N=1 SU(2) SYM Theory on the Lattice with Light Dynamical Wilson Gluinos

Kamel Demmouche

Institut für Theoretische Physik - WWU Münster.

In collaboration with:
F. Farchioni, A. Ferling, I. Montvay, G. Münster, E.E. Scholz, J. Wuilloud

Münster, March 27th 2008

WESTFÄLISCHE WILHELMS-UNIVERSITÄT MÜNSTER

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Outline:

- Intro & Low energy features of N=1 SYM in the continuum
- N=1 SYM on the Lattice
- Numerical results & SYM Spectrum
- Summary & Conclusion

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Motivations for SUSY

- Stabilization of the *hierarchy problem* in SM
- In SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ EW and Strong couplings do not match
- Discovery of SUSY in *LHC* is an "indirect" discovery of the *Higgs boson*
- SUSY is a necessary ingredient in *String Theory*
- What is the dark matter made of ? LSP (light susy particle)
- ...

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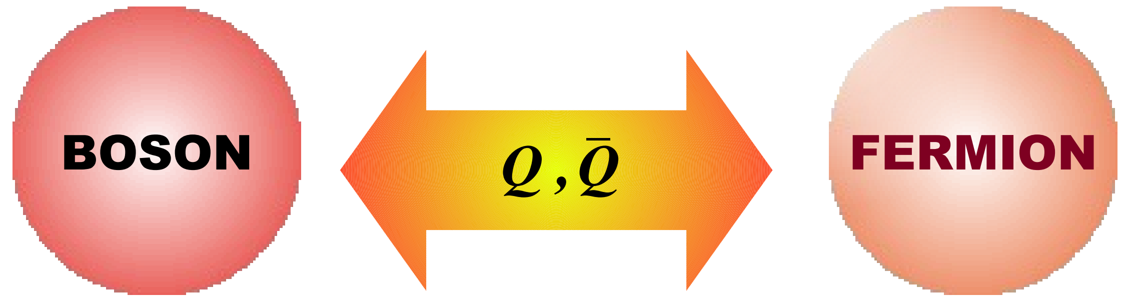
-->> the goal of this study <<--

Investigation of *low-energy* dynamics of strongly coupled SUSY gauge theories

SUSY is fascinating !



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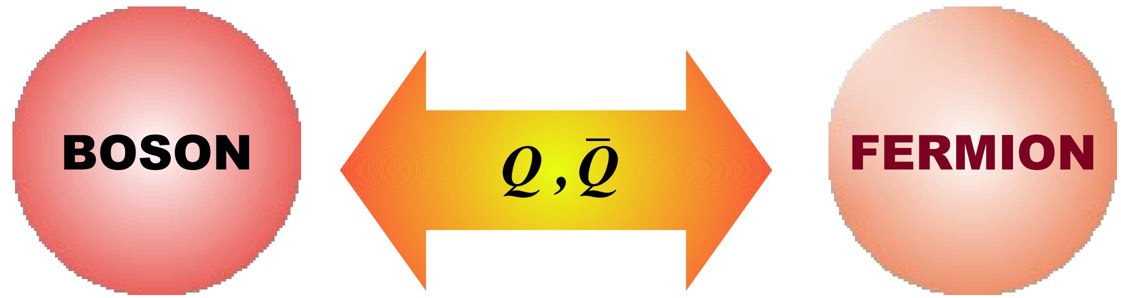
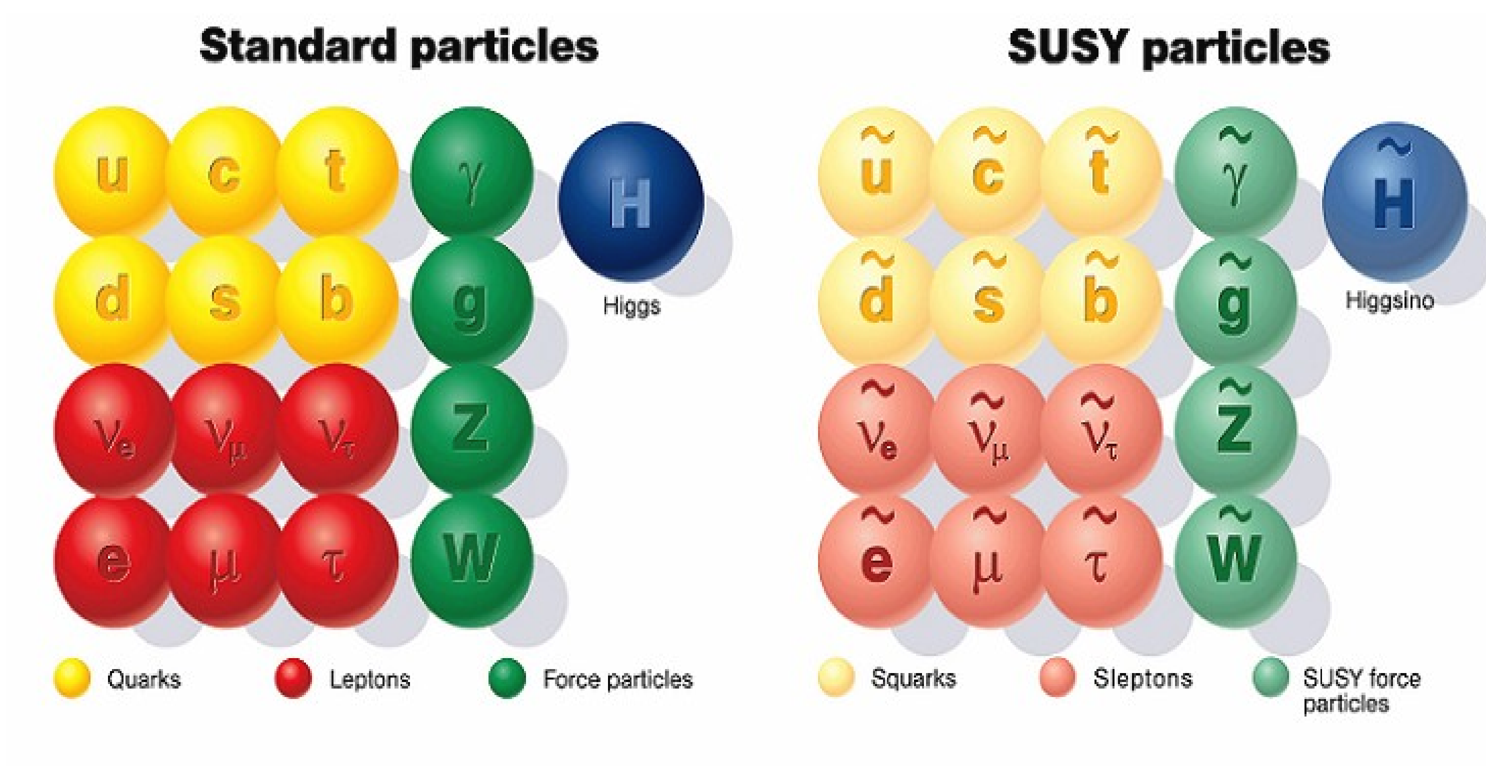
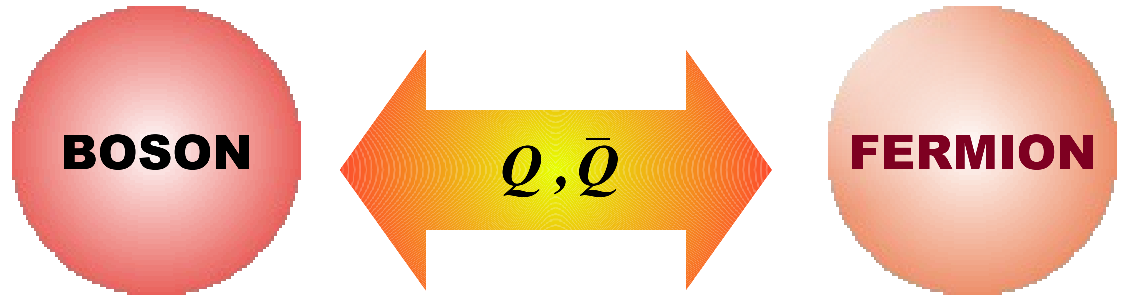


