# Combination and Reinterpretation of LHC SUSY Searches 

Submitted by<br>Alexander Feike

January 11, 2024

First examiner<br>Prof. Dr. Michael Klasen

Second examiner
Priv. Doz. Dr. Karol KovaŘík

University of Münster
Institute for Theoretical Physics
Working Group Klasen

## Contents

1. Introduction ..... 1
2. Supersymmetry ..... 3
2.1. Symmetries in Physics ..... 3
2.1.1. Group Theory ..... 3
2.1.2. Lorentz Group ..... 4
2.1.3. Poincaré Group ..... 6
2.1.4. Supersymmetry in the Poincaré Group ..... 7
2.2. Phenomenological Motivation ..... 8
2.3. Particle Content of the MSSM ..... 10
2.4. Mass Spectrum of the MSSM ..... 11
3. Motivation and Setup ..... 14
3.1. Simplified Scenario ..... 14
3.2. Toolchain ..... 16
4. Event Generation and Parton Shower ..... 18
4.1. Matrix Element Calculations ..... 18
4.2. Parton Shower ..... 19
4.3. Jet Merging ..... 22
5. Recasting analyses ..... 27
5.1. Cross Sections and K-Factors ..... 27
5.2. Experimental Analyses and Detector Simulation ..... 29
5.3. Confidence Level Calculation ..... 32
5.3.1. Likelihood Description ..... 32
5.3.2. Test Statistics ..... 34
5.3.3. Approximations and the Asimov Data Set ..... 35
5.3.4. Distribution of $\tilde{q}_{\mu}$ ..... 36
5.4. Combination ..... 37
6. Results ..... 42
6.1. Single Analysis ..... 42
6.2. Combined Analyses ..... 44
7. Conclusion ..... 47
A. Monte Carlo Method ..... 48
A.1. Monte Carlo Integration ..... 48
A.2. Monte Carlo Event Generator ..... 49
A.3. QCD Factorization ..... 50
A.4. Monte Carlo Generator in C++ ..... 51
B. MadGraph Commands ..... 53
C. Statistical Introduction ..... 56
C.1. Basic Introduction ..... 56
C.1.1. Primary Principles ..... 56
C.1.2. Relative Frequency and Bayesian Probability ..... 57
C.1.3. Probability Distribution Function ..... 58
C.2. Hypothesis Tests ..... 58
C.2.1. General ..... 58
C.2.2. $p$-Value and Confidence Interval ..... 59
C.2.3. Neyman-Pearson Lemma ..... 60
C.3. Likelihood ..... 61
C.3.1. Basics ..... 61
C.3.2. Maximum Likelihood ..... 61
D. Tables ..... 64
Bibliography ..... 67

## 1. Introduction

The standard model (SM) of particle physics as we know it today [1-3] has proven to be remarkably successful at describing experimentally observed features of nature. The discovery of the theoretically predicted [4, 5] Higgs boson in 2012 [6] undoubtedly represents one highlight in the recent history of the SM. However, observations like, for example, neutrino flavour oscillations, proving neutrinos are massive [7, [8] strongly indicate that the SM is not complete. Physicists thus try to find beyond standard model (BSM) theories explaining at least one open problem while agreeing with the SM in regions it demonstrates robust applicability.
One of the well-known BSM theories is supersymmetry (SUSY), a symmetry relating fermionic and bosonic degrees of freedom. After surpassing some mathematical obstacles 9,10 SUSY started to become prominent in the 1970s [11, 12]. Its significance in physical discussions increased because SUSY is able to resolve multiple limitations of the SM while also fitting well in the Poincaré group, the mathematical description of spacetime symmetries.
Consequently, high-energy particle colliders like the large hadron collider (LHC) started searching for SUSY particles. Along with the experimental efforts comes the need for precise theoretical predictions. State-of-the-art higher order calculations yield cross section, for example, for the strong squark pair, associated squark neutralino and neutralino pair production implemented and publicly available in, for example [13, 14]. Experimental collaborations like ATLAS and CMS base their analyses on these calculations, first focusing mainly on strong production and then including weak processes as well. However, no indication for SUSY signals was found in any of their numerous searches like 1520 . SUSY still remains a central topic of physical discussion and recent analyses 21,24 push the lower mass limits, depending on the considered model, into the TeV regime for squarks.
The data recorded from all the analyses can be reinterpreted in different scenarios. For the channels of interest in this thesis, so far, only squark pair production or squark pair production in combination with the associated squark neutralino channel has been included in single signal region (SR) interpretations 25. To further profit from the already collected data, it is necessary to add the weak neutralino pair production and furthermore combine different analyses in a statistical approach without double counting physical effects. The code TACO 26 tackles this problem by calculating the correlation of different SRS for a specific set of events in a recasted scenario and computing their best possible combination with a graph-based algorithm.
Within this discussion, we consider a simplified SUSY scenario restricted to one neutralino and one squark but containing all three previously mentioned processes. Using this scenario, we showcase the gain in exclusion resulting from taking all three processes into account and combining the most constraining uncorrelated SRs from multiple analyses. The results will also be made publicly available in 27 .
We start by introducing the basic concepts of SUSY in section 2, Afterwards, section 3 provides details on the considered scenario as well as an outline of the used toolchain. The detailed discussion of the individual steps is split between section 4, dealing with the event generation including the parton shower and jet merging, and section 5, explaining the cross sections and

