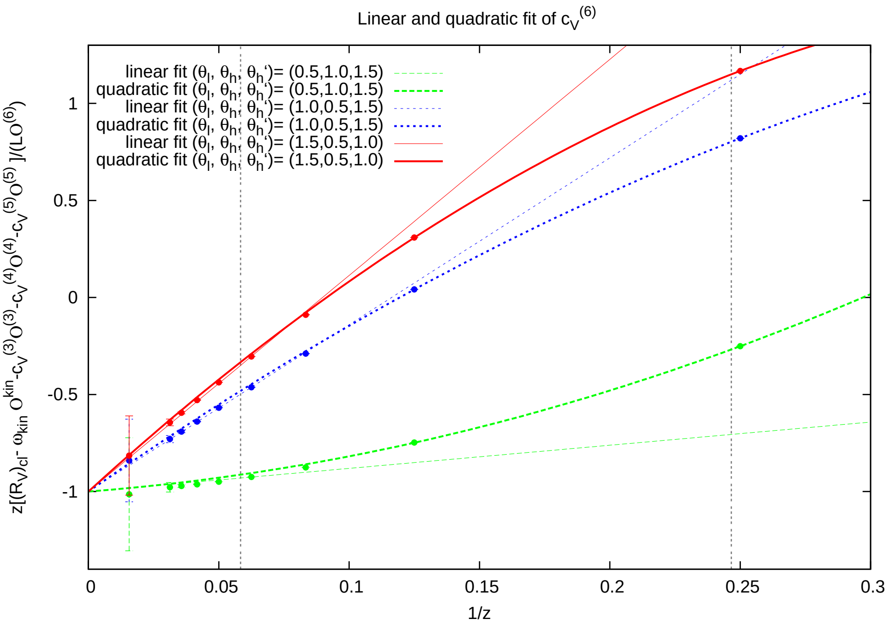


Figure 5.29: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.



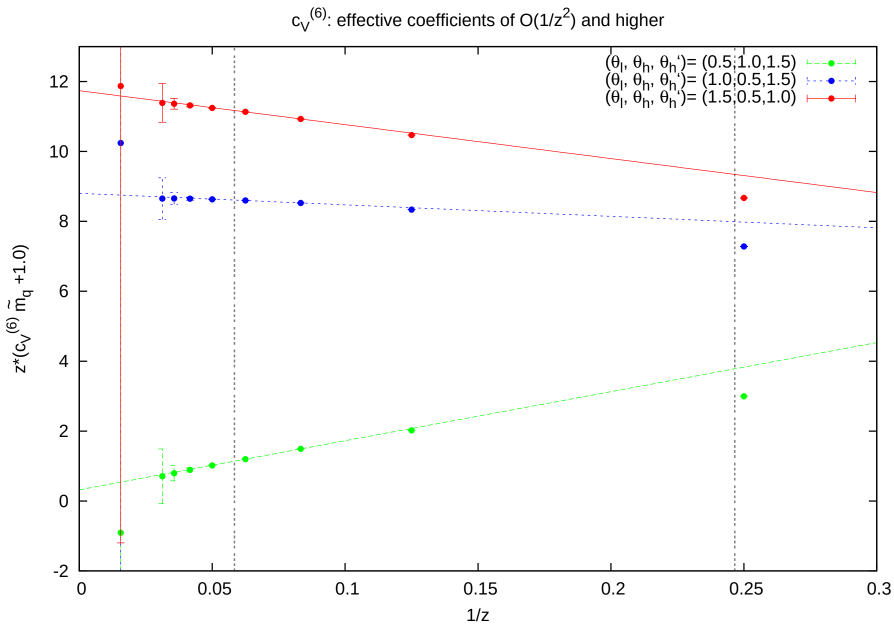


Figure 5.30: Linear and quadratic fit of the HQET parameters (top). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$. Effective coefficients of the HQET parameters (bottom). The linear fit is performed for the values $z = 12, 16, 20, 24, 28, 32, 64$.

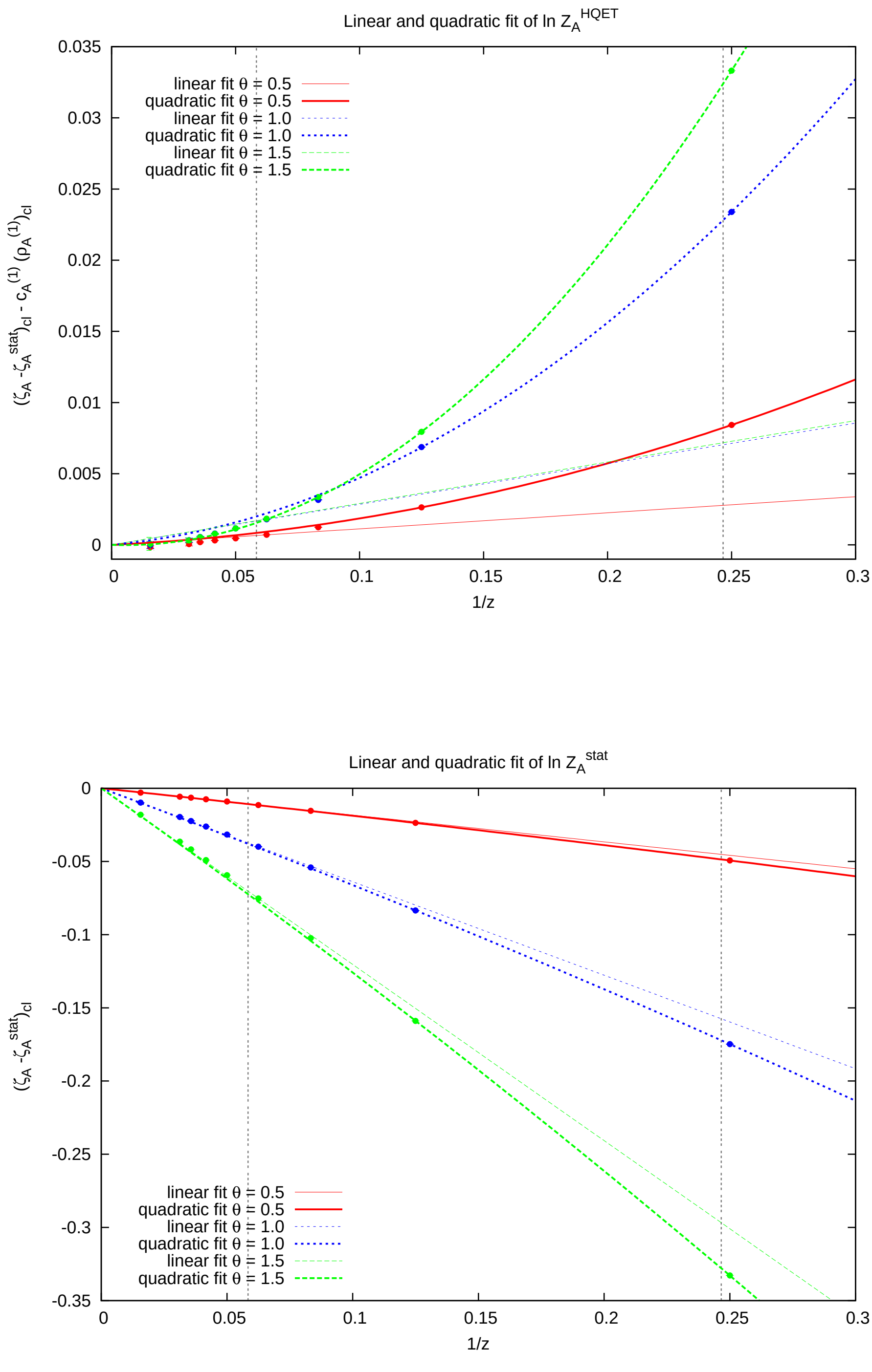


Figure 5.31: Linear and quadratic fit of the renormalization coefficient at HQET (top) and in the static approximation (bottom). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$.

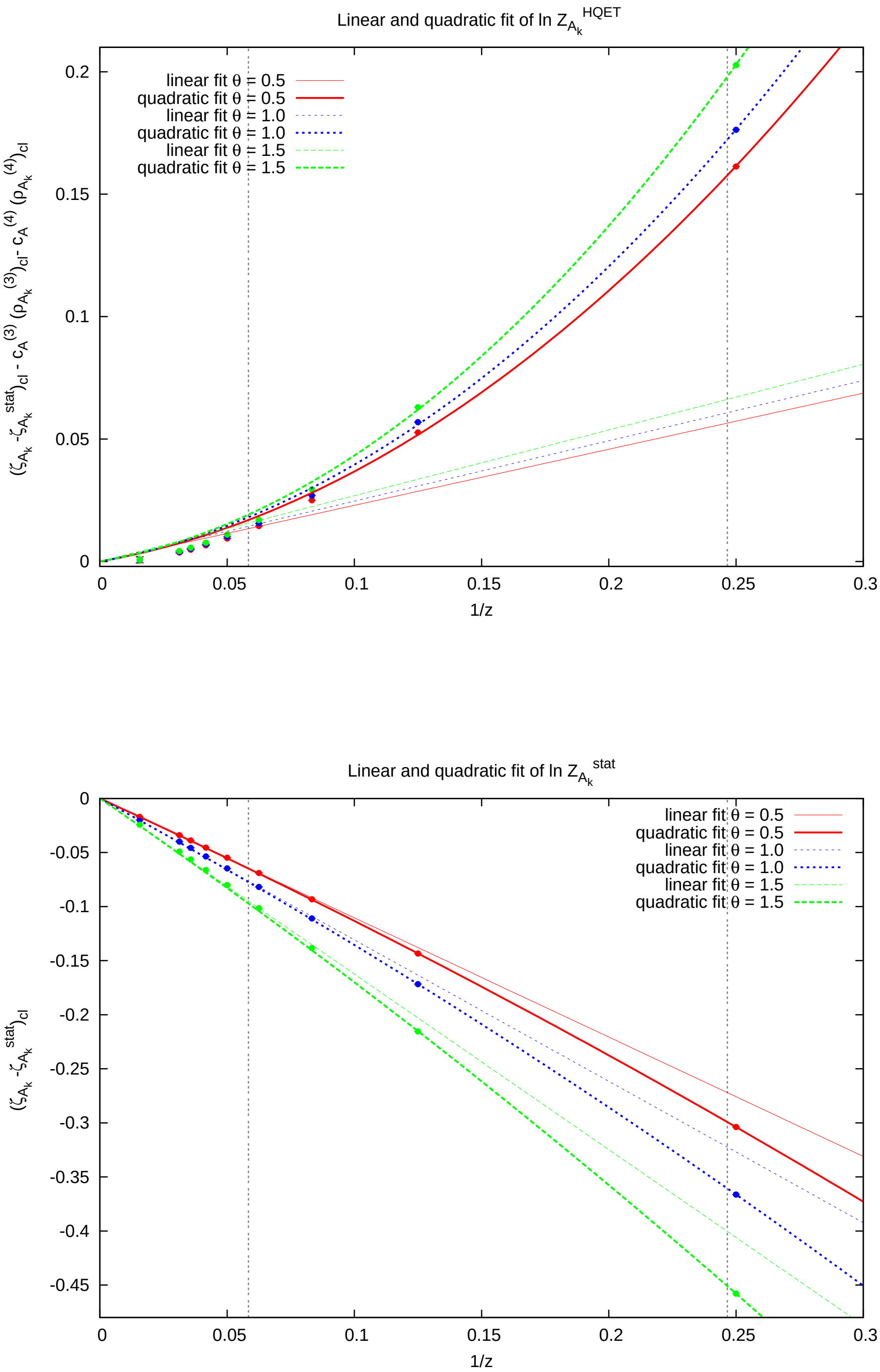
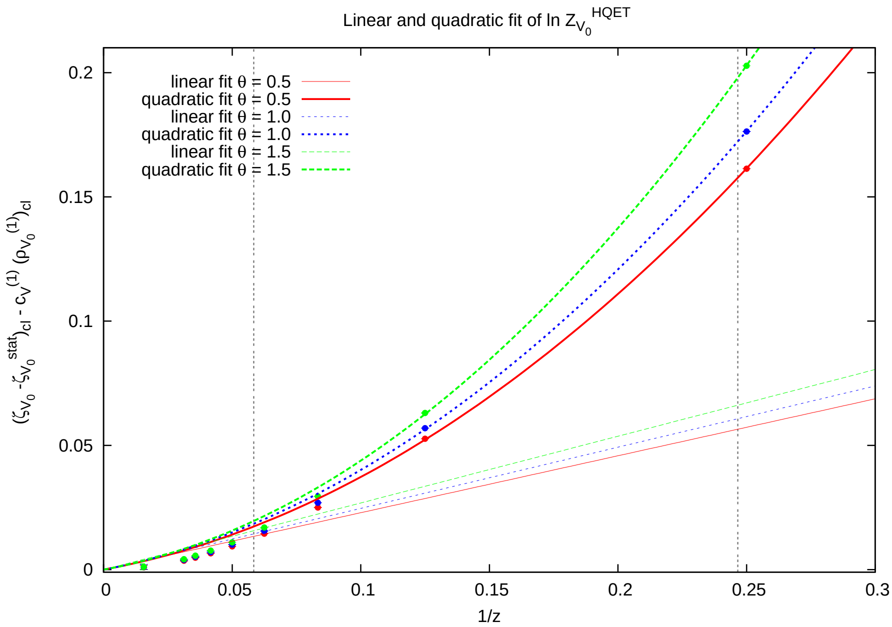


Figure 5.32: Linear and quadratic fit of the renormalization coefficient at HQET (top) and in the static approximation (bottom). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$.



