

Bachelor's thesis

**FeynRules implementation of minimal
dark matter and neutrino masses**

*FeynRules Implementierung von minimaler
dunkler Materie und Neutrinomassen*

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1 Introduction

While the Standard Model of particle physics is in good accordance with experimental data, there are still some problems left. Two of those problems are dark matter and the lack of neutrino masses.

Dark matter is a - so far not directly observed - kind of matter, which introduction was motivated by astrophysics. It is a large portion of the mass of the universe.

Neutrino masses are a problem in the Standard Model, because of the nonexistence of a right-handed neutrino, which we will see later.

In this thesis I will consider a minimal extension of the Standard Model, which tackles these two problems.

Minimal extensions usually make fewer assumptions and often add fewer parameters to the standard model. Another advantage of minimal extensions is, that they allow to study a certain area isolated of other changes in the model. So while supersymmetry for example might be a more elegant solution to the problems of the Standard Model, the model in consideration adds fewer particles, fewer parameters and can be useful to get a more general insight on certain aspects under consideration.

To start the thesis, I will give more background on the motivation for Dark Matter and then proceed with the theoretical background of the standard model. In the second part I will introduce the minimal extension and explain the implementation into FeynRules I made.

When the thesis was started, the plan was to analyse the model numerically using micrOMEGAs, but this was not possible, since the FeynRules implementation unfortunately does not work properly so far.

1.1 Motivation: Dark Matter

First hints on the existence of nonluminous matter were given in calculations done by Oort and Zwicky in 1932 and 1933 respectively [1]. Oort analyzed the velocities of nearby stars and came to the conclusion, that they have 30 – 50 % less mass than implied by their velocities.

Zwicky concluded further, that clusters and galaxies required 10 to 100 times the mass, that can be calculated by their luminosity [1].

In 1974 Ostriker, Yahil and Peebles as well as Einasto, Kraasik and Saar found galaxy mass distributions to increase as a function of the radius, providing more evidence for nonluminous matter. Continuing research found Dark Matter in many different objects on different scales of the universe. An overview can be found in [1].

There are various attempts to deal with the problem of Dark Matter. [2] is a collection of articles on the topic, giving a broad overview by experts of the different fields involved. We will anyhow only be concerned with "Weak Interacting Massive Particles" (WIMP) and therefore only need [2.1], [2.2] and [2.3]. WIMPs are good dark matter candidates, because they naturally yield a correct dark matter abundance.

In most models the WIMP production is before the big bang nucleosynthesis (BBN), which is the first epoch, we have any data from. Therefore assumptions have to be made for this phase, which can vary for different cosmologic models. The assumptions are, that the WIMPs were produced thermally in a plasma, in which the entropy of matter and radiation was conserved. The WIMPs were in a thermal equilibrium with the plasma and therefore, the annihilation rate

$$\Gamma_{ann} = \langle \sigma_{ann} v \rangle n_{eq} \quad (1)$$

was the same as the WIMP-production rate, where n_{eq} is the number of WIMPs in the equilibrium. $\langle \sigma_{ann} v \rangle$ is one important measure for dark matter candidates, combining the annihilation cross-section with the velocity distribution.

As the temperature of the universe lowered, the point was reached, where the plasma temperature is below the WIMP mass and therefore the WIMP production decreased with the Boltzmann factor

$$e^{-m_\chi/T}. \quad (2)$$

This process is anyhow slow enough to keep the WIMPs in thermal equilibrium, since a lower number of WIMPs leads to a lower annihilation rate, because Γ_{ann} is proportional to n_{WIMP} .

At some point the WIMP number "freezes out", that is, when the annihilation rate is lower than the Hubble constant H , describing the expansion of space.

After that point the dark matter abundance is approximately constant. We denote it by T_{fo} and it is at $T_{fo} \approx \frac{m_\chi}{20}$, where m_χ is the WIMP mass.

It is noticeable, that the relic abundance Ωh^2 is antiproportional to σv . Higher annihilation cross sections lead to a later freeze out, where (2) suppresses the WIMP number further.

The current WIMP density can be calculated, where one has to note, that in many models, such as the one considered, WIMPs are their own antiparticles. Therefore there is no antiWIMP density to calculate.

The calculation is done, using the equations for the WIMP number and for

entropy conservation, being

$$\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle(n^2 - n_{eq}^2) \quad (3)$$

and

$$\frac{ds}{dt} = -3Hs \quad (4)$$

respectively. One can combine both equations into a single one, with the variables $Y = \frac{n}{s}$ and $x = \frac{m}{T}$, where T is the photon temperature. The result is

$$\frac{dY}{dx} = \frac{1}{3H} \frac{ds}{dx} \langle\sigma v\rangle (Y^2 - Y_{eq}^2). \quad (5)$$

The Hubble parameter can be calculated, using the Friedman equation

$$H^2 = \frac{8\pi}{3M_P^2} \rho \quad (6)$$

with the Planck mass $M_P = 1.22 \cdot 10^{19}$ GeV and the energy density

$$\rho = \frac{\pi^2}{30} g_{eff}(T) T^4, \quad (7)$$

where g_{eff} is an effective degree of freedom. The entropy density is given by an equation using another effective degree of freedom h_{eff} . It is

$$s = \frac{2\pi^2}{45} h_{eff}(T) T^3. \quad (8)$$

One can combine g_{eff} and h_{eff} to get

$$g_*^{\frac{1}{2}} = \frac{h_{eff}}{g_{eff}} \left(1 + \frac{T}{3h_{eff}} \frac{dh_{eff}}{dT} \right) \quad (9)$$

and then we can rewrite (5) to be

$$\frac{dY}{dy} = - \left(\frac{45}{\pi M_P^2} \right)^{-\frac{1}{2}} \frac{g_*^{\frac{1}{2}} m}{x^2} \langle\sigma v\rangle (Y^2 - Y_{eq}^2). \quad (10)$$

This can be solved numerically. The program used here is micrOMEGAs, which does exactly that, as described in [3].

The relevant quantity, that is used as a measure for the amount of dark matter today and that is calculated by micrOMEGAs this way, is the relic density

$$\Omega_\chi h^2 = \frac{\rho_\chi^0}{\rho_c^0} h^2, \quad (11)$$

where ρ_c^0 is the present critical density. Its value is $\rho_c^0 = 10.537 h^2$ GeV/m³. An experimental value for $\Omega_\chi h^2$ is given by the WMAP experiment. It is

$$\Omega_\chi h^2 = 0.105(8). \quad (12)$$

Both values, the one for ρ_c^0 and for Ω_χ can be found in [3].

2 Theoretical Foundation

Quantum field theory is a broad field of theoretical physics, which is used to understand Quantum mechanics on its deepest level. In this chapter, I will provide the theoretical background needed to understand the model we are examining.

I will begin with basic aspects of quantum field theory and go on to explain the usage of symmetry in QFT. We will especially have a look at the theory of the electroweak force, which is of great importance for many models for dark matter and neutrino masses.

The literature used for most parts of this chapter is [4]. For 2.2.3 I also used [6] and for chapter 2.3 I used [5]. I will also use the same conventions and notations as in [4] for most of the time.

2.1 Basics of Quantum Field Theory

For a relativistic description of quantum mechanics it is not possible to keep a description of single particles. At high enough energies, particle-antiparticle pairs will form and even at low energies, there is the possibility of pair production because of the uncertainty $\Delta E \cdot \Delta t \geq \hbar$.

This problem can be solved by taking a field point of view on quantum mechanics, which is described in this chapter.

2.1.1 Classical Field Theory

Before having a look at quantum fields, we need to learn about the classical description of fields.

We will use the Lagrangian formalism, as used in classical mechanics. In a field theory the Lagrangian L is replaced by the Lagrangian density \mathcal{L} . They are related by

$$L = \int \mathcal{L} d^3x. \quad (13)$$

The Lagrangian density is a function of a number of fields ϕ_i and their derivatives $\partial_\mu \phi_i$. According to the principle of least action, the action S has to be extremal. It is defined as the integral over time of the Lagrangian L and is therefore

$$S = \int \mathcal{L} d^4x. \quad (14)$$

Evaluating the integral leads to the field version of the Euler-Lagrange equation

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0. \quad (15)$$

The transformation from the Lagrangian to the Hamiltonian formulation can be done by

$$\mathcal{H} = \pi(\mathbf{x}) \dot{\phi}(\mathbf{x}) - \mathcal{L}, \quad (16)$$

where $\pi(\mathbf{x}) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(\mathbf{x})}$ is the conjugated momentum density to the field ϕ . \mathcal{H} is the Hamiltonian density. By integration over space, we get the Hamiltonian

$$H = \int \mathcal{H} d^3x. \quad (17)$$

While the Hamiltonian formalism is more frequently used in quantum mechanics, the Lagrangian formulation has the advantage of being Lorentz invariant and is therefore preferred in quantum field theory.

Also it is usual to refer to the Lagrangian Density simply as the Lagrangian.

2.1.2 The Klein-Gordon Field

We start with a classical real field ϕ which follows the Klein-Gordon equation

$$(\partial^\mu \partial_\mu + m^2)\phi = 0. \quad (18)$$

We will also use the momentum density $\pi(x) = \dot{\phi}(x)$.

To quantize the system, ϕ and π are promoted to operators, which obey the commutation relation

$$[\phi(\mathbf{x}), \pi(\mathbf{y})] = i\delta^{(3)}(\mathbf{x} - \mathbf{y}) \quad (19)$$

in analogy to the relation

$$[q_i, p_j] = i\delta_{ij} \quad (20)$$

for the position and momentum of a particle in quantum mechanics. The only difference is, that we need a 3-dimensional delta distribution.

We can express the Klein-Gordon field using a Fourier transformation

$$\phi(\mathbf{x}, t) = \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{x}} \phi(\mathbf{p}, t) \quad (21)$$

and write down the Klein-Gordon equation in terms of $\phi(\mathbf{p}, t)$:

$$\left(\frac{\partial^2}{\partial t^2} + (|\mathbf{p}|^2 + m^2) \right) \phi(\mathbf{p}, t) = 0. \quad (22)$$

It is identical to the equation of motion of a simple harmonic oscillator with the frequency $\omega_{\mathbf{p}} = \sqrt{|\mathbf{p}|^2 + m^2}$ and we will treat every Fourier mode as a harmonic oscillator with annihilation and creation operators $a_{\mathbf{p}}$ and $a_{\mathbf{p}}^\dagger$.

The particle solution to the harmonic oscillator is

$$\phi = \frac{1}{\sqrt{2\omega}} (a + a^\dagger) \quad (23)$$

and the field solution is written as

$$\phi(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} (a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} + a_{\mathbf{p}}^\dagger e^{-i\mathbf{p}\cdot\mathbf{x}}) \quad (24)$$

with the commutation relation

$$[a_{\mathbf{p}}, a_{\mathbf{p}'}^\dagger] = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}'). \quad (25)$$

The commutation relation allows us to investigate the statistics of the created particles. Since the creation operators commute, we can write

$$a_{\mathbf{p}}^\dagger a_{\mathbf{q}}^\dagger |0\rangle = a_{\mathbf{q}}^\dagger a_{\mathbf{p}}^\dagger |0\rangle = |\mathbf{q}, \mathbf{p}\rangle = |\mathbf{p}, \mathbf{q}\rangle \quad (26)$$

and thereby see, that both states are identical. If we set $q = p$ it is obvious, that we can not distinguish the particles. We can also add any number of particles with the same momentum. Therefore they obey Bose-Einstein statistics.

2.1.3 The Dirac Field

Dirac found his equation, when investigating the behavior of a relativistic electron. Therefore we need to have a closer look at special relativity and Lorentz invariance first.

As suggested before, we will use the Lagrange formalism for relativistic quantum mechanics. The reason is, that, if the Lagrangian is a Lorentz scalar, the resulting equations of motions are Lorentz invariant.

To get started, we consider an arbitrary Lorentz transformation

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu_\nu x^\nu \quad (27)$$

in Klein-Gordon Theory. The fields are then transformed according to

$$\phi(x) \rightarrow \phi'(x) = \phi(\Lambda^{-1}x) \quad (28)$$

which leaves the Klein-Gordon equation unchanged.

The transformation law (28) is used for fields with just one component (scalar fields).

To investigate the electron on the other hand, one needs vector fields. Under a Lorentz transformation, they are transformed by

$$V^\mu(x) \rightarrow \Lambda^\mu_\nu V^\nu(\Lambda^{-1}x) \quad (29)$$

which can be further modified by adding additional indices and factors of Λ to transform tensor fields.

We can write down the Lorentz transformation as

$$V^\alpha \rightarrow (\delta^\alpha_\beta - \frac{i}{2}\omega_{\mu\nu}(\mathcal{J}^{\mu\nu})^\alpha_\beta)V^\beta, \quad (30)$$

with the antisymmetric parameter $\omega_{\mu\nu}$ and the generators

$$(\mathcal{J}^{\mu\nu})_{\alpha\beta} = i(\delta^\mu_\alpha \delta^\nu_\beta - \delta^\mu_\beta \delta^\nu_\alpha). \quad (31)$$

Because of its antisymmetry, $\omega_{\mu\nu}$ has six independent components, which are used to generate three boosts and three rotations.

Another way to construct these operators is the Dirac algebra, consisting of four different 4×4 matrices γ^μ fulfilling the anticommutation relation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \times I_{4 \times 4}, \quad (32)$$

with the anticommutator $\{a, b\} = ab + ba$ and the 4 dimensional unit matrix $I_{4 \times 4}$.

The operators generating our Lorentz transformations are now

$$S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]. \quad (33)$$

One can use these γ -matrices to write down a Lorentz invariant first-order equation, for a four component field ψ :

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0. \quad (34)$$

It is called Dirac equation and can be written in a more convenient way using $\not{\partial} = \gamma^\mu \partial_\mu$. By multiplication with $(-i\gamma^\mu \partial_\mu - m)$ we get the Klein-Gordon

equation.

Until now we suppressed some indices, called spinor indices, referring to the so called spinor space.

The four components of our field are components in spinor space, therefore $\psi = \psi_\alpha$. The gamma matrices also have two spinor components, therefore $\gamma^\mu = \gamma_{\alpha\beta}^\mu$.