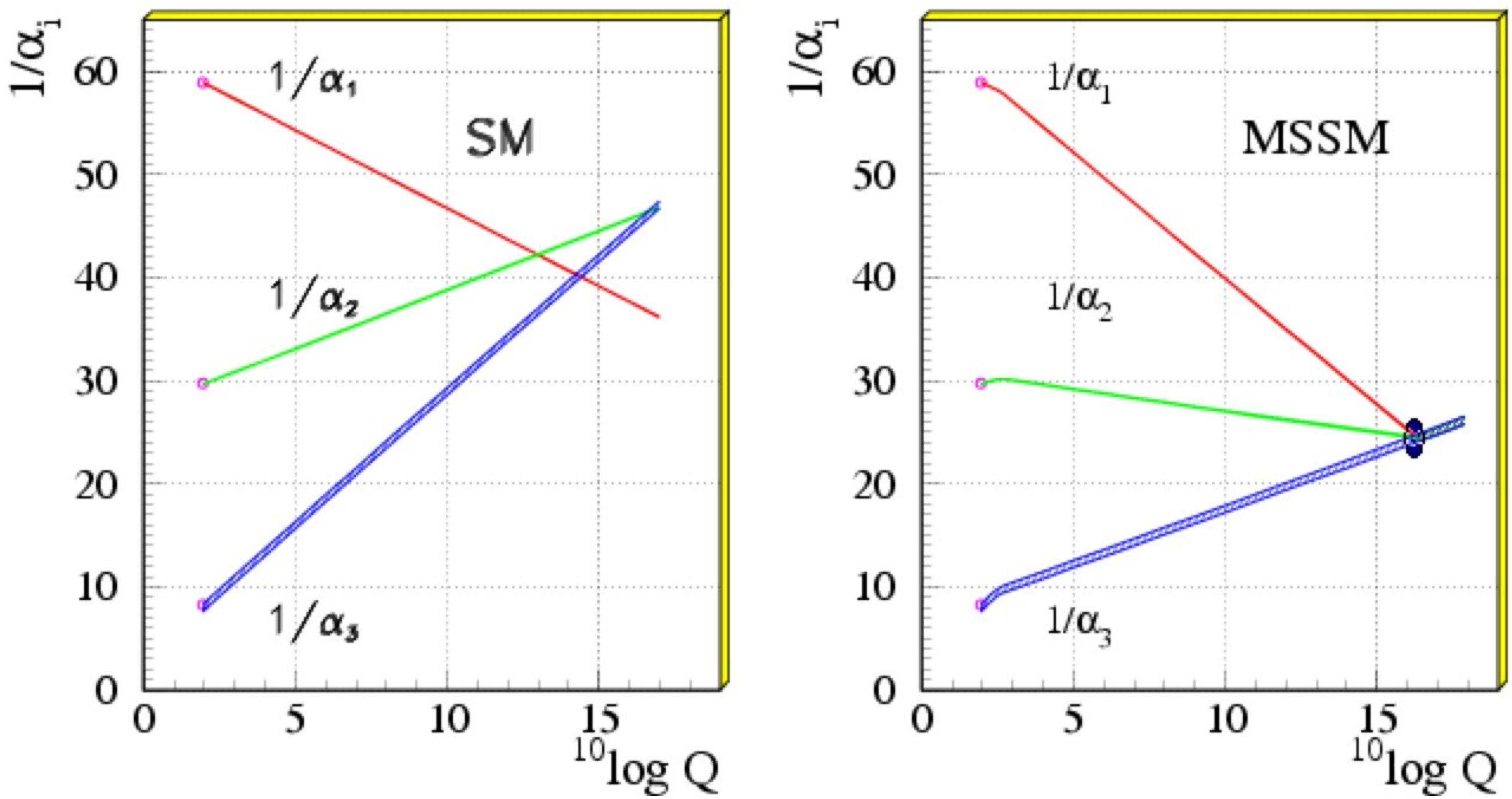
SUSY world:



Unification !

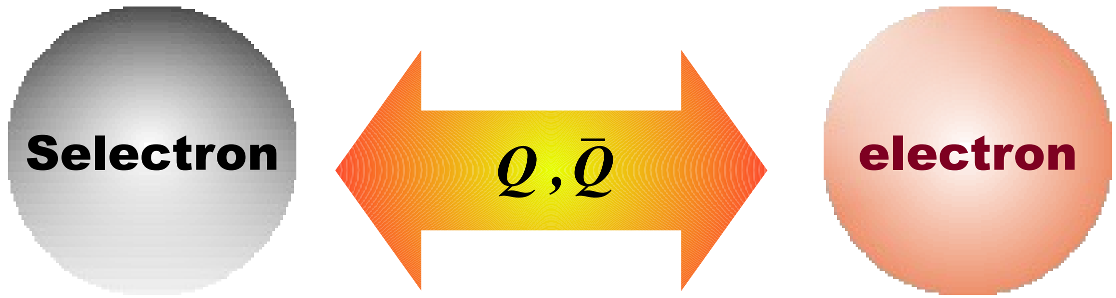


Running gauge Couplings:



[Particle Data Group]

SUSY breaking !

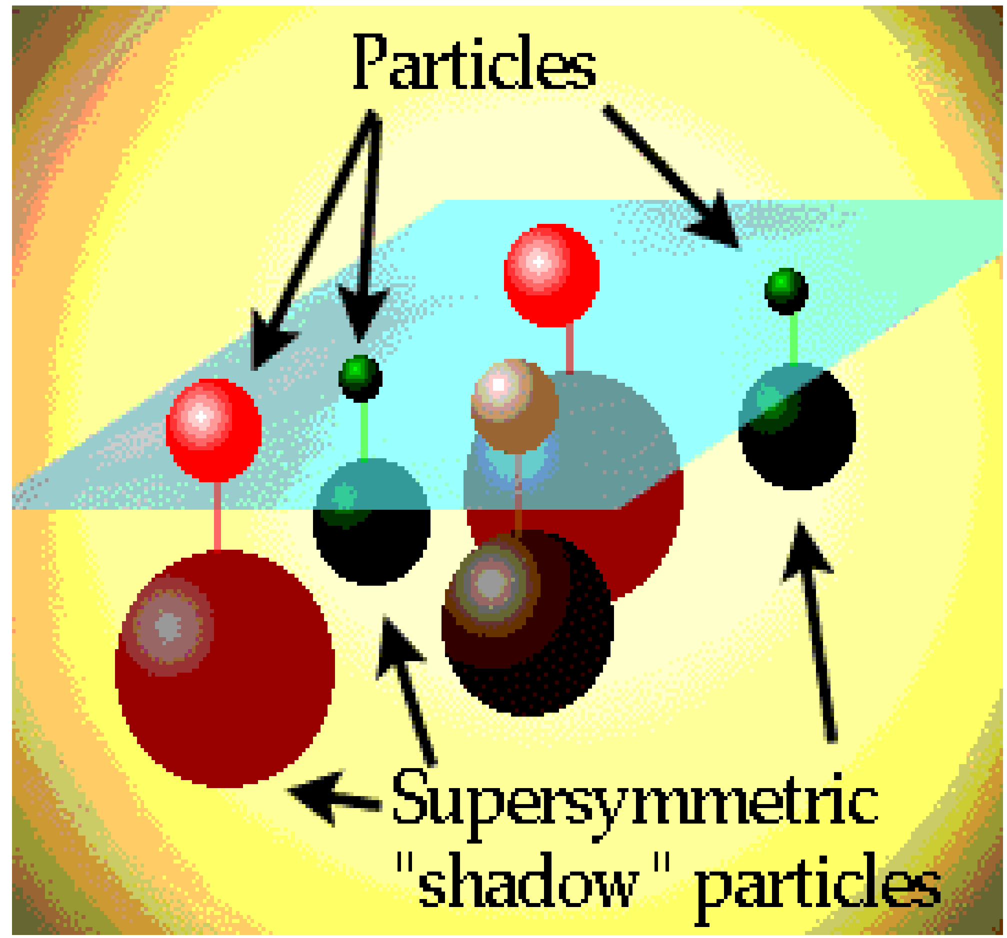


Is SUSY a symmetry of the nature ?