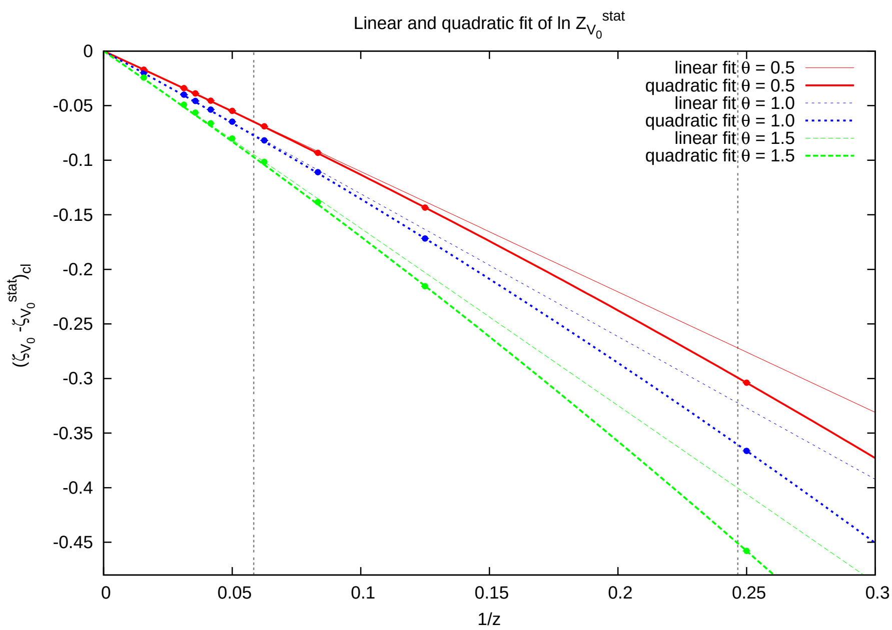


Figure 5.33: Linear and quadratic fit of the renormalization coefficient at HQET (top) and in the static approximation (bottom). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$.

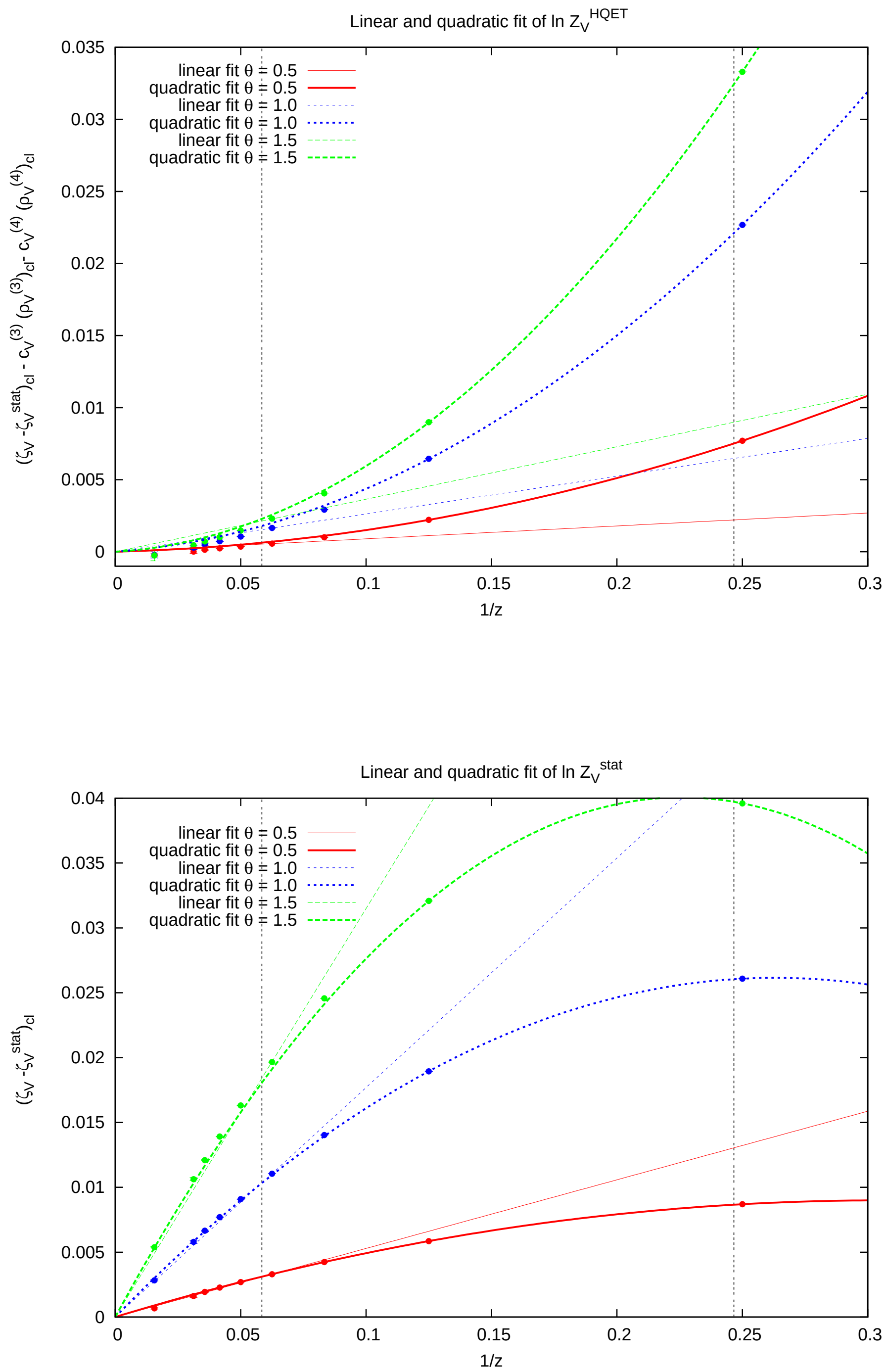


Figure 5.34: Linear and quadratic fit of the renormalization coefficient at HQET (top) and in the static approximation (bottom). The linear fit only considers the data for $z = 16, 20, 24, 28, 32, 64$.

Summary

In this work I determined a full set of HQET parameters at tree level only using two-point correlation functions. From the parameters and renormalization constants determined from the axial and vector matching a dependence in $1/z$ can be inferred. I evaluated the parameters of table (3.1) up to order $1/m^2$. All my continuum limit results approach the desired classically expected values. The behaviour of the coefficients was discussed in $1/z$ for different combinations of the angles for the light and the heavy quarks $\vec{\theta}_\mathrm{l}$, $\vec{\theta}_\mathrm{h}$.

To summarize my results I present in tables (5.1) and (5.2) the effects in $O(1/z)$ and $O(1/z^2)$. Furthermore the θ combinations, which show the most dependence on higher orders in $1/z$ are included. The magnitude of the effective coefficients of the $O(1/z^2)$ terms can be concluded from the y-intercept of the graphs in the lower figures of (5.18) to (5.30).

I conclude from the results that the sensitivity to $1/z$ increases with higher angles of the heavy quark $\vec{\theta}_\mathrm{h}$ for the most coefficients. I illustrated the behaviour of $c_\mathrm{X}^{(i)}\tilde{m}_\mathrm{q}$ in my graphs, where $X = A, V$ and $i = 1, \ldots, 6$, and I included fits to reveal the best approximated behaviour of the tree level coefficients in the region of the b-quark mass. A linear fit through the data points provides a satisfying approach. Therefore an feasible quadratically behaviour in $1/z$ in the region of the bottom quark mass of the coefficients can be concluded.

HQET parameter	$O(1/z)$	$O(1/z^2)$ in the range of	θ combination with the smallest and largest sensitivity to $1/z$
ω_kin	0.5	$[-0.5174(6), -0.5000(1)]$	$(0.5, 1.0),\ (1.0, 1.5)$
$c_\mathrm{A}^{(1)}$	-0.5	$[-1.3748(7), -0.5161(2)]$	$(0.5, 1.0),\ (1.0, 1.5)$
$c_\mathrm{A}^{(2)}$	-0.5	$[-4.9247(2), -2.2469(8)]$	$(1.0, 0.5, 1.5),\ (0.5, 1.0, 1.5)$
$c_\mathrm{A}^{(3)}$	0.5	$[-2.4583(8), -2.1962(8)]$	$(0.5, 1.0),\ (1.0, 1.5)$
$c_\mathrm{A}^{(4)}$	1.0	$[-0.0012(3), 0.2523(5)]$	$(1.0, 1.5),\ (0.5, 1.0)$
$c_\mathrm{A}^{(5)}$	0.5	$[-1.3260(9), 0.3053(2)]$	$(1.0, 0.5, 1.5),\ (0.5, 1.0, 1.5)$
$c_\mathrm{A}^{(6)}$	-1.0	$[0.034(2), 1.634(7)]$	$(1.0, 0.5, 1.5),\ (0.5, 1.0, 1.5)$

Table 5.1: $O(1/z)$ contributions of the axial HQET parameters.

HQET parameter	$O(1/z)$	$O(1/z^2)$ in the range of	θ combination with the smallest and largest sensitivity to $1/z$
$c_{\mathrm{V}}^{(1)}$	0.5	$[2.1950(6), 2.488(1)]$	$(0.5, 1.0)$, $(1.0, 1.5)$
$c_{\mathrm{V}}^{(2)}$	0.5	$[-0.077(1), 1.291(3)]$	$(0.5, 0.5, 1.0)$, $(0.5, 1.0, 1.5)$
$c_{\mathrm{V}}^{(3)}$	0.5	$[-1.1033(4), -0.7495(3)]$	$(0.5, 1.0)$, $(1.0, 1.5)$
$c_{\mathrm{V}}^{(4)}$	-1.0	$[-0.6574(4), 0.1929(3)]$	$(1.5, 0.5)$, $(1.5, 0.5)$
$c_{\mathrm{V}}^{(5)}$	0.5	$[-4.816(1), -1.0356(4)]$	$(1.5, 0.5, 1.0)$, $(0.5, 1.0, 1.5)$
$c_{\mathrm{V}}^{(6)}$	-1.0	$[0.321(2), 11.739(3)]$	$(0.5, 1.0, 1.5)$, $(1.5, 0.5, 1.0)$

Table 5.2: $O(1/z)$ contributions of the vector HQET parameters.

Furthermore the renormalization constants $\ln Z_{\mathrm{X}}$, where $X = A, A_k, V, V_0$, in HQET and in the static approximation are considered. One can see the reduction of higher $1/z$ contributions in going from $\ln Z_{\mathrm{X}}^{\mathrm{HQET}}$ to $\ln Z_{\mathrm{X}}^{\mathrm{stat}}$ for $X = A, A_k$ and V_0. Only the renormalization constant $\ln Z_{\mathrm{V}}^{\mathrm{stat}}$ reveals comparatively larger $O(1/z^2)$ corrections.

Previous calculations of matching parameters included in the action and time component of the axial current were performed in [28]. The parameters have an application in the non-perturbative determination of the b-quark mass, the hadronic decay constant and enable the mass splitting. Further main quantities in the CKM physics are form factors of semileptonic decays. These parameters can be extracted from three-point correlation functions involving the space components of the vector current. To get sensitivity to these parameters in the HQET expansion one can choose three-point correlation functions, which are inspired by semi-leptonic decays

$$B \to \pi l \nu, \quad B \to \rho l \nu. \tag{5.33}$$

For example correlation functions, which have a pseudo-scalar or vector current at the boundaries and the spatial component of the improved QCD vector current $(V_{\mathrm{bu}}^{\mathrm{I}})_\mu = \bar{\psi}_{\mathrm{u}} \gamma_\mu \psi_{\mathrm{b}} + a c_{\mathrm{V}} \tilde{\partial}_\mu \bar{\psi}_{\mathrm{u}} i \sigma_{\mu\nu} \psi_{\mathrm{b}}$ in the bulk:

$$f_{\mathrm{V}}(x_0) \propto \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z},k} \left\langle \bar{\zeta}_{\mathrm{d}}'(\mathbf{u}) \gamma_5 \zeta_{\mathrm{u}}'(\mathbf{v}) \left(V_{\mathrm{bu}}^{\mathrm{I}}\right)_k (x) \bar{\zeta}_{\mathrm{b}}(\mathbf{y}) \gamma_5 \zeta_{\mathrm{d}}(\mathbf{z}) \right\rangle, \tag{5.34}$$

$$h_{\mathrm{V}}(x_0) \propto \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z},k,i} \left\langle \bar{\zeta}_{\mathrm{d}}'(\mathbf{u}) \gamma_i \zeta_{\mathrm{u}}'(\mathbf{v}) \left(V_{\mathrm{bu}}^{\mathrm{I}}\right)_k (x) \bar{\zeta}_{\mathrm{b}}(\mathbf{y}) \gamma_5 \zeta_{\mathrm{d}}(\mathbf{z}) \right\rangle. \tag{5.35}$$

These three-point functions correspond to the transitions and it would be possible to include them in the calculations of the HQET parameters. This consideration can be a perspective for future work. Tree level QCD values already exist [38]. Although with two-point correlation functions a higher numerical precision of the coefficients is expected in the non-perturbative determination, it would be interesting to compare the two-point results with the results of the three-point correlation functions.