For the gamma matrices there is a relation between rotations in normal and rotations in spinor space. We can write

$$(\Lambda_{\frac{1}{2}}^{-1})_{\alpha\gamma} \gamma_{\gamma\delta}^\mu (\Lambda_{\frac{1}{2}})_{\delta\beta} = \Lambda_{\nu}^{\mu} \gamma_{\alpha\beta}^{\nu}, \quad (35)$$

where $(\Lambda_{\frac{1}{2}})_{\alpha\beta} = \exp(-\frac{i}{2}\omega_{\mu\nu} S_{\alpha\beta}^{\mu\nu})$ is the Lorentz transformation in spinor space, which is also used to transform ψ according to

$$\psi \rightarrow \Lambda_{\frac{1}{2}} \psi (\Lambda^{-1}x). \quad (36)$$

One can write down explicit forms of the γ matrices, e.g. the Weyl representation:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad (37)$$

The matrices are written in a 2×2 block form using the Pauli matrices σ^i . This representation is also called chiral representation, because it allows us to write the spinors ψ using a left- and a right-handed component, denoted by the respective indices L and R :

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \quad (38)$$

Defining a fifth γ matrix $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, we can write down the projection operators:

$$P_{\pm} \psi = \left(\frac{1 \pm \gamma^5}{2} \right) \psi = \psi_{\frac{R}{L}}. \quad (39)$$

Our next step is the quantization of the Dirac field. The Dirac field obeys the Klein-Gordon equation and therefore its free particle solutions can be written as

$$\psi(x) = u(p)e^{-ip \cdot x} \quad (40)$$

with the vector $u(p)$ representing the positive frequency solutions. One can find a more explicit form of $u(p)$ by analyzing the Dirac equation of the solution in rest frame $u(p^0)$ and then applying a Lorentz transformation to get the general solution

$$u^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \bar{\sigma}} \xi^s \end{pmatrix}. \quad (41)$$

Here we used the spinor ξ and the four component vector of Pauli matrices $\sigma = (I_{2 \times 2}, \boldsymbol{\sigma})$ with the complex conjugate $\bar{\sigma}$. $s = 1, 2$ is the spin of the solution. We can also write down negative frequency solutions

$$v^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \eta^s \\ -\sqrt{p \cdot \bar{\sigma}} \eta^s \end{pmatrix} \quad (42)$$

for the field $\psi(x) = v(p)e^{+ip \cdot x}$.

For the quantization we need be careful with the negative frequency solutions,

because imposing commutation relations on the creation operators for $u(p)$ and $v(p)$ leads to the possibility of negative energy.

The correct quantization uses anticommutation relations instead. The field operators are

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{E_{\mathbf{p}}}} \sum_s (a_{\mathbf{p}}^s u^s(p) e^{-ip \cdot x} + b_{\mathbf{p}}^{s\dagger} v^s(p) e^{ip \cdot x}) \quad (43)$$

and $\bar{\psi} = \psi^\dagger \gamma^0$ which is used instead of ψ^\dagger , because $\bar{\psi}\psi$ is a Lorentz scalar, while the $\psi^\dagger\psi$ is not.

The fields contain two different creation and annihilation operators for the solutions $u(p)$ and $v(p)$ and a sum over the possible spins.

The anticommutation rules are

$$\{a_{\mathbf{p}}^r, a_{\mathbf{p}'}^{s\dagger}\} = \{b_{\mathbf{p}}^r, b_{\mathbf{p}'}^{s\dagger}\} = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta^{rs}, \quad (44)$$

while all other anticommutators of a , b and their conjugates are zero.

The usage of anticommutation instead of commutation rules yields different many particle states. Following from $\{a_{\mathbf{p}}^{s\dagger}, a_{\mathbf{p}'}^{s\dagger}\} = 0$ we can consider

$$a_{\mathbf{p}'}^{s\dagger} a_{\mathbf{p}}^{s\dagger} |0\rangle = -a_{\mathbf{p}}^{s\dagger} a_{\mathbf{p}'}^{s\dagger} |0\rangle \quad (45)$$

and therefore

$$a_{\mathbf{p}}^{s\dagger} a_{\mathbf{p}}^{s\dagger} |0\rangle = -a_{\mathbf{p}}^{s\dagger} a_{\mathbf{p}}^{s\dagger} |0\rangle = 0. \quad (46)$$

Therefore we can deduce a version of the Pauli exclusion principle from the anticommutation rules. The Dirac particles obey Fermi-Dirac statistics.

The particles created by $a_{\mathbf{p}}^{s\dagger}$ are called fermions and the ones created by $b_{\mathbf{p}}^{s\dagger}$ are called antifermions. They have the momentum \mathbf{p} and the energy $E_{\mathbf{p}}$ as well as the spin s .

2.1.4 Propagators and Feynman Diagrams

In this section I want to provide a small insight into the calculation of feynman diagrams.

Define the Feynman propagator for the Klein-Gordon field:

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip \cdot (x-y)}, \quad (47)$$

which can be solved using contour integration around the poles $p^0 = \pm(E_{\mathbf{p}} - i\epsilon)$. It can be identified with the propagation amplitude $D_F(x-y) = \langle 0|T\phi(x)\phi(y)|0\rangle$ with the time ordering operator T . The propagator for the Dirac field is

$$\langle 0|\psi_\alpha(x)\bar{\psi}_\beta(y)|0\rangle = (i\cancel{\partial} + m)_{\alpha\beta} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}} e^{-ip \cdot (x-y)}. \quad (48)$$

If you swap $\psi_\alpha(x)$ and $\bar{\psi}_\beta(y)$ the propagator is the same, except for a sign change of the complete expression and of the argument of the exponential funtion.

The propagators are needed when calculating feynman diagrams, but first we need to have a closer look at perturbation theory.

Perturbation theory can be used for the calculation of interactions. We will have interaction terms in the Lagrangian such as

$$-\frac{\lambda}{4!}\phi^4, \quad (49)$$

which belongs to the ϕ^4 theory or

$$-e\bar{\psi}\gamma^\mu\psi A_\mu, \quad (50)$$

for different interactions of photons and electrons.

If the coupling constant of the interaction, being λ or e respectively, is small enough, we can use perturbation theory, thinking of the interactions as a correction to the result without them. We therefore divide the Hamiltonian into two parts

$$H = H_0 + H_{int}, \quad (51)$$

where H_{int} is the interaction Hamiltonian and H_0 is the part without interactions.

We can also use this division in the transition between the Schrödinger and the Heisenberg picture. There is a picture called interaction picture, which is equivalent to the Heisenberg picture for H_0 and therefore contains the leading part of the time evolution in the field.

The field in the interaction picture is

$$\phi(t, \mathbf{x}) = e^{iH_0(t-t_0)}\phi(t_0, \mathbf{x})e^{-iH_0(t-t_0)}, \quad (52)$$

with $t_0 = 0$ usually. We also define the interaction picture Hamiltonian to be

$$H_I(t) = e^{iH_0(t-t_0)}(H_{int})e^{-iH_0(t-t_0)} \quad (53)$$

We can then write the time evolution operator U as a power series

$$U(t, t_0) = 1 + (-i) \int_{t_0}^t dt_1 H_I(t_1) + (-i)^2 \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) + h.o.t. \quad (54)$$

or in a shorter form, using time ordering as

$$U(t, t_0) = T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\}. \quad (55)$$

Now we can have a look at our propagators again. The propagators (47) and (48) are using the interaction free vacuum state $|0\rangle$, but for an interacting theory, the vacuum state will be different in general. We will use $|\Omega\rangle$ as the vacuum state of the interacting theory.

The propagation amplitude in the interacting vacuum can be expressed using (55):

$$\langle \Omega | T \{ \phi(x) \phi(y) \} | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \left\{ \phi_I(x) \phi_I(y) \exp \left[-i \int_{-T}^T dt H_I(t) \right] \right\} | 0 \rangle}{\langle 0 | T \left\{ \exp \left[-i \int_{-T}^T dt H_I(t) \right] \right\} | 0 \rangle}, \quad (56)$$

with the time axis being slightly tilted into the complex and going from negative to positive infinity by $\lim_{T \rightarrow \infty(1-i\epsilon)}$.

To make things easier, we can use Wick's theorem, allowing us to rewrite time ordered terms to so called normal ordering, where we order all operators in a way, that all creation operators will be to the left of the annihilation operators. The vacuum expectation value of normal ordered terms is zero. The nonzero terms, that are left, are the contractions

$$\overline{\phi(x)\phi(y)} = \begin{cases} [\phi^+(x), \phi^-(y)] & \text{for } x^0 > y^0 \\ [\phi^+(y), \phi^-(x)] & \text{for } x^0 < y^0 \end{cases} = D_F(x-y), \quad (57)$$

where ϕ^\pm are the positive and negative frequency parts of ϕ .

So basically we will build up any process, that we want to calculate, from propagators going from one point to the other.

One can draw these processes in the form of so called Feynman diagrams and there are rules to translate the diagrams into the formulas that need to be calculated.

These rules are given in Momentum space, where in- and outgoing momenta are specified beforehand. The rules depend on the theory under consideration. In the analysis part a program is used, that calculates the Feynman rules for a given model.

Given the rules, one can calculate any process by summing up the probabilities of all constituent diagrams that are possible.

Any vertex of the diagram will give a factor $\propto g$, where g is the relevant coupling constant and since g is typically smaller than 1, diagrams with more vertices will give a smaller contribution to the resulting probability. This enables us to reduce the calculations to a small fraction of the possible diagrams, depending on the needed accuracy.

There are still lots of details left on how to calculate Feynman diagrams, especially, because there are certain divergences to be handled, but we will not go into further detail at this point.

2.2 Symmetry Groups and Gauge Symmetries

2.2.1 Noether's Theorem

According to Noether's theorem, there is a conserved quantity associated with any continuous symmetry of a system.

Continuous transformations can be build up from infinitesimal ones. An infinitesimal transformation of a field ϕ has the form

$$\phi(x) \rightarrow \phi(x) + \alpha \Delta\phi(x) \quad (58)$$

with α being an infinitesimal parameter and $\Delta\phi$ being the change of the field. A symmetry operation is an operation, that leaves the equations of motions of the system unchanged.

Therefore the condition for the transformation of the Lagrangian is

$$\mathcal{L}(x) \rightarrow \mathcal{L}(x) + \alpha \partial_\mu \mathcal{J}(x)^\mu(x) \quad (59)$$

with $\partial_\mu \mathcal{J}^\mu = \Delta\mathcal{L}$ being a 4-divergence, that does not change the resulting equations of motions.

To find the condition for the transformation of fields, we compare its effect on the Lagrangian with $\partial_\mu \mathcal{J}^\mu$ and find

$$\partial_\mu \mathcal{J}^\mu = \alpha \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial\phi)} \Delta\phi \right). \quad (60)$$

We can also formulate this condition as a conserved current

$$\partial_\mu j^\mu(x) = 0; \quad j^\mu(x) = \frac{\partial \mathcal{L}}{\partial(\partial\phi)} - \mathcal{J}^\mu \quad (61)$$

called Noether current.

Finally we can formulate this conservation as a conservation of a charge Q , defined by

$$Q = \int j^0 d^3x. \quad (62)$$

The Noether theorem is useful for quantum field theory, because it connects abstract notions of symmetry groups and transformations with more familiar conserved quantities, such as the conservation of electric charge.

2.2.2 Lie Groups

Lie Groups are continuous groups with the general infinitesimal element defined as

$$g(\alpha) = 1 + i\alpha^a T^a + \mathcal{O}(\alpha^2) \quad (63)$$

with α^a being the group parameters and T^a being the so called generators of the symmetry group.

The generators are hermitian operators and fulfill the closure relation

$$[T^a, T^b] = i f^{abc} T^c. \quad (64)$$

f^{abc} is the structure constant. If all components of f^{abc} vanish, the group is an abelian group.

We are interested in gauge theories, which are based on local unitary transformations of a set of fields.

Unitary transformations U_{ab} have the form

$$\eta_a \rightarrow U_{ab} \eta_b, \quad \xi_a \rightarrow U_{ab} \xi_b \quad (65)$$

where η and ξ are complex n -dimensional vectors. Unitary transformations preserve the inner product

$$\eta_a^* \xi_a. \quad (66)$$

The group of these transformations is called $U(n)$. One easy example is multiplication with the phase $e^{i\alpha}$, which forms the abelian group $U(1)$. The generator of $U(1)$ commutes with other unitary generators.

In a unitary group $U(n)$ with $n \geq 2$, the group $U(1)$ will be a subgroup. By removing the subgroup $U(1)$ from $U(n)$ we get the special unitary group $SU(n)$, which has $n^2 - 1$ generators.

2.2.3 Gauge Symmetries

We can promote a global symmetry to a local gauge symmetry. Let us have a look at the $U(1)$ symmetry

$$\phi \rightarrow e^{i\alpha} \phi. \quad (67)$$

Instead of having the same phase at every position of space, we can use the x -dependent transformation

$$\phi \rightarrow e^{i\alpha(x)} \phi, \quad (68)$$

but there is a problem with the derivative arising here. The definition of the derivative in the direction of a vector n^μ is

$$n^\mu \partial_\mu \psi = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (\psi(x + \epsilon n) - \psi(x)). \quad (69)$$

With the phase being arbitrarily different on neighboring points the definition of the derivative stops having a sensible geometrical interpretation.

To fix this problem, we can use the comparator $U(y, x)$ as a scalar quantity with the transformation law

$$U(y, x) \rightarrow e^{i\alpha(x)} U(y, x) e^{-i\alpha(x)}. \quad (70)$$

The transformation law is fulfilled by

$$U(x + \epsilon n, x) = 1 - i\epsilon n^\mu A_\mu(x) + \mathcal{O}(\epsilon^2). \quad (71)$$

Using the comparator we define the covariant derivative

$$n^\mu D_\mu \psi = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (\psi(x + \epsilon n) - U(x + \epsilon n, x) \psi(x)), \quad (72)$$

which can be written using (71) as

$$D_\mu \psi(x) = \partial_\mu \psi(x) + ie A_\mu \psi(x). \quad (73)$$

The so defined covariant derivative has sensible transformation properties, as we will see in (74) and (75). It has the usual geometric interpretation.

The vector field A_μ is called connection. For the case of quantum electrodynamics it is the vector potential of the electromagnetic field.

e is an arbitrary constant and in the electrodynamic case the electromagnetic coupling constant.

If we use the gauge transformation

$$A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x), \quad (74)$$

the covariant derivative is transformed according to

$$D_\mu \psi(x) \rightarrow e^{i\alpha(x)} D_\mu \psi(x). \quad (75)$$

We also need a kinetic term for our new vector field A_μ . One can show, that the term

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (76)$$

is locally invariant. We can combine it to get the kinetic term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$. We can use the same procedure for $SU(2)$ and will need three vector fields $A^i, i = 1, 2, 3$. One can even generalize the procedure further, using one gauge field per generator of the group, but we just need $SU(2)$ and $U(1)$. The $SU(2)$ local transformation can be written as

$$\psi \rightarrow \exp\left(-i\frac{\tau^a}{2}\alpha^a(x)\right)\psi, \quad (77)$$

where ψ is a doublet $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$. τ^a are the matrices representing the generators and α^a are the components of the parameter of the transformation, where $a = 1, 2, 3$.