## 1 Introduction

the statistical analysis. The final results for single and combined processes and for single and combined $5 R$ analyses are given in section 6 . Finally, we summarise and give a conclusion in section 7

## 2. Supersymmetry

Supersymmetry, the symmetry that connects bosonic and fermionic degrees of freedom came up in the early 1970s, among others pushed through publications like [12]. A real SUSY hype started as evidenced by many hundreds and sometimes up to around two thousand publications regarding this topic per year. This was motivated by the solutions SUSY could provide for still unanswered questions regarding the SM and thus the universe we live in while being nicely integrated in the fundamental spacetime symmetry described by the Poincaré group. In the following section we want to motivate and introduce the concepts of SUSY briefly. We will first address the group theoretical structure [28], give a general motivation based on observations and introduce the new particle content 29. However, a complete mathematical description and derivation of SUSY would exceed the scope of this thesis and therefore, this section aims just to give an introductory overview.

### 2.1. Symmetries in Physics

One of the fundamental concepts in physics are symmetries, transformations that leave specific properties of a physical system unchanged. There are continuous transformations like rotations or translations as well as discrete symmetries like parity or time reversal, characterising the universe we live in. Considering the term supersymmetry it appears natural to review symmetries and their mathematical description, group theory, in this thesis. In the following, we will briefly repeat the basics of group theory, introduce the Lorentz and the Poincaré group and discuss how SUSY fits into this picture.

### 2.1.1. Group Theory

Generally a group is a set of elements and an operation which is associative, includes a one and an inverse element, and is complete, or expressed in mathematical terms

$$
\begin{align*}
& \forall a, b, c \in G, \quad a \cdot(b \cdot c)=(a \cdot b) \cdot c,  \tag{1}\\
& \exists e \in G: \forall a \in G, \quad a \cdot e=e \cdot a=a  \tag{2}\\
& \forall a \in G, \quad \exists a^{-1} \in G: a \cdot a^{-1}=a^{-1} \cdot a=e  \tag{3}\\
& \forall a, b \in G, \quad a \cdot b \in G \tag{4}
\end{align*}
$$

with $G$ representing the group. Every transformation fulfilling these axioms is called a group. Lie theory treats continuous symmetries, so transformations that are parameterised by one or multiple parameters which can be tuned continuously. For example rotations of angle $\Phi$ which can take any value. In contrast to discrete symmetries like spatial reflection, continuous symmetries incorporate elements arbitrarily close to the identity transformation. Mathematically they can be written as

$$
\begin{equation*}
g(\epsilon)=I+\epsilon X \tag{5}
\end{equation*}
$$

## 2 Supersymmetry

with a small parameter $\epsilon$ and a so called generator $X$. Repeatingly applying those infinitesimal transformations results in a finite transformation, for example, for a rotation around an angle $\theta$

$$
\begin{equation*}
h(\theta)=\lim _{N \rightarrow \infty}\left(I+\frac{\theta}{N} X\right)^{N}=e^{\theta X} \tag{6}
\end{equation*}
$$

