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Trabajo Fin de Grado

Spin and colour correlations for prompt photon production in heavy-ion collisions

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1 Introduction

According to our current knowledge, in the early universe all the known particles were in an extremely hot, dense state [1]. Under these conditions, coloured particles would interact less strongly with each other and be deconfined from hadrons. This "gas" of elementary particles is what we call the quark-gluon plasma. It would only last a few microseconds before it starts cooling down, then the interaction would become stronger, resulting in the combination of partons to form hadrons.

In the last few decades this hypothetical new state of matter has been intensively studied by particle and nuclear physicists. It is believed that the QGP would be involved in heavy-ion collisions [2]. In this context, experiments have been conducted at high-energy particle colliders aimed at arriving at a solid evidence of its existence and at examining its properties. However, because of its short lifetime the QGP has never been observed directly; the information has to be obtained indirectly, *e.g.* through the photons coming from it.

When two heavy ions collide, our theoretical model predicts that partons (either a quark, an antiquark or a gluon) emerge from each ion and interact with each other. The rest of the nuclei would overlap in a highly excited state in which the partons could move freely. Therefore, the partonic interaction of those selected outgoing components of the ions would occur immersed in this QGP, rather than in vacuum or in air. The idea is the following: we calculate theoretically some quantity, normally the distribution of final photons, as if the process had happened in vacuum. We compare this result with the experimental data and atribute possible deviations to the presence of the QGP. It is clear, that we need high precision calculations if we want to proceed in this way. In the framework of perturbative quantum chromodynamics this would mean including the contribution from the next-to-leading order (NLO). This can only partly be performed analitically and the use of numerical methods is required.

The goal of this work is the computation of some quantities that are needed in high-precission calculations of the cross section of photon production in quantum chromodynamics. In Section 2 some concepts of quantum chromodynamics are reviewed. The detection of the QGP through photons is shortly discussed in Section 3. Section 4 includes a global description of how general purpose event generators work, a definition of the quantities to be evaluated and a full calculation of them.

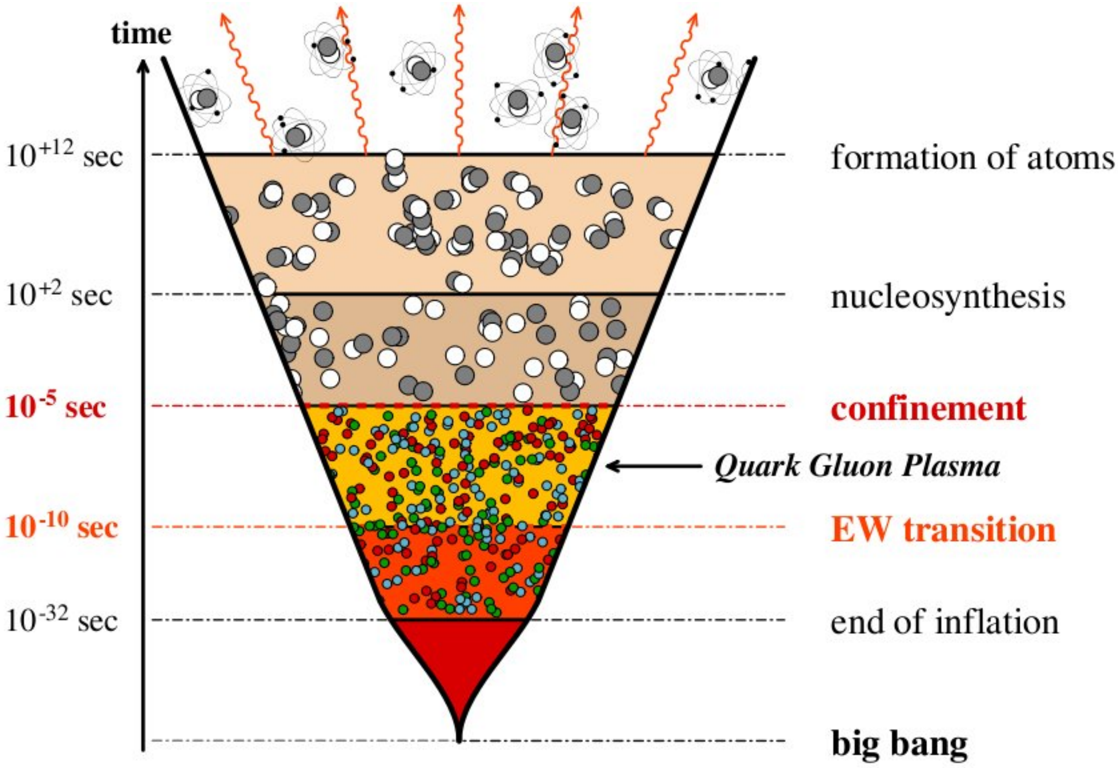


Figure 1: The Quark-Gluon Plasma in the early universe. Image taken from [3]

2 Quantum Chromodynamics

It is well known that there are four fundamental interactions in the universe: gravity, electromagnetic, weak and strong interactions. At microscopic level the first of them can be neglected and the other three are formulated in terms of quantum field theories (QFT's).

Quantum chromodynamics is the theory of the strong interaction. It combines the principles of QFT with a non-abelian gauge symmetry group SU(3) to describe the interactions between quarks and gluons, collectively named as partons [4]. The quarks are point-like, massive, spin-1/2 particles and are represented by a four-component Dirac spinor ψ. Since they are not able to interact directly with each other, the gluons are required for mediating the force, analogously to the photons in quantum electrodynamics. These force carriers are massless bosons (spin 1) and are represented by four-vector fields A^a_μ.

The charge of strongly interacting particles is called colour. Unlike the electric charge in the electromagnetic force, colour is somewhat more complicated. The interaction of quarks according to their colours is due to an exact SU(3) symmetry, leading to three different colours: red, blue and green. Also gluons carry colour charge, so when a quark emits a gluon, its colour changes. There are in total eight different gluonic colour states, a gluonic octet. In addition, quarks have six flavour degrees of freedom, normally denoted as: u (up), d (down), s (strange), c (charm), b (bottom) and t (top) [5].

The QCD Lagrangian is:

$$\mathcal{L} = \sum_{\text{flavours}} \bar{\psi}(D^\mu \gamma_\mu - m)\psi - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} \tag{1}$$

with the gluon field tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu \tag{2}$$

and the covariant derivative

$$D^\mu = \partial^\mu - i g_s A^{a\mu}\, t^a \tag{3}$$

The Einstein summation convention is assumed (summation over repeated indices (a)), also the usual covariant/contravariant tensor notation. g_s is the strong coupling constant and f^{abc} are the structure constants of SU(3) (see App. A).

One can deduce *a priori* some couplings of the particles. Notice that the term $g_s f^{abc} A^b_\mu A^c_\nu$ in equation (2) does not appear in the electromagnetic tensor of QED. It has its origin in the non-Abelian gauge symmetry and implies couplings between the gauge bosons, *i.e.* the gluons. More precisely, terms of the form $\partial_\mu A^a_\nu g_s f^{abc} A^{b\,\mu} A^{c\,\nu}$ when expanding $F^a_{\mu\nu} F^{a\,\mu\nu}$ in equation (1) will lead to 3-gluon vertex and the term $g_s^2 f^{abc} f^{ade} A^b_\mu A^c_\nu A^{d\,\mu} A^{e\,\nu}$ to a 4-gluon vertex [6].

However, the Lagrangian is a theoretical and abstract quantity. We need an empirical quantity, that allows us to compare the theoretical calculations with the experiments: this is the cross section. In particle physics we deal with scattering experiments, in which some initial particles interact to produce a final state. The differential cross section indicates how likely a particular final state is. In a partonic process with two incoming particles, denoted as a and b, scattering into f final particles $1, 2, ..., f$, the differential cross section can be written as:

$$d\hat{\sigma} = \frac{(2\pi)^4}{4E_a E_b v_{ab}} \prod_{i=1}^{f} \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \delta^{(4)} \left(\sum_{i=1}^{f} p_i - p_a - p_b \right) \overline{|\mathcal{M}_{fi}|^2} \tag{4}$$

The derivation of this formula step by step along with a definition of the cross section can be found in some introductory books on QFT as [7, 8]

p_a and p_b, E_a and E_b are respectively the momenta and energies of the incoming particles and p_i and E_i are those of the outgoing ones. v_{ab} is the modulus of the relative velocity of particle a with respect to b and the four-δ function assures four-momentum conservation in the process. The total cross section $\hat{\sigma}$ is the integral of equation (4) over the final momenta [9].