The commutation relation is

$$\left[\frac{\tau^a}{2}, \frac{\tau^b}{2}\right] = i\epsilon^{abc}\frac{\tau^c}{2}. \quad (78)$$

The structure constant of $SU(2)$ is (up to a proportionality factor) the anti-symmetric tensor ϵ^{abc} . We can therefore think of the $SU(2)$ transformations as rotations, having the same commutation relations as the generators of angular momentum in quantum mechanics.

The covariant derivative in this case is

$$D_\mu = \partial_\mu + i\frac{g}{2}\tau^a A_\mu^a(x) \quad (79)$$

with the already mentioned gauge fields $A_\mu^a(x)$ and the coupling constant g . The gauge transformation we need to use on A_μ^a when using an $SU(2)$ transformation is

$$A_\mu \rightarrow UA_\mu U^{-1} - \frac{i}{g}U\partial_\mu U^{-1} \quad (80)$$

with the transformation U used in (77) on the field $A_\mu = \frac{1}{2}\tau^a A_\mu^a$. We will also need the field strength tensor for $SU(2)$, which is

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon^{abc}A_\mu^b A_\nu^c \quad (81)$$

and can also be written using $F_{\mu\nu} = \frac{1}{2}\tau^a F_{\mu\nu}^a$ and the field A_μ as above

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \quad (82)$$

2.3 Spontaneous Symmetry Breaking

Spontaneous symmetry breaking is an important idea in today's quantum field theories, enabling the usage of higher symmetries, which are not manifest in the theory.

My examination of this subject will be close to the one in [5], starting with the Goldstone model as an example of global symmetry breaking and going on with the Abelian Higgs model and the breaking of $SU(2) \times U(1)$.

2.3.1 The Goldstone Model

The Goldstone model uses a complex valued scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2). \quad (83)$$

Its Lagrangian is defined as

$$\mathcal{L}_G = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi); \quad V = \frac{1}{4}\lambda(\phi^\dagger \phi)^2 + m^2 \phi^\dagger \phi \quad (84)$$

and is invariant under the global $U(1)$ transformation (67).

We need to have a look at the ground state of the system and will therefore take a semiclassical approach, by considering the minimum of the Hamiltonian

$$\mathcal{H}_G = \dot{\phi}^\dagger \dot{\phi} + \nabla \phi^\dagger \cdot \nabla \phi + V(\phi) \quad (85)$$

which is at $\phi = \text{constant} = 0$. To progress to the quantum picture, we expand the field around the minimum by small oscillations.

To make things easier, we set interactions to be zero ($\lambda = 0$) and can therefore use plane wave solutions

$$\phi = \int \frac{d^3 \mathbf{k}}{(2\pi)^3 \sqrt{2E_{\mathbf{k}}}} \left(a_{\mathbf{k}} e^{-ik \cdot x} + b_{\mathbf{k}}^\dagger e^{ik \cdot x} \right) \quad (86)$$

with momentum \mathbf{k} and the energy $E = \sqrt{m^2 + \mathbf{k}^2}$. $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ are annihilation operators and their Hermitian conjugates are creation operators for a particle with momentum \mathbf{k} .

The Vacuum state $|0\rangle$ is defined by

$$a_{\mathbf{k}}|0\rangle = 0 = b_{\mathbf{k}}|0\rangle. \quad (87)$$

Therefore the vacuum expectation value of the field is

$$\langle 0|\phi|0\rangle = 0, \quad (88)$$

which is a reasonable result, if we interpret the expectation value as we would interpret a classical field, which has its minimum at $\phi = 0$ for the given potential. We will change the potential by altering the sign of the mass $m^2 =: -\mu^2 < 0$ and thereby get the potential

$$V = V_{SB} = \frac{\lambda}{4}(\phi^\dagger \phi)^2 - \mu^2 \phi^\dagger \phi. \quad (89)$$

The extremal point at $\phi = 0$ is now a local maximum and therefore unstable. The minimum of V_{SB} is at $|\phi| = v/\sqrt{2}$, where $v = \frac{2|\mu|}{\sqrt{\lambda}}$. Since the minimum just gives us a condition for the absolute value of ϕ and ϕ is a complex valued field, we still have a full degree of freedom left, being the rotation in the complex plane.

Therefore the minimum state is infinitely degenerate at

$$\phi = \frac{v}{\sqrt{2}} e^{i\theta/v}, \quad (90)$$

where θ can take any value.

As done before, we want to expand the field around its minimum. There are two possible directions for oscillations: The one in radial direction, around $\rho = v$, where ρ is the radial parameter and the one in angular direction changing θ .

The states with different θ are degenerate states with the same energy. Therefore

no energy is needed to excite this oscillation and it is massless. The oscillations in radial direction on the other hand change the energy of the field and we therefore suppose, that they are massive particles. The expanded field can be expressed by

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))e^{-i\theta(x)/v}, \quad (91)$$

where $h(x)$ and $\theta(x)$ are now considered to be fields and represent the oscillations in the ρ - and θ -direction respectively.

The combination of the Lagrangian in (84) with the potential in (89) and our ansatz for the field (91) gives the Lagrangian

$$\mathcal{L}_G = \frac{1}{2}\partial_\mu h \partial^\mu h - \mu^2 h^2 + \frac{1}{2}\partial_\mu \theta \partial^\mu \theta + \frac{\mu^4}{\lambda} + \dots, \quad (92)$$

where we left out cubic and quartic interaction terms.

The Lagrangian now contains a mass term $-\mu^2 h^2$, giving the mass $\sqrt{2}\mu$ to h , but no mass term for θ .

Using the annihilation operators a_θ and a_h for the respective modes on the vacuum $|0\rangle_B$ should give 0. Therefore the vacuum expectation value of the field ϕ for the vacuum $|0\rangle_B$ is

$${}_B\langle 0|\phi|0\rangle_B = \frac{v}{\sqrt{2}}. \quad (93)$$

The main observations we made in the Goldstone model for spontaneous symmetry breaking are, that we get a non-zero vacuum expectation value (VEV) and have one massive and one massless field instead of two fields with the same mass.

Also following from the non-zero VEV, the vacuum is not invariant under the symmetry transformation $U(1)$ anymore, while the Lagrangian still is invariant.

2.3.2 $U(1)$ Higgs Model

For the $U(1)$ Higgs Model we promote the global $U(1)$ symmetry of the Goldstone model to a gauge symmetry and therefore replace the derivatives ∂^μ by covariant derivatives (73) and add the kinetic term for the A^μ field.

The resulting Lagrangian is

$$\mathcal{L}_{U(1)} = [(\partial^\mu + ieA^\mu)\phi]^\dagger [(\partial_\mu + ieA_\mu)\phi] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\lambda}{4}(\phi^\dagger\phi)^2 + \mu^2(\phi^\dagger\phi). \quad (94)$$

As done before in the Goldstone model we want to expand the field ϕ around a vacuum expectation value:

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))e^{-i\theta(x)/v}. \quad (95)$$

Since the Lagrangian is invariant under the local $U(1)$ transformation (68) (74) we can gauge away the θ field, using the gauge transformation. It is therefore an unphysical field.

To see what happens to the degree of freedom of the Goldstone mode θ , when we gauge it away, we will have a look at the field equation for A^μ being

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) = j_{em}^\nu \quad (96)$$

with the 4-current $j_{em}^\nu = ie(\phi^\dagger \partial^\nu \phi - (\partial^\nu \phi^\dagger)\phi) - 2e^2 A^\nu \phi^\dagger \phi$. We now use the expansion around the minimum as it is done in (95) and get

$$j_{em}^\nu = -v^2 e^2 \left(A^\nu - \frac{\partial^\nu \theta}{ve} \right), \quad (97)$$

where we left out higher orders again.

Having this we can insert it into the equation of motion again and then gauge the field θ away, to see what happens. At first we get

$$\partial^\mu \partial_\mu A^\nu - \partial^\nu \partial_\mu A^\mu = -v^2 e^2 \left(A^\nu - \frac{\partial^\nu \theta}{ve} \right) \quad (98)$$

and the $U(1)$ gauge transformation (74) with the parameter $\alpha = -\frac{\theta}{v}$ gets us to

$$(\partial^\mu \partial_\mu + v^2 e^2) A^\nu - \partial^\nu \partial_\mu A^\mu = 0 \quad (99)$$

which is the equation for a free vector field with the mass $m = ve$. Thus the Higgs mechanism gave mass to our vector field A^μ .

The degree of freedom of θ can now be found in the additional degree of freedom a massive vector field has, i.e. the longitudinal polarization of a plane-wave solution.

2.3.3 $SU(2) \times U(1)$ Higgs Model

For the $SU(2) \times U(1)$ Higgs model we need three $SU(2)$ and one $U(1)$ gauge field, as mentioned in the gauge symmetries section.

The three $SU(2)$ fields are named $W_i^\mu(x)$, the $U(1)$ gauge field is $B^\mu(x)$ and we also have the scalar Higgs doublet

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (100)$$

the covariant derivative is

$$D^\mu = \partial^\mu + ig\boldsymbol{\tau} \cdot \mathbf{W}^\mu / 2 + ig' B^\mu / 2 \quad (101)$$

and the Lagrangian is

$$\mathcal{L}_{SU(2) \times U(1)} = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2 - \frac{1}{4} \mathbf{F}^{\mu\nu} \cdot \mathbf{F}_{\mu\nu} - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} \quad (102)$$

with the field strength tensors $G_{\mu\nu}$ as in (76) and $\mathbf{F}_{\mu\nu}$ as in (81) (Where you have to keep in mind, that we renamed the connections now).

When giving the Higgs field a VEV, we keep in mind, that we want to end up with a massless photon. Therefore we should not give a VEV to the charged field ϕ^+ . Instead the chargeless field ϕ^0 gets a VEV, as done before:

$$\langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}. \quad (103)$$

With this choice the vacuum is still invariant under the transformation

$$\langle 0 | \phi | 0 \rangle \rightarrow \exp \left(i\alpha \left(\frac{1}{2} + \frac{\tau^3}{2} \right) \right) \langle 0 | \phi | 0 \rangle. \quad (104)$$

As done before we will expand the field ϕ around the VEV to get

$$\phi = \exp(-i\boldsymbol{\theta}(x) \cdot \boldsymbol{\tau}/2v) \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h(x)) \end{pmatrix} \quad (105)$$

and gauge away the phase fields $\boldsymbol{\theta} = 0$. By replacing the resulting field ϕ into the Lagrangian we get

$$\begin{aligned} \mathcal{L}_{SSB} &= \frac{1}{2} \partial_\mu h \partial^\mu h - \mu^2 h^2 \\ &\quad - \frac{1}{4} (\partial^\mu W^{1\nu} - \partial^\nu W^{1\mu}) (\partial_\mu W_{1\nu} - \partial_\nu W_{1\mu}) + \frac{1}{8} g^2 v^2 W^{1\mu} W_{1\mu} \\ &\quad - \frac{1}{4} (\partial^\mu W^{2\nu} - \partial^\nu W^{2\mu}) (\partial_\mu W_{2\nu} - \partial_\nu W_{2\mu}) + \frac{1}{8} g^2 v^2 W^{2\mu} W_{2\mu} \\ &\quad - \frac{1}{4} (\partial^\mu W^{3\nu} - \partial^\nu W^{3\mu}) (\partial_\mu W_{3\nu} - \partial_\nu W_{3\mu}) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \\ &\quad + \frac{1}{8} (gW^{3\mu} - g'B^\mu) (gW_{3\mu} - g'B_\mu). \end{aligned} \quad (106)$$

The particle spectrum contains the massive field H , two massive fields W_1 and W_2 , which have the same mass $M_W = gv/2$ and a mixed field $(gW_{3\mu} - g'B_\mu)$. One can unmix the last fields using the weak mixing angle θ_W defined by

$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \quad \text{and} \quad \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}. \quad (107)$$

The resulting two fields are

$$Z^\mu = \cos \theta_W W_3^\mu - \sin \theta_W B^\mu \quad (108)$$

and

$$A^\mu = \sin \theta_W W_3^\mu + \cos \theta_W B^\mu \quad (109)$$

with the Lagrangian terms

$$- \frac{1}{4} (\partial^\mu Z^\nu - \partial^\nu Z^\mu) (\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \frac{1}{8} (g^2 + g'^2) v^2 Z^\mu Z_\mu \quad (110)$$

for the Z^μ field, acquiring mass $M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$, and

$$- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad (111)$$

for the usual field strength Tensor for the electromagnetic potential $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, being massless $M_A = 0$.

So to sum things up we used the mechanism of SSB to give mass to the gauge fields. We also chose the VEV in a way, that keeps the photon massless, by keeping the part of the vacuum, that the photon couples to, invariant under the symmetry transformation. One can also state this fact by identifying the eigenvalue of $(1 + \tau^3)$ with the electromagnetic charge q and stating, that the vacuum carries no electric charge.

2.4 The Electroweak Theory

Using the theoretic background of the previous chapters we can now finish the theory section with a consideration of the electroweak theory.

We can start with the particle content of the $SU(2) \times U(1)$ Higgs model, being 4 gauge fields and one scalar doublet. 3 of these gauge fields acquire mass via the Higgs mechanism.

What still needs to be added is the electron and the neutrino.

We previously saw that we can split fermion fields into a left- and a right-handed part, by using the projection operator (39). This splitting is very relevant for the electroweak theory. If we were to write down an electron mass term connecting a left-handed to a left-handed electron, we would end up with

$$m\bar{e}_L e_L = m\bar{e}\bar{P}_L P_L e = m\bar{e}P_R P_L e = 0. \quad (112)$$

Therefore we need mass terms, that connect the right- to the left-handed electron and vice versa. One way of thinking about this is the so called "zig-zag"-picture (See [7]), where the left- and the right-handed electron are thought of as two different particles (called "zig" and "zag"), that interact with each other by an interaction with the coupling constant m :

$$m\bar{e}_R e_L. \quad (113)$$

Thinking of left- and right-handed particles in this way is useful in the electroweak theory, since the weak force is inherently chiral. The W boson only couples to the left-handed helicity states of fermions.

This is incorporated into the theory by assigning the left-handed fermions to $SU(2)$ doublets, while making their right-handed versions singlets. This way mass terms like (113) will not be gauge invariant anyhow, since the different helicity states have different quantum numbers T^3 and Y , which are the relevant conserved charges for $SU(2)$ and $U(1)$ respectively.