$$m_{\tilde{e}} \neq m_e$$

SUSY must be *broken* -> soft breaking

$$m_{\tilde{g}} \sim \mathcal{O}(1\,TeV)$$



N=1 SUSY Yang-Mills with *SU(N_c)* gauge group

$$\mathcal{L}_{SYM} = -\frac{1}{4} F^a_{\mu\nu}(x) F^{a\mu\nu}(x) + \frac{i}{2} \bar{\lambda}^a(x) \gamma^\mu \mathcal{D}_\mu \lambda^a(x) - \frac{m_{\tilde{g}}}{2} \bar{\lambda}^a \lambda^a$$

Soft breaking term

$$A^a_\mu(x) \Longleftrightarrow \lambda^a(x)$$

g $\tilde{g}$

Equivalence to
One flavor QCD
At large N_c

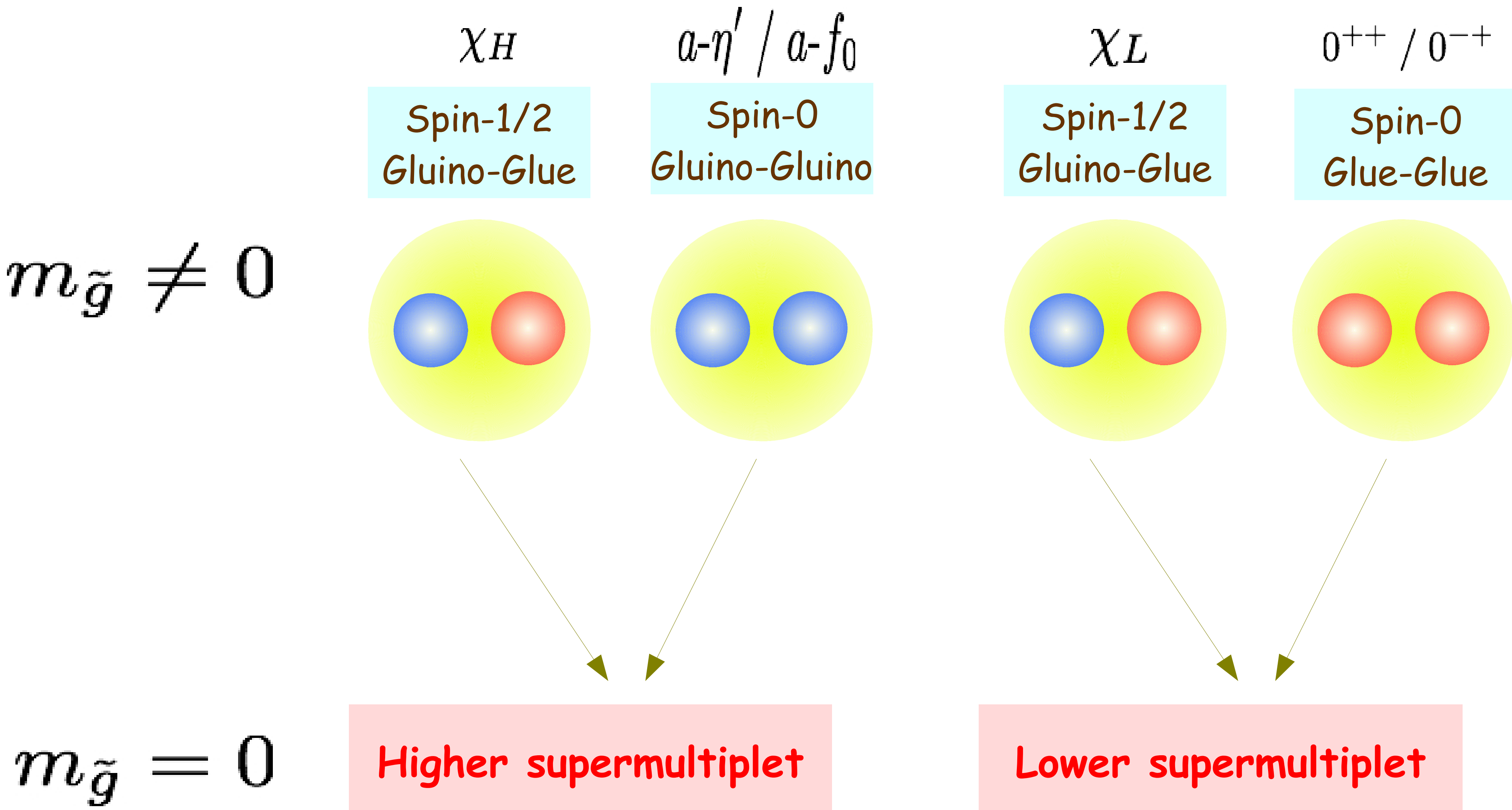
$a \in \{1, \ldots, N_c^2 - 1\}$	Adjoint representation
$\lambda^a(x)$:	Majorana spinor field $N_f = 1/2$
$F^a_{\mu\nu}(x)$:	Field strenght tensor
$\mathcal{D}_\mu$:	Covariant derivative

- Presence of anomalous global chiral symmetry: $U(1)_\lambda \Longleftrightarrow R\text{-}symmetry$
- Spontaneous discrete chiral symmetry breaking: $Z_{2N_c} \longrightarrow Z_2$