Therefore, the finite transformation $h$ is generated with the generator $X$ given as

$$
\begin{equation*}
X=\left.\frac{\mathrm{d} h(\theta)}{\mathrm{d} \theta}\right|_{\theta=0} \tag{7}
\end{equation*}
$$

The set of generators of a Lie group $G$ together with a Lie bracket [, ] is called Lie algebra $\mathfrak{g}$. For matrix Lie groups, the Lie bracket is the commutator, under which the Lie algebra is closed:

$$
\begin{equation*}
\forall X, Y \in \mathfrak{g}, \quad[X, Y] \in \mathfrak{g} \tag{8}
\end{equation*}
$$

In order to relate a group, which describes transformations without a connection to any physical object, to the real world, a representation is needed. Representations map any group element $g$ to a linear transformation $R(g)$ of a vector space, while preserving the group properties listed in eqs. (1) to (4). Thus, the abstract transformation described by the group is mapped to a specific vector space. For example $S U(2)$ can act on objects in $\mathbb{C}^{2}$ as $2 \times 2$ matrices while also being able to describe rotations of objects in $\mathbb{C}^{3}$ as $3 \times 3$ matrices.

In particle physics special attention is paid to representations with no invariant subspace but the zero space, so called irreducible representations. This means the matrix cannot be rewritten in a block diagonal form. These irreducible representations are used to describe elementary particles.

### 2.1.2. Lorentz Group

The Lorentz group includes all transformations that preserve

$$
\begin{equation*}
x^{\mu} x_{\mu}=x^{\mu} \eta_{\mu \nu} x^{\nu}=\left(x^{0}\right)^{2}-\left(x^{1}\right)^{2}-\left(x^{2}\right)^{2}-\left(x^{3}\right)^{2} \tag{9}
\end{equation*}
$$

with the Minkowski metric $\eta_{\mu \nu}=\operatorname{diag}(1,-1,-1,-1)$ which also gives the Lorentz group the name $O(1,3)$. Here and in the following, we use greek letters as indices to denote all four space time directions like $\mu, \nu \in\{0,1,2,3\}$, latin letters for solely space directions like $i, j \in\{1,2,3\}$ and employ Einsteins summation convention. The group is set up by considering a general transformation $\Lambda$ with $x^{\mu} \rightarrow x^{\mu}=\Lambda_{\nu}^{\mu} x^{\nu}$ that preserves the inner product. This implies that the relation

$$
\begin{equation*}
\Lambda_{\mu}{ }^{\sigma} \eta_{\sigma \tau} \Lambda_{\nu}^{\tau}=\eta_{\mu \nu} \quad \text { or } \quad \Lambda^{T} \eta \Lambda=\eta \tag{10}
\end{equation*}
$$

in index and matrix form respectively, must hold. The previous relation defines the Lorentz transformation, however, considering the determinant

$$
\begin{equation*}
\operatorname{det}(\Lambda) \underbrace{\operatorname{det}(\eta)}_{=-1} \operatorname{det}(\Lambda)=\underbrace{\operatorname{det}(\eta)}_{=-1} \Longrightarrow \operatorname{det}(\Lambda)= \pm 1, \tag{11}
\end{equation*}
$$

and the $(0,0)$ element of the metric

$$
\begin{equation*}
1=\eta_{00}=\Lambda_{0}{ }^{\sigma} \eta_{\sigma \tau} \Lambda_{0}^{\tau}=\left(\Lambda_{0}^{0}\right)^{2}-\sum_{i=1}^{3}\left(\Lambda_{i}^{0}\right)^{2} \Longrightarrow \Lambda_{0}^{0}= \pm \sqrt{1+\sum_{i}\left(\Lambda_{i}^{0}\right)^{2}} \tag{12}
\end{equation*}
$$