Instead of (113) we incorporate the doublets into the mass terms. The electroweak fermion doublet is

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad (114)$$

and has the weak hypercharge $Y = -1/2$. Actually there have to be three doublets L_i , $i = 1, 2, 3$, because there are three different generations of leptons, being the electron, muon, tauon and their respective neutrinos ν_e , ν_μ , ν_τ , but we will consider only the electron in this discussion.

The neutrino has the T^3 charge $T^3 = +1/2$ and the electron $T^3 = -1/2$, resulting in the correct charges $q = 0$ and $q = -1$ respectively. An $SU(2)$ invariant coupling to the Higgs doublet, resulting in a mass term for the electron is

$$- \lambda_e \bar{L} \cdot \phi e_R + \text{h.c.} . \quad (115)$$

In principle one could do the same with the neutrino, but the problem is, that there doesn't seem to be a right-handed neutrino field ν_R . The Standard Model treats the neutrino as massless, while they actually seem to have at least very small masses, which motivates the extension to the Standard Model, that we are about to introduce.

3 The Model

3.1 Basic Outline and Particle Content

The model that I will analyse is a minimal extension to the standard model, that is based on [8], [9] and [10].

We add two $SU(2)$ fermion doublets

$$D_1 = \begin{pmatrix} \psi_1 \\ E_1 \end{pmatrix} \text{ with } Y = -\frac{1}{2} \quad (116)$$

and

$$D_2 = \begin{pmatrix} -E_2 \\ \psi_2 \end{pmatrix} \text{ with } Y = +\frac{1}{2}, \quad (117)$$

where the E_1 and E_2 fields have the respective charges $q_{E_1} = -1$ and $q_{E_2} = +1$ and the ψ fields are neutral.

We also add a neutral Majorana Fermion singlet S . Majorana fermions are their own antiparticles and therefore can not carry a lepton number. This allows for lepton number nonconserving interactions, which are useful for mechanisms, that provide neutrino masses.

An additional scalar field is also added. It is neutral as well and we will denote it by η .

We also add a new symmetry, that is called \mathbb{Z}_2 . A \mathbb{Z}_2 is just a symmetry under sign-inversion. All particles of the standard model will be even under this symmetry, therefore not changing their sign.

All particles of the extension are odd. Therefore any term in the Lagrangian will need an even number of new particles, to be invariant under this \mathbb{Z}_2 symmetry. This is commonly used in dark matter models, because it ensures the stability of the dark matter candidate. Being odd under \mathbb{Z}_2 makes it impossible for a particle to just decay into standard model particles. Instead annihilation will only be possible, when two odd particles annihilate into one standard model particle.

An equivalent of this \mathbb{Z}_2 symmetry is the R-parity used in supersymmetry [11]. The additional Lagrangian terms $\Delta\mathcal{L}$ are all renormalizable terms, that are invariant under $SU(2)$ -, $U(1)$ -, Lorentz- and \mathbb{Z}_2 -transformations.

To ensure $SU(2)$ invariance the epsilon tensor ϵ_{ij} is used to contract two doublets. Therefore any term connecting two doublets will have an implicit epsilon. We also use \tilde{D} to denote $i\sigma_2 D$, where σ_2 is the second pauli matrix. D^c denotes the charge conjugate of D . The combination \tilde{D}^c is needed to ensure charge conservation in some vertices.

The resulting Lagrangian is

$$\begin{aligned} \Delta\mathcal{L} = & -\lambda_1 \bar{D}_{1L} \tilde{D}_{2L}^c - \lambda_2 \bar{D}_{1R} \tilde{D}_{2R}^c - \lambda_3 \bar{D}_{1R} \tilde{D}_{1L} - \lambda_4 \bar{D}_{2R} \tilde{D}_{2L} \\ & - \beta_1 \bar{D}_{1R} \phi^* S_L - \beta_2 \bar{D}_{2R} \tilde{\phi} S_L - \beta_3 \bar{D}_{1L} \phi^* S_R - \beta_4 \bar{D}_{2L} \tilde{\phi} S_R \\ & - \alpha_i \bar{D}_{1R} \tilde{L}_i \eta - \alpha'_i \bar{D}_{2L} \tilde{L}_i^c \eta - \gamma \phi^\dagger \tilde{\phi} \eta^2 \\ & - \frac{1}{2} m_\eta \eta^2 - \frac{1}{2} m_s \bar{S} S - \gamma' \eta^4 + \text{h.c.} \end{aligned} \quad (118)$$

The model makes use of spontaneous symmetry breaking, giving a VEV to the

Higgs.

Therefore $\beta_1 \bar{D}_{1R} \phi^\dagger S_L$, $\beta_2 \bar{D}_{2R} \tilde{\phi} S_L$, $\beta_3 \bar{D}_{1L} \phi^\dagger S_R$ and $\beta_4 \bar{D}_{2L} \tilde{\phi} S_R$ have to be thought of as mass terms with respective masses $\frac{\beta_i v}{\sqrt{2}}$.

One can combine all massterms in the Lagrangian, using the mass matrix \mathcal{M}

$$(\bar{\psi}_{1L}, \bar{\psi}_{2L}, \bar{\psi}_{1R}^c, \bar{\psi}_{2R}^c, \bar{S}) \begin{pmatrix} 0 & \lambda_1 & \lambda_3 & 0 & \frac{\beta_3 v}{\sqrt{2}} \\ \lambda_1 & 0 & 0 & \lambda_4 & \frac{\beta_4 v}{\sqrt{2}} \\ \lambda_3 & 0 & 0 & \lambda_2 & \frac{\beta_1 v}{\sqrt{2}} \\ 0 & \lambda_4 & \lambda_2 & 0 & \frac{\beta_2 v}{\sqrt{2}} \\ \frac{\beta_3 v}{\sqrt{2}} & \frac{\beta_4 v}{\sqrt{2}} & \frac{\beta_1 v}{\sqrt{2}} & \frac{\beta_2 v}{\sqrt{2}} & m_s \end{pmatrix} \begin{pmatrix} \psi_{1L}^c \\ \psi_{2L}^c \\ \psi_{1R} \\ \psi_{2R} \\ S \end{pmatrix}, \quad (119)$$

where the charge conjugate fields ψ^c are used to make each vector contain only particles of same chirality.

Diagonalisation of this matrix gives the mass eigenbasis χ , where the resulting eigenvalues are the masses of the physical particles. The diagonalisation is done numerically and provides the mixing matrix, mixing the gauge basis fields $\psi_{1L}, \psi_{2L}, \psi_{1R}, \psi_{2R}$ and S into five physical fields χ_i .

Same is done for the fields E_1 and E_2 , having corresponding mass eigenstates ξ_1 and ξ_2 . This way definitions for the mass basis fields in terms of the gauge basis fields are obtained, which can be put in by FeynRules to achieve the Feynman rules in the mass basis.

The possible dark matter candidates in the model are either the Majorana fermions χ_i or the scalar field η . micrOMEGAs obtains the dark matter candidate in its calculation.

Any parameter combination, where ξ_i is the dark matter candidate is excluded, since dark matter does not couple to light and is therefore chargeless.

3.2 The Radiative Seesaw Mechanism

As mentioned in the section on the electroweak theory, neutrinos are massless in the Standard model. Experiments anyhow show, that neutrinos have a mass [12].

One way to solve this issue is the so called seesaw mechanism, which I will examine, before going to the special case, relevant to the model, being radiative seesaw.

I use [13] as the main source for seesaw in general in this chapter. [9] is used for the more specific information about radiative seesaw and [2.3] is used to complement some details.

It is possible to add an operator to the standard model, providing neutrino masses, which is

$$O = \lambda L_i \phi L_j \phi, \quad (120)$$

using the lepton doublets L_i and the Higgs doublet ϕ . This creates Majorana neutrino masses $\propto \langle \phi \rangle^2$, which are massterms, connecting left- to left- and right- to right-handed fermions.

These Majorana fermion fields can then be combined to form dirac fermions (see [13]).

For the neutrinos to obtain small enough masses, this operator has to be sup-

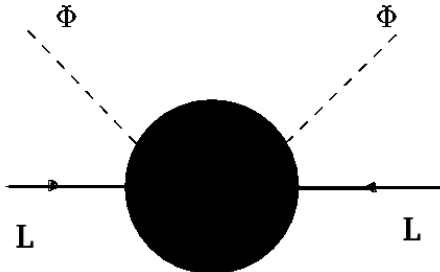


Figure 1: Feynman diagram corresponding to the operator O [13].

pressed. Considering a resulting mass

$$m_\nu = \lambda_0 \frac{\langle \phi \rangle^2}{\Lambda}, \quad (121)$$

there are two ways to achieve this suppression. One is a large scale Λ , such as the GUT scale, where grand unified theories expect new particles [2,3].

Another option is, that λ_0 is suppressed. Inducing the operator O with loop interactions is a way that leads to a natural small λ_0 .

The seesaw mechanism takes the first path. We will have a look at the general framework of seesaw in form of an effective theory, not containing any explicit dynamics.

Three left- and three right-handed chargeless fermions $\nu_{iL}, \nu_{iL}^c =: n_a$; $a = 1, \dots, 6$ form a basis with the general mass matrix

$$\mathcal{M}_\nu = \begin{pmatrix} M_1 & D \\ D^T & M_2 \end{pmatrix}, \quad (122)$$

where D, M_1, M_2 are 3×3 matrices, containing the flavor degrees of freedom. \mathcal{M}_ν can be diagonalised by a unitary transformation U_ν to get the mass eigenstates

$$\nu_i = \sum_{a=1}^6 (U_\nu)_{ia} n_a \quad (123)$$

with the resulting mass matrix

$$U_\nu^T \mathcal{M}_\nu U_\nu = \text{diag}(m_i, M_i). \quad (124)$$

To get small neutrino masses, one assumes $M_2 \gg D \gg M_1$, where $M_1 = 0$ is also possible.

This leads to three light neutrinos of masses m_i and three heavy leptons of masses M_i .

There are three types of seesaw mechanism, using different dynamics on a high energy scale, to achieve the matrix given in (122).

Since the model considered here uses a different mechanism we won't go into detail here. A short overview is given in [13], where further reference is provided.

The radiative seesaw mechanism as it is used in the model under consideration makes the massive leptons N_i odd under \mathbb{Z}_2 . It also adds an odd scalar doublet

$$\begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix}. \quad (125)$$

The operator O is realised in a one loop diagram. As mentioned before, the

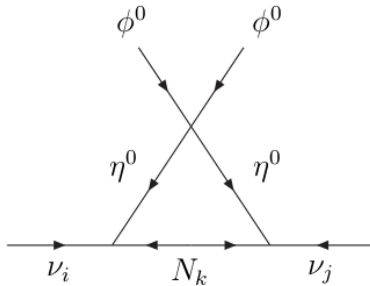


Figure 2: One loop diagram corresponding to O [9]

realisation in a one loop diagram suppresses the contribution of O to the neutrino masses.

As a result, the particles added for a radiative seesaw can have lower masses and “may be in fact light enough to be detected” [9].

The Yukawa Lagrangian used in [9] is

$$\Delta\mathcal{L}_Y = f_{ij}(\phi^- \nu_i + \bar{\phi}^0 l_i) l_j^c + h_{ij}(\nu_i \eta^0 - l_j \eta^+) N_j + \text{h.c.}, \quad (126)$$

where the different electron flavors are denoted by $l_i = \{e^-, \mu^-, \tau^-\}$. In our model η is not a doublet, but only the neutral scalar η^0 . The equivalent of N_i are the mass eigenstates χ .

η also needs a coupling to the Higgs field for the mechanism. In our model it is

$$-\gamma \phi^\dagger \tilde{\phi} \eta^2. \quad (127)$$

Combining the Lagrangian terms coupling ν_i to η and N_j and the quartic coupling between η and ϕ one achieves the diagram shown in Fig.2.

The calculations in [9] differ in some aspects from the model presented here, since the Lagrangian (118) is given in gauge basis and the mechanism as provided here is done in mass basis.

Anyhow common for both is the reduction by the seesaw scale by the factor $\approx \gamma/16\pi^2$. With $\gamma \sim 10^{-4}$ one ends up with particle masses in the order of TeV, making the model verifiable at experiments e.g. at the LHC.

3.3 Implementation in FeynRules

One major part of my work was to implement the model into FeynRules. FeynRules is a Mathematica package, that can calculate the Feynman rules for a given model.

Once the user specified the needed information for the model, e.g. field-definitions, the Lagrangian and parameters, FeynRules can produce output for different interfaces used to calculate Feynman diagrams. In our case CalcHEP output is

needed for micrOMEGAs.

More information about FeynRules can be found in the manual [14], which is also the source of reference for this section. I also want to thank Benjamin Fuks for his support concerning the issues I had with my code.

In this section I will explain the code part by part. The complete code can be found in appendix A, but I will copy some relevant parts into this section, to improve readability.

A complete implementation of the standard model can be found on <http://feynrules.irmp.ucl.ac.be/wiki/StandardModel> and is the basis for this implementation.

The first thing to add to the implementation are the additional field specifications. Particle classes in FeynRules are labeled by their spin, where S stands for scalar, F for Dirac and Majorana fermions and V for vector bosons.

The first field we are going to have a look at is ψ_1 , which is implemented as

```
F[21] == {
  ClassName -> Psi1,
  (* This is the field \psi_1, as it occurs in the doublets *)
  SelfConjugate -> False,
  Unphysical -> True,
  (* The fields Psi1, Psi2 and S are fields in the gauge basis.
  They are later mixed into Chi and therefore declared unphysical *)
  Definitions -> { Psi1[sp1_] :> Module[{sp2},
    ProjM[sp1,sp2] (AlphaAngleL[1] Chi[sp2,1]+
    I*AlphaAngleL[2] Chi[sp2,2]+AlphaAngleL[3] Chi[sp2,3]+
    I*AlphaAngleL[4] Chi[sp2,4]+AlphaAngleL[5] Chi[sp2,5])+
    ProjP[sp1,sp2] (AlphaAngleR[1] Chi[sp2,1]+
    I*AlphaAngleR[2] Chi[sp2,2]+AlphaAngleR[3] Chi[sp2,3]+
    I*AlphaAngleR[4] Chi[sp2,4]+AlphaAngleR[5] Chi[sp2,5])) }
},
```

Each field has a number, where the fermions in the extension range from $F[21]$ to $F[32]$. As said before, the particles ψ_1 , ψ_2 and S in gauge basis mix into the mass states χ_i , $i = 1, \dots, 5$.