Low energy features of N=1 SYM

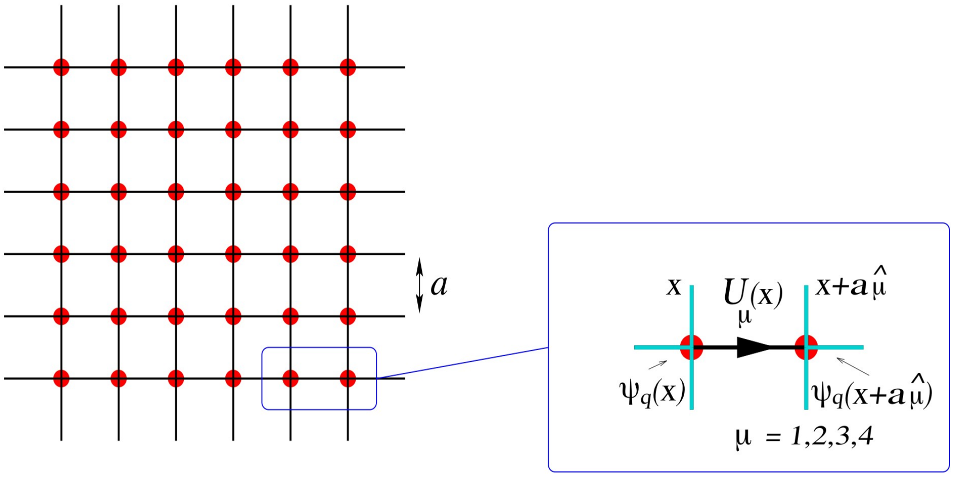
Confinement → Colorless bound states

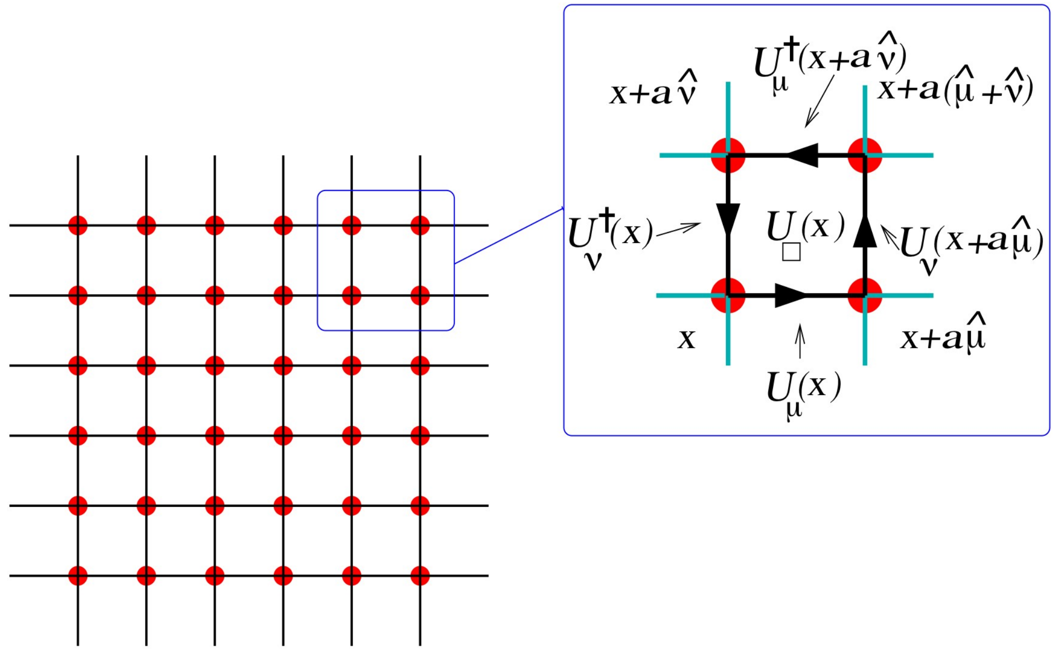
(perturbation theory cannot be applied !)



[Veneziano & Yankielowicz 82, Farrar et al. 98]

Non-perturbative methods, Lattice regularization

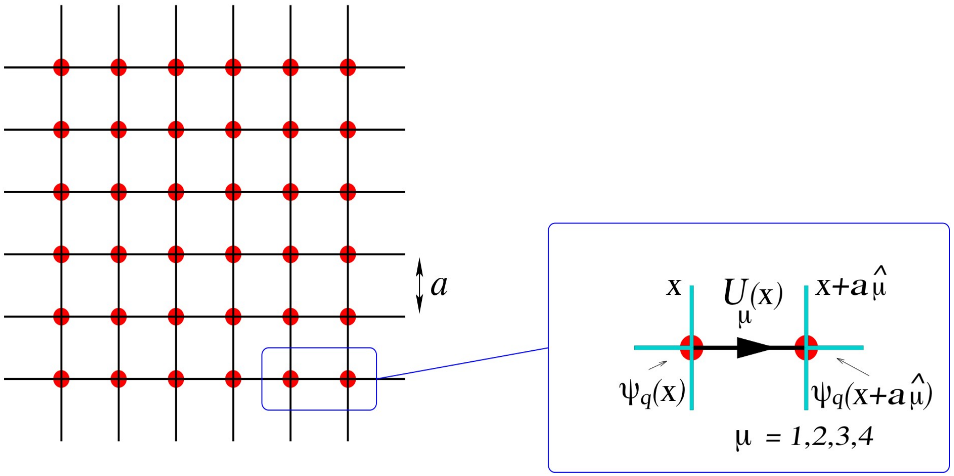


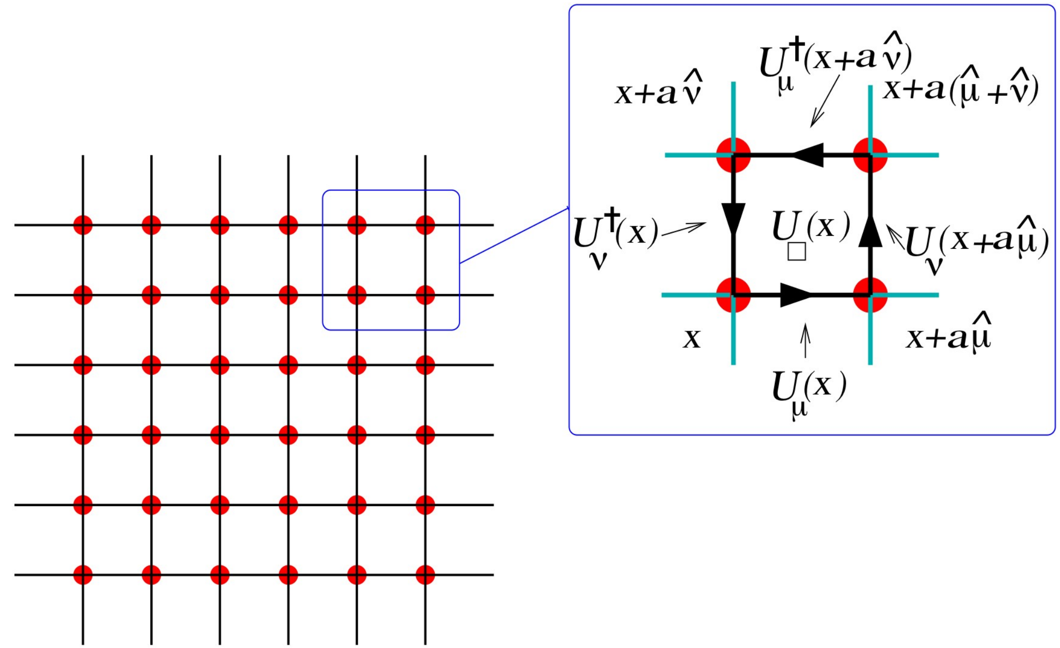


a :	lattice spacing
x :	lattice site
$U_\mu(x)$:	gauge fields (links)
$\psi(x)$:	fermion fields

Lattice volume: L x L x L x T

Non-perturbative methods, Lattice regularization





a : lattice spacing

x : lattice site

$U_\mu(x)$: gauge fields (links)

$\psi(x)$: fermion fields

Lattice volume: L × L × L × T

Problems !!

$$\{Q, \bar{Q}\} \sim P_\mu$$

-> No infinitesimal translations

-> SUSY is broken by space-time discretization

-> *Naive* discretization of SYM action leads to ***The doubling problem:*** no balance between Fermionic and bosonic degrees of freedom

Lattice action

$$S_{lattice} = S_g + S_{\tilde{g}} \stackrel{a\to 0}{\longrightarrow} S_{SYM} + \mathcal{O}(a)$$

Alternatives: Improved actions
Rectangular Wilson loops

$$S_g^W = \frac{2N_c}{g_0^2}\sum_x\sum_{\mu\nu}\left[1-\frac{1}{N_c}\mathrm{ReTr}U_{\mu\nu}\right]$$

$$S_{\tilde{g}}^W = a^4\sum_x \frac{1}{4a}\sum_{\mu=\pm 1}^{\pm 4}\left[r\bar{\lambda}_x^a\lambda_x^a - \bar{\lambda}_{x+a\hat{\mu}}^a(r+\gamma_\mu)V_\mu^{ab}(x)\lambda^b(x)\right]$$

$$+\frac{m_0}{2}\bar{\lambda}_x^a\lambda_x^a$$

$$= -\frac{1}{2}\sum_{xy} a^4\bar{\lambda}_y Q_{yx}\lambda_x$$

Relevant operator

SUSY and chiral limit

$$\text{Tuning } m_0 \to m_{0cr}(g_0) \Longleftrightarrow m_{\tilde{g}} = 0$$

[Curci & Veneziano 87]

Path integral (euclidean):

$$sgn(\mathrm{Pf}[CQ])$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int [\mathcal{D}U] \mathcal{O}[U] \sigma e^{-S_g(U) + \underbrace{\frac{1}{2} \ln[\det Q]}}$$

Importance Sampling (Monte Carlo)

$$\{U^{(i)},\ i = 1 \ldots N\}$$

$$\approx \frac{1}{N} \frac{\sum_{i=1}^{N} \sigma^{(i)} \mathcal{O}[U^{(i)}]}{\sum_{i=1}^{N} \sigma^{(i)}}$$

Polynomial approximation

$$|\det(Q)|^{1/2} = \{\det(Q^\dagger Q)\}^{1/4} \simeq \frac{1}{\det P_{n_1}^{(1)}(Q^\dagger Q)}$$

Algorithms:

- **TSMB** [Montvay 96]
- **TS-PHMC** [Montvay & Scholz 05]

$$m_u = 5 \text{ MeV} \quad m_s = 175 \text{ MeV}$$

$$m_{gluino} = 115 - 126 \text{ MeV}$$

Problems !!