$\mathcal{M}_{fi}$ is called the invariant matrix element and can obtained with the help of the Feynman rules, which can be derived from the Lagrangian (for more information see *e.g.* [7]).$\overline{|\mathcal{M}_{fi}|^2}$ is the averaged matrix element squared. An example of a detailed calculation of $\overline{|\mathcal{M}_{fi}|^2}$ is done in Section 4.3. The Feynman rules of quantum chromodynamics required for those calculations are shown in the Appendix A, together with their respective diagrams.

Two further essential concepts of quantum chromodynamics are *asymptotic freedom* and *confinement*. Asymptotic freedom tells us, that at high energies the coupling constant decreases, *i.e.* that the interaction between partons becomes very weak. Confinement explains why we do not observe coloured particles in nature: quarks and gluons have to combine constructing colourless bound states. Mathematically this is translated into a very fast growth of the coupling constant at increasing distances.

2.1 Factorization Theorem

In perturbation theory the perturbative parameter has to be smaller than one for the expansion to be sensible. This will not be the case for the calculation of hadronic wavefunctions; the coupling constant, which is our perturbative parameter, is not necessarily small enough. Thus, when dealing with hadronic scattering a purely perturbative approach is no longer valid and it has to be combined it with a non-perturbative method [10].

The factorization theorem states that the hadron scattering cross section can be factorized or split into two parts. One of them can be calculated pertubatively, and the other can not. The former can be calculated from perturbative quantum chromodynamics (hereafter, pQCD) and can be viewed as the cross section of the partonic process. The latter one absorbes the non-perturbative aspects in a function that accounts for how quarks and gluons are arranged inside the hadrons. Namely, the probability of finding a parton a in a hadron A with momentum fraction between x and $x+dx$ is the parton distribution function (short, PDF) and will be denoted as $G_{a/A}(x)$. Furthermore the probability of obtaining a hadron C from a parton c is called fragmentation function, $D_{C/c}(z)$ (notation from [10]). Altogether, the cross section can be expressed as

$$
\begin{aligned}
d\sigma(AB \to C + X) = \sum_{abcd} \int & dx_a dx_b dz_c G_{a/A}(x_a, \mu_f) G_{b/B}(x_b, \mu_f) \\
& D_{C/c}(z_c, \mu_D) d\hat{\sigma}(ab \to cd)
\end{aligned}
\tag{5}
$$

Here, capital letters refer to hadrons and small letters to partons. $\mathrm{d}\hat{\sigma}$ is the differential cross section of the partonic process and the momentum fractions x_a, x_b and z_c are integrated in the interval from zero to one. The sum over $abcd$ considers all the possible partons that may participate in the hard process. μ_f and μ_D are the factorization scales. Furthermore, since we are focused on the process when two particular hadrons A and B collide to yield a hadron C, obtained from the parton c, we do not worry about which other hadrons are to be found in the final state. These unmeasured final state hadrons are represented by X.

2.2 Parton Distribution Functions

As mentioned above, the parton distribution functions can not be determined in the framework of perturbative theory, but rather, data coming from the measurement of some reference processes has to be analysed to that end [10]. In this section we review these processes and their connection to the parton distribution functions.

To determine the quark PDF, deep inelastic lepton-nucleon scattering is a suitable process. Given an initial lepton with four-momentum $k = (E, \mathbf{k})$

that gets scattered with a four-momentum $k' = (E', \mathbf{k}')$, the momentum of the exchanged photon will be $q = k - k'$ (see Figure 2). Due to q^2 normally being smaller than zero, it is usual to define $Q^2 = -q^2 > 0$. A basic review of deep inelastic scattering and its consequences in the parton model is found in [11].

By analogue to the Heisenberg uncertainty relation:

$$\Delta x \Delta p \gtrsim h \tag{6}$$

we can say that:

$$\Delta x \sim \frac{h}{\Delta p} \tag{7}$$

It is easy to see from here that if we are interested in resolving small distances, we need large values of the momentum, *i.e* large values of Q^2. Deep inelastic scattering refers to the processes in which Q^2 is sufficiently high to probe the inner structure of hadrons. This happens for $Q^2 > 0,5$ GeV^2, approximately [11].

We briefly analyse a general lepton-nucleon scattering from the point of view of quantum field theory. At leading order (LO), a lepton emits a photon which interacts with a quark coming from the nucleon (see Figure 2). The Born squared amplitude is here proportional to the electromagnetic coupling constant squared (α^2). Thus, in the formula of the cross section, the quark PDF will be present. One can extract it by comparing the LO calculation with experimental results. However, there is no LO contribution to this process that will provide us information about the gluon PDF. A gluon has to produce a quark-antiquark pair, which will be able to interact with the emitted photon. This is a next to leading order calculation and the squared amplitude is now proportional to the electromagnetic coupling constant squared times the strong coupling constant ($\alpha^2\alpha_s$).

It is convenient, at this point, to search for a process involving the gluon distribution function at LO. A possibility for this is a hadron-hadron collision. If a quark and a gluon come out of the hadrons, they can interact producing an outgoing quark and a direct photon, corresponding to a squared amplitude proportional to $\alpha\alpha_s$. In this diagram, the quark PDF can be obtained from the deep inelastic scattering, and the cross section of the hard process can be computed in perturbation theory. The only remaining unknown is then the gluon PDF, which can be determined by comparing with the data. This also has experimental advantages as photons are generally easier to measure than, for example, jets [12].

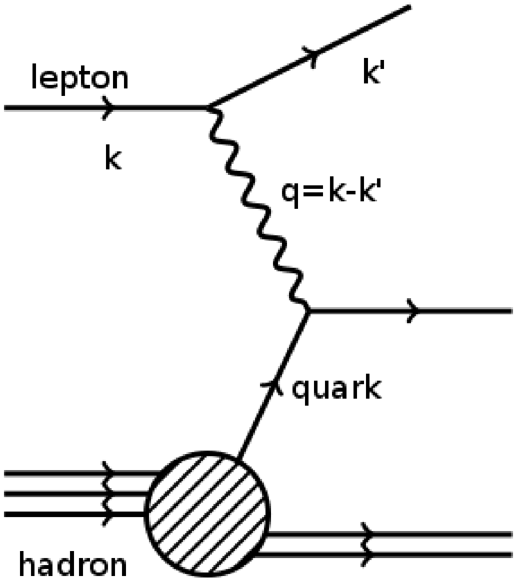


Figure 2: Lepton-nucleon scattering at high energies involves the quark PDF at leading order. The amplitude is proportional to α^2, coming from the lepton-photon vertex and the quark-photon vertex.

Going one step further, we may think of a hadron-nucleus collision. In this case, there is an important aspect to be considered: as the strong interaction is also the responsible for the nucleons being attached in the nucleus, it is not assured that the quark PDFs will be the same as for a single hadron. In fact, inside a nucleus, the presence of the other nucleons will alter the distribution of the partons inside some particular nucleon. As a consequence, we have to introduce the nuclear parton distribution functions (nPDFs). Constraints on nPDFs could be deduced from experiments as proton-heavy ion collisions (pA collisions) [13]

3 The Quark-Gluon Plasma

In the last section we discussed the effects of *confinement* and *asymptotic freedom*. We said that at increasing energies the strong coupling constant goes to zero. At sufficiently high energies the strong interaction does not force partons to be coupled in coulourless bound states: they are then said to be deconfined [5]. In the limit of very large energies, much larger than the rest mass of the particles, this state might be considered as a hot relativistic free gas. In general, the QGP is a medium of coloured components floating around with a relatively low interaction. This medium has been studied in terms of thermodynamics and lattice QCD [1]. When it cools down, the coupling constant becomes larger and confinement starts making the partons group in hadrons. After this phase transition we are left with a hadron gas.