allows the transformations to be sorted into subcategories. They are denoted as $L_{+}^{\uparrow}, L_{-}^{\uparrow}$, $L_{+}^{\downarrow}$ and $L_{-}^{\downarrow}$, encoding if the determinant is +1 or -1 in the subscript and if $\Lambda_{0}^{0} \geq 1$ or $\Lambda_{0}^{0} \leq-1$ with up and down arrows respectively. The identity has $\operatorname{det}(I)=1$ and $I_{0}^{0}=$ 1 and thus belongs to $L_{+}^{\uparrow}$. This is crucial since the definition of the categories incorporate gaps, such that there are no transformations with a determinant between +1 and -1 and similarly no transformations with the first element being in this interval either. Thus, only $L_{+}^{\uparrow}$, the proper orthochonous Lorentz group, can be smoothly connected to the identity and built up from infinitesimal transformations. Therefore, we will concentrate on this category in the following, however, the other categories can be entered by employing parity and time reversal transformations:

$$
\begin{equation*}
O(1,3)=\left\{L_{+}^{\uparrow}, L_{-}^{\uparrow}, L_{+}^{\downarrow}, L_{-}^{\downarrow}\right\}=\left\{L_{+}^{\uparrow}, \Lambda_{P} L_{+}^{\uparrow}, \Lambda_{T} L_{+}^{\uparrow}, \Lambda_{T} \Lambda_{P} L_{+}^{\uparrow}\right\} \tag{13}
\end{equation*}
$$

with $\Lambda_{P}=\operatorname{diag}(1,-1,-1,-1)$ and $\Lambda_{T}=\operatorname{diag}(-1,1,1,1)$.
One explicit representation of the Lorentz group acting on four-vectors is given by $4 \times 4$ matrices. The condition in eq. 10 reduces the free parameters of the transformation matrices from 16 to six, so six independent generators build the basis of the Lie algebra. First of, one finds that rotation in three-dimensional space, so transformations involving only space and no time components fulfill the condition. Therefore, three generators are given by

$$
J_{i}=\left(\begin{array}{cc}
0 &  \tag{14}\\
& J_{i}^{3 \mathrm{~d}}
\end{array}\right) \quad \text { with } \quad\left(J_{i}^{3 \mathrm{~d}}\right)_{j k}=-i \epsilon_{i j k}
$$

as known from $S O(3)$ incorporating totally antisymmetric tensor $\epsilon_{i j k}$. Transformations involving space and time can be constructed by plugging an infinitesimal transformation

$$
\begin{equation*}
\Lambda_{\rho}^{\mu} \approx \delta_{\rho}^{\mu}+\epsilon K_{\rho}^{\mu} \tag{15}
\end{equation*}
$$

## 2 Supersymmetry

into the defining condition eq. 10. This yields $K^{T} \eta=-\eta K$ which is fulfilled by the generators

$$
K_{1}=\left(\begin{array}{cccc}
0 & i & 0 & 0  \tag{16}\\
i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right), \quad K_{2}=\left(\begin{array}{cccc}
0 & 0 & i & 0 \\
0 & 0 & 0 & 0 \\
i & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right), \quad K_{3}=\left(\begin{array}{cccc}
0 & 0 & 0 & i \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
i & 0 & 0 & 0
\end{array}\right)
$$

which generate the boosts, so the transformation into a coordinate system moving with a different constant velocity. The complete Lie algebra is therefore given by

$$
\begin{align*}
& {\left[J_{i}, J_{j}\right]=i \epsilon_{i j k} J_{k}}  \tag{17}\\
& {\left[K_{i}, K_{j}\right]=-i \epsilon_{i j k} J_{k}}  \tag{18}\\
& {\left[J_{i}, K_{j}\right]=i \epsilon_{i j k} K_{k}} \tag{19}
\end{align*}
$$

generating a general Lorentz transformation of the form $\Lambda=e^{i \vec{\theta} \cdot \vec{J}+i \vec{\phi} \cdot \vec{K}}$. They can be rotated such that the different types of generators commute:

$$
\begin{align*}
N_{i}^{ \pm}= & \frac{1}{2}\left(J_{i} \pm i K_{i}\right)  \tag{20}\\
\Longrightarrow & {\left[N_{i}^{+}, N_{j}^{+}\right]=i \epsilon_{i j k} N_{k}^{+}, }  \tag{21}\\
& {\left[N_{i}^{-}, N_{j}^{-}\right]=i \epsilon_{i j k} N_{k}^{-} }  \tag{22}\\
& {\left[N_{i}^{+}, N_{j}^{-}\right]=0 . } \tag{23}
\end{align*}
$$

Comparing with the $S U(2)$ shows that the Lie algebra of $L_{+}^{\uparrow}$ can be built from two copies of $\mathfrak{s u}(2)$. Therefore, we can rely on the irreducible representations of $S U(2)$, but beware, similar to $S O(3)$ the Lorentz group is not simply connected. The representations derived from $S U(2)$ considerations are thus representations of the covering group of the Lorentz group. We will keep this in mind but for brevity denote the representations of the covering group simply as the representations of the Lorentz and Poincareé group in the following.
The irreducible representations are denoted by $\left(j_{1}, j_{2}\right)$, two integer or half integer numbers.