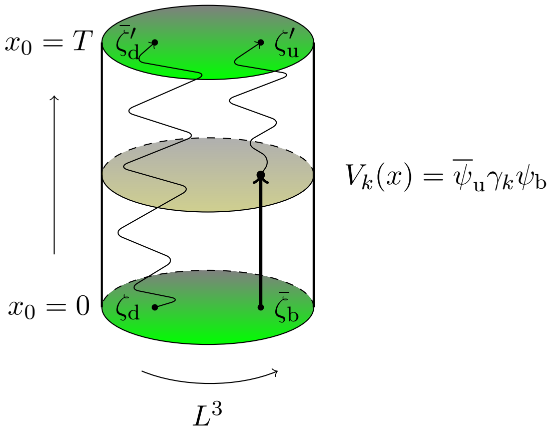


Figure 5.35: Three-point Schrödinger Functional $f_{\rm V}$ correlation function with pseudo-scalar boundary fields.

One could consider to carry on with this work by including the 1-loop calculations, or even directly start with the non-perturbative matching including order $1/m$-corrections.

The tree level study is a guidance to the non-perturbative calculations. With the successful matching at tree level at hand, one now is in the position to extend the non-perturbative programme by the observables introduced in 3.3. Those observables were introduced based on the already existing observables involving the time component of the axial current. They are chosen to provide useful information about the HQET parameters. A further essential current, the vector current, was introduced to make the set of coefficients complete. The expectation that the matching of the tree level observables provide the classically expected parameters in the continuum, was confirmed.

Appendix A

Notations

A.1 Index conventions

In this work, the Greek letters $\mu, \nu, \ldots$ label the space-time components and run from 0 to 3. Latin indices $k, l, \ldots$ run from 1 to 3 and represent the space components of spatial vectors. The unit vectors are marked with a hat, $\hat{\mu}, \hat{k}, \ldots$. Bold printed letters represent a vector with three spatial components. The $N^2 - 1$ generators of the gauge group SU(N) are indicated by $a, b, \ldots$. I used the Einstein convention in my work implying summation over repeated indices.

A.2 Dirac matrices

I used the chiral representation for the Dirac matrices

$$\gamma_\mu = \begin{pmatrix} 0 & e_\mu \\ e_\mu^\dagger & 0 \end{pmatrix}, \tag{A.1}$$

where the 2×2-matrices e_μ are

$$e_0 = -1, \quad e_k = -i\sigma_k. \tag{A.2}$$

The Pauli matrices σ_k are given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.3}$$

In this representation the Dirac matrices have the following properties

$$\gamma_\mu = \gamma_\mu^\dagger, \quad \{\gamma_\mu, \gamma_\nu\} = \gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu} \tag{A.4}$$

One can define the γ_5 matrix as $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$ and it has the features

$$\gamma_5 = \gamma_5^\dagger, \quad \gamma_5^2 = \mathbb{1}, \quad \{\gamma_5, \gamma_\mu\} = 0. \tag{A.5}$$

The hermitian σ matrices are defined through

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \tag{A.6}$$

and explicitly

$$\sigma_{0k} = \begin{pmatrix} \sigma_k & 0 \\ 0 & -\sigma_k \end{pmatrix}, \quad \sigma_{ij} = i\epsilon_{ijk} \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} = \epsilon_{ijk}\sigma_k. \tag{A.7}$$

A.3 Lattice conventions

The forward and backward derivatives on the lattice acting on colour singlet functions are

$$\begin{aligned} \partial_\mu \psi(x) &= \tfrac{1}{a}[\psi(x + a\hat{\mu}) - \psi(x)], \\ \partial^*_\mu \psi(x) &= \tfrac{1}{a}[\psi(x) - \psi(x - a\hat{\mu})], \end{aligned} \tag{A.8}$$

and acting on the left

$$\begin{aligned} \overline{\psi}(x)\overleftarrow{\partial}_\mu &= \tfrac{1}{a}[\overline{\psi}(x + a\hat{\mu}) - \overline{\psi}(x)], \\ \overline{\psi}(x)\overleftarrow{\partial}^*_\mu &= \tfrac{1}{a}[\overline{\psi}(x) - \overline{\psi}(x - a\hat{\mu})]. \end{aligned} \tag{A.9}$$

The gauge covariant derivatives in the SF acting on a quark field are

$$\begin{aligned} \nabla_\mu \psi(x) &= \tfrac{1}{a}[\lambda_\mu U_\mu(x)\psi(x + a\hat{\mu}) + \psi(x)], \\ \nabla^*_\mu \psi(x) &= \tfrac{1}{a}[\psi(x) - \lambda_\mu^{-1} U_\mu(x - a\hat{\mu})^{-1}\psi(x - a\hat{\mu})], \end{aligned} \tag{A.10}$$

with the constant phase factor

$$\lambda_\mu = e^{ia\theta_\mu/L}, \quad \theta_0 = 0, \quad -\pi < \theta_k \leq \pi. \tag{A.11}$$

They act on the left as

$$\begin{aligned} \overline{\psi}(x)\overleftarrow{\nabla}_\mu &= \tfrac{1}{a}[\overline{\psi}(x + a\hat{\mu})U_\mu(x)^{-1}\lambda_\mu^{-1} + \overline{\psi}(x)], \\ \overline{\psi}(x)\overleftarrow{\nabla}^*_\mu &= \tfrac{1}{a}[\overline{\psi}(x) - \overline{\psi}(x - a\hat{\mu})U_\mu(x - a\hat{\mu})\lambda_\mu]. \end{aligned} \tag{A.12}$$

The lattice version of the δ functions are

$$\delta(x_\mu) = \frac{1}{a}\delta_{x_\mu 0}, \quad \delta(\mathbf{x}) = \prod_{k=1}^{3}\delta(x_k), \quad \delta(x) = \prod_{\mu=0}^{3}\delta(x_\mu) \tag{A.13}$$

and for the Heaviside functions

$$\begin{aligned} \theta(x_\mu) &= 1 \quad \text{for} \quad x_\mu \geq 0, \\ \theta(x_\mu) &= 0 \quad \text{otherwise}. \end{aligned} \tag{A.14}$$

A.4 Renormalization group functions

The renormalization group (RG) functions

$$\begin{aligned} \mu\frac{\partial\bar{g}}{\partial\mu} &= \beta(\bar{g}), \\ \frac{\mu}{\bar{m}}\frac{\partial\bar{m}}{\partial\mu} &= \tau(\bar{g}), \\ \frac{\mu}{\Phi}\frac{\partial\Phi}{\partial\mu} &= \gamma(\bar{g}), \end{aligned} \tag{A.15}$$

are defined in terms of the running coupling $\bar{g}$, the running quark mass $\bar{m}$ and a matrix element of a composite field Φ. The perturbative expansion of the RG functions is

$$\begin{aligned} \beta(\bar{g}) &\sim -\bar{g}^3\{b_0 + b_1\bar{g}^2 + \ldots\}, \\ \tau(\bar{g}) &\sim -\bar{g}^2\{d_0 + d_1\bar{g}^2 + \ldots\}, \\ \gamma(\bar{g}) &\sim -\bar{g}^2\{\gamma_0 + \gamma_1\bar{g}^2 + \ldots\}. \end{aligned} \tag{A.16}$$

The coefficients $b_0 = \frac{1}{(4\pi)^2}(11 - \frac{2}{3}N_f)$, $b_1 = \frac{1}{(4\pi)^2}(103 - \frac{38}{3}N_f)$ and $d_0 = \frac{8}{(4\pi)^2}$ are independent of the renormalization scale. The RG invariants are defined by the integration constants of the solution of the RG equations as

$$\begin{aligned} \Lambda &= \mu(b_0\bar{g}^2)^{-b_1/(2b_0^2)}\mathrm{e}^{-1/(2b_0\bar{g}^2)} \exp\left\{-\int_0^{\bar{g}} \mathrm{d}x \left[\frac{1}{\beta(x)} + \frac{1}{b_0x^3} - \frac{b_1}{b_0^2x}\right]\right\}, \\ M &= \bar{m}(2b_0\bar{g}^2)^{-d_1/(2b_0)} \exp\left\{-\int_0^{\bar{g}} \mathrm{d}x \left[\frac{\tau(x)}{\beta(x)} - \frac{d_0}{b_0x}\right]\right\}, \\ \Phi_{\mathrm{RGI}} &= \Phi(2b_0\bar{g}^2)^{-\gamma_0/(2b_0)} \exp\left\{-\int_0^{\bar{g}} \mathrm{d}x \left[\frac{\gamma(x)}{\beta(x)} - \frac{\gamma_0}{b_0x}\right]\right\}, \end{aligned} \tag{A.17}$$

whereas $\bar{g}$, $\bar{m}$ and Φ depend on the renormalization scheme μ.

Appendix B

Tree level calculations for the correlation functions

B.1 The tree level quark propagator

The tree level quark propagator can be obtained from [32]. For simplicity I use the abbreviations

$$A(\mathbf{q}) = 1 + a(m_0 + \frac{1}{2}a\hat{\mathbf{q}}^2) \tag{B.1}$$

and

$$R(q) = M(q)\left(1 - e^{-2\omega(\mathbf{q})T}\right) - i\mathring{q}_0\left(1 + e^{-2\omega(\mathbf{q})T}\right). \tag{B.2}$$

I obtain for $x_0 > y_0$

$$\begin{aligned}
\tilde{S}_{\mathrm{l}}^{(0)}(x_0, y_0; \mathbf{p}) = &-\left(2i\,\mathring{p}_0^{+}\,A(\mathbf{p}^+)R(p^+)\right)^{-1} \\
&\times \Big\{ (M(p^+) - i\gamma_\mu \mathring{p}_\mu^{+})\,(M(p^+) - i\mathring{p}_0^{+})\,e^{-\omega(\mathbf{p}^+)(x_0-y_0)} \\
&+ (M(p^+) + i\gamma_0\mathring{p}_0^{+} - i\gamma_k\mathring{p}_k^{+})\,(M(p^+) + i\mathring{p}_0^{+})\,e^{-\omega(\mathbf{p}^+)(2T-x_0+y_0)} \\
&- (M^2(p^+) + (\mathring{p}_0^{+})^2 - i\gamma_k M(p^+)\mathring{p}_k^{+} - \gamma_0\gamma_k\mathring{p}_0^{+}\mathring{p}_k^{+})\,e^{-\omega(\mathbf{p}^+)(x_0+y_0)} \\
&- (M^2(p^+) + (\mathring{p}_0^{+})^2 - i\gamma_k M(p^+)\mathring{p}_k^{+} + \gamma_0\gamma_k\mathring{p}_0^{+}\mathring{p}_k^{+})\,e^{-\omega(\mathbf{p}^+)(2T-x_0-y_0)} \Big\}
\end{aligned} \tag{B.3}$$

and for $x_0 < y_0$

$$\begin{aligned}
\tilde{S}_{\mathrm{l}}^{(0)}(x_0, y_0; \mathbf{p}) = &-\left(2i\,\mathring{p}_0^{+}\,A(\mathbf{p}^+)R(p^+)\right)^{-1} \\
&\times \Big\{ (M(p^+) + i\gamma_0\mathring{p}_0^{+} - i\gamma_k\mathring{p}_k^{+})\,(M(p^+) - i\mathring{p}_0^{+})\,e^{-\omega(\mathbf{p}^+)(y_0-x_0)} \\
&+ (M(p^+) - i\gamma_\mu \mathring{p}_\mu^{+})\,(M(p^+) + i\mathring{p}_0^{+})\,e^{-\omega(\mathbf{p}^+)(2T-y_0+x_0)} \\
&- (M^2(p^+) + (\mathring{p}_0^{+})^2 - i\gamma_k M(p^+)\mathring{p}_k^{+} - \gamma_0\gamma_k\mathring{p}_0^{+}\mathring{p}_k^{+})\,e^{-\omega(\mathbf{p}^+)(y_0+x_0)} \\
&- (M^2(p^+) + (\mathring{p}_0^{+})^2 - i\gamma_k M(p^+)\mathring{p}_k^{+} + \gamma_0\gamma_k\mathring{p}_0^{+}\mathring{p}_k^{+})\,e^{-\omega(\mathbf{p}^+)(2T-y_0-x_0)} \Big\}
\end{aligned} \tag{B.4}$$