3.1 Detection of the QGP

In heavy-ion collisions, the QGP exists for a very short time before decaying in the hadron gas. For that reason, its detection is not trivial. Electromagnetic radiation is a good candidate for analysing the QGP . Because photons only interact electromagnetically, and this interaction is much weaker than the strong interaction, they can travel through this strongly interacting region without being rescattered. The size of their mean path is longer than the dimensions of the QGP and, therefore, they will carry information about the point in which they were produced [14]. We will see, however, that not every photon will be useful in the description of the QGP; we are only interested in the so-called direct photons. In this section we will define this term, along with the related terms prompt photon and thermal photon [15].

Direct photons are those who are directly created in particle collisions, in contrast with decay photons which emerge from the electromagnetic decay of baryons or hadrons. In relativistic hadron collisions the contribution to the radiation coming from decay photons is much larger, and it is difficult to separate the direct photons from them. There are some experimental procedures to solve this problem (for further information see [15]).

The Figure 3 shows four ways of direct photon production in relativistic hadron collisions. Attending to this, it is possible to classify the direct photons in two groups:

1. **Prompt photons:** are represented in the panels A and B. Panel A exhibits the two processes at leading order of creation of a photon from the hard scattering of partons: the QCD Compton scattering (upper diagram) and the quark-antiquark annihilation (lower diagram). These

processes will be studied in more detail in the next section. In panel B, above we see the emission of a photon from an outgoing quark; and below, this same emission induced by the interactions of such quark with a dense strongly interacting medium, when travelling through it. These are known as fragmentation photons. The rate of prompt photon production can be calculated from perturbative QCD and, accordingly, they are often called pQCD photons.

2. **Thermal photons:** denotes the radiation proceding from a medium in thermal equilibrium. In panel C, this medium is the QGP, the degrees of freedom are partons and the possible processes are analogue to those discussed above. If the medium is composed by hadrons instead, like for example in the hadron gas, photon production will occur in hadron scattering, as can be seen in panel D.

As already mentioned, QGP is believed to appear in heavy-ion collisions (AA collisions). This has been researched in Pb-Pb collisions [16] and in Au-Au collisions [17]. Subtracting the contribution from the decay photons to the measured photons, we would end up with spectrum of only direct photons. On the other hand, the yield of prompt photons can be calculated by making use of nPDFs (obtained from pA collisions) and pQCD. Comparing these two results, one should see that more direct photons are measured than expected from pQCD. This difference could be due to the emission of thermal photons from a thermalized medium, as the QGP or a hadron gas [16].

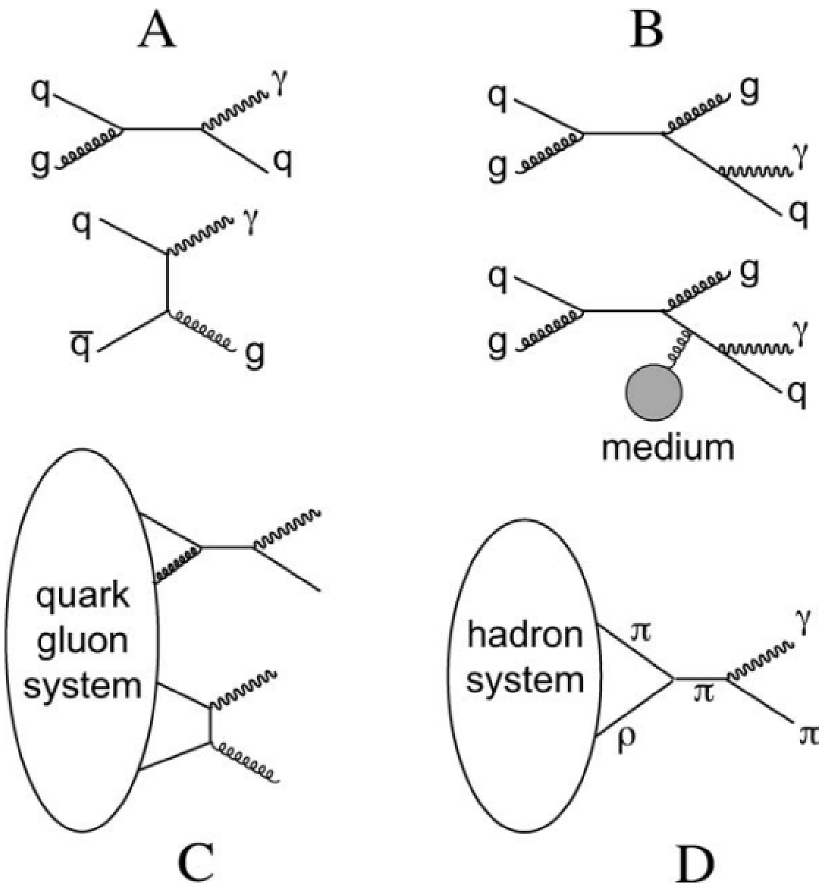


Figure 3: Ways of production of photons in QCD in Heavy-Ion Collisions. (A) Direct photon production at tree level at leading order in hard scattering. (B) Photon emitted by an outgoing quark spontaneously (above) and as a result of the interaction with the medium (below). (C) Interaction of the partons in the QGP to produce direct photons. (D) Hadron interactions from a thermalized hadron gas with direct photon in the final state. Image from [15]

4 Calculations

4.1 General Purpose Event Generators

Event generators simulate hypothetical processes with the distribution predicted by the theory. We discuss in this section how this is done. The differential cross section of a partonic process can be calculated with the Eq.(4) and this is connected to the hadronic cross section via Eq.(5). The final differential cross section depends on many variables: at first sight, on the integration variables, that is, on the momenta of the final and initial particles. But also, it accounts for the process that is taking place at the parton level and, therefore, changes from one process to another.

In the explanation of how General Purpose Event Generators (GPEG) work, the processes of direct photon production at tree level will be taken as an example. At LO we find two processes: quark-antiquark annihilation (process 1) and QCD Compton scattering (process 2), each of them with a differential cross section, namely $d\sigma_1$ and $d\sigma_2$. The differential cross section of photon production will be $d\sigma = d\sigma_1 + d\sigma_2$ and will have a maximum, $d\sigma_{max}$, that can be estimated with numerical methods. The GPEG selects randomly an event: a process (1 or 2) and the momenta of the intial and final partons. Then it computes its corresponding differential cross section $d\sigma_{event}$, or event weight, which will be proportional to the probability that such an event occurs. If we divide this by the maximum weight, $d\sigma_{max}$, we are left with a number in the interval [0,1]. Generating a random number g in this interval allows to decide if the candidate event will be accepted: if the ratio $d\sigma_{event}/d\sigma_{max}$ is larger than g, the event is accepted; if not, it is rejected [18]. After a large number of produced events, they will be statistically distributed according to their probability assigned by the cross section.

Up to the present, several event generators have been developed, like PYTHIA or HERWIG. In this work we deal with POWHEG (Positive Weighted Hardest Emission Generator) which is a framework to implement NLO calculations in known general purpose event generators [19]. The package POWHEG-BOX is a user-friendly computer framework for implementing the POWHEG method. A detailed description of its code is far away from the goal of this work. Many processes have already been encoded in this framework, like Z pair hadroproduction, $e^+ \, e^-$ annihilation into hadrons or Higgs boson production via gluon fusion [19].

4.2 Ingredients for POWHEG Implementation

In this work we provide some of the ingredients needed for the implementation of prompt photon production: the Born squared amplitude and the spin- and colour-correlated Born squared amplitudes. All these quantities are neatly described in [19] and we will keep the same notation. The Born squared amplitude is simply the average of the squared matrix element of the partonic process and will be denoted as $\mathcal{B}$.

The colour correlated Born amplitude is defined as:

$$2s_b\mathcal{B}_{ij} = -N \sum_{\substack{\text{spins}\\ \text{colours}}} \mathcal{M}_{\{c_k\}} \left(\mathcal{M}^\dagger_{\{c_k\}}\right)_{\substack{c_i\to c_i'\\ c_j\to c_j'}} T^d_{c_i,c_i'} T^d_{c_j,c_j'} \tag{8}$$

$\mathcal{M}_{\{c_k\}}$ is the matrix element, $\{c_k\}$ refers to the colour indexes of the external particles and the arrows in $\mathcal{M}^\dagger_{\{c_k\}}$ indicate that the colour index c_i has to be replaced by c_i' and c_j by c_j'. The N in front of the summation counts for the average over initial spin and colour. For incoming gluons, $T^d_{\alpha\alpha'} = if_{\alpha d\alpha'}$, for incoming quarks (as well as for outgoing antiquarks) $T^d_{aa'} = t^d_{aa'}$ and for incoming antiquarks (and outgoing quarks) $T^d_{aa'} = -t^d_{a'a}$. Colour conservation demands:

$$\sum_{i\neq j} \mathcal{B}_{ij} = C_{f_j}\mathcal{B} \tag{9}$$

with the summation over all coloured particles. C_{f_j} is the Casimir constant for the colour representation of particle j, in our case, C_F for quarks and antiquarks and C_A for gluons. Eq.(9) will be used as a consistency check of our results.