Therefore the particle is defined as *unphysical*. This will lead FeynRules to not include it in the particle content, when writing output for CalcHEP.

Unphysical particles need the definitions in terms of physical fields given by numerical diagonalisation of the mass matrix.

$ProjM$ and $ProjP$ are the projection operators (39), giving only the left- and right-handed parts of the following particles respectively.

$sp1$ is a spinor index, where the underscore in the definition makes Mathematica running through all values for the index. $Module[sp2, \dots]$ provides an environment in which $sp2$ is a local variable.

Furthermore $CC[\dots]$ denotes the charge conjugate and I is the imaginary unit i , that is used in the definition, to keep the used parameters real. $AlphaAngleL[i]$ and $AlphaAngleR[i]$ are the needed mixing angles, mixing the χ fields to ψ fields.

ψ_2 is in complete analogy to ψ_1 . E_1 and E_2 work the same way, but we have to add information about the charge. The corresponding line for E_1 is

```
QuantumNumbers -> {Q -> -1},
```

The definitions in terms of physical fields for E_1 and E_2 is also different and consists only of slight field redefinitions.

The next field to define is S . S is a Majorana fermion and therefore we have

```
SelfConjugate -> True,
```

Except for that it is in complete analogy to the ψ_i with the definition differing only in the fact, that the left- and right-handed parts are not different from each other. The fields ψ_i and E_i are arranged into $SU(2)$ doublets in the model. The doublets are just defined in FeynRules as any other field is. D_{1L} is given by

```
F[26] == {
  ClassName      -> D1L,
  Unphysical     -> True,
  Indices        -> {Index[SU2D]},
  FlavorIndex    -> SU2D,
  SelfConjugate  -> False,
  QuantumNumbers -> {Y -> -1/2},
  Definitions    -> {
    D1L[sp1_,1] := Module[{sp2}, ProjM[sp1,sp2] Psi1[sp2]],
    D1L[sp1_,2] := Module[{sp2}, ProjM[sp1,sp2] E1[sp2]] }
},
```

The doublets are also declined as unphysical. The *Indices* option is used to provide the doublet with an $SU(2)$ index. The needed definition for the gauge group is done in the beginning of the standard model implementation.

In the definition the $SU(2)$ index is the second one given, therefore $D_{1L}[sp1_,i]$ defines the i -th component of the doublet. *ProjM* is again used to obtain the left-handed parts of the fields.

The doublet also carries the quantum number Y , which is specified as done before for E_1 . The other doublets are in analogy to this one, where one has to keep in mind, that the second doublet has $Y = +\frac{1}{2}$.

The last fields that need to be implemented are the five χ_i and the two ξ_i . There is just one class for all five χ_i . It is

```
F[30] == {
  ClassName -> Chi,
  (* The 5 Chi fields are the result of
  the diagonalisation of the mass matrix. *)
  ClassMembers -> {Chi1,Chi2,Chi3,Chi4,Chi5},
  ParticleName -> {"~Chi1", "~Chi2", "~Chi3", "~Chi4", "~Chi5"},
  (* The "~" in front of the name tells micrOMEGAS,
  that this field is odd unter a Z2 symmetry *)
  SelfConjugate -> True,
  (* The Chi fields are Majorana
  fermions and therefore selfconjugate *)
  Unphysical -> False,
  Indices -> {Index[DMGen]},
  FlavorIndex -> DMGen,
  Mass ->
  {MChi, {MChi1, 105.419},
  {MChi2, 241.321}, {MChi3, 278.388},
  {MChi4, 424.311}, {MChi5, 481.825}},
  PropagatorLabel ->
  {"Chi", "Chi1", "Chi2", "Chi3", "Chi4", "Chi5"},
  PropagatorType -> Straight,
  PropagatorArrow -> None
},
```

Here a new index *DMGen* was used, which is also defined in the beginning of the file. For a class with more than one member, we have to define all members using *ClassMembers*.

Furthermore a particle name was assigned, which would be the name of the classmember per default, but micrOMEGAs uses \sim in front of the name to identify particles, that are odd under \mathbb{Z}_2 .

The χ fields are Majorana fermions and therefore *SelfConjugate* was set to *True*. Each classmember has a mass assigned. Furthermore one can define labels and types for the propagator used to draw Feynman diagrams.

Here the labels are just the classmembers, where one label for the complete class is given in the beginning. Type straight means, that it is just a normal straight line and no propagator arrow is given, since it is a Majorana fermion, therefore not having a fermion flow in the usual sense.

ξ_1 and ξ_2 are defined in analogy to E_1 and E_2 , being their physical counterparts. Finally what is left is the chargeless scalar singlet η . Its definition uses only parts, we know from the other ones, where the propagator type *Dash* is a dashed line, as usually used for scalar particles.

```
S[21] == {
  ClassName -> eta0,
  ParticleName -> "~eta0",
  SelfConjugate -> True,
  Mass -> {Meta0, 100},
  PropagatorLabel -> "eta0",
  PropagatorType -> Dash,
  PropagatorArrow -> None
  (* The chargeless scalar field eta *)
}
```

Having all particles of the model, we proceed to the parameter section. As described in [13] there are different types of parameters. One division to make is between internal and external parameters. Internal parameters are given by definitions in terms of other parameters, while external parameters have a numerical value to be entered.

This way external parameters can be changed and are not (manifestly) dependent on each other.

The parameters defined for the model are all defined as external parameters, since the dependencies between them are quite complex e.g. the dependencies between the mixing matrix elements are results of the diagonalisation of a 5×5 matrix.

The other division to make is between scalar and tensor parameters. Scalar parameters do not carry any indices, while tensor parameters carry at least one and can in principle carry any number of indices.

One subtlety is, that tensor parameters are by default complex valued. External parameters are anyhow real valued and complex external parameters have to be split in their real and imaginary part.

Therefore all tensor parameters used for the Model use the option

```
ComplexParameter -> False,
```

to prevent arising problems from this ambiguity.

Any parameter names are the names used in (118). The coupling constants anyhow use *Coupling* and then A , B or C instead of α , β and γ .

The last part of the implementation is the Lagrangian. The Lagrangian of the model is given in (118). In the code it is

```
(* The extension Lagrangian is given in the gauge basis.
It is built from two parts with one being the h.c. of the other *)
LMDM1 := Block[{sp1,k1,k2,f,feynmangaugerules},
feynmangaugerules = If[Not[FeynmanGauge], {G0|GP|GPbar ->0}, {}];
ExpandIndices[
```

```

+ Lambda1 IndexDelta[k1,k2] D1Lbar[sp1,k1].CC[D2L[sp1,k2]]
+ Lambda2 IndexDelta[k1,k2] D1Rbar[sp1,k1].CC[D2R[sp1,k2]]
+ Lambda3 IndexDelta[k1,k2] D1Lbar[sp1,k1].[D1R[sp1,k2]]
+ Lambda4 IndexDelta[k1,k2] D2Lbar[sp1,k1].[D2R[sp1,k2]]
+ CouplingA1[f] IndexDelta[k1,k2] D1Rbar[sp1,k1].LL[sp1,k2,f] eta0
- CouplingA2[f] Eps[k1,k2] D2Rbar[sp1,k1].CC[LL[sp1,k2,f]] eta0
- CouplingB1 Eps[k1,k2] Phibar[k2] ProjM[sp1,sp2] D1Rbar[sp1,k1].S[sp2]
+ CouplingB2 IndexDelta[k1,k2] Phi[k2] ProjM[sp1,sp2] D2Rbar[sp1,k1].CC[S[sp2]]
- CouplingB3 Eps[k1,k2] Phibar[k2] ProjP[sp1,sp2] D1Lbar[sp1,k1].S[sp2]
+ CouplingB4 IndexDelta[k1,k2] Phi[k2] ProjP[sp1,sp2] D2Lbar[sp1,k1].CC[S[sp2]]
+ CouplingC1 IndexDelta[k1,k2] Phibar[k1] Phi[k2] HC[eta0] eta0
- CouplingC2 HC[eta0] eta0 HC[eta0] eta0
- 1/2 Meta0 HC[eta0] eta0
- 1/2 MassS Sbar[sp1] S[sp1]
,FlavorExpand->{SU2D,Generation}]/.feynmangaugerules
];

```

The first line of the Lagrangian starts with the *Block*[{*a, b, ...*}, ...] command, that is much like the *Module* command, in the sense, that it provides an environment, in which *a, b, ...* are local variables.

The second line is the realisation of gauging away the Goldstone bosons. If the Feynman gauge is not chosen, the gauge, in which there are no Goldstone bosons, is chosen. *G0, GP* and *GPbar* are those bosons.

ExpandIndices restores all suppressed indices. *FlavorExpand* expands flavor indices in the sense, that the rules are not given for the complete class, i.e. *vl* for the left handed neutrinos, but instead for all its constituents *ve, vm, vt*, being ν_e, ν_μ and ν_τ .

The first four vertices are the mass mixings of the doublets. \bar{D}_{1L} is denoted by *D1Lbar*. *IndexDelta* is the Kronecker delta δ_{ij} , which is the result of the combination of ϵ_{ij} coming from the doublet contraction and $i\sigma_2$ from \bar{D}_{2L} . *k1* and *k2* are *SU(2)* indices. One subtlety here is that the *Dot* command has to be used here. It is used to multiply anticommuting objects and will keep their order.

The next two vertices are the couplings between the doublets in the model, the fermion doublets of the standard model and η , where *LL* denotes the electroweak fermionic doublet (114) with the flavor index *f*.

The vertices with *CouplingBi* are the couplings between the Higgs doublet ϕ , the model doublets and the singlet *S*, which correspond to mixing terms in the mass matrix, when spontaneous symmetry breaking takes place.

The subtlety here is, that only those objects, which are anticommuting, are multiplied using *Dot*. *ProjM[sp1, sp2]* anyhow denotes a matrix element instead of a matrix and is therefore commuting.

The remaining vertices are the coupling between η and the Higgs, an η^4 coupling and mass terms for η and *S*.

4 Conclusion and Outlook

As mentioned in the beginning of the thesis, the FeynRules implementation does not work properly so far.

The implementation can be used to achieve the vertices in the mass basis. Problems occur, when it is used to generate CalcHEP output, which is the output needed for micrOMEGAs. In that case FeynRules gives many lines of errors and produces output, that can not be used in micrOMEGAs.

Therefore as a next step before using the code to do numerical analysis, it has to be searched through for errors. One point to start is surely the change from gauge- to mass-eigenbasis, since the relevant problems came in, when this change was imple-

mented. Parallel to this, the implementation has to be cross checked with an up to date version of the model. Chances are, that there are still subtleties in the model making problems.

I hope, that the explanation of the code in this thesis will make it accessible to whoever is going to continue to work with it and will make it possible to find the remaining errors and therefore make the implementation work.

Once working, the implementation can be cross-checked with the implementation in SARAH, which was done by David Lamprea.

From there the next step would be to use a suitable algorithm to examine the parameter space of the model, such as a Monte Carlo algorithm.

The constraints given by the relic density can then be combined with the constraints given by neutrino masses.

A FeynRules code

The following code is the FeynRules implementation of the model. Most of the file is an implementation of the standard model, which I used as a basis for the extension and which is mostly unchanged.

I left the model information the way it was to provide information about the original authors.

```
(*****  
(***** This is the FeynRules mod-file for the Standard model *****)  
(***** including a minimal extension for dark matter and *****)  
(***** neutrino masses *****)  
(***** Authors: N. Christensen, C. Duhr, B. Fuks, *****)  
(***** The extension was implemented by Philipp Rüter *****)  
(***** Choose whether Feynman gauge is desired. *****)  
(***** If set to False, unitary gauge is assumed. *****)  
(***** Feynman gauge is especially useful for CalcHEP/CompHEP *****)  
(***** where the calculation is 10-100 times faster. *****)  
(***** Feynman gauge is not supported in MadGraph and Sherpa. *****)  
(*****  
  
(* ***** *)  
(* ***** Information ***** *)  
(* ***** *)  
M$ModelName = "Minimal Dark Matter";  
  
M$Information = {  
  Authors      -> {"N. Christensen", "C. Duhr", "B. Fuks"},  
  Version      -> "1.4.1",  
  Date         -> "03. 11. 2011",  
  Institutions -> {"Michigan State University",  
                  "Universite catholique de Louvain (CP3)",  
                  "IPHC Strasbourg / University of Strasbourg"},  
  Emails       -> {"neil@pa.msu.edu",  
                  "claudeduhr@uclouvain.be",  
                  "benjamin.fuks@cnrs.in2p3.fr"},  
  URLs        ->  
  "http://feynrules.phys.ucl.ac.be/view/Main/StandardModel"  
};  
  
FeynmanGauge = False;  
  
(* ***** *)  
(* ***** Change log ***** *)  
(* ***** *)  
  
(* v1.4: Added SU(2) representation. *)  
(* -> Modification in the field declarations (doublets are added) *)  
(* -> Modification in the Lagrangian (much simpler). *)  
(* v1.3: Added yukawa couplings for all fermions for gauge invariance. *)  
(* Added yukawa couplings for 1st generation fermions to Massless.rst. *)  
(* Updated parameters to PDG 2010. *)  
(* v1.2: Set FeynmanGauge=True as default. *)  
(* Set Gluonic ghosts to be included in both gauges. *)  
(* v1.1: Fixed yukawa couplings in Feynman gauge. *)
```

```

(* Changed yd[n] CKM[n,m] to yd[m] CKM[n,m]. *)
(* Changed yu[n] Conjugate[CKM[m,n]] to yu[m] Conjugate[CKM[m,n]]. *)

(* ***** *)
(* ***** Gauge groups ***** *)
(* ***** *)
M$GaugeGroups = {
  U1Y == {
    Abelian      -> True,
    CouplingConstant -> g1,
    GaugeBoson   -> B,
    Charge       -> Y
  },
  SU2L == {
    Abelian      -> False,
    CouplingConstant -> gw,
    GaugeBoson   -> Wl,
    StructureConstant -> Eps,
    Representations -> {Ta,SU2D},
    Definitions   ->
    {Ta[a_,b_,c_]->PauliSigma[a,b,c]/2, FSU2L[i_,j_,k_]:> I Eps[i,j,k]}
  },
  SU3C == {
    Abelian      -> False,
    CouplingConstant -> gs,
    GaugeBoson   -> G,
    StructureConstant -> f,
    Representations -> {T,Colour},
    DTerm        -> dSUN
  }
};

(* ***** *)
(* ***** Indices ***** *)
(* ***** *)

IndexRange[Index[SU2W    ]] = Unfold[Range[3]];
IndexRange[Index[SU2D    ]] = Unfold[Range[2]];
IndexRange[Index[Gluon   ]] = NoUnfold[Range[8]];
IndexRange[Index[Colour  ]] = NoUnfold[Range[3]];
IndexRange[Index[Generation]] = Range[3];
IndexRange[Index[DMGen   ]] = Range[5];
(* Index for the 5 different mass eigenstates *)

IndexStyle[SU2W,    j];
IndexStyle[SU2D,    k];
IndexStyle[Gluon,   a];
IndexStyle[Colour,  m];
IndexStyle[Generation, f];
IndexStyle[DMGen,   l];
(* Index for the 5 different mass eigenstates *)