while for $x_0 = y_0$ I obtain

$$\tilde{S}_{\mathrm{l}}^{(0)}(x_0, y_0; \mathbf{p}) = \frac{1}{A(\mathbf{p}^+)} P_- - \Big(2i\, \mathring{p}_0^+\, A(\mathbf{p}^+) R(p^+)\Big)^{-1} \tag{B.5}$$
$$\times \Big\{ (M(p^+) - i\gamma_\mu \mathring{p}_\mu^+)\,(M(p^+) - i\mathring{p}_0^+)$$
$$+ (M(p^+) + i\gamma_0 \mathring{p}_0^+ - i\gamma_k \mathring{p}_k^+)\,(M(p^+) + i\mathring{p}_0^+)\, e^{-\omega(\mathbf{p}^+)2T}$$
$$- (M^2(p^+) + (\mathring{p}_0^+)^2 - i\gamma_k M(p^+)\mathring{p}_k^+ - \gamma_0\gamma_k \mathring{p}_0^+ \mathring{p}_k^+)\, e^{-\omega(\mathbf{p}^+)2x_0}$$
$$- (M^2(p^+) + (\mathring{p}_0^+)^2 - i\gamma_k M(p^+)\mathring{p}_k^+ + \gamma_0\gamma_k \mathring{p}_0^+ \mathring{p}_k^+)\, e^{-\omega(\mathbf{p}^+)(2T-2x_0)} \Big\}\,.$$

B.2 Perturbative expansion of the correlation functions

I present the tree level calculation of the the correlation functions including the time component of the vector current as well as the space components. The tree level results for $k_{\mathrm{V}_{11}}$ can be obtained by using $k = 1$ and changing the normalisation. Furthermore the tree level results for the correlation functions concerning the space components of the axial current are shown. The tree level results for $f_{\mathrm{AV}_{21}}$ are determined in the same way, thus I only give the results. The calculations follow the structure in chapter 4.

B.2.1 Tree level results for k_{V_0} and $k_{\mathrm{V}_0}^{\mathrm{stat}}$

The correlation function is given by

$$k_{\mathrm{V}_0} = i\frac{a^6}{6} \sum_{\mathbf{y},\mathbf{z},k} \left\langle V_0(x)\, \bar{\zeta}_{\mathrm{b}}(\mathbf{y}) \gamma_k \zeta_{\mathrm{l}}(\mathbf{z}) \right\rangle \tag{B.6}$$

with $V_0(x) = \overline{\psi}_{\mathrm{l}}(x)\gamma_0\psi_{\mathrm{b}}(x)$ at $x_0 = T/2$. Applying Wick's theorem and the heavy and light quark matrices (4.49) provides

$$k_{\mathrm{V}_0} = -i\frac{a^6}{6} \sum_{\mathbf{y},\mathbf{z},k} \left\langle \mathrm{tr}\left\{ \gamma_k [\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)]_{\mathrm{F}} \gamma_0 [\psi_{\mathrm{b}}(x)\bar{\zeta}_{\mathrm{b}}(\mathbf{y})]_{\mathrm{F}} \right\}\right\rangle_{\mathrm{G}}$$
$$= -i\frac{a^6}{6} \sum_{\mathbf{y},\mathbf{z},k} \Big\langle \mathrm{tr}\Big\{ \gamma_k\gamma_5 [\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^\dagger \gamma_5\gamma_0 [\psi_{\mathrm{b}}(x)\bar{\zeta}_{\mathrm{b}}(\mathbf{y})]_{\mathrm{F}} \Big\}\Big\rangle_{\mathrm{G}}$$
$$= -\frac{i}{6} \sum_k \Big\langle \mathrm{tr}\Big\{ \gamma_k\gamma_5 H_{\mathrm{l}}(x)^\dagger \gamma_5\gamma_0 H_{\mathrm{b}}(x) \Big\}\Big\rangle_{\mathrm{G}}\,. \tag{B.7}$$

With the tree level expressions for the heavy and light quark matrices I obtain for the QCD correlation function

$$k_{\mathrm{V}_0}^{(0)} = -\frac{i}{6} \sum_k \Big\langle \mathrm{tr}\Big\{ \gamma_k\gamma_5 \chi_{\mathrm{l}}(x_0)^\dagger \gamma_5\gamma_0 \chi_{\mathrm{b}}(x_0) \Big\}\Big\rangle_{\mathrm{G}} \tag{B.8}$$

and for the static correlation function

$$k_{\mathrm{V}_0}^{\mathrm{stat}\,(0)} = -\frac{i}{6}\sum_{k}\left\langle \mathrm{tr}\left\{\gamma_k\gamma_5\chi_{\mathrm{l}}(x_0)^{\dagger}\gamma_5 P_+\right\}\right\rangle_{\mathrm{G}}. \tag{B.9}$$

B.2.2 Tree level results for $k^{\mathrm{stat}}_{\mathrm{V}_0^{(1)}}$ and $k^{\mathrm{stat}}_{\mathrm{V}_0^{(2)}}$

The correlation functions are given by

$$k^{\mathrm{stat}}_{\mathrm{V}_0^{(1)}} = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle V_0^{(1)}(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_k\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{B.10}$$

$$k^{\mathrm{stat}}_{\mathrm{V}_0^{(2)}} = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle V_0^{(2)}(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_k\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle \tag{B.11}$$

with

$$\begin{array}{l} V_0^{(1)}(x) = \overline{\psi}_{\mathrm{l}}(x)\tfrac{1}{2}(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\psi_{\mathrm{h}}(x) \\ V_0^{(2)}(x) = \overline{\psi}_{\mathrm{l}}(x)\tfrac{1}{2}(\nabla_i^{\mathrm{S}} + \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\psi_{\mathrm{h}}(x) \end{array} \qquad \text{at } x_0 = T/2. \tag{B.12}$$

Applying Wick's theorem yields

$$\begin{aligned} k^{\mathrm{stat}}_{\mathrm{V}_0^{(1)}} &= -i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{\gamma_k\gamma_5[\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}\gamma_5\gamma_i[\nabla_i\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}} \\ &\quad + i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{\gamma_k\gamma_5[\nabla_i\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}\gamma_5\gamma_i[\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}} \end{aligned} \tag{B.13}$$

and

$$\begin{aligned} k^{\mathrm{stat}}_{\mathrm{V}_0^{(2)}} &= -i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{\gamma_k\gamma_5[\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}\gamma_5\gamma_i[\nabla_i\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}} \\ &\quad - i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{\gamma_k\gamma_5[\nabla_i\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}^{\dagger}\gamma_5\gamma_i[\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}}. \end{aligned} \tag{B.14}$$

The contractions $[\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]_{\mathrm{F}}$ and $[\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}$ can be expressed through the quark matrices. Whereas the contractions, containing covariant gauge derivatives, are at tree level

$$
\begin{aligned}
[\nabla_i\psi_{\rm h}(x)\bar\zeta_{\rm h}(\mathbf{y})]_{\rm F} &= \frac{1}{2a}\left(\lambda_{{\rm h},i}-\lambda_{{\rm h},i}^{-1}\right)\left(\frac{\delta\psi^{(0)}_{\rm h,cl}(x_0)}{\delta\rho_{\rm h}(\mathbf{y})}\right)\\
&= \frac{i}{a}\sin(a\frac{\theta_{{\rm h},i}}{L})\left(\frac{\delta\psi^{(0)}_{\rm h,cl}(x_0)}{\delta\rho_{\rm h}(\mathbf{y})}\right) \qquad \text{(B.15)}\\
[\nabla_i\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]_{\rm F} &= \frac{1}{2a}\left(\lambda_{{\rm l},i}-\lambda_{{\rm l},i}^{-1}\right)\left(\frac{\delta\psi^{(0)}_{\rm l,cl}(x_0)}{\delta\rho_{\rm h}(\mathbf{z})}\right)\\
&= \frac{i}{a}\sin(a\frac{\theta_{{\rm l},i}}{L})\left(\frac{\delta\psi^{(0)}_{\rm l,cl}(x_0)}{\delta\rho_{\rm l}(\mathbf{z})}\right). \qquad \text{(B.16)}
\end{aligned}
$$

From this I obtain the tree level correlation functions

$$
\begin{aligned}
&k^{{\rm stat}\,(0)}_{{\rm V}_0^{(1)}}(x_0,\vec\theta_{\rm l},\vec\theta_{\rm h})\\
&= \frac{1}{12a}\sum_{k,i}\left(\sin(a\frac{\theta_{{\rm h},i}}{L})+\sin(a\frac{\theta_{{\rm l},i}}{L})\right)\left\langle{\rm tr}\left\{\gamma_k\gamma_5\chi_{\rm l}(x_0)^\dagger\gamma_5\gamma_iP_+\right\}\right\rangle_{\rm G} \qquad \text{(B.17)}
\end{aligned}
$$

and

$$
\begin{aligned}
&k^{{\rm stat}\,(0)}_{{\rm V}_0^{(2)}}(x_0,\vec\theta_{\rm l},\vec\theta_{\rm h})\\
&= \frac{1}{12a}\sum_{k,i}\left(\sin(a\frac{\theta_{{\rm h},i}}{L})-\sin(a\frac{\theta_{{\rm l},i}}{L})\right)\left\langle{\rm tr}\left\{\gamma_k\gamma_5\chi_{\rm l}(x_0)^\dagger\gamma_5\gamma_iP_+\right\}\right\rangle_{\rm G}. \qquad \text{(B.18)}
\end{aligned}
$$

B.2.3 Tree level results for $k^{\rm stat}_{{\rm V}^{(3)}}$ and $k^{\rm stat}_{{\rm V}^{(4)}}$

The correlation functions are given by

$$
k^{\rm stat}_{{\rm V}^{(3)}} = -\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle V_k^{(3)}(x)\,\bar\zeta_{\rm h}(\mathbf{y})\gamma_k\zeta_{\rm l}(\mathbf{z})\right\rangle, \qquad \text{(B.19)}
$$

$$
k^{\rm stat}_{{\rm V}^{(4)}} = -\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle V_k^{(4)}(x)\,\bar\zeta_{\rm h}(\mathbf{y})\gamma_k\zeta_{\rm l}(\mathbf{z})\right\rangle \qquad \text{(B.20)}
$$

with

$$
\left.\begin{aligned}
V_k^{(3)}(x) &= \overline{\psi}_{\rm l}(x)\tfrac{1}{2}(\nabla_i^{\rm S}-\overleftarrow{\nabla}_i^{\rm S})\gamma_i\gamma_k\psi_{\rm h}(x)\\
V_k^{(4)}(x) &= \overline{\psi}_{\rm l}(x)\tfrac{1}{2}(\nabla_k^{\rm S}-\overleftarrow{\nabla}_k^{\rm S})\psi_{\rm h}(x)
\end{aligned}\right.\quad \text{at } x_0=T/2. \qquad \text{(B.21)}
$$

Applying Wick's theorem I obtain the contractions

$$
\begin{aligned}
k^{\rm stat}_{{\rm V}^{(3)}} &= \frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle{\rm tr}\left\{\gamma_k\gamma_5[\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]_{\rm F}^\dagger\gamma_5\gamma_i\gamma_k[\nabla_i^{\rm S}\psi_{\rm h}(x)\bar\zeta_{\rm l}(\mathbf{y})]_{\rm F}\right\}\right\rangle_{\rm G}\\
&-\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle{\rm tr}\left\{\gamma_k\gamma_5[\nabla_i^{\rm S}\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]_{\rm F}^\dagger\gamma_5\gamma_i\gamma_k[\psi_{\rm h}(x)\bar\zeta_{\rm h}(\mathbf{y})]_{\rm F}\right\}\right\rangle_{\rm G}. \qquad \text{(B.22)}
\end{aligned}
$$

and

$$k^{\rm stat}_{\rm V^{(4)}} = \frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle {\rm tr}\left\{\gamma_k\gamma_5[\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]^\dagger_{\rm F}\gamma_5[\nabla^{\rm S}_k\psi_{\rm h}(x)\bar\zeta_{\rm l}(\mathbf{y})]_{\rm F}\right\}\right\rangle_{\rm G} - \frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle {\rm tr}\left\{\gamma_k\gamma_5[\nabla^{\rm S}_k\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]^\dagger_{\rm F}\gamma_5[\psi_{\rm h}(x)\bar\zeta_{\rm h}(\mathbf{y})]_{\rm F}\right\}\right\rangle_{\rm G}. \tag{B.23}$$