The spin-correlated Born amplitude is defined to be only different from zero for processes involving gluons in the intial or final state. If the j^{th} leg is a gluon:

$$\mathcal{B}^{\mu\nu}_j = N \sum_{\{i\},s_j,s_j'} \mathcal{M}\left(\{i\}, s_j\right) \mathcal{M}^\dagger\left(\{i\}, s_j'\right) \left(\epsilon^\mu_{s_j}\right)^* \epsilon^\nu_{s'j} \tag{10}$$

Here, s_j is the polarization index of the j^{th} that has to be replace by s_j' in $\mathcal{M}^\dagger$, $\{i\}$ are all the other indices of colours, spins, etc. The $\epsilon^\mu_{s_j}$ and $\epsilon^\nu_{s'j}$ are polarization vectors, normalized as:

$$\sum_{\mu,\nu} g_{\mu\nu} \left(\epsilon^\mu_{s_j}\right)^* \epsilon^\nu_{s'j} = -\delta_{s_j s_j'} \tag{11}$$

The condition:

$$\sum_{\mu,\nu} g_{\mu\nu} \mathcal{B}_j^{\mu\nu} = -\mathcal{B} \tag{12}$$

has to be fulfilled. Later, Eq.(12) will be used to verify the results obtained for spin-correlated Born amplitudes.

4.3 Calculations and Results

In this section we analyse the processes of direct photon production at tree level in QCD. There are two processes at leading order: quark- antiquark annihilation and QCD Compton scattering.

Quark- antiquark annihilation

$$q(p_a) + \bar{q}(p_b) \longrightarrow g(p_1) + \gamma(p_2)$$

In this process we find an incoming quark and antiquark with momenta p_a and p_b, spins s_a and s_b and colours c_a and c_b respectively. The final particles are a gluon with momentum p_1 and a photon with p_2. The quark propagator carries a momentum q. There are two contributing diagrams: a t-channel and an u-channel (see Figure 4).

1. **Born amplitude**

   We start calculating the squared matrix element. The non-squared matrix element of the process is not simply the sum of the matrix elements of the two diagrams: we have to be aware that these diagrams have a relative phase, that is, we do not know beforehand if we have to add or substract them. The relative sign of the diagrams was deduced in the following way. We drew the squared diagrams $|\mathcal{M}_t|^2$, $|\mathcal{M}_u|^2$, $\mathcal{M}_t\mathcal{M}_u^*$ and $\mathcal{M}_t^*\mathcal{M}_u$. We drew lines joining the external fermion lines, incoming fermions of one diagram with outgoing fermions of the other and viceversa and we counted the number of loops in each case. This number has to be the same for $|\mathcal{M}_t|^2$ and $|\mathcal{M}_u|^2$ on one hand and for $\mathcal{M}_t\mathcal{M}_u^*$ and $\mathcal{M}_t^*\mathcal{M}_u$ on the other. If the difference between these number of loops is odd, the relative sign is minus and if it is even, the sign is plus. This is schematically explained in [20]. All the same, as we only have one fermion line, in our case there will always be one loop and the relative sign is then positive:

$$\mathcal{M} = \mathcal{M}_t + \mathcal{M}_u \tag{13}$$

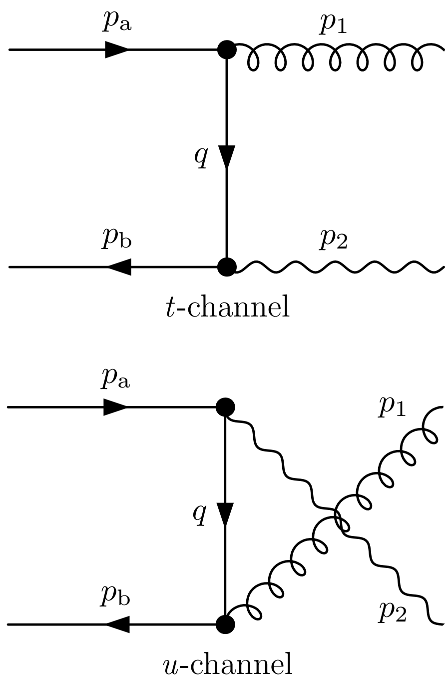


Figure 4: Contributing Feynman diagrams at tree level to the process of quark-antiquark annihilation

Squaring this, yields:

$$|\mathcal{M}|^2 = |\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 + 2\mathcal{M}_t\mathcal{M}_u^* \tag{14}$$

Where we used that $\mathcal{M}_t^*\mathcal{M}_u = \mathcal{M}_t\mathcal{M}_u^*$, which is easy to check. In general, computing $|\mathcal{M}_t|^2$, $|\mathcal{M}_u|^2$ and $\mathcal{M}_t\mathcal{M}_u^*$ is more convenient and will create less confusion than computing the total matrix element and then squaring it. So we will proceed in that way.

The average over spins and colours of the squared matrix element is usually named Born amplitude and represented by $\mathcal{B}$. We will keep this notation from now on:

$$\mathcal{B} = \overline{|\mathcal{M}|^2} = \overline{|\mathcal{M}_t|^2} + \overline{|\mathcal{M}_u|^2} + 2\overline{\mathcal{M}_t\mathcal{M}_u^*} \tag{15}$$

From the Feynman rules (see App. A) and following the fermion lines in direction opposite to the arrows, we get for the t-channel:

$$\mathcal{M}_t = \bar{v}^{(s_b)}(p_b)c_b^\dagger iee_q\gamma^\mu \, \frac{i\not{q}}{q^2} \left(-ig_s t^\alpha \gamma^\nu\right) u^{(s_a)}(p_a)c_a\epsilon_\mu^*(p_2)\epsilon_\nu^*(p_1)\chi^\alpha \tag{16}$$

Rearranging factors:

$$\mathcal{M}_t = \frac{iee_q g_s}{q^2} \left[\bar{v}^{(s_b)}(p_b)\gamma^\mu \not{\epsilon}\gamma^\nu u^{(s_a)}(p_a)\right] \left[c_b^\dagger t^\alpha c_a \chi^\alpha\right] \epsilon_\mu^*(p_2)\epsilon_\nu^*(p_1) \tag{17}$$

The meaning of the variables in this equation is collected in Appendix A. In addition, the approximation of massless quarks was made, setting $m = 0$.

Now, the complex conjugate of $\mathcal{M}_t$ has to be computed. But before, notice that from Eq.(61), $c_b^\dagger t^\alpha c_a$ is the element ba of the matrix t^α: t^α_{ba}. The complex conjugate of the pre-factor and the polarization vectors is trivial. We will leave the first square bracket for the end, because it is a bit more complicated. The second bracket:

$$[t^\alpha_{ba}\chi^\alpha]^* = \chi^\alpha \left(t^\alpha_{ba}\right)^* \tag{18}$$

as χ^α is a real number. Knowing that t^α are hermitian, $t^\alpha = (t^\alpha)^\dagger$, then $(t^\alpha_{ba})^* = (t^\alpha_{ba})^T = t^\alpha_{ab}$.