```

```

(* ***** *)
(* *** Interaction orders *** *)
(* *** (as used by mg5) *** *)
(* ***** *)

M$InteractionOrderHierarchy = {
  {QCD, 1},
  {QED, 2}
};

(* ***** *)
(* **** Particle classes **** *)
(* ***** *)
M$ClassesDescription = {

(* Gauge bosons: physical vector fields *)
V[1] == {
  ClassName      -> A,
  SelfConjugate  -> True,
  Mass           -> 0,
  Width          -> 0,
  ParticleName   -> "a",
  PDG            -> 22,
  PropagatorLabel -> "a",
  PropagatorType -> W,
  PropagatorArrow -> None,
  FullName       -> "Photon"
},
V[2] == {
  ClassName      -> Z,
  SelfConjugate  -> True,
  Mass           -> {MZ, 91.1876},
  Width          -> {WZ, 2.4952},
  ParticleName   -> "Z",
  PDG            -> 23,
  PropagatorLabel -> "Z",
  PropagatorType -> Sine,
  PropagatorArrow -> None,
  FullName       -> "Z"
},
V[3] == {
  ClassName      -> W,
  SelfConjugate  -> False,
  Mass           -> {MW, Internal},
  Width          -> {WW, 2.085},
  ParticleName   -> "W+",
  AntiParticleName -> "W-",
  QuantumNumbers -> {Q -> 1},
  PDG            -> 24,
  PropagatorLabel -> "W",
  PropagatorType -> Sine,
  PropagatorArrow -> Forward,
  FullName       -> "W"
}
}

```



```

},
V[4] == {
  ClassName      -> G,
  SelfConjugate  -> True,
  Indices        -> {Index[Gluon]},
  Mass          -> 0,
  Width         -> 0,
  ParticleName   -> "g",
  PDG           -> 21,
  PropagatorLabel -> "G",
  PropagatorType -> C,
  PropagatorArrow -> None,
  FullName       -> "G"
},

(* Ghosts: related to physical gauge bosons *)
U[1] == {
  ClassName      -> ghA,
  SelfConjugate  -> False,
  Ghost          -> A,
  QuantumNumbers -> {GhostNumber -> 1},
  Mass          -> 0,
  PropagatorLabel -> "uA",
  PropagatorType -> GhostDash,
  PropagatorArrow -> Forward
},
U[2] == {
  ClassName      -> ghZ,
  SelfConjugate  -> False,
  Ghost          -> Z,
  QuantumNumbers -> {GhostNumber -> 1},
  Mass          -> {MZ,91.1876},
  PropagatorLabel -> "uZ",
  PropagatorType -> GhostDash,
  PropagatorArrow -> Forward
},
U[31] == {
  ClassName      -> ghWp,
  SelfConjugate  -> False,
  Ghost          -> W,
  QuantumNumbers -> {GhostNumber -> 1, Q -> 1},
  Mass          -> {MW,Internal},
  PropagatorLabel -> "uWp",
  PropagatorType -> GhostDash,
  PropagatorArrow -> Forward
},
U[32] == {
  ClassName      -> ghWm,
  SelfConjugate  -> False,
  Ghost          -> Wbar,
  QuantumNumbers -> {GhostNumber -> 1, Q -> -1},
  Mass          -> {MW,Internal},
  PropagatorLabel -> "uWm",
  PropagatorType -> GhostDash,

```

```

    PropagatorArrow -> Forward
  },
  U[4] == {
    ClassName      -> ghG,
    SelfConjugate  -> False,
    Indices        -> {Index[Gluon]},
    Ghost          -> G,
    QuantumNumbers -> {GhostNumber -> 1},
    Mass           -> 0,
    PropagatorLabel -> "uG",
    PropagatorType -> GhostDash,
    PropagatorArrow -> Forward
  },
(* Gauge bosons: unphysical vector fields *)
  V[11] == {
    ClassName      -> B,
    Unphysical     -> True,
    SelfConjugate  -> True,
    Definitions    -> { B[mu_] -> -sw Z[mu]+cw A[mu]}
  },
  V[12] == {
    ClassName      -> Wi,
    Unphysical     -> True,
    SelfConjugate  -> True,
    Indices        -> {Index[SU2W]},
    FlavorIndex    -> SU2W,
    Definitions    -> { Wi[mu_,1] -> (Wbar[mu]+W[mu])/Sqrt[2],
                       Wi[mu_,2] -> (Wbar[mu]-W[mu])/(I*Sqrt[2]),
                       Wi[mu_,3] -> cw Z[mu] + sw A[mu]}
  },
(* Ghosts: related to unphysical gauge bosons *)
  U[11] == {
    ClassName      -> ghB,
    Unphysical     -> True,
    SelfConjugate  -> False,
    Ghost          -> B,
    Definitions    -> { ghB -> -sw ghZ + cw ghA}
  },
  U[12] == {
    ClassName      -> ghWi,
    Unphysical     -> True,
    SelfConjugate  -> False,
    Ghost          -> Wi,
    Indices        -> {Index[SU2W]},
    FlavorIndex    -> SU2W,
    Definitions    -> { ghWi[1] -> (ghWp+ghWm)/Sqrt[2],
                       ghWi[2] -> (ghWm-ghWp)/(I*Sqrt[2]),
                       ghWi[3] -> cw ghZ+sw ghA}
  },
(* Fermions: physical fields *)
  F[1] == {

```

```

ClassName      -> vl,
ClassMembers   -> {ve,vm,vt},
Indices        -> {Index[Generation]},
FlavorIndex    -> Generation,
SelfConjugate  -> False,
Mass           -> 0,
Width          -> 0,
PropagatorLabel -> {"v", "ve", "vm", "vt"} ,
PropagatorType -> S,
PropagatorArrow -> Forward,
PDG            -> {12,14,16},
ParticleName   -> {"ve","vm","vt"},
AntiParticleName -> {"ve~","vm~","vt~"},
FullName       -> {"Electron-neutrino",
                  "Mu-neutrino",
                  "Tau-neutrino"}
},
F[2] == {
  ClassName      -> l,
  ClassMembers   -> {e, mu, ta},
  Indices        -> {Index[Generation]},
  FlavorIndex    -> Generation,
  SelfConjugate  -> False,
  Mass           -> {Ml, {Me,5.11*^-4}, {MM,0.10566}, {MTA,1.777}},
  Width          -> 0,
  QuantumNumbers -> {Q -> -1},
  PropagatorLabel -> {"l", "e", "mu", "ta"},
  PropagatorType -> Straight,
  PropagatorArrow -> Forward,
  PDG            -> {11, 13, 15},
  ParticleName   -> {"e-", "mu-", "ta-"},
  AntiParticleName -> {"e+", "mu+", "ta+"},
  FullName       -> {"Electron", "Muon", "Tau"}
},
F[3] == {
  ClassName      -> uq,
  ClassMembers   -> {u, c, t},
  Indices        -> {Index[Generation], Index[Colour]},
  FlavorIndex    -> Generation,
  SelfConjugate  -> False,
  Mass           -> {Mu, {MU, 2.55*^-3}, {MC,1.27}, {MT,172}},
  Width          -> {0, 0, {WT,1.50833649}},
  QuantumNumbers -> {Q -> 2/3},
  PropagatorLabel -> {"uq", "u", "c", "t"},
  PropagatorType -> Straight,
  PropagatorArrow -> Forward,
  PDG            -> {2, 4, 6},
  ParticleName   -> {"u", "c", "t" },
  AntiParticleName -> {"u~", "c~", "t~"},
  FullName       -> {"u-quark", "c-quark", "t-quark"}
},
F[4] == {
  ClassName      -> dq,
  ClassMembers   -> {d, s, b},

```

```

Indices      -> {Index[Generation], Index[Colour]},
FlavorIndex  -> Generation,
SelfConjugate -> False,
Mass         -> {Md, {MD,5.04*^-3}, {MS,0.101}, {MB,4.7}},
Width        -> 0,
QuantumNumbers -> {Q -> -1/3},
PropagatorLabel -> {"dq", "d", "s", "b"},
PropagatorType -> Straight,
PropagatorArrow -> Forward,
PDG          -> {1,3,5},
ParticleName -> {"d", "s", "b" },
AntiParticleName -> {"d~", "s~", "b~"},
FullName     -> {"d-quark", "s-quark", "b-quark"}
},

(* Fermions: unphysical fields *)
F[11] == {
  ClassName    -> LL,
  Unphysical   -> True,
  Indices      -> {Index[SU2D], Index[Generation]},
  FlavorIndex  -> SU2D,
  SelfConjugate -> False,
  QuantumNumbers -> {Y -> -1/2},
  Definitions  -> {
    LL[sp1_,1,ff_] := Module[{sp2}, ProjM[sp1,sp2] vl[sp2,ff]],
    LL[sp1_,2,ff_] := Module[{sp2}, ProjM[sp1,sp2] l[sp2,ff]] }
},
F[12] == {
  ClassName    -> lR,
  Unphysical   -> True,
  Indices      -> {Index[Generation]},
  FlavorIndex  -> Generation,
  SelfConjugate -> False,
  QuantumNumbers -> {Y -> -1},
  Definitions  -> {
    lR[sp1_,ff_] := Module[{sp2}, ProjP[sp1,sp2] l[sp2,ff]] }
},
F[13] == {
  ClassName    -> QL,
  Unphysical   -> True,
  Indices      -> {Index[SU2D], Index[Generation], Index[Colour]},
  FlavorIndex  -> SU2D,
  SelfConjugate -> False,
  QuantumNumbers -> {Y -> 1/6},
  Definitions  -> {
    QL[sp1_,1,ff_,cc_] := Module[{sp2}, ProjM[sp1,sp2] uq[sp2,ff,cc]],
    QL[sp1_,2,ff_,cc_] := Module[{sp2,ff2}, CKM[ff,ff2] ProjM[sp1,sp2]
    dq[sp2,ff2,cc]] }
},
F[14] == {
  ClassName    -> uR,
  Unphysical   -> True,
  Indices      -> {Index[Generation], Index[Colour]},
  FlavorIndex  -> Generation,

```

```

SelfConjugate -> False,
QuantumNumbers -> {Y -> 2/3},
Definitions -> {
  uR[sp1_,ff_,cc_] := Module[{sp2}, ProjP[sp1,sp2] uq[sp2,ff,cc] ]
},
F[15] == {
  ClassName      -> dR,
  Unphysical     -> True,
  Indices        -> {Index[Generation], Index[Colour]},
  FlavorIndex    -> Generation,
  SelfConjugate  -> False,
  QuantumNumbers -> {Y -> -1/3},
  Definitions    -> {
    dR[sp1_,ff_,cc_] := Module[{sp2}, ProjP[sp1,sp2] dq[sp2,ff,cc] ]
  },
},
(* Higgs: physical scalars *)
S[1] == {
  ClassName      -> H,
  SelfConjugate  -> True,
  Mass           -> {MH,120},
  Width         -> {WH,0.00575308848},
  PropagatorLabel -> "H",
  PropagatorType -> D,
  PropagatorArrow -> None,
  PDG           -> 25,
  ParticleName   -> "H",
  FullName      -> "H"
},
(* Higgs: physical scalars *)
S[2] == {
  ClassName      -> G0,
  SelfConjugate  -> True,
  Goldstone      -> Z,
  Mass           -> {MZ, 91.1876},
  Width         -> WGo,
  PropagatorLabel -> "Go",
  PropagatorType -> D,
  PropagatorArrow -> None,
  PDG           -> 250,
  ParticleName   -> "G0",
  FullName      -> "G0"
},
S[3] == {
  ClassName      -> GP,
  SelfConjugate  -> False,
  Goldstone      -> W,
  Mass           -> {MW, Internal},
  QuantumNumbers -> {Q -> 1},
  Width         -> WGP,
  PropagatorLabel -> "GP",
  PropagatorType -> D,
  PropagatorArrow -> None,

```

```

    PDG          -> 251,
    ParticleName -> "G+",
    AntiParticleName -> "G-",
    FullName     -> "GP"
  },

(* Higgs: unphysical scalars *)
S[11] == {
  ClassName     -> Phi,
  Unphysical    -> True,
  Indices       -> {Index[SU2D]},
  FlavorIndex   -> SU2D,
  SelfConjugate -> False,
  QuantumNumbers -> {Y -> 1/2},
  Definitions   -> {
    Phi[1] -> -I GP, Phi[2] -> (vev + H + I GO)/Sqrt[2] }
  },

(* New Particles: Fermion Fields *)
F[21] == {
  ClassName -> Psi1,
  (* This is the field \psi_1, as it occurs in the doublets *)
  SelfConjugate -> False,
  Unphysical -> True,
  (* The fields Psi1, Psi2 and S are fields in the gauge basis.
  They are later mixed into Chi and therefore declared unphysical *)
  Definitions -> { Psi1[sp1_] :> Module[{sp2},
    ProjM[sp1,sp2] (AlphaAngleL[1] Chi[sp2,1]+
    I*AlphaAngleL[2] Chi[sp2,2]+AlphaAngleL[3] Chi[sp2,3]+
    I*AlphaAngleL[4] Chi[sp2,4]+AlphaAngleL[5] Chi[sp2,5])+
    ProjP[sp1,sp2] (AlphaAngleR[1] Chi[sp2,1]+
    I*AlphaAngleR[2] Chi[sp2,2]+AlphaAngleR[3] Chi[sp2,3]+
    I*AlphaAngleR[4] Chi[sp2,4]+AlphaAngleR[5] Chi[sp2,5])) }
  (* See the section on the implementation *)
  },
F[22] == {
  ClassName -> Psi2,
  (* This is the field \psi_2, as it occurs in the doublets *)
  SelfConjugate -> False,
  Unphysical -> True,
  (* The fields Psi1, Psi2 and S are fields in the gauge basis.
  They are later mixed into Chi and therefore declared unphysical *)
  Definitions -> { Psi2[sp1_] :> CC[Module[{sp2},
    ProjM[sp1,sp2] (BetaAngleL[1] Chi[sp2,1]+
    I*BetaAngleL[2] Chi[sp2,2]+BetaAngleL[3] Chi[sp2,3]+
    I*BetaAngleL[4] Chi[sp2,4]+BetaAngleL[5] Chi[sp2,5])+
    ProjP[sp1,sp2] (BetaAngleR[1] Chi[sp2,1]+
    I*BetaAngleR[2] Chi[sp2,2]+BetaAngleR[3] Chi[sp2,3]+
    I*BetaAngleR[4] Chi[sp2,4]+BetaAngleR[5] Chi[sp2,5]))] }
  (* See the section on the implementation *)
  },
F[23] == {
  ClassName -> E1,
  (* This is the field E_1, as it occurs in the doublets *)