At tree level the correlation functions provide with (B.15) and (B.16)

$$k^{\rm stat\,(0)}_{\rm V^{(3)}}(x_0,\vec\theta_{\rm l},\vec\theta_{\rm h}) = \frac{i}{12a}\sum_{i,k}\left(\sin(a\frac{\theta_{{\rm h},i}}{L})+\sin(a\frac{\theta_{{\rm l},i}}{L})\right)\left\langle {\rm tr}\left\{\gamma_k\gamma_5\chi_{\rm l}^\dagger(x_0)\gamma_5\gamma_i\gamma_k P_+\right\}\right\rangle_{\rm G} \tag{B.24}$$

and

$$k^{\rm stat\,(0)}_{\rm V^{(4)}}(x_0,\vec\theta_{\rm l},\vec\theta_{\rm h}) = \frac{i}{12a}\sum_{k}\left(\sin(a\frac{\theta_{{\rm h},k}}{L})+\sin(a\frac{\theta_{{\rm l},k}}{L})\right)\left\langle {\rm tr}\left\{\gamma_k\gamma_5\chi_{\rm l}^\dagger(x_0)\gamma_5 P_+\right\}\right\rangle_{\rm G}. \tag{B.25}$$

B.2.4 Tree level results for $k^{\rm stat}_{\rm V^{(5)}}$ and $k^{\rm stat}_{\rm V^{(6)}}$

The correlation functions are given by

$$k^{\rm stat}_{\rm V^{(5)}} = -\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle V^{(5)}_k(x)\,\bar\zeta_{\rm h}(\mathbf{y})\gamma_k\zeta_{\rm l}(\mathbf{z})\right\rangle, \tag{B.26}$$

$$k^{\rm stat}_{\rm V^{(6)}} = -\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle V^{(6)}_k(x)\,\bar\zeta_{\rm h}(\mathbf{y})\gamma_k\zeta_{\rm l}(\mathbf{z})\right\rangle \tag{B.27}$$

with

$$\begin{array}{l} V^{(5)}_k(x) = \overline{\psi}_{\rm l}(x)\frac{1}{2}(\nabla^{\rm S}_i+\overleftarrow{\nabla}^{\rm S}_i)\gamma_i\gamma_k\psi_{\rm h}(x) \\ V^{(6)}_k(x) = \overline{\psi}_{\rm l}(x)\frac{1}{2}(\nabla^{\rm S}_k+\overleftarrow{\nabla}^{\rm S}_k)\psi_{\rm h}(x)\end{array} \qquad \text{at } x_0 = T/2. \tag{B.28}$$

Applying Wick's theorem I obtain the contractions

$$k^{\rm stat}_{\rm V^{(5)}} = \frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle {\rm tr}\left\{\gamma_k\gamma_5[\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]^\dagger_{\rm F}\gamma_5\gamma_i\gamma_k[\nabla^{\rm S}_i\psi_{\rm h}(x)\bar\zeta_{\rm l}(\mathbf{y})]_{\rm F}\right\}\right\rangle_{\rm G} + \frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle {\rm tr}\left\{\gamma_k\gamma_5[\nabla^{\rm S}_i\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]^\dagger_{\rm F}\gamma_5\gamma_i\gamma_k[\psi_{\rm h}(x)\bar\zeta_{\rm h}(\mathbf{y})]_{\rm F}\right\}\right\rangle_{\rm G}. \tag{B.29}$$

and

$$k^{\rm stat}_{\rm V^{(6)}} = \frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle {\rm tr}\left\{\gamma_k\gamma_5[\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]^\dagger_{\rm F}\gamma_5[\nabla^{\rm S}_k\psi_{\rm h}(x)\bar\zeta_{\rm l}(\mathbf{y})]_{\rm F}\right\}\right\rangle_{\rm G} + \frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle {\rm tr}\left\{\gamma_k\gamma_5[\nabla^{\rm S}_k\psi_{\rm l}(x)\bar\zeta_{\rm l}(\mathbf{z})]^\dagger_{\rm F}\gamma_5[\psi_{\rm h}(x)\bar\zeta_{\rm h}(\mathbf{y})]_{\rm F}\right\}\right\rangle_{\rm G}. \tag{B.30}$$

At tree level the correlation functions provide with (B.15) and (B.16)

$$k_{\mathrm{V}^{(5)}}^{\mathrm{stat}\,(0)}(x_0,\vec{\theta}_\mathrm{l},\vec{\theta}_\mathrm{h}) = \frac{i}{12a}\sum_{i,k}\left(\sin(a\frac{\theta_{\mathrm{h},i}}{L}) - \sin(a\frac{\theta_{\mathrm{l},i}}{L})\right)\left\langle \mathrm{tr}\left\{\gamma_k\gamma_5\chi_\mathrm{l}^\dagger(x_0)\gamma_5\gamma_i\gamma_k P_+\right\}\right\rangle_\mathrm{G} \tag{B.31}$$

and

$$k_{\mathrm{V}^{(6)}}^{\mathrm{stat}\,(0)}(x_0,\vec{\theta}_\mathrm{l},\vec{\theta}_\mathrm{h}) = \frac{i}{12a}\sum_{k}\left(\sin(a\frac{\theta_{\mathrm{h},k}}{L}) - \sin(a\frac{\theta_{\mathrm{l},k}}{L})\right)\left\langle \mathrm{tr}\left\{\gamma_k\gamma_5\chi_\mathrm{l}^\dagger(x_0)\gamma_5 P_+\right\}\right\rangle_\mathrm{G}. \tag{B.32}$$

B.2.5 Tree level results for f_{A_k} and $f_{\mathrm{A}_k}^{\mathrm{stat}}$

The correlation function is given by

$$f_{\mathrm{A}_k} = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle A_k(x)\,\bar{\zeta}_\mathrm{b}(\mathbf{y})\gamma_5\zeta_\mathrm{l}(\mathbf{z})\right\rangle \tag{B.33}$$

with $A_k(x) = \overline{\psi}_\mathrm{l}(x)\gamma_k\gamma_5\psi_\mathrm{b}(x)$ at $x_0 = T/2$. Applying Wick's theorem and the heavy and light quark matrices (4.49) provides

$$\begin{aligned} f_{\mathrm{A}_k} &= -i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{\gamma_5[\zeta_\mathrm{l}(\mathbf{z})\overline{\psi}_\mathrm{l}(x)]_\mathrm{F}\gamma_k\gamma_5[\psi_\mathrm{b}(x)\bar{\zeta}_\mathrm{b}(\mathbf{y})]_\mathrm{F}\right\}\right\rangle_\mathrm{G} \\ &= i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{[\psi_\mathrm{l}(x)\bar{\zeta}_\mathrm{l}(\mathbf{z})]_\mathrm{F}^\dagger\gamma_k[\psi_\mathrm{b}(x)\bar{\zeta}_\mathrm{b}(\mathbf{y})]_\mathrm{F}\right\}\right\rangle_\mathrm{G} \\ &= \frac{i}{6}\sum_k\left\langle \mathrm{tr}\left\{H_\mathrm{l}(x)^\dagger\gamma_k H_\mathrm{b}(x)\right\}\right\rangle_\mathrm{G}. \end{aligned} \tag{B.34}$$

With the tree level expressions for the heavy and light quark matrices I obtain for the QCD correlation function

$$f_{\mathrm{A}_k}^{(0)} = \frac{i}{6}\sum_k\left\langle \mathrm{tr}\left\{\chi_\mathrm{l}(x_0)^\dagger\gamma_k\chi_\mathrm{b}(x_0)\right\}\right\rangle_\mathrm{G} \tag{B.35}$$

and for the static correlation function

$$f_{\mathrm{A}_k}^{\mathrm{stat}\,(0)} = \frac{i}{6}\sum_k\left\langle \mathrm{tr}\left\{\chi_\mathrm{l}(x_0)^\dagger\gamma_k P_+\right\}\right\rangle_\mathrm{G}. \tag{B.36}$$

B.2.6 Tree level results for $f^{\text{stat}}_{\mathrm{A}_k^{(3)}}$ and $f^{\text{stat}}_{\mathrm{A}_k^{(4)}}$

The correlation functions are given by

$$f^{\text{stat}}_{\mathrm{A}_k^{(3)}} = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle A_k^{(3)}(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{B.37}$$

$$f^{\text{stat}}_{\mathrm{A}_k^{(4)}} = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle A_k^{(4)}(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle \tag{B.38}$$

with

$$\begin{aligned} A_k^{(3)}(x) &= \overline{\psi}_{\mathrm{l}}(x)\tfrac{1}{2}(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\gamma_5\gamma_k\psi_{\mathrm{h}}(x) \\ A_k^{(4)}(x) &= \overline{\psi}_{\mathrm{l}}(x)\tfrac{1}{2}(\nabla_k^{\mathrm{S}} - \overleftarrow{\nabla}_k^{\mathrm{S}})\gamma_5\psi_{\mathrm{h}}(x) \end{aligned} \qquad \text{at } x_0 = T/2. \tag{B.39}$$

Applying Wick's theorem I obtain the contractions

$$\begin{aligned} f^{\text{stat}}_{\mathrm{A}_k^{(3)}} =& -i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle \mathrm{tr}\left\{\gamma_5[\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)]_{\mathrm{F}}\gamma_i\gamma_5\gamma_k[\nabla_i^{\mathrm{S}}\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}} \\ & + i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle \mathrm{tr}\left\{\gamma_5[\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)\overleftarrow{\nabla}_i^{\mathrm{S}}]_{\mathrm{F}}\gamma_i\gamma_5\gamma_k[\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}}. \end{aligned} \tag{B.40}$$

and

$$\begin{aligned} f^{\text{stat}}_{\mathrm{A}_k^{(4)}} =& -i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{\gamma_5[\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)]_{\mathrm{F}}\gamma_5[\nabla_k^{\mathrm{S}}\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}} \\ & + i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{\gamma_5[\zeta_{\mathrm{l}}(\mathbf{z})\overline{\psi}_{\mathrm{l}}(x)\overleftarrow{\nabla}_k^{\mathrm{S}}]_{\mathrm{F}}\gamma_5[\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}}. \end{aligned} \tag{B.41}$$

At tree level the correlation functions provide with the hermiticity property (1.92) and (B.15) and (B.16)

$$\begin{aligned} & f^{\text{stat}\,(0)}_{\mathrm{A}_k^{(3)}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) \\ & = -\frac{1}{12a}\sum_{i,k}\left(\sin(a\frac{\theta_{\mathrm{h},i}}{L}) + \sin(a\frac{\theta_{\mathrm{l},i}}{L})\right)\left\langle \mathrm{tr}\left\{\chi_{\mathrm{l}}^{\dagger}(x_0)\gamma_i\gamma_k P_+\right\}\right\rangle_{\mathrm{G}} \end{aligned} \tag{B.42}$$

and

$$\begin{aligned} & f^{\text{stat}\,(0)}_{\mathrm{A}_k^{(4)}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) \\ & = \frac{1}{12a}\sum_{k}\left(\sin(a\frac{\theta_{\mathrm{h},k}}{L}) + \sin(a\frac{\theta_{\mathrm{l},k}}{L})\right)\left\langle \mathrm{tr}\left\{\chi_{\mathrm{l}}^{\dagger}(x_0)P_+\right\}\right\rangle_{\mathrm{G}}. \end{aligned} \tag{B.43}$$

B.2.7 Tree level results for $f^{\mathrm{stat}}_{\mathrm{A}_k^{(5)}}$ and $f^{\mathrm{stat}}_{\mathrm{A}_k^{(6)}}$

The correlation functions are given by

$$f^{\mathrm{stat}}_{\mathrm{A}_k^{(5)}} = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle A_k^{(5)}(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle, \tag{B.44}$$

$$f^{\mathrm{stat}}_{\mathrm{A}_k^{(6)}} = i\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k}\left\langle A_k^{(6)}(x)\,\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z})\right\rangle \tag{B.45}$$

with

$$\begin{aligned} A_k^{(5)}(x) &= \overline{\psi}_{\mathrm{l}}(x)\tfrac{1}{2}(\nabla_i^{\mathrm{S}} + \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\gamma_5\gamma_k\psi_{\mathrm{h}}(x) \\ A_k^{(6)}(x) &= -\overline{\psi}_{\mathrm{l}}(x)\tfrac{1}{2}(\nabla_k^{\mathrm{S}} + \overleftarrow{\nabla}_k^{\mathrm{S}})\gamma_5\psi_{\mathrm{h}}(x) \end{aligned} \quad \text{at } x_0 = T/2. \tag{B.46}$$