- Finite a (lattice spacing) effects: O(a)-improvements --> $O(a^2)$
- Finite volume effects L
- Small dynamical fermion mass --> slowing down

Observables

Update

- Extremal eigenvalues
- Autocorrelation-times
- Wilson loops, Polyakov loops
- Physical scale r_0=0.5 fm
- Static quark potential

Analysis

- Correction factors
- Pfaffian sign
- Correlation functions

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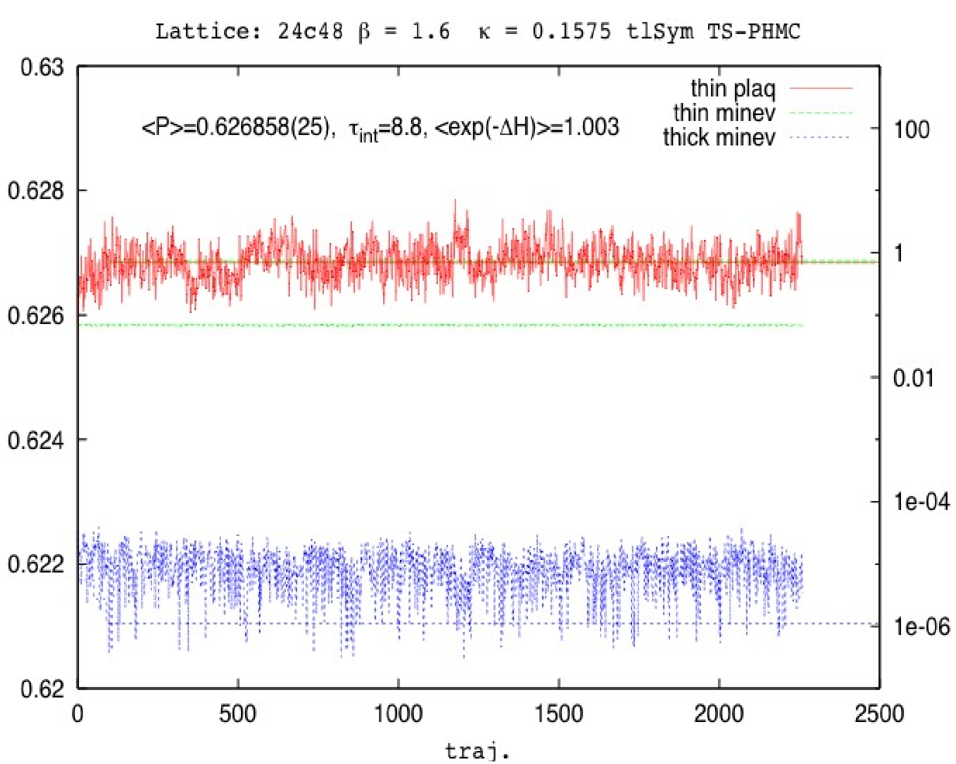
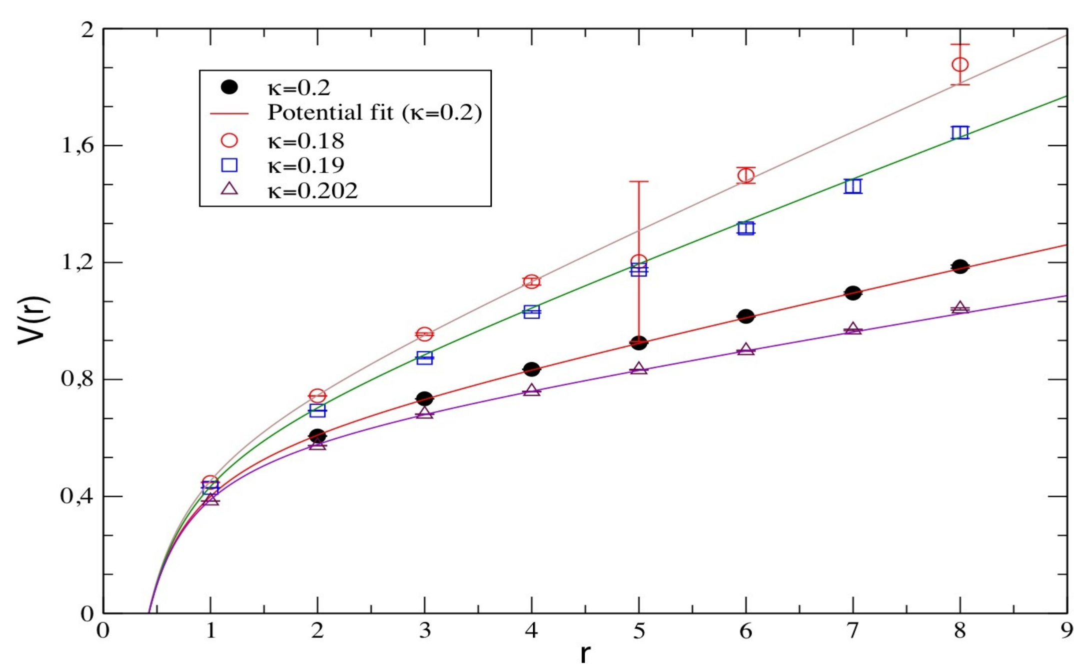
Run history



Static quark potential

Lattice: $16^3.32$ $\beta=1.6$ PHMC-tlsym



Mass determination

- Construct interpolating operators:
- Smearing techniques: Jacobi/APE
  (variational method)
- Time-slice correlation functions
- Effective mass plateaux: *t*-range
- Fitting function
- Error from jackknife/linearization
- X^2 correlated fit
- Choose *t*-range to minimize X^2

$$\text{spin:0,1/2}\ \textit{bound states}:\ \mathcal{O}_{JPC}$$

$$S_t = \frac{1}{\sqrt{V_s}} \sum_{\vec{x}} \mathcal{O}[U](\vec{x}, t)$$

$$C(\Delta t) = \langle S_t S_{t+\Delta t} \rangle - \langle S_t \rangle \langle S_{t+\Delta t} \rangle$$

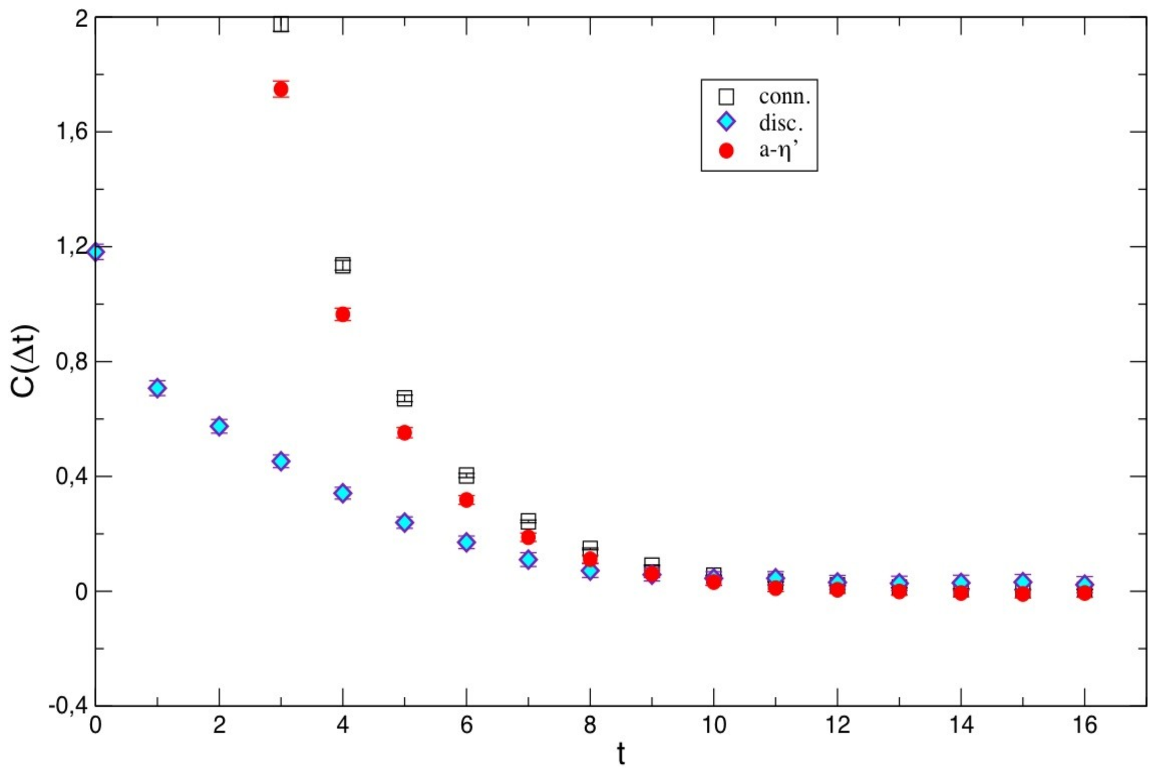
$$am_{\text{eff}} = \log \frac{C(t)}{C(t+1)}$$