```

```

SelfConjugate -> False,
Unphysical -> True,
(* The fields E1 and E2 are redefined into Xi1 and Xi2 *)
QuantumNumbers -> {Q -> -1},
Definitions -> {E1[sp1_] :> I*Xi2[sp1]}
(* See the section on the implementation *)
},
F[24] == {
  ClassName -> E2,
  (* This is the field E_2, as it occurs in the doublets *)
  SelfConjugate -> False,
  Unphysical -> True,
  (* The fields E1 and E2 are redefined into Xi1 and Xi2 *)
  QuantumNumbers -> {Q -> 1},
  Definitions -> {E1[sp1_] :> CC[Xi1[sp1]]}
  (* See the section on the implementation *)
},
F[25] == {
  ClassName -> S,
  (* The S field, being a singlet Majorana fermion *)
  SelfConjugate -> True,
  (* S is a Majorana fermion and therefore selfconjugate *)
  Unphysical -> True,
  (* The fields Psi1, Psi2 and S are fields in the gauge basis.
  They are later mixed into Chi and therefore declared unphysical *)
  Definitions -> {
    S[sp1_] :> (ThetaAngle[1] Chi[sp1,1]+
    I*ThetaAngle[2] Chi[sp1,2]+ThetaAngle[3] Chi[sp1,3]+
    I*ThetaAngle[4] Chi[sp1,4]+ThetaAngle[5] Chi[sp1,5]) }
  (* See the section on the implementation *)
},
F[26] == {
  ClassName -> D1L,
  Unphysical -> True,
  Indices -> {Index[SU2D]},
  FlavorIndex -> SU2D,
  SelfConjugate -> False,
  QuantumNumbers -> {Y -> -1/2},
  Definitions -> {
    D1L[sp1_,1] :> Module[{sp2}, ProjM[sp1,sp2] Psi1[sp2]],
    D1L[sp1_,2] :> Module[{sp2}, ProjM[sp1,sp2] E1[sp2]] }
  (* See the section on the implementation *)
},
F[27] == {
  ClassName -> D1R,
  Unphysical -> True,
  Indices -> {Index[SU2D]},
  FlavorIndex -> SU2D,
  SelfConjugate -> False,
  QuantumNumbers -> {Y -> -1/2},
  Definitions -> {
    D1R[sp1_,1] :> Module[{sp2}, ProjP[sp1,sp2] Psi1[sp2]],
    D1R[sp1_,2] :> Module[{sp2}, ProjP[sp1,sp2] E1[sp2]] }
  (* See the section on the implementation *)
}

```

```

    },
F[28] == {
  ClassName      -> D2L,
  Unphysical     -> True,
  Indices        -> {Index[SU2D]},
  FlavorIndex    -> SU2D,
  SelfConjugate  -> False,
  QuantumNumbers -> {Y -> 1/2},
  Definitions    -> {
    D2L[sp1_,1] :=> Module[{sp2}, -ProjM[sp1,sp2] E2[sp2]],
    D2L[sp1_,2] :=> Module[{sp2}, ProjM[sp1,sp2] Psi2[sp2]] }
  (* See the section on the implementation *) },
F[29] == {
  ClassName      -> D2R,
  Unphysical     -> True,
  Indices        -> {Index[SU2D]},
  FlavorIndex    -> SU2D,
  SelfConjugate  -> False,
  QuantumNumbers -> {Y -> 1/2},
  Definitions    -> {
    D2R[sp1_,1] :=> Module[{sp2}, -ProjP[sp1,sp2] E2[sp2]],
    D2R[sp1_,2] :=> Module[{sp2}, ProjP[sp1,sp2] Psi2[sp2]] }
  (* See the section on the implementation *) },
F[30] == {
  ClassName -> Chi,
  (* The 5 Chi fields are the result of
  the diagonalisation of the mass matrix. *)
  ClassMembers -> {Chi1,Chi2,Chi3,Chi4,Chi5},
  ParticleName  -> {"~Chi1", "~Chi2", "~Chi3", "~Chi4", "~Chi5"},
  (* The "~" in front of the name tells micrOMEGAS,
  that this field is odd unter a Z2 symmetry *)
  SelfConjugate -> True,
  (* The Chi fields are Majorana
  fermions and therefore selfconjugate *)
  Unphysical -> False,
  Indices -> {Index[DMGen]},
  FlavorIndex -> DMGen,
  Mass ->
  {MChi, {MChi1, 105.419},
  {MChi2, 241.321}, {MChi3, 278.388},
  {MChi4, 424.311}, {MChi5, 481.825}},
  PropagatorLabel ->
  {"Chi", "Chi1", "Chi2", "Chi3", "Chi4", "Chi5"},
  PropagatorType -> Straight,
  PropagatorArrow -> None
},
F[31] == {
  ClassName -> Xi1,
  (* The Xi fields are the equivalent
  of the E fields in mass eigenbasis *)
  ParticleName  -> "~Xi1",
  (* The "~" in front of the name tells micrOMEGAS,
  that this field is odd unter a Z2 symmetry *)
  AntiParticleName -> "~Xi1~",

```



```

SelfConjugate -> False,
Unphysical -> False,
Mass -> {ME, 5000},
QuantumNumbers -> {Q -> -1},
PropagatorLabel -> "Xi1",
PropagatorType -> Straight,
PropagatorArrow -> Forward
},
F[32] == {
  ClassName -> Xi2,
  (* The Xi fields are the equivalent
  of the E fields in mass eigenbasis *)
  ParticleName -> "~Xi2",
  (* The "~" in front of the name tells micrOMEGAS,
  that this field is odd under a Z2 symmetry *)
  AntiParticleName -> "~Xi2~",
  SelfConjugate -> False,
  Unphysical -> False,
  Mass -> {ME, 5000},
  QuantumNumbers -> {Q -> -1},
  PropagatorLabel -> "Xi2",
  PropagatorType -> Straight,
  PropagatorArrow -> Forward
},

(* New Particles: Physical Scalar Fields *)
S[21] == {
  ClassName -> eta0,
  ParticleName -> "~eta0",
  SelfConjugate -> True,
  Mass -> {Meta0, 100},
  PropagatorLabel -> "eta0",
  PropagatorType -> Dash,
  PropagatorArrow -> None
  (* The chargeless scalar field eta *)
}

};

(* ***** *)
(* ***** Gauge ***** *)
(* ***** Parameters ***** *)
(* ***** (FeynArts) ***** *)
(* ***** *)

GaugeXi[ V[1] ] = GaugeXi[A];
GaugeXi[ V[2] ] = GaugeXi[Z];
GaugeXi[ V[3] ] = GaugeXi[W];
GaugeXi[ V[4] ] = GaugeXi[G];
GaugeXi[ S[1] ] = 1;
GaugeXi[ S[2] ] = GaugeXi[Z];
GaugeXi[ S[3] ] = GaugeXi[W];
GaugeXi[ U[1] ] = GaugeXi[A];
GaugeXi[ U[2] ] = GaugeXi[Z];

```

```
GaugeXi[ U[31] ] = GaugeXi[W];
GaugeXi[ U[32] ] = GaugeXi[W];
GaugeXi[ U[4] ] = GaugeXi[G];
```

```
(* ***** *)
(* ***** Parameters ***** *)
(* ***** *)
M$Parameters = {

  (* External parameters *)
  aEWm1 == {
    ParameterType -> External,
    BlockName     -> SMINPUTS,
    OrderBlock    -> 1,
    Value         -> 127.9,
    InteractionOrder -> {QED,-2},
    Description    ->
    "Inverse of the EW coupling constant at the Z pole"
  },
  Gf == {
    ParameterType -> External,
    BlockName     -> SMINPUTS,
    OrderBlock    -> 2,
    Value         -> 1.16637*^-5,
    InteractionOrder -> {QED,2},
    TeX           -> Subscript[G,f],
    Description    -> "Fermi constant"
  },
  aS == {
    ParameterType -> External,
    BlockName     -> SMINPUTS,
    OrderBlock    -> 3,
    Value         -> 0.1184,
    InteractionOrder -> {QCD,2},
    TeX           -> Subscript[\[Alpha],s],
    Description    -> "Strong coupling constant at the Z pole"
  },
  ymdo == {
    ParameterType -> External,
    BlockName     -> YUKAWA,
    OrderBlock    -> 1,
    Value         -> 5.04*^-3,
    Description    -> "Down Yukawa mass"
  },
  ymup == {
    ParameterType -> External,
    BlockName     -> YUKAWA,
    OrderBlock    -> 2,
    Value         -> 2.55*^-3,
    Description    -> "Up Yukawa mass"
  },
  yms == {
    ParameterType -> External,
```

```

    BlockName      -> YUKAWA,
    OrderBlock     -> 3,
    Value          -> 0.101,
    Description    -> "Strange Yukawa mass"
  },
  ymc == {
    ParameterType -> External,
    BlockName     -> YUKAWA,
    OrderBlock    -> 4,
    Value         -> 1.27,
    Description   -> "Charm Yukawa mass"
  },
  ymb == {
    ParameterType -> External,
    BlockName     -> YUKAWA,
    OrderBlock    -> 5,
    Value         -> 4.7,
    Description   -> "Bottom Yukawa mass"
  },
  ymt == {
    ParameterType -> External,
    BlockName     -> YUKAWA,
    OrderBlock    -> 6,
    Value         -> 172,
    Description   -> "Top Yukawa mass"
  },
  yme == {
    ParameterType -> External,
    BlockName     -> YUKAWA,
    OrderBlock    -> 11,
    Value         -> 5.11*^-4,
    Description   -> "Electron Yukawa mass"
  },
  ymm == {
    ParameterType -> External,
    BlockName     -> YUKAWA,
    OrderBlock    -> 13,
    Value         -> 0.10566,
    Description   -> "Muon Yukawa mass"
  },
  ymtau == {
    ParameterType -> External,
    BlockName     -> YUKAWA,
    OrderBlock    -> 15,
    Value         -> 1.777,
    Description   -> "Tau Yukawa mass"
  },
  cabi == {
    ParameterType -> External,
    BlockName     -> CKMBLOCK,
    OrderBlock    -> 1,
    Value         -> 0.227736,
    TeX          -> Subscript[\[Theta], c],
    Description   -> "Cabibbo angle"
  }