Writing $\bar{v} = v^\dagger\gamma^0$ and making use of Eq.(67) and Eq.(68), the complex conjugate of the first square bracket is:

$$\begin{aligned}\left[\left(v^{(s_b)}(p_b)\right)^\dagger \gamma^0\gamma^\mu\not{\epsilon}\gamma^\nu u^{(s_a)}(p_a)\right]^* &= \left(u^{(s_a)}\right)^\dagger (\gamma^\nu)^\dagger \left(\not{\epsilon}\right)^\dagger (\gamma^\mu)^\dagger \gamma^0 v^{(s_b)} \\ &= \left(u^{(s_a)}\right)^\dagger \gamma^0\gamma^\nu\not{\epsilon}\gamma^\mu v^{(s_b)} = \bar{u}^{(s_a)}\gamma^\nu\not{\epsilon}\gamma^\mu v^{(s_b)}\end{aligned} \tag{19}$$

Altogether, the complex conjugate of the matrix element reads:

$$\mathcal{M}_t^* = -\frac{iee_q g_s}{q^2} \left[\bar{u}^{(s_a)}\gamma^\sigma\not{\epsilon}\gamma^\rho v^{(s_b)}\right] \left[t^\beta_{ab}\chi^\beta\right] \epsilon_\rho(p_2)\epsilon_\sigma(p_1) \tag{20}$$

We changed the indices to avoid confusion when multiplying this with Eq.(17).

At this point, it is convenient to introduce the Mandelstam variables:

$$s = (p_a + p_b)^2 = (p_1 + p_2)^2 \tag{21a}$$

$$t = (p_a - p_1)^2 = (p_b - p_2)^2 \tag{21b}$$

$$u = (p_a - p_2)^2 = (p_b - p_1)^2 \tag{21c}$$

If all initial and final particles are massless, $p_a^2 = p_b^2 = p_1^2 = p_2^2 = 0$ and:

$$s = 2p_a p_b = 2p_1 p_2 \tag{22a}$$

$$t = -2p_a p_1 = -2p_b p_2 \tag{22b}$$

$$u = -2p_a p_2 = -2p_b p_1 \tag{22c}$$

Back to our diagram, the momentum of the propagator, q, can be deduced imposing momentum conservation at each vertex:

$$q = p_a - p_1 \Longrightarrow q^2 = t \tag{23}$$

This is the reason why it is often called t-channel. We are now in conditions to compute the average matrix element squared. In order to do so, we have to sum over final colours and polarizations and average over intial colours and spins. As quarks and antiquarks have two possible spins, we have to multiply by 1/4, also, as they come in three colours, we would have to multiply by 1/9, but we will leave the number of colours as a parameter, N, which is general for a symmetry SU(N), and multiply by $1/N^2$ instead:

$$\overline{|\mathcal{M}_t|^2} = \frac{1}{4N^2}\frac{e^2 e_q^2 g_s^2}{t^2} \sum_{\substack{\text{spins}\\ \text{colours}\\ \text{polar.}}} \left[\bar{v}^{(s_b)}\gamma^\mu(\not{p}_a - \not{p}_1)\gamma^\nu u^{(s_a)}\bar{u}^{(s_a)}\gamma^\sigma(\not{p}_a - \not{p}_1)\gamma^\rho v^{(s_b)}\right] \times$$

$$\times \left[t^\alpha_{ba} t^\beta_{ab} \chi^\alpha \chi^\beta\right] \epsilon^*_\mu(p_2)\epsilon_\rho(p_2)\epsilon^*_\nu(p_1)\epsilon_\sigma(p_1) \tag{24}$$

We will evaluate this expression step by step. First of all, from Eq.(11), the sum over the polarization vectors will yield $(-g_{\mu\rho})(-g_{\nu\sigma}) = g_{\mu\rho}g_{\nu\sigma}$. The indices σ and ρ of these tensors contract with those of the γ-matrices from the first square bracket.

In the second bracket we find the product $\chi^\alpha\chi^\beta$ where χ is a vector with one of its components equal to one, and the others equal to zero, especifying the colour of the gluon. It is straightforward that this product will yield a Kronecker delta: $\delta_{\alpha\beta}$. Also, notice that:

$$t^\alpha_{ba} t^\beta_{ab} = Tr(t^\alpha t^\beta) \tag{25}$$

Consequently, the second bracket is often called "colour trace" or also "colour factor" [23]. This gives (using Eq.(63)):

$$Tr(t^\alpha t^\beta)\,\delta_{\alpha\beta} = Tr(t^\alpha t^\alpha) = Tr\left(\frac{N^2-1}{2N}\cdot\mathbf{1}\right) = \frac{N^2-1}{2} \tag{26}$$

It only remains then, to evaluate the first bracket. Writing this scalar as its trace and taking into account that the trace is cyclic ($Tr(ABC) = Tr(BCA) = Tr(CAB)$), we arrive at:

$$Tr\left[v^{(s_b)}\bar{v}^{(s_b)}\gamma^\mu(\not{p}_a - \not{p}_1)\gamma^\nu u^{(s_a)}\bar{u}^{(s_a)}\gamma_\nu(\not{p}_a - \not{p}_1)\gamma_\mu\right] \tag{27}$$

Summing over spins (Eq.(60)) for massless quarks:

$$\begin{aligned} &Tr\left[\not{p}_b\gamma^\mu(\not{p}_a - \not{p}_1)\gamma^\nu\not{p}_a\gamma_\nu(\not{p}_a - \not{p}_1)\gamma_\mu\right] \\ &= 4Tr\left[\not{p}_b(\not{p}_a - \not{p}_1)\not{p}_a(\not{p}_a - \not{p}_1)\right] \end{aligned} \tag{28}$$

Where we used Eq.(69) from Appendix B. Knowing that $p_a^2 = m_a^2 = 0$ and using Eq.(70), this is equal to:

$$4Tr\left[\not{p}_b\not{p}_1\not{p}_a\not{p}_1\right] \tag{29}$$

Commuting $\not{p}_a$ with $\not{p}_1$ according to the commutation relation (Eq.(66)):

$$4Tr\left[\not{p}_b\not{p}_1\left(2g^{\mu\nu}p_{a\,\mu}p_{1\,\nu} - \not{p}_1\not{p}_a\right)\right] = 8(p_ap_1)Tr\left[\not{p}_b\not{p}_1\right] = 32(p_ap_1)(p_bp_1) \tag{30}$$

In the last equality, the property Eq.(71) was applied. This is more compact in terms of the Mandelstam variables: $8ut$. Bringing all the pieces together:

$$\overline{|\mathcal{M}_t|^2} = \frac{N^2-1}{N^2}e^2e_q^2g_s^2\,\frac{u}{t} \tag{31}$$

Following these steps, $\overline{|\mathcal{M}_u|^2}$ and $\overline{2\mathcal{M}_t\mathcal{M}_u^*}$ can be calculated. The second one will imply a somewhat more cumbersome trace but the procedure is the same. We are not repeating all these calculations again, and just exhibit the final results:

$$\overline{|\mathcal{M}_u|^2} = \frac{N^2-1}{N^2} e^2 e_q^2 g_s^2 \, \frac{t}{u} \tag{32}$$

$$2\overline{\mathcal{M}_t \mathcal{M}_u^*} = \frac{N^2-1}{N^2} e^2 e_q^2 g_s^2 \frac{2s(s+t+u)}{ut} = 0 \tag{33}$$

In the massless quarks approximation, $s + t + u = 0$. Hence, the Born squared amplitude reads:

$$\mathcal{B} = \frac{N^2-1}{N^2} e^2 e_q^2 g_s^2 \frac{t^2+u^2}{tu} \tag{34}$$

2. **Colour-correlated Born amplitudes**

   There are three coloured particles in this process, the quark (a), the antiquark (b) and the gluon(1), so in total there will be six colour-correlated Born amplitudes, corresponding to all possible combinations of these particles in the indices i, j of $\mathcal{B}_{ij}$. However, as $T^a_{c_i,c'_i}$ and $T^a_{c_j,c'_j}$ are simply scalars, they commute and $\mathcal{B}_{ij} = \mathcal{B}_{ji}$. Therefore, we only have to compute three of them. As in the previous case, we show how this is done for one of them; for the rest we will write the result we got.