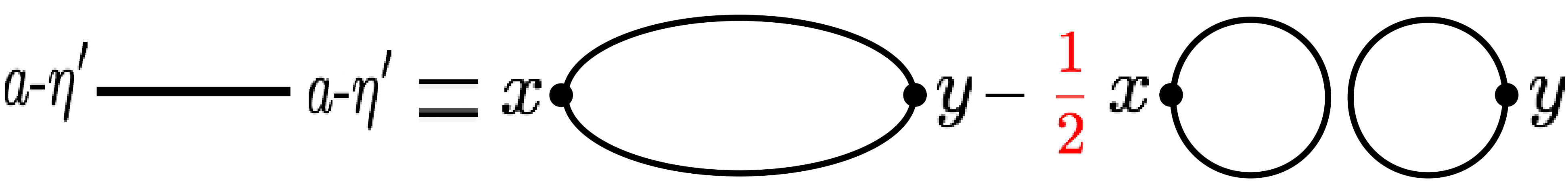
$$C(t) \to a_0^2 + \sum_{n=1} a_n^2 e^{-E_n t} \pm a_n^2 e^{-E_n (T-t)}$$

$$\boxed{m = E_1}$$

Correlators

Lattice: $16^3.32$ $\beta=1.6$ $\kappa=0.2$ TS-PHMC





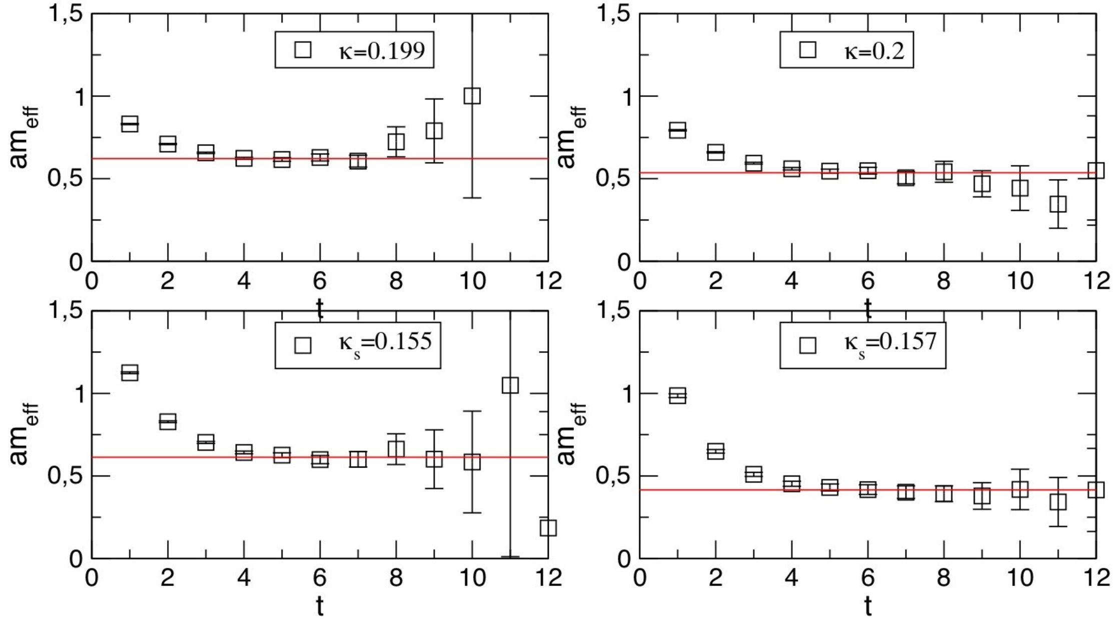
Connected (a-π)

Disconnected
SET / IVST

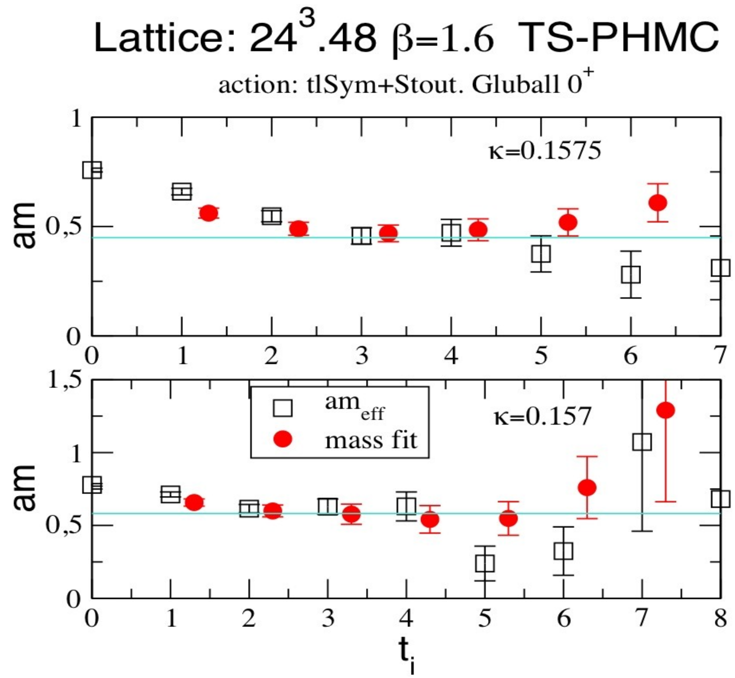
Bound states masses

Spin-0
Pseudo-scalar
Adjoint meson
a-η'

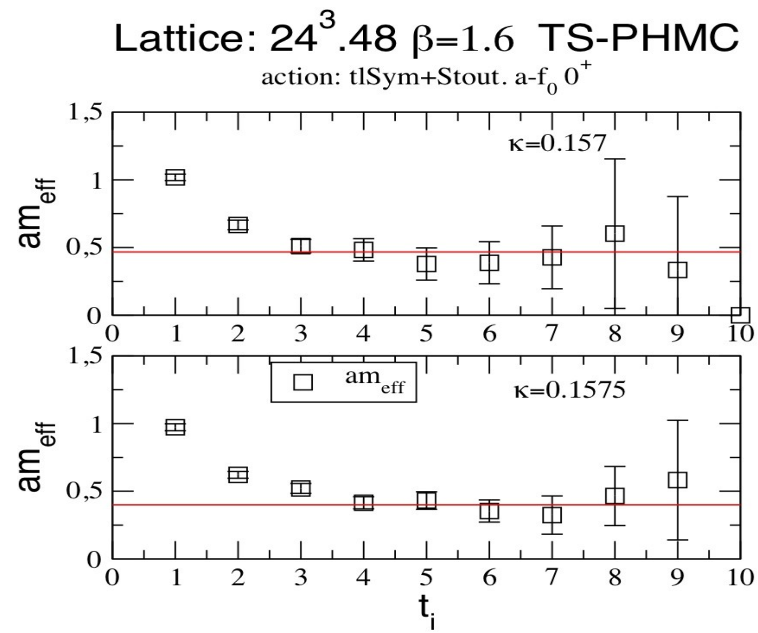
a-η'