Applying Wick's theorem I obtain the contractions

$$\begin{aligned} f^{\mathrm{stat}}_{\mathrm{A}_k^{(5)}} &= i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle \mathrm{tr}\left\{[\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]^{\dagger}_{\mathrm{F}}\gamma_i\gamma_k[\nabla_i^{\mathrm{S}}\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}} \\ &\quad + i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k,i}\left\langle \mathrm{tr}\left\{[\nabla_i^{\mathrm{S}}\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]^{\dagger}_{\mathrm{F}}\gamma_i\gamma_k[\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}}. \end{aligned} \tag{B.47}$$

and

$$\begin{aligned} f^{\mathrm{stat}}_{\mathrm{A}_k^{(6)}} &= i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{[\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]^{\dagger}_{\mathrm{F}}[\nabla_k^{\mathrm{S}}\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}} \\ &\quad + i\frac{a^6}{12}\sum_{\mathbf{y},\mathbf{z},k}\left\langle \mathrm{tr}\left\{[\nabla_k^{\mathrm{S}}\psi_{\mathrm{l}}(x)\bar{\zeta}_{\mathrm{l}}(\mathbf{z})]^{\dagger}_{\mathrm{F}}[\psi_{\mathrm{h}}(x)\bar{\zeta}_{\mathrm{h}}(\mathbf{y})]_{\mathrm{F}}\right\}\right\rangle_{\mathrm{G}}. \end{aligned} \tag{B.48}$$

At tree level the correlation functions provide with (B.15) and (B.16)

$$\begin{aligned} &f^{\mathrm{stat}\,(0)}_{\mathrm{A}_k^{(5)}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) \\ &= -\frac{1}{12a}\sum_{i,k}\left(\sin(a\frac{\theta_{\mathrm{h},i}}{L}) - \sin(a\frac{\theta_{\mathrm{l},i}}{L})\right)\left\langle \mathrm{tr}\left\{\chi_{\mathrm{l}}^{\dagger}(x_0)\gamma_i\gamma_k P_+\right\}\right\rangle_{\mathrm{G}}. \end{aligned} \tag{B.49}$$

and

$$\begin{aligned} &f^{\mathrm{stat}\,(0)}_{\mathrm{A}_k^{(6)}}(x_0,\vec{\theta}_{\mathrm{l}},\vec{\theta}_{\mathrm{h}}) \\ &= -\frac{1}{12a}\sum_{k}\left(\sin(a\frac{\theta_{\mathrm{h},k}}{L}) - \sin(a\frac{\theta_{\mathrm{l},k}}{L})\right)\left\langle \mathrm{tr}\left\{\chi_{\mathrm{l}}^{\dagger}(x_0)P_+\right\}\right\rangle_{\mathrm{G}}. \end{aligned} \tag{B.50}$$

B.2.8 Tree level results for $f_{\mathrm{AV}_{21}}$ and $f^{\mathrm{stat}}_{\mathrm{AV}_{21}}$

The correlation function is given by

$$f_{\mathrm{AV}_{21}} = i\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}}\langle A_2(x)\,\bar{\zeta}_{\mathrm{b}}(\mathbf{y})\gamma_1\zeta_(\mathbf{z})\rangle \tag{B.51}$$

With the same calculation as for f_{A_k} only with a different γ structure I obtain

$$f^{(0)}_{\mathrm{AV}_{21}} = \frac{i}{2}\left\langle \mathrm{tr}\left\{\gamma_1\gamma_5\chi_{\mathrm{l}}(x_0)^\dagger\gamma_2\chi_{\mathrm{b}}(x_0)\right\}\right\rangle_{\mathrm{G}} \tag{B.52}$$

and for the static correlation function

$$f^{\mathrm{stat}\,(0)}_{\mathrm{AV}_{21}} = \frac{i}{2}\left\langle \mathrm{tr}\left\{\gamma_1\gamma_5\chi_{\mathrm{l}}(x_0)^\dagger\gamma_2 P_+\right\}\right\rangle_{\mathrm{G}}. \tag{B.53}$$

B.2.9 Tree level results for $f^{\mathrm{stat}}_{\mathrm{AV}^{(3)}_{21}}$ to $f^{\mathrm{stat}}_{\mathrm{AV}^{(6)}_{21}}$

The tree level results of the correlation functions can be obtained similarly to the above calculations. I only present the results.

$$f^{\mathrm{stat}\,(0)}_{\mathrm{AV}^{(3)}_{21}} = -\frac{1}{4}\sum_i\left(\sin(a\frac{\theta_{\mathrm{h},i}}{L}) + \sin(a\frac{\theta_{\mathrm{l},i}}{L})\right)\left\langle \mathrm{tr}\left\{\gamma_1\gamma_5\chi_{\mathrm{l}}(x_0)^\dagger\gamma_i\gamma_2 P_+\right\}\right\rangle_{\mathrm{G}} \tag{B.54}$$

$$f^{\mathrm{stat}\,(0)}_{\mathrm{AV}^{(4)}_{21}} = \frac{1}{4}\left(\sin(a\frac{\theta_{\mathrm{h},y}}{L}) + \sin(a\frac{\theta_{\mathrm{l},y}}{L})\right)\left\langle \mathrm{tr}\left\{\gamma_1\gamma_5\chi_{\mathrm{l}}(x_0)^\dagger P_+\right\}\right\rangle_{\mathrm{G}} \tag{B.55}$$

$$f^{\mathrm{stat}\,(0)}_{\mathrm{AV}^{(5)}_{21}} = -\frac{1}{4}\sum_i\left(\sin(a\frac{\theta_{\mathrm{h},i}}{L}) - \sin(a\frac{\theta_{\mathrm{l},i}}{L})\right)\left\langle \mathrm{tr}\left\{\gamma_1\gamma_5\chi_{\mathrm{l}}(x_0)^\dagger\gamma_i\gamma_2 P_+\right\}\right\rangle_{\mathrm{G}} \tag{B.56}$$

$$f^{\mathrm{stat}\,(0)}_{\mathrm{AV}^{(6)}_{21}} = -\frac{1}{4}\left(\sin(a\frac{\theta_{\mathrm{h},y}}{L}) - \sin(a\frac{\theta_{\mathrm{l},y}}{L})\right)\left\langle \mathrm{tr}\left\{\gamma_1\gamma_5\chi_{\mathrm{l}}(x_0)^\dagger P_+\right\}\right\rangle_{\mathrm{G}} \tag{B.57}$$

Appendix C

Numerical results

C.1 Determination of the uncertainty of the HQET parameters

Since the expressions are analytically known at tree level, but need to be evaluated numerically, there is no statistical error, only machine precision. I assume an error of all correlations functions of $O(10^{-8})$. The errors of the observables are determined by error propagation. For the errors of the continuum limit I extrapolate the data from $L/a = 170, \ldots, 256$ [1] and obtain the error by the difference to the continuum limit values of the coefficients. The continuum limit of all observables was taken and is not mentioned in the formula. For a better readability I label different combinations of the angels by θ_1 and θ_2 and omit other arguments of the correlation functions. The uncertainty of the kinetic quantity is zero because it was determined analytically and only depends on the angles in the continuum limit and not on the correlation functions anymore.

$$\Delta\omega_{\mathrm{kin}}\tilde{m}_{\mathrm{q}} = \frac{z}{(LR_1^{\mathrm{kin}})}\Delta(R_1 - R_1^{\mathrm{stat}}) \tag{C.1}$$

$$\Delta c_{\mathrm{A}}^{(1)}\tilde{m}_{\mathrm{q}} = \left[\left(\frac{z}{LR^{\mathrm{stat}}_{\mathrm{A}^{(1)}}}\Delta(R_{\mathrm{A}} - R_{\mathrm{A}}^{\mathrm{stat}})\right)^2 + \left(\frac{LR_{\mathrm{A}}^{\mathrm{kin}}}{LR^{\mathrm{stat}}_{\mathrm{A}^{(1)}}}\Delta\omega_{\mathrm{kin}}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{c_{\mathrm{A}}^{(1)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{A}^{(1)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{A}^{(1)}})\right)^2\right]^{1/2} \tag{C.2}$$

$$\Delta c_{\mathrm{A}}^{(2)}\tilde{m}_{\mathrm{q}} = \left[\left(\frac{z}{LR^{\mathrm{stat}}_{\mathrm{A}^{(2)}}}\Delta R_{\mathrm{A}}\right)^2 + \left(\frac{LR_{\mathrm{A}}^{\mathrm{kin}}}{LR^{\mathrm{stat}}_{\mathrm{A}^{(2)}}}\Delta\omega_{\mathrm{kin}}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{LR^{\mathrm{stat}}_{\mathrm{A}^{(1)}}}{LR^{\mathrm{stat}}_{\mathrm{A}^{(2)}}}\Delta c_{\mathrm{A}}^{(1)}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{c_{\mathrm{A}}^{(1)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{A}^{(2)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{A}^{(1)}})\right)^2 + \left(\frac{c_{\mathrm{A}}^{(2)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{A}^{(2)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{A}^{(2)}})\right)^2\right]^{1/2} \tag{C.3}$$

[1] for $z = 64$ only the finest lattices are considered, that means half of the lattices which are used for the continuum limit extrapolation.

$$\Delta \ln Z_{\rm A}^{\rm HQET} = \Big[(\Delta\zeta_{\rm A})^2 + (\Delta\zeta_{\rm A}^{\rm stat})^2 + \left(\frac{L\rho_{\rm A^{(1)}}^{\rm stat}}{z}\Delta c_{\rm A}^{(1)}\tilde{m}_{\rm q}\right)^2 + \left(\frac{c_{\rm A}^{(1)}\tilde{m}_{\rm q}}{z}\Delta L\rho_{\rm A^{(1)}}^{\rm stat}\right)^2\Big]^{1/2}. \tag{C.4}$$

The errors of the observables are given by the error propagation of the correlation functions:

$$\Delta(R_1 - R_1^{\rm stat}) = \sqrt{(\Delta R_1)^2 + (\Delta R_1^{\rm stat})^2}, \tag{C.5}$$

$$\Delta R_1 = \sqrt{\left(\frac{\Delta f_1(\theta_1)}{f_1(\theta_1)}\right)^2 + \left(\frac{\Delta f_1(\theta_2)}{f_1(\theta_2)}\right)^2}, \tag{C.6}$$

$$\Delta R_1^{\rm stat} = \sqrt{\left(\frac{\Delta f_1^{\rm stat}(\theta_1)}{f_1^{\rm stat}(\theta_1)}\right)^2 + \left(\frac{\Delta f_1^{\rm stat}(\theta_2)}{f_1^{\rm stat}(\theta_2)}\right)^2}, \tag{C.7}$$

$$\Delta(R_{\rm A} - R_{\rm A}^{\rm stat}) = \sqrt{(\Delta R_{\rm A})^2 + (\Delta R_{\rm A}^{\rm stat})^2}, \tag{C.8}$$

$$\Delta R_{\rm A} = \sqrt{\left(\frac{\Delta f_{\rm A}(\theta_1)}{f_{\rm A}(\theta_1)}\right)^2 + \left(\frac{\Delta f_{\rm A}(\theta_2)}{f_{\rm A}(\theta_2)}\right)^2}, \tag{C.9}$$

$$\Delta R_{\rm A}^{\rm stat} = \sqrt{\left(\frac{\Delta f_{\rm A}^{\rm stat}(\theta_1)}{f_{\rm A}^{\rm stat}(\theta_1)}\right)^2 + \left(\frac{\Delta f_{\rm A}^{\rm stat}(\theta_2)}{f_{\rm A}^{\rm stat}(\theta_2)}\right)^2} \tag{C.10}$$

and

$$\Delta R_{\rm A^{(1)}}^{\rm stat} = \Bigg[\left(\frac{\Delta f_{\rm A^{(1)}}^{\rm stat}(\theta_1)}{f_{\rm A}^{\rm stat}(\theta_1)}\right)^2 + \left(\frac{\Delta f_{\rm A^{(1)}}^{\rm stat}(\theta_2)}{f_{\rm A}^{\rm stat}(\theta_2)}\right)^2 + \left(\frac{f_{\rm A^{(1)}}^{\rm stat}(\theta_1)}{f_{\rm A}^{\rm stat}(\theta_1)}\Delta f_{\rm A}^{\rm stat}(\theta_1)\right)^2 + \left(\frac{f_{\rm A^{(1)}}^{\rm stat}(\theta_2)}{f_{\rm A}^{\rm stat}(\theta_2)}\Delta f_{\rm A}^{\rm stat}(\theta_2)\right)^2\Bigg]^2. \tag{C.11}$$

The error of $R_{\rm A^{(2)}}^{\rm stat}$ can be obtained analogous to (C.11).

$$\Delta\zeta_{\rm A} = \sqrt{\left(\frac{\Delta f_{\rm A}}{f_{\rm A}}\right)^2 + \left(\frac{1}{2}\frac{\Delta f_1}{f_1}\right)^2}, \tag{C.12}$$

$$\Delta\zeta_{\rm A}^{\rm stat} = \sqrt{\left(\frac{\Delta f_{\rm A}^{\rm stat}}{f_{\rm A}^{\rm stat}}\right)^2 + \left(\frac{1}{2}\frac{\Delta f_1^{\rm stat}}{f_1^{\rm stat}}\right)^2} \tag{C.13}$$

and

$$\Delta\rho_{\rm A^{(1)}}^{\rm stat} = \sqrt{\left(\frac{\Delta f_{\rm A^{(1)}}^{\rm stat}}{f_{\rm A}^{\rm stat}}\right)^2 + \left(\frac{f_{\rm A^{(1)}}^{\rm stat}}{(f_{\rm A}^{\rm stat})^2}\Delta f_{\rm A}^{\rm stat}\right)^2}. \tag{C.14}$$

For the time component of the vector current the uncertainties are determined in the same way by replacing the axial correlation functions with the vector correlation functions.