   If one writes the full expression for $\mathcal{M}$, one sees directly that $t^\alpha_{ba}\chi^\alpha$ is an overall factor. This makes perfectly sense: it does not really matter if the gluon was emitted "before" or "after" the propagator, because it just involves the multiplication by a scalar. Moreover, this is the only part that has some colour dependence. This means that when we square and average $\mathcal{M}$ we will have:

$$\mathcal{B} = \begin{bmatrix}\text{Things that do not}\\ \text{depend on colours}\end{bmatrix} \times t^\alpha_{ba} t^\beta_{ab} \chi^\alpha \chi^\beta \tag{35}$$

   Keeping that in mind and taking a closer look at Eq.(8) it becomes clear that changing the colour indices does not affect the first square bracket of the equation above. That is, the colour indices have to be replaced only in the colour trace:

$$\mathcal{B}_{ij} = -\begin{bmatrix}\text{Things that do not}\\ \text{depend on colours}\end{bmatrix} \times \left(t^\alpha_{ba} t^\beta_{ab} \chi^\alpha \chi^\beta \right)_{\substack{c_i \to c'_i \\ c_j \to c'_j}} T^d_{c_i,c'_i} T^d_{c_j,c'_j} \tag{36}$$

   The first bracket is known from Eq.(35):

$$\mathcal{B}_{ij} = -\frac{\mathcal{B}}{t^{\alpha}_{ba}t^{\beta}_{ab}\chi^{\alpha}\chi^{\beta}} \times \left(t^{\alpha}_{ba}t^{\beta}_{ab}\chi^{\alpha}\chi^{\beta}\right)_{\substack{c_i\to c_i' \\ c_j\to c_j'}} T^{d}_{c_i,c_i'}T^{d}_{c_j,c_j'}$$

$$= -\frac{2\mathcal{B}}{N^2-1} \times \left(t^{\alpha}_{ba}t^{\beta}_{ab}\chi^{\alpha}\chi^{\beta}\right)_{\substack{c_i\to c_i' \\ c_j\to c_j'}} T^{d}_{c_i,c_i'}T^{d}_{c_j,c_j'} \tag{37}$$

Let's compute $\mathcal{B}_{a1}$ as an example. Here i is the quark and j is the gluon. Following the rules given in Eq.(8), we have to evaluate:

$$t^{\alpha}_{ba}t^{\beta}_{a'b}\chi^{\alpha'}\chi^{\beta}t^{d}_{aa'}if_{\alpha d\alpha'} \tag{38}$$

Where a' and α' denote the new colour indices of the quark and the gluon. Writing $\chi^{\alpha'}\chi^{\beta}$ as a Kronecker delta, suming over α' and using the definition of trace, this is equal to:

$$t^{\alpha}_{ba}t^{\beta}_{a'b}t^{d}_{aa'}if_{\alpha d\beta} = Tr\left(t^{\alpha}t^{d}t^{\beta}\right)if_{\alpha d\beta} \tag{39}$$

Change the names of the indices for convenience $\alpha \to a$, $d \to b$, $\beta \to c$ and use the commutation relation Eq.(62) to commute t^b and t^c

$$if_{abc}Tr\left(t^{a}t^{b}t^{c}\right) = if_{abc}Tr\left[t^{a}\left(if_{bcd}t^{d}+t^{c}t^{b}\right)\right]$$

$$= -f_{abc}f_{bcd}Tr\left[t^{a}t^{d}\right] + if_{abc}Tr\left(t^{a}t^{c}t^{b}\right) \tag{40}$$

The f_{abc} is a totally antisymmetric tensor, this means that it changes it sign under the switch of two indices. Then, writing in the second term $f_{abc} = -f_{acb}$ and renaming the indices $b \leftrightarrow c$ we are left with what we had on the left of the equal sign. In addition, we can shift $b \leftrightarrow d$ and then $c \leftrightarrow b$ in the tensor of the first term with no change in its sign.

$$if_{abc}Tr\left(t^{a}t^{b}t^{c}\right) = -f_{abc}f_{dbc}Tr\left[t^{a}t^{d}\right] + if_{abc}Tr\left(t^{a}t^{b}t^{c}\right)$$

$$\Longrightarrow if_{abc}Tr\left(t^{a}t^{b}t^{c}\right) = -\frac{1}{2}f_{abc}f_{dbc}Tr\left[t^{a}t^{d}\right] \tag{41}$$

And from Eq.(65) and Eq.(64):

$$if_{abc}Tr\left(t^{a}t^{b}t^{c}\right) = -\frac{1}{2}N\delta_{ad}Tr\left(t^{a}t^{d}\right) = -\frac{1}{2}\,N\,\frac{N^2-1}{2} \tag{42}$$

Plugging this into Eq.(37) gives $\mathcal{B}_{a1}$, which is equal, as previously discussed, to $\mathcal{B}_{1a}$:

$$\mathcal{B}_{a1} = \mathcal{B}_{1a} = \frac{N}{2}\mathcal{B} \tag{43}$$

The other four colour-correlated Born amplitudes are:

$$\mathcal{B}_{ab} = \mathcal{B}_{ba} = -\frac{1}{2N}\mathcal{B} \tag{44}$$

$$\mathcal{B}_{b1} = \mathcal{B}_{1b} = \frac{N}{2}\mathcal{B} \tag{45}$$

At this point, Eq.(9) can be used to check the results. For the quark and for the antiquark:

$$\mathcal{B}_{ba} + \mathcal{B}_{1a} = \mathcal{B}_{ab} + \mathcal{B}_{1b} = \left(\frac{N}{2} - \frac{1}{2N}\right) = \frac{N^2-1}{2N}\,\mathcal{B} = C_F\,\mathcal{B} \tag{46}$$

And for the gluon:

$$\mathcal{B}_{a1} + \mathcal{B}_{b1} = \left(\frac{N}{2} + \frac{N}{2}\right)\mathcal{B} = N\mathcal{B} = C_A\,\mathcal{B} \tag{47}$$

3. **Spin-correlated Born amplitude**

The calculation of the spin-correlated Born amplitude turns out to be more complicated than the Born amplitude and the colour-correlated one. Although there is only one gluon and, therefore only one spin-correlated amplitude $\mathcal{B}_1^{\mu\nu}$, no simplification was found in this case (as we did with the colour-correlated amplitude). In addition the squared matrix element can not split into pieces (the $|\mathcal{M}_t|^2$, $|\mathcal{M}_u|^2$ and the $2\mathcal{M}_t\mathcal{M}_u^*$) anymore. We need to write the full matrix element:

$$\mathcal{M} = -\frac{e e_q g_s}{2}\bar{v}^{(s_b)}(p_b)\left[\frac{\gamma^\delta\left(\not{p}_a - \not{p}_1\right)\gamma^\eta}{p_a p_1} + \frac{\gamma^\eta\left(\not{p}_a - \not{p}_2\right)\gamma^\delta}{p_a p_2}\right]u^{(s_a)}(p_a)\times$$

$$\times\, t^\alpha_{ba}\chi^\alpha\left(\epsilon_\eta^{(s_1)}(p_1)\right)^*\left(\epsilon_\delta^{(s_2)}(p_2)\right)^* \tag{48}$$

And its complex conjugate:

$$\mathcal{M}^* = -\frac{ee_q g_s}{2}\bar{u}^{(s_a)}(p_a)\left[\frac{\gamma^\sigma\left(\not{p}_a - \not{p}_1\right)\gamma^\rho}{p_a p_1} + \frac{\gamma^\rho\left(\not{p}_a - \not{p}_2\right)\gamma^\sigma}{p_a p_2}\right]v^{(s_b)}(p_b)\times$$
$$\times\, t^\beta_{ab}\chi^\beta \epsilon^{(s_1')}_\sigma(p_1)\epsilon^{(s_2)}_\rho(p_2) \tag{49}$$

Notice that we changed the index of the spin of the gluon $s_1 \to s_1'$. Bringing all together in Eq.(10), including the average over initial spins and colours, computing the colour traces and summing the polarization vectors one arrives at:

$$\mathcal{B}_1^{\mu\nu} = -\frac{e^2 e_q^2 g_s^2}{32}\frac{N^2-1}{N^2}\times$$
$$\sum_{\{i\},s_j,s_j'}\left\{\frac{1}{p_a p_1}\left[\bar{v}^{(s_b)}\gamma_\rho(\not{p}_a - \not{p}_1)\gamma^\mu u^{(s_a)}\right] + \frac{1}{p_a p_2}\left[\bar{v}^{(s_b)}\gamma^\mu(\not{p}_a - \not{p}_2)\gamma_\rho u^{(s_a)}\right]\right\}$$
$$\times\left\{\frac{1}{p_a p_1}\left[\bar{u}^{(s_a)}\gamma^\nu(\not{p}_a - \not{p}_1)\gamma^\rho v^{(s_b)}\right] + \frac{1}{p_a p_2}\left[\bar{v}^{(s_a)}\gamma^\rho(\not{p}_a - \not{p}_2)\gamma^\nu v^{(s_b)}\right]\right\} \tag{50}$$