Spin-0
Scalar
Adjoint meson
a-f_0

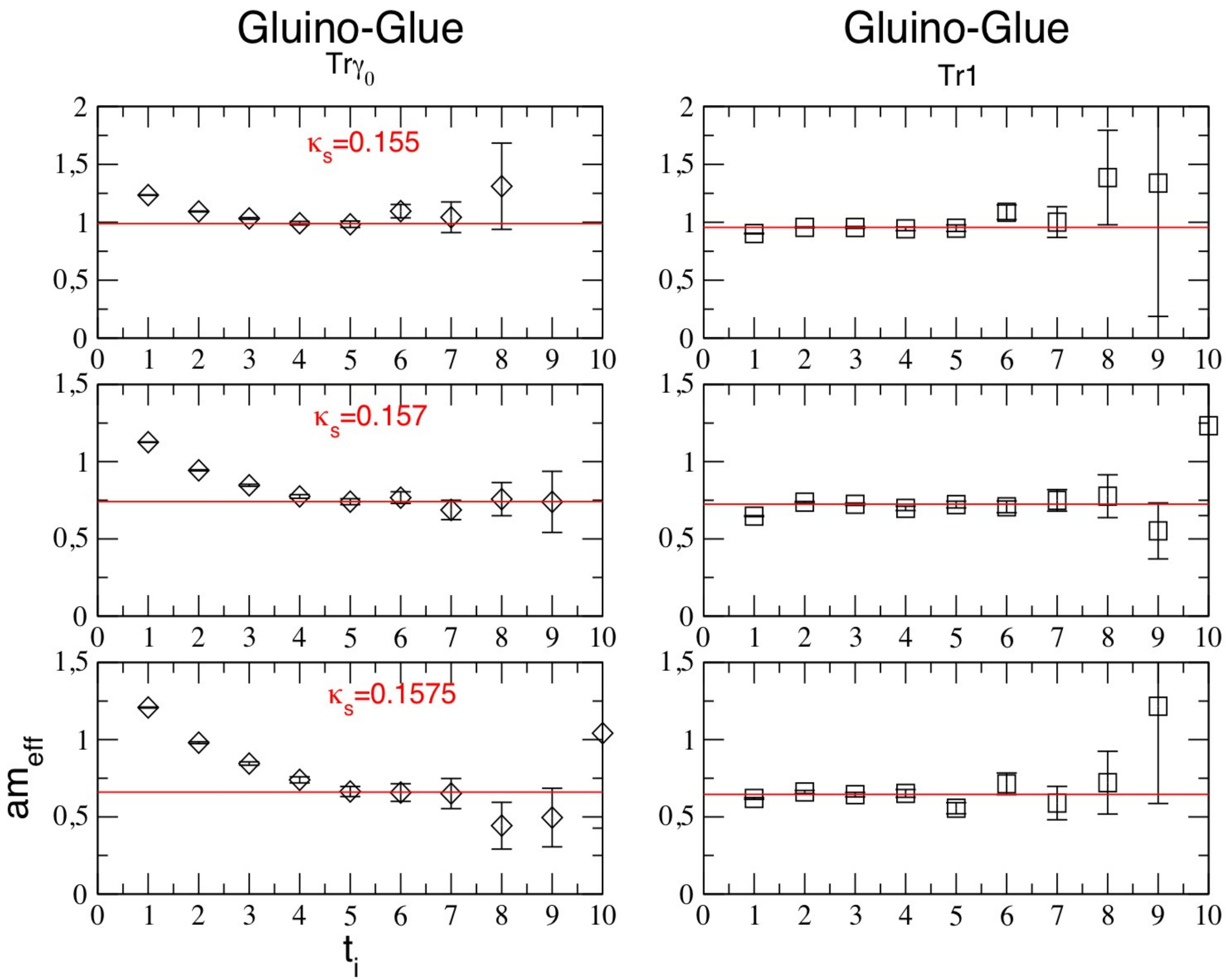


Spin-0
Scalar glueball

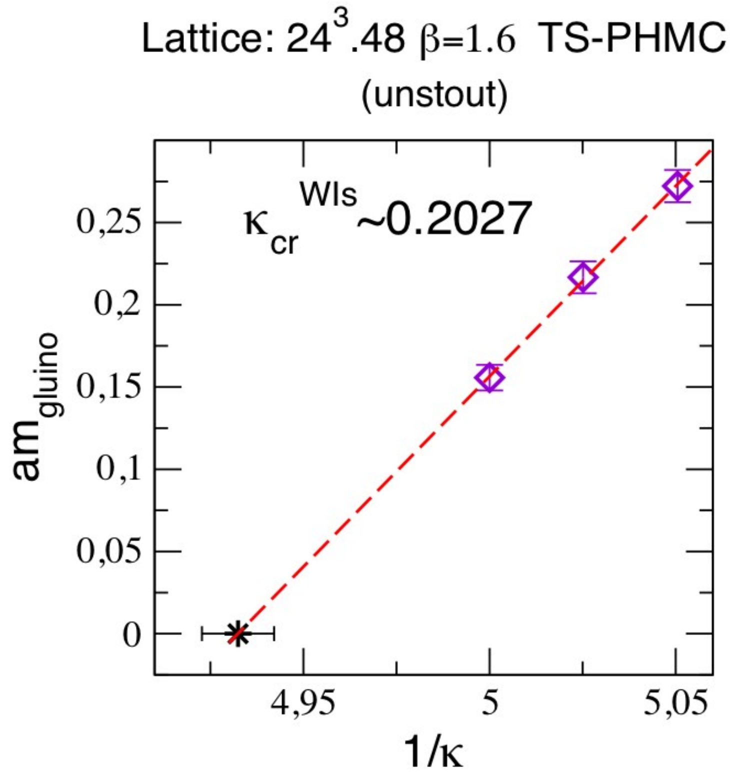


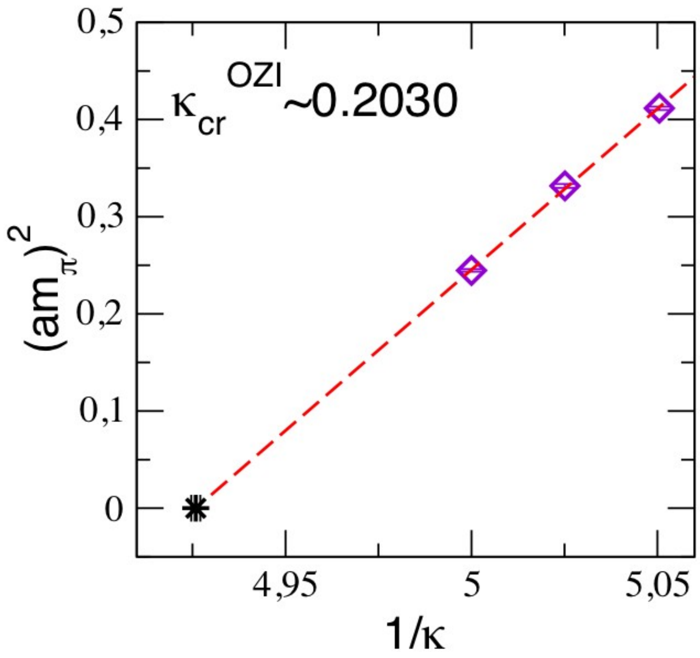
Bound states masses

Spin-1/2
Gluino-Glue



Chiral ~ SUSY limit





SUSY Ward-Identities

(renormalized gluino mass)