```

```

},

(* Internal Parameters *)
aEW == {
  ParameterType -> Internal,
  Value         -> 1/aEWM1,
  InteractionOrder -> {QED,2},
  TeX           -> Subscript[\[Alpha], EW],
  Description    -> "Electroweak coupling constant"
},
MW == {
  ParameterType -> Internal,
  Value         -> Sqrt[MZ^2/2+Sqrt[MZ^4/4-Pi/Sqrt[2]*aEW/Gf*MZ^2]],
  TeX           -> Subscript[M,W],
  Description    -> "W mass"
},
sw2 == {
  ParameterType -> Internal,
  Value         -> 1-(MW/MZ)^2,
  Description    -> "Squared Sin of the Weinberg angle"
},
ee == {
  ParameterType -> Internal,
  Value         -> Sqrt[4 Pi aEW],
  InteractionOrder -> {QED,1},
  TeX           -> e,
  Description    -> "Electric coupling constant"
},
cw == {
  ParameterType -> Internal,
  Value         -> Sqrt[1-sw2],
  TeX           -> Subscript[c,w],
  Description    -> "Cosine of the Weinberg angle"
},
sw == {
  ParameterType -> Internal,
  Value         -> Sqrt[sw2],
  TeX           -> Subscript[s,w],
  Description    -> "Sine of the Weinberg angle"
},
gw == {
  ParameterType -> Internal,
  Definitions    -> {gw->ee/sw},
  InteractionOrder -> {QED,1},
  TeX           -> Subscript[g,w],
  Description    -> "Weak coupling constant at the Z pole"
},
g1 == {
  ParameterType -> Internal,
  Definitions    -> {g1->ee/cw},
  InteractionOrder -> {QED,1},
  TeX           -> Subscript[g,1],
  Description    -> "U(1)Y coupling constant at the Z pole"
},

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gs == {
  ParameterType -> Internal,
  Value         -> Sqrt[4 Pi aS],
  InteractionOrder -> {QCD,1},
  TeX           -> Subscript[g,s],
  ParameterName  -> G,
  Description    -> "Strong coupling constant at the Z pole"
},
vev == {
  ParameterType -> Internal,
  Value         -> 2*MW*sw/ee,
  InteractionOrder -> {QED,-1},
  Description    -> "Higgs vacuum expectation value"
},
lam == {
  ParameterType -> Internal,
  Value         -> MH^2/(2*vev^2),
  InteractionOrder -> {QED, 2},
  Description    -> "Higgs quartic coupling"
},
muH == {
  ParameterType -> Internal,
  Value         -> Sqrt[vev^2 lam],
  TeX           -> \[Mu],
  Description    -> "Coefficient of the quadratic piece of the Higgs potential"
},
yl == {
  ParameterType -> Internal,
  Indices       -> {Index[Generation], Index[Generation]},
  Definitions   ->
  {yl[i_?NumericQ, j_?NumericQ] :> 0 /; (i != j)},
  Value        ->
  {yl[1,1] -> Sqrt[2] yme / vev, yl[2,2] ->
  Sqrt[2] ymm / vev, yl[3,3] -> Sqrt[2] ymtau / vev},
  InteractionOrder -> {QED, 1},
  ParameterName  ->
  {yl[1,1] -> ye, yl[2,2] -> ym, yl[3,3] -> ytau},
  TeX           -> Superscript[y, 1],
  Description    -> "Lepton Yukawa couplings"
},
yu == {
  ParameterType -> Internal,
  Indices       -> {Index[Generation], Index[Generation]},
  Definitions   ->
  {yu[i_?NumericQ, j_?NumericQ] :> 0 /; (i != j)},
  Value        ->
  {yu[1,1] -> Sqrt[2] ymup/vev, yu[2,2] ->
  Sqrt[2] ymc/vev, yu[3,3] -> Sqrt[2] ymt/vev},
  InteractionOrder -> {QED, 1},
  ParameterName  ->
  {yu[1,1] -> yup, yu[2,2] -> yc, yu[3,3] -> yt},
  TeX           -> Superscript[y, u],
  Description    -> "Up-type Yukawa couplings"
}

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},
yd == {
  ParameterType -> Internal,
  Indices -> {Index[Generation], Index[Generation]},
  Definitions ->
  {yd[i_?NumericQ, j_?NumericQ] :> 0 /; (i != j)},
  Value ->
  {yd[1,1] -> Sqrt[2] ymdo/vev, yd[2,2] ->
  Sqrt[2] yms/vev, yd[3,3] -> Sqrt[2] ymb/vev},
  InteractionOrder -> {QED, 1},
  ParameterName ->
  {yd[1,1] -> ydo, yd[2,2] -> ys, yd[3,3] -> yb},
  TeX -> Superscript[y, d],
  Description -> "Down-type Yukawa couplings"
},
(* New Parameters *)
ThetaAngle == {
  ParameterType -> External,
  Indices -> {Index[DMGen]},
ComplexParameter -> False,
  Description -> "S Mixing Angle",
Value -> {ThetaAngle[1] -> 0.784557,
          ThetaAngle[2] -> -0.133177,
          ThetaAngle[3] -> 0.425457,
          ThetaAngle[4] -> 0.023141,
          ThetaAngle[5] -> 0.430331}
},
AlphaAngleL == {
  ParameterType -> External,
ComplexParameter -> False,
  Indices -> {Index[DMGen]},
Value -> {AlphaAngleL[1] -> -0.286260,
          AlphaAngleL[2] -> 0.717589,
          AlphaAngleL[3] -> 0.621443,
          AlphaAngleL[4] -> -0.006843,
          AlphaAngleL[5] -> 0.129936},
  Description -> "Psi1L Mixing Angle"
},
AlphaAngleR == {
  ParameterType -> External,
ComplexParameter -> False,
  Indices -> {Index[DMGen]},
Value -> {AlphaAngleR[1] -> -0.222173,
          AlphaAngleR[2] -> 0.023840,
          AlphaAngleR[3] -> -0.252651,
          AlphaAngleR[4] -> 0.704590,
          AlphaAngleR[5] -> 0.624333},
  Description -> "Psi1R Mixing Angle"
},
BetaAngleL == {
  ParameterType -> External,
ComplexParameter -> False,
  Indices -> {Index[DMGen]},
Value -> {BetaAngleL[1] -> -0.464243,

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        BetaAngleL[2] -> -0.683126,
        BetaAngleL[3] -> 0.557506,
        BetaAngleL[4] -> 0.002520,
        BetaAngleL[5] -> 0.083646},
    Description -> "PsiL Mixing Angle"
  },
  BetaAngleR == {
    ParameterType -> External,
    ComplexParameter -> False,
    Indices -> {Index[DMGen]},
    Value -> {BetaAngleR[1] -> -0.194017,
              BetaAngleR[2] -> 0.009989,
              BetaAngleR[3] -> -0.241142,
              BetaAngleR[4] -> -0.709200,
              BetaAngleR[5] -> 0.633359},
    Description -> "PsiR Mixing Angle"
  },
  Lambda1 == {
    ParameterType -> External,
    Value -> 0.00001,
    InteractionOrder -> {QED,1},
    Description -> "Dark Matter Coupling"
  },
  Lambda2 == {
    ParameterType -> External,
    Value -> 0.00001,
    InteractionOrder -> {QED,1},
    Description -> "Dark Matter Coupling"
  },
  Lambda3 == {
    ParameterType -> External,
    Value -> 0.00001,
    InteractionOrder -> {QED,1},
    Description -> "Dark Matter Coupling"
  },
  Lambda4 == {
    ParameterType -> External,
    Value -> 0.00001,
    InteractionOrder -> {QED,1},
    Description -> "Dark Matter Coupling"
  },
  CouplingA1 == {
    ParameterType -> External,
    ComplexParameter -> False,
    Indices -> {Index[Generation]},
    InteractionOrder -> {QED,1},
    Value -> {CouplingA1[1] -> 0.00001,
              CouplingA1[2] -> 0.00001,
              CouplingA1[3] -> 0.00001},
    Description ->
    "Coupling between D1R, the Fermion Doublet and the eta field"
  },
  CouplingA2 == {
    ParameterType -> External,

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ComplexParameter -> False,
Indices -> {Index[Generation]},
InteractionOrder -> {QED,1},
Value -> {CouplingA2[1] -> 0.00001,
          CouplingA2[2] -> 0.00001,
          CouplingA2[3] -> 0.00001},
Description ->
"Coupling between D2R, the Fermion Doublet and the eta field"
},
CouplingB1 == {
ParameterType -> External,
InteractionOrder -> {QED,1},
Value -> 0.00001,
Description ->
"Coupling between D1R, the Higgs Doublet and the S field"
},
CouplingB2 == {
ParameterType -> External,
InteractionOrder -> {QED,1},
Value -> 0.00001,
Description ->
"Coupling between D2R, the Higgs Doublet and the S field"
},
CouplingB3 == {
ParameterType -> External,
InteractionOrder -> {QED,1},
Value -> 0.00001,
Description ->
"Coupling between D1L, the Higgs Doublet and the S field"
},
CouplingB4 == {
ParameterType -> External,
InteractionOrder -> {QED,1},
Value -> 0.00001,
Description ->
"Coupling between D2L, the Higgs Doublet and the S field"
},
CouplingC1 == {
ParameterType -> External,
InteractionOrder -> {QED,2},
Value -> 0.00001,
Description ->
"Coupling between the Higgs Doublet and the eta field"
},
CouplingC2 == {
ParameterType -> External,
InteractionOrder -> {QED,2},
Value -> 0.00001,
Description -> "Quartic eta coupling"
},
MassS == {
ParameterType -> External,
Value -> 250,
Description -> "S Mass"
}

```



```

},

(* N. B. : only Cabibbo mixing! *)
CKM == {
  ParameterType -> Internal,
  Indices       -> {Index[Generation], Index[Generation]},
  Unitary       -> True,
  Value        -> {
    CKM[1,1] -> Cos[cabi],  CKM[1,2] -> Sin[cabi],  CKM[1,3] -> 0,
    CKM[2,1] -> -Sin[cabi], CKM[2,2] -> Cos[cabi],  CKM[2,3] -> 0,
    CKM[3,1] -> 0,         CKM[3,2] -> 0,         CKM[3,3] -> 1},
  TeX          -> Superscript[V,CKM],
  Description   -> "CKM-Matrix"}
};

(* ***** *)
(* ***** Lagrangian ***** *)
(* ***** *)

(* ***** Extension ***** *)

(* The extension Lagrangian is given in the gauge basis.
It is built from too parts with one being the h.c. of the other *)
LMDM1 := Block[{sp1,k1,k2,f,feynmangaugerules},
feynmangaugerules = If[Not[FeynmanGauge], {G0|GP|GPbar ->0}, {}];
ExpandIndices[
+ Lambda1 IndexDelta[k1,k2] D1Lbar[sp1,k1].CC[D2L[sp1,k2]]
+ Lambda2 IndexDelta[k1,k2] D1Rbar[sp1,k1].CC[D2R[sp1,k2]]
+ Lambda3 IndexDelta[k1,k2] D1Lbar[sp1,k1].[D1R[sp1,k2]]
+ Lambda4 IndexDelta[k1,k2] D2Lbar[sp1,k1].[D2R[sp1,k2]]
+ CouplingA1[f] IndexDelta[k1,k2] D1Rbar[sp1,k1].LL[sp1,k2,f] eta0
- CouplingA2[f] Eps[k1,k2] D2Rbar[sp1,k1].CC[LL[sp1,k2,f]] eta0
- CouplingB1 Eps[k1,k2] Phibar[k2] ProjM[sp1,sp2] D1Rbar[sp1,k1].S[sp2]
+ CouplingB2 IndexDelta[k1,k2] Phi[k2] ProjM[sp1,sp2] D2Rbar[sp1,k1].CC[S[sp2]]
- CouplingB3 Eps[k1,k2] Phibar[k2] ProjP[sp1,sp2] D1Lbar[sp1,k1].S[sp2]
+ CouplingB4 IndexDelta[k1,k2] Phi[k2] ProjP[sp1,sp2] D2Lbar[sp1,k1].CC[S[sp2]]
+ CouplingC1 IndexDelta[k1,k2] Phibar[k1] Phi[k2] HC[eta0] eta0
- CouplingC2 HC[eta0] eta0 HC[eta0] eta0
- 1/2 Meta0 HC[eta0] eta0
- 1/2 MassS Sbar[sp1] S[sp1]
,FlavorExpand->{SU2D,Generation}]/.feynmangaugerules
];

LMDM2 := HC[LMDM1];
LMDM := LMDM1 + LMDM2;

(* ***** Standard Model ***** *)
LGauge := Block[{mu,nu,ii,aa},
ExpandIndices[-1/4 FS[B,mu,nu] FS[B,mu,nu]
-1/4 FS[Wi,mu,nu,ii] FS[Wi,mu,nu,ii]
-1/4 FS[G,mu,nu,aa] FS[G,mu,nu,aa], FlavorExpand->SU2W]];

```

```

LFermions := Block[{mu},
  ExpandIndices[I*(
    QLbar.Ga[mu].DC[QL, mu] + LLbar.Ga[mu].DC[LL, mu]
    +uRbar.Ga[mu].DC[uR, mu] + dRbar.Ga[mu].DC[dR, mu]
    +lRbar.Ga[mu].DC[lR, mu]),
  FlavorExpand->{SU2W,SU2D} /. {CKM[a_, b_] Conjugate[CKM[a_, c_]]
->IndexDelta[b, c], CKM[b_, a_] Conjugate[CKM[c_, a_]]
->IndexDelta[b, c]};

LHiggs := Block[{ii, mu, feynmangaugerules},
  feynmangaugerules = If[Not[FeynmanGauge], {G0|GP|GPbar ->0}, {}];

  ExpandIndices[DC[Phibar[ii], mu] DC[Phi[ii], mu]
+muH^2 Phibar[ii] Phi[ii]
-lam Phibar[ii] Phi[ii] Phibar[jj] Phi[jj],
  FlavorExpand->{SU2D, SU2W} /. feynmangaugerules
];

LYukawa := Block[{sp, ii, jj, cc, ff1, ff2, ff3, yuk, feynmangaugerules},
  feynmangaugerules = If[Not[FeynmanGauge], {G0|GP|GPbar ->0}, {}];

  yuk = ExpandIndices[
  -yd[ff2, ff3] CKM[ff1, ff2] QLbar[sp, ii, ff1, cc].dR [sp, ff3, cc] Phi[ii] -
  yl[ff1, ff3] LLbar[sp, ii, ff1].lR [sp, ff3] Phi[ii] -
  yu[ff1, ff2] QLbar[sp, ii, ff1, cc].uR [sp, ff2, cc] Phibar[jj] Eps[ii, jj]
  , FlavorExpand -> SU2D];
  yuk = yuk /. { CKM[a_, b_] Conjugate[CKM[a_, c_]]
-> IndexDelta[b, c], CKM[b_, a_] Conjugate[CKM[c_, a_]]
-> IndexDelta[b, c]};
  yuk+HC[yuk] /. feynmangaugerules
];

LGhost := Block[{LGh1, LGhw, LGhs, LGhphi, mu, generators, gh, ghbar,
  Vectorize, phi1, phi2, togoldstones, doublet, doublet0},
  (* Pure gauge piece *)
  LGh1 = -ghBbar.del[DC[ghB, mu], mu];
  LGhw = -ghWibar.del[DC[ghWi, mu], mu];
  LGhs = -ghGbar.del[DC[ghG, mu], mu];

  (* Scalar pieces: see Peskin pages 739-742 *)
  (* phi1 and phi2 are the real degrees of freedom of GP *)
  (* Vectorize transforms a doublet in a vector in the phi-basis,
  i.e. the basis of real degrees of freedom *)
  gh = {ghB, ghWi[1], ghWi[2], ghWi[3]};
  ghbar = {ghBbar, ghWibar[1], ghWibar[2], ghWibar[3]};
  generators = {-I/2 g1 IdentityMatrix[2], -I/2 gw PauliSigma[1],
  -I/2 gw PauliSigma[2], -I/2 gw PauliSigma[3]};
  doublet = Expand[{-I phi1 - phi2/Sqrt[2], Phi[2]} /. MR$Definitions
  /. vev -> 0];
  doublet0 = {0, vev/Sqrt[2]};
  Vectorize[{a_, b_}] := Simplify[{Sqrt[2] Re[Expand[a]],
  Sqrt[2] Im[Expand[a]], Sqrt[2] Re[Expand[b]],
  Sqrt[2] Im[Expand[b]]} /. {Im[_]->0, Re[num_]->num}];
  togoldstones := {phi1 -> (GP + GPbar)/Sqrt[2],

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phi2 -> (-GP + GPbar)/(I Sqrt[2]);
LGhphi=Plus@@Flatten[Table[-ghbar[[kkk]].gh[[111]]
Vectorize[generators[[kkk]].doublet0]
.Vectorize[generators[[111]].(doublet+doublet0)]
,{kkk,4},{111,4}]] /.tgoldstones;

ExpandIndices[ LGhs + If[FeynmanGauge, LGh1 + LGhw + LGhphi,0]
, FlavorExpand->SU2W]];

(* The Lagrangian of the Standard model *)
LSM:= LGauge + LFermions + LHiggs + LYukawa + LGhost;

(* The Standard model with added extension *)
LSME:= LSM + LMDM;

```

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Plagiatserklärung des Studierenden

Hiermit versichere ich, dass die vorliegende Arbeit

“FeynRules implementation of minimal dark matter and neutrino masses”

bzw.

“FeynRules Implementierung von minimaler dunkler Materie und Neutrinomassen”

selbstständig verfasst worden ist, dass keine anderen Quellen und Hilfsmittel als die

angegebenen benutzt worden sind und dass die Stellen der Arbeit, die anderen Werken

- auch elektronischen Medien - dem Wortlaut oder Sinn nach entnommen wurden, auf

jeden Fall unter Angabe der Quelle als Entlehnung kenntlich gemacht worden sind.

(Datum, Unterschrift)

Ich erkläre mich mit einem Abgleich der Arbeit mit anderen Texten zwecks Auffind-

ung von Übereinstimmungen, sowie mit einer zu diesem Zweck vorzunehmenden Spe-

icherung der Arbeit in einer Datenbank einverstanden.

(Datum, Unterschrift)