Writing this as a trace and summing over spins yields:

$$\mathcal{B}_1^{\mu\nu} = -\frac{e^2 e_q^2 g_s^2}{32}\frac{N^2-1}{N^2}\left[\mathcal{A}_{11}^{\mu\nu} + \mathcal{A}_{12}^{\mu\nu} + \mathcal{A}_{21}^{\mu\nu} + \mathcal{A}_{22}^{\mu\nu}\right] \tag{51}$$

With:

$$\mathcal{A}_{11}^{\mu\nu} = \frac{1}{(p_a p_1)^2}Tr\left[\not{p}_b\gamma_\rho(\not{p}_a - \not{p}_1)\gamma^\mu\not{p}_a\gamma^\nu(\not{p}_a - \not{p}_1)\gamma^\rho\right]$$
$$\mathcal{A}_{12}^{\mu\nu} = \frac{1}{(p_a p_1)(p_a p_2)}Tr\left[\not{p}_b\gamma_\rho(\not{p}_a - \not{p}_1)\gamma^\mu\not{p}_a\gamma^\rho(\not{p}_a - \not{p}_2)\gamma^\nu\right]$$
$$\mathcal{A}_{21}^{\mu\nu} = \frac{1}{(p_a p_2)(p_a p_1)}Tr\left[\not{p}_b\gamma^\mu(\not{p}_a - \not{p}_2)\gamma_\rho\not{p}_a\gamma^\nu(\not{p}_a - \not{p}_1)\gamma^\rho\right]$$
$$\mathcal{A}_{22}^{\mu\nu} = \frac{1}{(p_a p_2)^2}Tr\left[\not{p}_b\gamma^\mu(\not{p}_a - \not{p}_2)\gamma_\rho\gamma^\rho(\not{p}_a - \not{p}_2)\gamma^\nu\right]$$

Both the traces and the verification of the results (Eq.(12)) were computed in *Mathematica*. Further details are provided in Appendix C. Here we show the final result obtained:

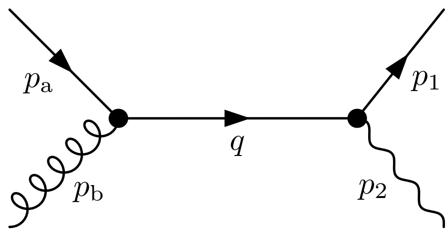
$$\mathcal{B}_1^{\mu\nu} = -\frac{N^2-1}{2N^2}\,\frac{e^2 e_q^2 g_s^2}{tu}\{2p_1^\nu\left[up_2^\mu + (t-u)\,p_a^\mu\right]$$

$$+ 2p_1^\mu\left[2up_1^\nu + up_2^\nu + (t-u)\,p_a^\nu\right] + \left(t^2+u^2\right)g^{\mu\nu}\} \quad (52)$$

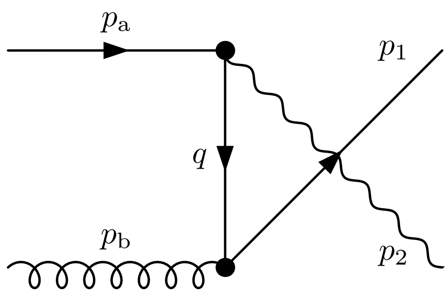
QCD Compton scattering

$$q(p_a) + g(p_b) \longrightarrow q(p_1) + \gamma(p_2)$$

Two contributing diagrams:



s-channel



u-channel

Figure 5: Contributing Feynman diagrams at tree level to the process of quark QCD Compton scattering

1. **Born amplitude**

   To evaluate the Born amplitude of the QCD Compton scattering, we could proceed as in the quark-antiquark annihilation case, but one can also perform a "trick" that makes it much easier. This is known as *crossing symmetry* and is based on Feynman's interpretation of particles and antiparticles. Namely, an emitted antiparticle with four-momentum p^μ is physically equivalent to an absorbed particle with four-momentum $-p^\mu$ (and, analogously, an absorbed antiparticle with four-momentum p^μ is so to an emitted particle with four-momentum $-p^\mu$). This idea is proposed in [8] in Chapter 4.5.4 and is applied to a particular process in Chapter 8.5.

An additional aspect to be taken into account is that every time we rotate a fermion line, we need to add a minus to the amplitude. The reason for this can also be found in [8]. We are now in conditions to apply the crossing symmetry to our processes. In quark-antiquark annihilation we had an initial antiquark (p_b) and a final gluon (p_1); and now we have an initial gluon (p_b) and a final quark (p_1). We have to perform the change in the momenta:

$$p_b \longrightarrow -p_1 \tag{53a}$$

$$p_1 \longrightarrow -p_b \tag{53b}$$

Switching these momenta is translated in the following transformation in the Mandelstam variables (Eq.(22)):

$$s \longrightarrow t \tag{54a}$$

$$t \longrightarrow s \tag{54b}$$

$$u \longrightarrow u \tag{54c}$$

Moreover, as we want to evaluate the averaged squared amplitude, we have to consider that the factor when averaging over initial spins and colours is not the same as before. The last time we divided by 4, due to the 2 possible spins and by N^2 from the N colours of the quarks. The new Born amplitude has to be multiplied by this and divided by the new average factor which is 4 (2 spins of the quark and 2 polarizations of the gluon) times N (colours of the quark) times $N^2 - 1$ (colours of the gluon). The last step is to include the minus sign, as we are rotating a fermion line.

Mathematically, the Born amplitude of the QCD Compton process $\mathcal{B}_{Co}$ can be directly calculated from the one of the quark-antiquark annilation $\mathcal{B}_{An}$ as:

$$\mathcal{B}_{Co} = -(1)\ \left(\mathcal{B}_{An}\right)_{\binom{s\to t}{t\to s}} \times \frac{4N^2}{4N(N^2-1)} \tag{55}$$

Substituting in this equation, one gets for the QCD Compton process:

$$\mathcal{B} = -\frac{1}{N} e^2 e_q^2 g_s^2 \frac{s^2+u^2}{su} \tag{56}$$

2. **Colour-correlated Born amplitudes**

   Once again, if the $\mathcal{B}$ is known the calculation of these amplitudes is reduced to the computation of the colour traces. In these traces is does not make a difference the fact that now the gluon is in the initial state (remember $(\chi^\alpha)^* = \chi^\alpha$) and that there is an outgoing quark instead of an incoming antiquark (see in Appendix A that both have the same "colour part": $c_i^\dagger$).

$$\mathcal{B}_{ab} = \mathcal{B}_{ba} = \mathcal{B}_{b1} = \mathcal{B}_{1b} = \frac{N}{2}\mathcal{B} \tag{57}$$

$$\mathcal{B}_{a1} = \mathcal{B}_{1a} = -\frac{1}{2N}\mathcal{B} \tag{58}$$

3. **Spin-correlated Born amplitude**

   This calculation was implemented in *Mathematica* and is explained in Appendix C

$$\mathcal{B}_b^{\mu\nu} = \frac{1}{2N}\frac{e^2 e_q^2 g_s^2}{su}\{-2p_b^\nu\left[up_2^\mu + (s-u)\,p_a^\mu\right]$$

$$-\,2p_b^\mu\left[-2up_b^\nu + up_2^\nu + (s-u)\,p_a^\nu\right] + \left(s^2+u^2\right)g^{\mu\nu}\} \tag{59}$$

5 Summary and Outlook

In these pages a review of the importance of direct photon production in QCD has been given. We started introducing the hypothetical highly excited state of QGP as motivation for the study of heavy-ion collisions. Afterwards some basics of QCD were discussed: the QCD Lagrangian, the factorization theorem, confinement and asymptotic freedom. These allowed a more accurate description of the QGP in Section 3 along with a comment of its detection techniques. In Section 4 we dealt with GPEGs and presented the ingredients of the implementation in POWHEG that have been computed. We see how some ideas of QFT and QCD are extremely useful in these calculations and make them much simpler. On the other hand, the spin-correlated amplitudes turned out to be too cumbersome to be computed by hand and had to be implemented in *Mathematica*.

To end with, I would like to highlight that, in spite of having these ingredients, the implementation of direct photon production in POWHEG is far from trivial. Further problems arise and this has not been achieved yet [28].