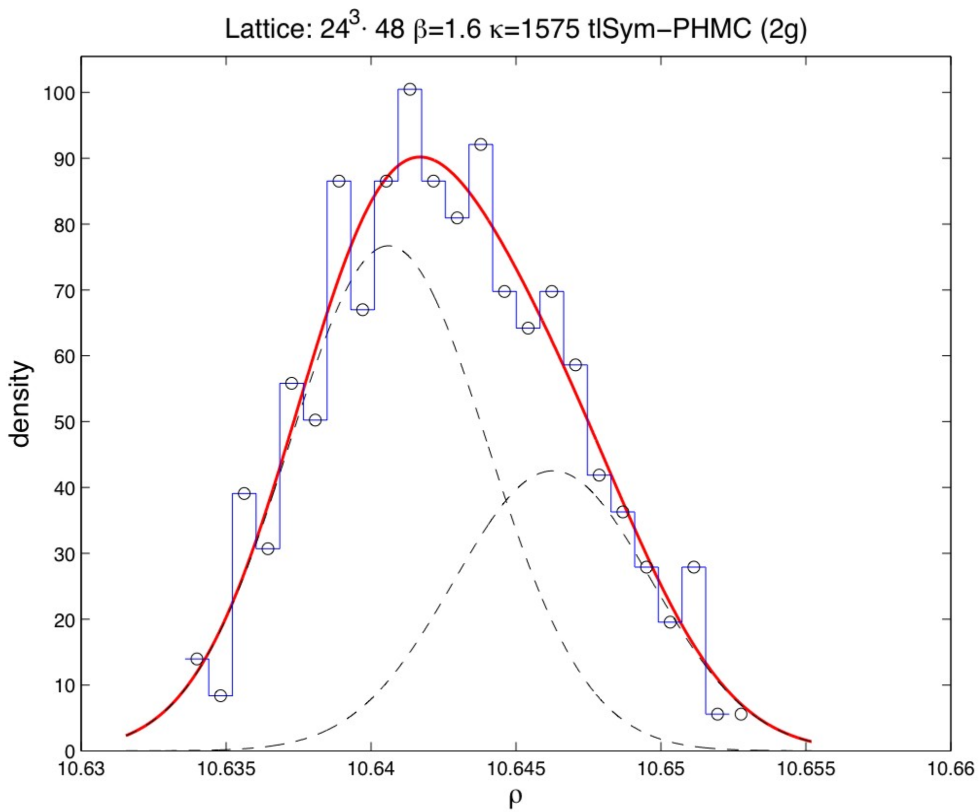
OZI-arguments

$$am_{\tilde{g}} Z_S = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_{cr}}\right)$$

$$(am_\pi)^2 \simeq A\left(\frac{1}{\kappa} - \frac{1}{\kappa_{cr}}\right)$$

κ_{cr} : Critical hopping parameter.

Chiral transition



Spontaneous discrete chiral symmetry breaking

$Z_4 \longrightarrow Z_2$

Two vacua:

$$\langle\bar\lambda\lambda\rangle_+ > 0\,;\ \langle\bar\lambda\lambda\rangle_- < 0$$

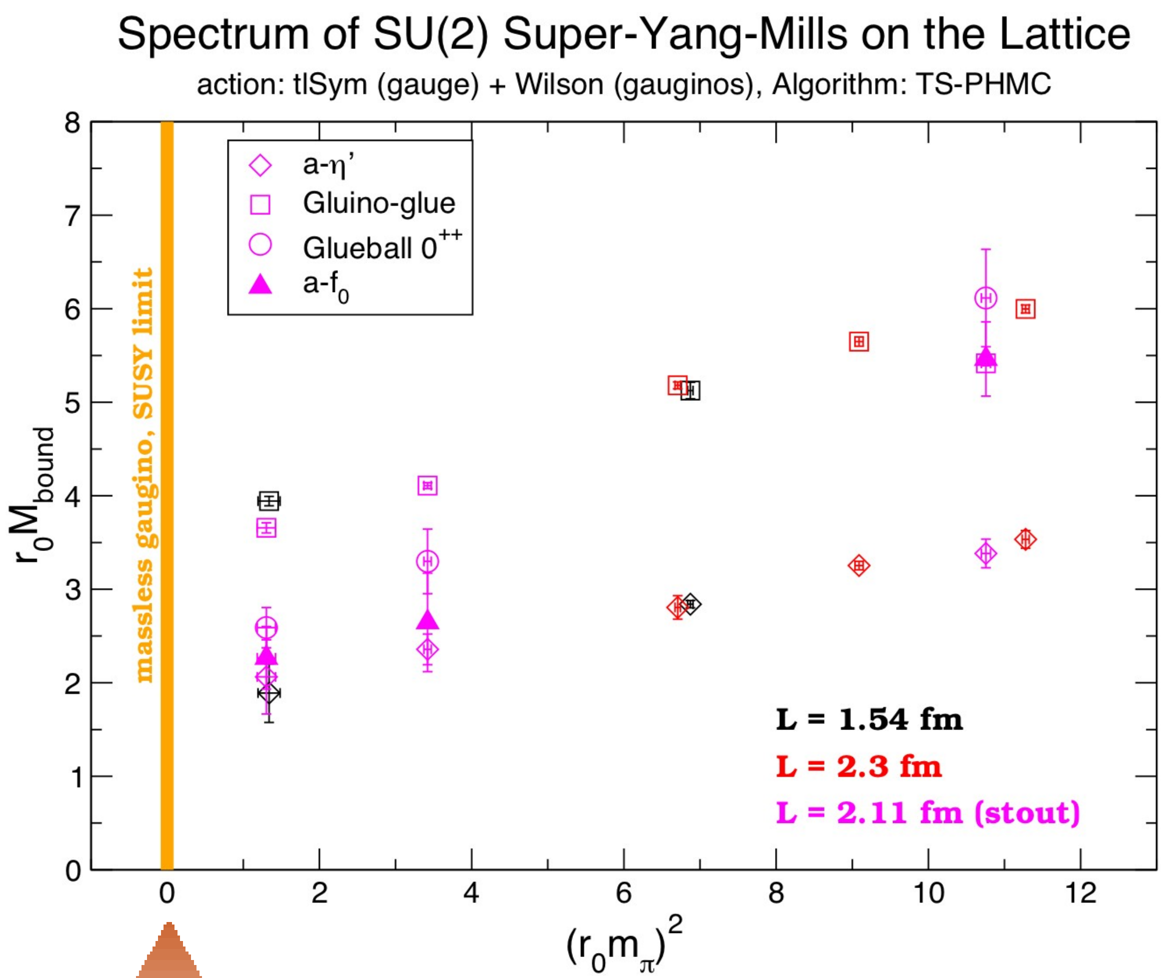
At zero gluino mass

First order phase transition

Renormalized condensate:

$$\langle\bar\lambda\lambda\rangle_{R(\mu)} = Z(a\mu)[\langle\bar\lambda\lambda\rangle - b_0(a\mu)]$$

Bound states spectrum



$\propto m_{\tilde{g}}$

SUSY + O(a) effects

- Lattice simulation $16^3.32$ and $24^3.48$, $a \sim 0.1$ fm

$$m_{a\text{-}\pi} \sim 460 \text{ MeV}$$

Spin-1/2
Gluino-Glue

$$m_{\tilde{g}g} = 1580 \text{ MeV}$$

Spin-0
Gluino-Gluino

$$m_{\tilde{g}\tilde{g}} = 760 \text{ MeV}$$

- Possible mixing in the Scalar channel (f_0 - 0^+)

Summary & outlook

- The first ***quantitative*** results of low-energy spectrum of SU(2) SYM
- Large physical volume ***L > 2 fm*** (required for spectrum studies)
- ***Finite size effects*** under control
- ***Higher statistics*** for analysis were collected
- Efficient algorithm for ***dynamical*** simulations: ***TS-PHMC***
- Small gluino mass ***m ~ 126 MeV***
- Is Gluino-Gluino and Gluino-Glue ***mass splitting*** an ***O(a) effect ! ?***
- Answer: **extrapolation** to continuum limit
- Next: apply recent **QCD** methods of spectroscopy to SYM

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THANK YOU !