The uncertainties of the HQET parameters from the matching of the space component of the vector current can be obtained by the following formula:

$$\Delta c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}} = \Bigg[\left(\frac{z}{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(3)}}}\Delta(R_{\mathrm{V}_{11}} - R^{\mathrm{stat}}_{\mathrm{V}_{11}})\right)^2 + \left(\frac{LR^{\mathrm{kin}}_{\mathrm{V}_{11}}}{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(3)}}}\Delta\omega_{\mathrm{kin}}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(1)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(3)}})\right)^2\Bigg]^{1/2}, \tag{C.15}$$

$$\Delta c_{\mathrm{V}}^{(4)}\tilde{m}_{\mathrm{q}} = \Bigg[\left(\frac{z}{LR^{\mathrm{stat}}_{\mathrm{V}^{(4)}}}\Delta(R_{\mathrm{V}} - R^{\mathrm{stat}}_{\mathrm{V}})\right)^2 + \left(\frac{LR^{\mathrm{kin}}_{\mathrm{V}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(4)}}}\Delta\omega_{\mathrm{kin}}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{LR^{\mathrm{stat}}_{\mathrm{V}^{(3)}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(4)}}}\Delta c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(4)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}^{(3)}})\right)^2 + \left(\frac{c_{\mathrm{V}}^{(4)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(4)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}^{(4)}})\right)^2\Bigg]^{1/2}, \tag{C.16}$$

$$\Delta c_{\mathrm{V}}^{(5)}\tilde{m}_{\mathrm{q}} = \Bigg[\left(\frac{z}{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(5)}}}\Delta R_{\mathrm{V}_{11}}\right)^2 + \left(\frac{LR^{\mathrm{kin}}_{\mathrm{V}_{11}}}{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(5)}}}\Delta\omega_{\mathrm{kin}}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(3)}}}{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(5)}}}\Delta c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(5)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(3)}})\right)^2 + \left(\frac{c_{\mathrm{V}}^{(5)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(5)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}_{11}^{(5)}})\right)^2\Bigg]^{1/2}, \tag{C.17}$$

$$\Delta c_{\mathrm{V}}^{(6)}\tilde{m}_{\mathrm{q}} = \Bigg[\left(\frac{z}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta R_{\mathrm{V}}\right)^2 + \left(\frac{LR^{\mathrm{kin}}_{\mathrm{V}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta\omega_{\mathrm{kin}}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{LR^{\mathrm{stat}}_{\mathrm{V}^{(3)}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{LR^{\mathrm{stat}}_{\mathrm{V}^{(4)}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta c_{\mathrm{V}}^{(4)}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{LR^{\mathrm{stat}}_{\mathrm{V}^{(5)}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta c_{\mathrm{V}}^{(5)}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}^{(3)}})\right)^2 + \left(\frac{c_{\mathrm{V}}^{(4)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}^{(4)}})\right)^2 + \left(\frac{c_{\mathrm{V}}^{(5)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}^{(5)}})\right)^2 + \left(\frac{c_{\mathrm{V}}^{(6)}\tilde{m}_{\mathrm{q}}}{LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}}}\Delta(LR^{\mathrm{stat}}_{\mathrm{V}^{(6)}})\right)^2\Bigg]^{1/2}, \tag{C.18}$$

$$\begin{aligned}\Delta \ln Z_{\mathrm{V}}^{\mathrm{HQET}} = \Big[&(\Delta\zeta_{\mathrm{V}})^2 + (\Delta\zeta_{\mathrm{V}}^{\mathrm{stat}})^2 \\ &+ \left(\frac{L\rho_{\mathrm{V}^{(3)}}^{\mathrm{stat}}}{z}\Delta c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{c_{\mathrm{V}}^{(3)}\tilde{m}_{\mathrm{q}}}{z}\Delta(L\rho_{\mathrm{V}^{(3)}}^{\mathrm{stat}})\right)^2 \\ &+ \left(\frac{L\rho_{\mathrm{V}^{(4)}}^{\mathrm{stat}}}{z}\Delta c_{\mathrm{V}}^{(4)}\tilde{m}_{\mathrm{q}}\right)^2 + \left(\frac{c_{\mathrm{V}}^{(4)}\tilde{m}_{\mathrm{q}}}{z}\Delta(L\rho_{\mathrm{V}^{(4)}}^{\mathrm{stat}})\right)^2 \Bigg]^{1/2} . \qquad \text{(C.19)}\end{aligned}$$

The errors of the observables can be obtained as in equations (C.5) to (C.14) with the corresponding correlation functions.

C.2 Tables of the tree level HQET parameters

	$\omega_{\mathrm{kin}} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_1 = 0.5$ $\theta_2 = 1.0$	$\theta_1 = 1.0$ $\theta_2 = 1.5$	$\theta_1 = 1.5$ $\theta_2 = 0.5$
4	0.36556825(6)	0.34881285(3)	0.35509612(5)
8	0.43257614(8)	0.42500505(8)	0.42784421(8)
12	0.4557813(3)	0.4517860(4)	0.4532843(3)
16	0.467216(2)	0.464790(2)	0.465700(2)
20	0.47399(1)	0.47237(1)	0.47298(1)
24	0.47848(4)	0.47732(4)	0.47776(4)
28	0.4817(1)	0.4809(1)	0.4812(1)
32	0.4844(4)	0.4837(4)	0.4839(4)
64	0.498(5)	0.498(5)	0.498(5)

Table C.1: Continuum limit results for ω_{kin} at tree level.

	$c_{\mathrm{A}}^{(1)} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_1 = 0.5$ $\theta_2 = 1.0$	$\theta_1 = 1.0$ $\theta_2 = 1.5$	$\theta_1 = 1.5$ $\theta_2 = 0.5$
4	-0.6543665(2)	-0.6969177(2)	-0.6775476(2)
8	-0.5965653(3)	-0.6402569(4)	-0.620371(4)
12	-0.567092(1)	-0.601238(2)	-0.58569(1)
16	-0.55118(7)	-0.578434(9)	-0.566027(8)
20	-0.54133(3)	-0.56385(4)	-0.55360(4)
24	-0.5346(1)	-0.5538(1)	-0.5451(1)
28	-0.5300(5)	-0.5466(4)	-0.5390(4)
32	-0.527(1)	-0.541(1)	-0.535(1)
64	-0.52(1)	-0.52(2)	-0.52(1)

Table C.2: Continuum limit results for $c_{\mathrm{A}}^{(1)}$ at tree level.

	$c_{\mathrm{A}}^{(2)} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_{\mathrm{l}} = 0.5$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.0$	$\theta_{\mathrm{l}} = 0.5$ $\theta_{\mathrm{h}} = 1.0$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.0$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.5$
4	-1.0025023(8)	-1.028164(1)	-0.7796(2)
8	-0.800364(1)	-0.930200(2)	-0.7099(2)
12	-0.707375(5)	-0.827761(8)	-0.6559(2)
16	-0.65773(3)	-0.76130(4)	-0.6228(2)
20	-0.6271(1)	-0.7164(2)	-0.6011(2)
24	-0.6065(4)	-0.6845(7)	-0.5858(4)
28	-0.591(1)	-0.661(2)	-0.575(1)
32	-0.581(5)	-0.643(7)	-0.566(4)
64	-0.55(5)	-0.58(8)	-0.54(4)

Table C.3: Continuum limit results for $c_{\mathrm{A}}^{(2)}$ at tree level.

	$\ln Z_{\mathrm{A}}^{\mathrm{HQET}}$		
z	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$
4	0.00842120(2)	0.02339116(7)	0.0333164(1)
8	0.00263566(2)	0.00687239(6)	0.0079446(1)
12	0.00123573(2)	0.0031576(3)	0.0033582(4)
16	0.000710(1)	0.001799(1)	0.001838(2)
20	0.000457(4)	0.001156(5)	0.001154(6)
24	0.00031(1)	0.00079(2)	0.00078(2)
28	0.00019(4)	0.00055(5)	0.00054(6)
32	0.0001(1)	0.0003(1)	0.0003(2)
64	-0.00015(9)	-0.00001(25)	0.00007(46)

Table C.4: Continuum limit results for $\ln Z_{\mathrm{A}}^{\mathrm{HQET}}$ at tree level. For $z = 64$ I used the last 15 lattices for the continuum limit extrapolation.

	$\ln Z_{\mathrm{A}}^{\mathrm{stat}}$		
z	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$
4	-0.04935870(1)	-0.17476900(4)	-0.3328310(7)
8	-0.02370240(2)	-0.08345580(4)	-0.1589580(6)
12	-0.0154555(2)	-0.0540861(2)	-0.1024130(3)
16	-0.011457(1)	-0.039929(1)	-0.075264(1)
20	-0.009103(4)	-0.031630(5)	-0.059426(5)
24	-0.00756(1)	-0.02619(2)	-0.04908(2)
28	-0.00649(4)	-0.02237(5)	-0.04182(5)
32	-0.0058(1)	-0.0196(1)	-0.0365(2)
64	0.00005(5)	0.00006(6)	0.00006(6)

Table C.5: Continuum limit results for $\ln Z_{\mathrm{A}}^{\mathrm{stat}}$ at tree level. For $z = 64$ I used the last 15 lattices for the continuum limit extrapolation.

	$c_{\mathrm{A}}^{(3)} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_1 = 0.5$ $\theta_2 = 1.0$	$\theta_1 = 1.0$ $\theta_2 = 1.5$	$\theta_1 = 1.5$ $\theta_2 = 0.5$
4	-0.0337595(3)	0.0216039(3)	-0.0003084(3)
8	0.2181728(5)	0.2185915(4)	0.2184276(4)
12	0.312890(2)	0.305841(1)	0.308631(2)
16	0.360427(9)	0.35237(1)	0.35556(1)
20	0.38883(4)	0.38105(7)	0.38413(5)
24	0.40769(2)	0.4005(2)	0.4033(2)
28	0.4212(5)	0.4145(6)	0.4172(6)
32	0.431(2)	0.425(2)	0.428(2)
64	0.47(2)	0.47(2)	0.47(2)

Table C.6: Continuum limit results for $c_{\mathrm{A}}^{(3)}$ at tree level.

	$c_{\mathrm{A}}^{(4)} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_1 = 0.5$ $\theta_2 = 1.0$	$\theta_1 = 1.0$ $\theta_2 = 1.5$	$\theta_1 = 1.5$ $\theta_2 = 0.5$
4	0.8425237(5)	0.74269443(5)	0.7822060(5)
8	0.9566813(8)	0.9387427(8)	0.9458444(8)
12	0.980624(3)	0.979234(3)	0.979772(3)
16	0.98908(2)	0.99225(2)	0.99099(2)
20	0.99303(7)	0.99763(8)	0.99581(8)
24	0.9952(3)	1.0002(3)	0.9982(3)
28	0.9967(9)	1.0017(9)	0.9997(9)
32	0.9981(3)	1.003(3)	1.001(3)
64	1.00(3)	1.01(3)	1.00(3)

Table C.7: Continuum limit results for $c_{\mathrm{A}}^{(4)}$ at tree level. For $z = 64$ I used the last 45 lattices for the continuum limit extrapolation.

	$c_{\mathrm{A}}^{(5)} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_{\mathrm{l}} = 0.5$ $\theta_{\mathrm{h}} = 1.0$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.0$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.5$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.0$
4	0.41797852(3)	0.5070717(3)	0.5731924(3)
8	0.4101258(5)	0.4901318(5)	0.5432664(5)
12	0.424391(2)	0.488243(2)	0.528081(2)
16	0.43676(1)	0.48907(1)	0.52057(1)
20	0.44611(5)	0.49023(5)	0.51619(5)
24	0.4532(2)	0.4913(2)	0.5134(2)
28	0.4588(6)	0.4923(6)	0.5114(6)
32	0.463(1)	0.493(2)	0.510(2)
64	0.49(2)	0.50(2)	0.51(2)

Table C.8: Continuum limit results for $c_{\mathrm{A}}^{(5)}$ at tree level. For $z = 64$ I used the last 28 lattices for the continuum limit extrapolation.