A Feynman Rules in QCD

We show the Feynman rules for QCD with their tree level diagram representation (from [21, 22]), using a similar notation to [23].

- Incoming quark: $u^{(s_i)}(p)c_i$

- Outgoing quark: $\bar{u}^{(s_i)}(p)c_i^\dagger$

- Incoming antiquark: $\bar{v}^{(s_i)}(p)c_i^\dagger$

- Outgoing antiquark: $v^{(s_i)}(p)c_i$

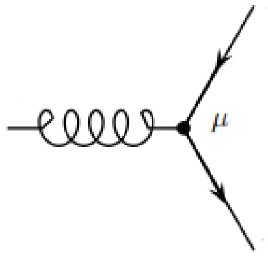
- Incoming gluon: $\epsilon_\mu(p)\chi^\alpha$

- Outgoing gluon: $\epsilon^*_\mu(p)(\chi^\alpha)^*$

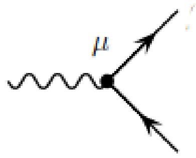
- Quark propagator: $\frac{i(\not{q}+m)}{q^2-m^2}$

- Gluon propagator: $\frac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}$

- Quark-Gluon vertex: $-ig_s t^\alpha \gamma^\mu$



- Photon-Quark vertex: $ie_q e\gamma^\mu$



- Incoming photon: $\epsilon_\mu(k)$

- Outgoing photon: $\epsilon^*_\mu(k)$

Here, $u^{(s)}(p)$ and $v^{(s)}(p)$ are the Dirac spinors of the quark and the anti-quark, satisfying:

$$\sum_{\text{spins}} u^{(s)}(p)\bar{u}^{(s)}(p) = \not{p} + m \tag{60a}$$

$$\sum_{\text{spins}} v^{(s)}(p)\bar{v}^{(s)}(p) = \not{p} - m \tag{60b}$$

c_i refers to the colour of the quark and is a vector of three components (corresponding to the three colours red, blue and green) [23]. The index i runs from one to three:

$$c_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad c_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad c_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{61}$$

ϵ^μ is the μ component of a polarization vector, normalized according to Eq.(11). Analogously as for the quarks, the colours of the gluons are indicated by χ^α, but here χ is an eight component vector. The component of this vector is denoted with a greek letter, to distinguish it from the quark case.

e is the charge of the electron and e_q is the fractional charge of the quark (2/3 for the u-, c- and t-quarks and -1/3 for the d-, s- and b-quarks). g_s is related to the coupling constant through $g_s = \sqrt{4\pi\alpha_s}$ [23].

γ^μ are the Dirac matrices and the "slashed" notation is used: $\not{q} = \gamma^\mu q_\mu$.

t^i are the generators of the SU(N) group in the fundamental representation, satisfying the commutation relation:

$$[t_a, t_b] = i f_{abc} t_c \tag{62}$$

being f_{abc} the structure constant, a totally antisymmetric tensor. The normalization conditions read:

$$tr(t_a t_b) = \frac{1}{2}\delta_{ab} \tag{63}$$

$$t_a t_a = C_F \cdot \mathbf{1} = \frac{N^2-1}{2N} \cdot \mathbf{1} \tag{64}$$

$$f_{abc} f_{dbc} = C_A \cdot \delta_{ad} = N \cdot \delta_{ad} \tag{65}$$

B Properties of the Gamma Matrices

Some properties of the gamma matrices that have been used in this work:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \cdot \mathbf{1} \tag{66}$$

$$\left(\gamma^0\right)^\dagger = \gamma^0 \quad \left(\gamma^0\right)^2 = \mathbf{1} \tag{67}$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \tag{68}$$

$$\gamma^\mu \not{p} \gamma_\mu = -2\not{p} \tag{69}$$

$$\not{p}\not{p} = 4p^2 \cdot \mathbf{1} \tag{70}$$

$$Tr\left(\not{p}_a \not{p}_b\right) = 4(p_a p_b) \tag{71}$$

C Mathematica

Some packages and codes for *Mathematica* that are usefull when working with Feynman diagrams are:

- *FeynArts:* generates Feynman diagrams and amplitudes. A full guide of the package is given in [24].
- *FormCalc:* is able to compute tree level Feynman diagrams generated by *FeynArts.* It combines the higher speed of FORM with the user friendly interface of *Mathematica.* Combined with *FeynArts* it provides a fast computation of the Born amplitude. The user's guide is [25]
- *Tracer* and *FeynCalc*: the first [26] is capable of evaluating gamma traces and *FeynCalc* [27] allows to work in SU(N) algebra in the calculation of colour traces.

All these codes are free software and can be downloaded in internet.

We are not going to discuss the computation of Born amplitudes with *FeynArts* and *FormCalc*, that can be learnt from the guides. We will only evaluate the traces in Eq.(51) with the package *Tracer.*

First, run the package. If the PATH is defined, this can be done typing:

```
<< Tracer'
```

Define the dimension of the space. We work with 4 dimensions:

```
VectorDimension[4]
```

Give a name to the trace that will be computed, we called it “1”:

```
Spur[1]
```

A message appears, saying that the matrix “1” will be traced. Now we write this matrix and it will be traced automatically:

```
GammaTrace[1,pb,{rho},pa-p1,{mu},pa,{nu},pa-p1,{rho}]
```

The “1” in the front names the matrix, the “pi” represent $\not{p}_i = p_i^\mu \gamma_\mu$ and “{mu}” is γ^μ.

We get a very long result that can be simplified in the massless quarks approximation:

```
Simiplify[%,Assumptions→ {pa.pa==0,p1.p1==0,p2.p2==0}]
```

We called this “Tr11”:

```
Tr11 = %
```

Following this procedure, we calculated the other three traces and called them “Tr12”, “Tr21” and “Tr22”. They have to be divided by the momentum products and summed:

```
Tr11/((pa.p1)∧2) + (Tr12+Tr21)/((pa.p1)(pa.p2)) +
+ Tr22/((pa.p2)∧2)
```

Simplify this:

```
Simplify [ %/.{pb→p1+p2-pa},Assumptions→ {pa.pa==0, p1.p1==0,
p2.p2==0}]
```

We called this “Result”.

```
Result = %
```

It will be useful to express it in terms of the Mandelstam variables, name it “FinalResult”, for instance:

```
FinalResult = Simplify[Result/.{p1.p2→-T/2-U/2, p1.pa→-T/2,
pa.p2→-U/2}]
```

This is the result we showed in Section 4.3. We will now check that the condition given by Eq.(60) is satisfied.

The metric tensor $g^{\mu\nu}$ is denoted in *Tracer* as `S[{mu},{nu}]`. Write:

```
%*S[{mu},{nu}]
```

Then we contract the indices:

```
ContractEpsGamma[%]
```

And simplify:

```
Simplify [ %/.,Assumptions→ {pa.pa==0, p1.p1==0, p2.p2==0}]
```

```
Simplify[%/.{p1.p2→S/2, p1.pa→-T/2, p2.pa→-U/2},
Assumptions→ {S+T+U==0}]
```

One last simplification:

```
Simplify [%/.{S→-T-U}]
```

If everything is done properly, one gets here:

$$\frac{32\left(T^2+U^2\right)}{TU} \tag{72}$$

This is what we called $\mathcal{A}_{11}^{\mu\nu}+\mathcal{A}_{12}^{\mu\nu}+\mathcal{A}_{21}^{\mu\nu}+\mathcal{A}_{22}^{\mu\nu}$ in Eq.(51) contracted with $g_{\mu\nu}$. Plugging this into the Eq.(51) yields:

$$g_{\mu\nu}\mathcal{B}_1^{\mu\nu} = -\frac{N^2-1}{N}e^2 e_q^2 g_s^2 \frac{t^2+u^2}{tu} = -\mathcal{B} \tag{73}$$

The Born spin-correlated amplitude for the QCD Compton scattering process is obtained performing the substitutions of Eq.(53) and Eq.(54) in "FinalResult" and changing its sign, due to the rotation of the fermion line.

```
FinalResultCompton = Simplify[-FinalResult/.{T→S,pb→-p1,
p1→-pb}]
```

The comprobation was done following the same steps as before, getting the expected result.

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Erklärung des Studierenden

I hereby declare that the script with the title:

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