	$c^{(6)}_{\mathrm{A}_k} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_{\mathrm{l}} = 0.5$ $\theta_{\mathrm{h}} = 1.0$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.0$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.5$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.0$
4	-1.839959(1)	-0.9790563(8)	-0.8994113(7)
8	-1.167297(1)	-0.940228(1)	-0.887626(1)
12	-1.071434(4)	-0.945054(4)	-0.905741(4)
16	-1.03962(2)	-0.95265(3)	-0.92151(2)
20	-1.0252(1)	-0.9591(1)	-0.9334(1)
24	-1.0175(4)	-0.9642(4)	-0.9423(4)
28	-1.013(1)	-0.968(1)	-0.949(1)
32	-1.011(4)	-0.972(4)	-0.955(4)
64	-1.003(4)	-0.98(4)	-0.97(4)

Table C.9: Continuum limit results for $c^{(6)}_{\mathrm{A}_k}$ at tree level. For $z = 64$ I used the last 22 lattices for the continuum limit extrapolation.

	$\ln Z^{\mathrm{HQET}}_{\mathrm{A}_k}$		
z	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$
4	0.1613271(3)	0.1762989(3)	0.2027507(4)
8	0.0526762(2)	0.0569133(3)	0.0630299(4)
12	0.0249970(4)	0.0269194(5)	0.0294803(6)
16	0.014455(3)	0.015544(3)	0.016941(4)
20	0.00939(1)	0.01009(1)	0.01097(1)
24	0.00657(3)	0.00706(4)	0.00766(4)
28	0.00483(9)	0.0052(1)	0.0056(1)
32	0.0036(3)	0.0039(3)	0.0042(3)
64	0.0006(11)	0.0007(13)	0.0007(16)

Table C.10: Continuum limit results for $\ln Z^{\mathrm{HQET}}_{\mathrm{A}_k}$ at tree level.

	$\ln Z_{\mathrm{A}_k}^{\mathrm{stat}}$		
z	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$
4	-0.3038610(1)	-0.3662650(1)	-0.4579340(1)
8	-0.1433480(1)	-0.1717160(1)	-0.2153740(1)
12	-0.0931618(3)	-0.110893(3)	-0.1383350(3)
16	-0.0689775(1)	-0.081766(1)	-0.101554(1)
20	-0.054761(5)	-0.064732(5)	-0.08014(6)
24	-0.04541(2)	-0.05357(2)	-0.06617(2)
28	-0.03882(5)	-0.04572(5)	-0.05637(6)
32	-0.0340(2)	-0.03997(2)	-0.0492(2)
64	-0.01701(6)	-0.01989(6)	-0.02431(6)

Table C.11: Continuum limit results for $\ln Z_{\mathrm{A}_k}^{\mathrm{stat}}$ at tree level.

	$c_{\mathrm{V}}^{(1)} \cdot \tilde{m}_{\mathrm{q}}$		
	$\theta_1 = 0.5$	$\theta_1 = 1.0$	$\theta_1 = 1.5$
z	$\theta_2 = 1.0$	$\theta_2 = 1.5$	$\theta_2 = 0.5$
4	0.8762832(4)	0.8347205(4)	0.8511706(4)
8	0.7385085(6)	0.7468982(7)	0.7435794(6)
12	0.667734(2)	0.680238(3)	0.675276(3)
16	0.62865(1)	0.64027(1)	0.63566(1)
20	0.60420(2)	0.61444(7)	0.61039(6)
24	0.5875(2)	0.5966(2)	0.5930(2)
28	0.5756(7)	0.5836(8)	0.5804(7)
32	0.5677(2)	0.574(2)	0.571(2)
64	0.54(3)	0.54(3)	0.54(3)

Table C.12: Continuum limit results for $c_{\mathrm{V}}^{(1)}$ at tree level.

	$c_V^{(2)} \cdot \tilde{m}_q$		
z	$\theta_l = 0.5$ $\theta_h = 0.5$ $\theta'_h = 1.0$	$\theta_l = 0.5$ $\theta_h = 1.0$ $\theta'_h = 1.5$	$\theta_l = 1.0$ $\theta_h = 0.5$ $\theta'_h = 1.5$
4	0.7336588(6)	1.421981(1)	0.5610778(6)
8	0.5486209(8)	0.7571710(9)	0.5301029(9)
12	0.518642(3)	0.647043(3)	0.520664(3)
16	0.50928(2)	0.60286(2)	0.51589(2)
20	0.50530(7)	0.57910(8)	0.51296(9)
24	0.5033(3)	0.5643(3)	0.5110(3)
28	0.5023(8)	0.5543(9)	0.5010(1)
32	0.502(3)	0.547(3)	0.509(3)
64	0.51(3)	0.53(3)	0.51(4)

Table C.13: Continuum limit results for $c_V^{(2)}$ at tree level.

	$\ln Z_{V_0}^{\mathrm{HQET}}$		
z	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$
4	0.1613271(2)	0.1762989(3)	0.2027507(3)
8	0.0526762(2)	0.0569133(2)	0.0630299(2)
12	0.0249970(5)	0.0269194(5)	0.0294803(6)
16	0.014455(2)	0.015544(3)	0.016941(3)
20	0.009389(8)	0.010088(9)	0.01097(1)
24	0.00657(2)	0.00706(3)	0.00766(3)
28	0.00483(7)	0.00519(8)	0.0056(1)
32	0.0036(2)	0.0039(2)	0.0042(3)
64	0.0009(9)	0.001(1)	0.001(1)

Table C.14: Continuum limit results for $\ln Z_{V_0}^{\mathrm{HQET}}$ at tree level. For $z = 64$ I used the last 15 lattices for the continuum limit extrapolation.

	$\ln Z_{\mathrm{V}_0}^{\mathrm{stat}}$		
z	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$
4	-0.3038610(6)	-0.36626500(8)	-0.4579340(1)
8	-0.1433480(5)	-0.17171600(6)	-0.21537400(8)
12	-0.0931618(3)	-0.1108930(2)	-0.1383350(3)
16	-0.068978(1)	-0.081766(1)	-0.101554(2)
20	-0.054761(5)	-0.064732(5)	-0.080143(6)
24	-0.04541(2)	-0.05357(2)	-0.06617(2)
28	-0.03882(5)	-0.04572(540)	-0.05637(6)
32	-0.0340(2)	-0.0400(2)	-0.0492(2)
64	-0.01701(6)	-0.01989(6)	-0.02431(6)

Table C.15: Continuum limit results for $\ln Z_{\mathrm{V}_0}^{\mathrm{stat}}$ at tree level. For $z = 64$ I used the last 15 lattices for the continuum limit extrapolation.

	$c_{\mathrm{V}}^{(3)} \cdot \tilde{m}_{\mathrm{q}}$		
	$\theta_1 = 0.5$	$\theta_1 = 1.0$	$\theta_1 = 1.5$
z	$\theta_2 = 1.0$	$\theta_2 = 1.5$	$\theta_2 = 0.5$
4	0.30685691(7)	0.2515114(1)	0.27559450(9)
8	0.4029903(1)	0.3657687(1)	0.3819653(1)
12	0.4358646(4)	0.4093880(6)	0.4209094(5)
16	0.452210(3)	0.431788(4)	0.440675(3)
20	0.46196(1)	0.44536(1)	0.45259(2)
24	0.46844(5)	0.45448(6)	0.46055(6)
28	0.4731(2)	0.4611(2)	0.4663(2)
32	0.4768(5)	0.4662(7)	0.4708(6)
64	0.494(6)	0.489(8)	0.491(7)

Table C.16: Continuum limit results for $c_{\mathrm{V}}^{(3)}$ at tree level.

	$c_{\mathrm{V}}^{(4)} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_1 = 0.5$ $\theta_2 = 1.0$	$\theta_1 = 1.0$ $\theta_2 = 1.5$	$\theta_1 = 1.5$ $\theta_2 = 0.5$
4	-0.8867710(4)	-0.9144693(5)	-0.9018590(4)
8	-0.9613954(6)	-1.0167051(8)	-0.9915286(7)
12	-0.977529(2)	-1.025460(3)	-1.003639(3)
16	-0.98434(1)	-1.02459(2)	-1.00626(1)
20	-0.98807(7)	-1.02235(9)	-1.00674(8)
24	-0.9905(2)	-1.0202(3)	-1.0066(3)
28	-0.9922(8)	-1.018(1)	-1.0065(9)
32	-0.994(2)	-1.017(3)	-1.007(3)
64	-1.00(2)	-1.01(3)	-1.00(3)

Table C.17: Continuum limit results for $c_{\mathrm{V}}^{(4)}$ at tree level. For $z = 64$ I used the last 40 lattices for the continuum limit extrapolation.

	$c_{\mathrm{V}}^{(5)} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_{\mathrm{l}} = 0.5$ $\theta_{\mathrm{h}} = 1.0$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.0$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.5$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.0$
4	-0.4287803(8)	0.0541141(4)	0.2360660(2)
8	-0.047947(1)	0.253984(5)	0.3658552(2)
12	0.121362(5)	0.333501(2)	0.4113223(9)
16	0.21175(3)	0.37454(1)	0.433935(6)
20	0.2675(1)	0.39945(6)	0.44741(3)
24	0.3053(5)	0.4162(2)	0.4564(1)
28	0.333(2)	0.4282(7)	0.4628(4)
32	0.353(5)	0.437(2)	0.468(1)
64	0.43(7)	0.47(3)	0.49(1)

Table C.18: Continuum limit results for $c_{\mathrm{V}}^{(5)}$ at tree level.

	$c_{\mathrm{V}}^{(6)} \cdot \tilde{m}_{\mathrm{q}}$		
z	$\theta_{\mathrm{l}} = 0.5$ $\theta_{\mathrm{h}} = 1.0$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.0$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.5$	$\theta_{\mathrm{l}} = 1.5$ $\theta_{\mathrm{h}} = 0.5$ $\theta'_{\mathrm{h}} = 1.0$
4	-0.251135(4)	0.820080(3)	1.167135(3)
8	-0.747630(6)	0.041778(4)	0.308730(4)
12	-0.87575(2)	-0.28934(2)	-0.08921(2)
16	-0.9251(1)	-0.4627(1)	-0.3042(1)
20	-0.9492(4)	-0.5685(5)	-0.4377(5)
24	-0.9629(2)	-0.640(2)	-0.5285(2)
28	-0.972(8)	-0.691(6)	-0.594(6)
32	-0.98(2)	-0.73(2)	-0.64(2)
64	-1.0(3)	-0.8(2)	-0.8(2)

Table C.19: Continuum limit results for $c_{\mathrm{V}}^{(6)}$ at tree level. For $z = 64$ and the combinations $(1.0, 0.5, 1.5)$ and $(1.5, 0.5, 1.0)$ I used the last 40 lattices for the continuum limit extrapolation.

	$\ln Z_{\mathrm{V}}^{\mathrm{HQET}}$		
z	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$
4	0.00770240(1)	0.02267275(5)	0.03329674(8)
8	0.00221097(1)	0.00644784(3)	0.00898783(6)
12	0.0009998(2)	0.0029216(2)	0.0040528(2)
16	0.0005638(9)	0.0016524(9)	0.002296(1)
20	0.000358(4)	0.001057(4)	0.001473(4)
24	0.00024(1)	0.00072(1)	0.00102(1)
28	0.00014(4)	0.00050(4)	0.00071(5)
32	0.0001(1)	0.0003(1)	0.0005(1)
64	-0.00022(8)	-0.0002(2)	-0.0003(4)

Table C.20: Continuum limit results for $\ln Z_{\mathrm{V}}^{\mathrm{HQET}}$ at tree level. For $z = 64$ I used the last 15 lattices for the continuum limit extrapolation.

	$\ln Z_{\mathrm{V}}^{\mathrm{stat}}$		
z	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$
4	0.00869723(1)	0.02608480(1)	0.03960090(2)
8	0.00585441(8)	0.0189434(1)	0.03207610(9)
12	0.00423805(15)	0.0140275(1)	0.0245735(1)
16	0.0033032(9)	0.0110474(8)	0.0196550(8)
20	0.002700(4)	0.009088(4)	0.016312(3)
24	0.00227(1)	0.00770(1)	0.01391(1)
28	0.00194(4)	0.00666(4)	0.01210(4)
32	0.0016(1)	0.0058(1)	0.0106(1)
64	0.00067(5)	0.00283(6)	0.00538(5)

Table C.21: Continuum limit results for $\ln Z_{\mathrm{V}}^{\mathrm{stat}}$ at tree level. For $z = 64$ I used the last 15 lattices for the continuum limit extrapolation.

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