### 2.1.3. Poincaré Group

In addition to the rotations and boosts from the Lorentz group, translations in space also leave the speed of light constant. This is generated by $P_{\mu}$ which can be related to the momentum operator. The Poincaré group includes rotations, boosts and translations, thus the algebra consists of $J_{i}, K_{i}$ and $P_{\mu}$ and their commutators. However, it is conventional to use

$$
\begin{equation*}
J_{i}=\frac{1}{2} \epsilon_{i j k} M_{j k} \quad \text { and } \quad K_{i}=M_{0 i} \tag{24}
\end{equation*}
$$

which sorts the Poincaré algebra to

$$
\begin{align*}
& {\left[P_{\mu}, P_{\nu}\right]=0}  \tag{25}\\
& {\left[M_{\mu \nu}, P_{\rho}\right]=i\left(\eta_{\mu \rho} P_{\nu}-\eta_{\nu \rho} P_{\mu}\right)}  \tag{26}\\
& {\left[M_{\mu \nu}, M_{\rho \sigma}\right]=i\left(\eta_{\mu \rho} M_{\nu \sigma}-\eta_{\mu \sigma} M_{\nu \rho}-\eta_{\nu \rho} M_{\mu \sigma}+\eta_{\nu \sigma} M_{\mu \rho}\right)} \tag{27}
\end{align*}
$$

The different representation of this group are labelled by the Casimir operators $P_{\mu} P^{\mu} \equiv m^{2}$ and $W_{\mu} W^{\mu} \equiv j=j_{1}+j_{2}$ with the Pauli-Lubanski four-vector

$$
\begin{equation*}
W^{\mu}=\frac{1}{2} \epsilon^{\mu \nu \rho \sigma} P_{\nu} M_{\rho \sigma} \tag{28}
\end{equation*}
$$

They denote the freely varying mass and the half integer or integer spin. Following the previously defined $\left(j_{1}, j_{2}\right)$ representations, spin- 0 , spin- $1 / 2$ and spin- 1 elementary particles transform following the $(0,0),(1 / 2,0) \oplus(0,1 / 2)$ and $(1 / 2,1 / 2)$ representation respectively.

### 2.1.4. Supersymmetry in the Poincaré Group

The previous sections introduced the Lie algebra of the Poincare group, or more precisely, the double cover of the Poincaré group. The fundamental particles are embedded in different representations, from which the free Lagrangians can be derived. Hence, the double cover of the Poincaré group can be seen as the fundamental symmetry group of nature and consequently new spacetime symmetries incorporated in BSM models need to be embedded in it as well. However, the Coleman-Mandula theorem [30], stating the only symmetry group of a scattering matrix is the direct product of the Poncaré group and an internal symmetry group, strongly restricts the choice of new symmetries. But, following Haag, Lopuszanski and Sohnius [31] the extension of the algebra with

$$
\begin{align*}
& \left\{Q, Q^{\dagger}\right\}=P^{\mu}  \tag{29}\\
& \{Q, Q\}=\left\{Q^{\dagger}, Q^{\dagger}\right\}=0  \tag{30}\\
& {\left[Q, P^{\mu}\right]=\left[Q^{\dagger}, P^{\mu}\right]=0} \tag{31}
\end{align*}
$$

with fermionic operators $Q$ and $Q^{\dagger}$ which are in general complex and the known translation operator $P^{\mu}$ is possible. Here, all spinor indices are suppressed. The anti-commuting operators generate transformations that turn bosonic into fermionic states and vice versa

$$
\begin{equation*}
Q \mid \text { boson }\rangle \sim \mid \text { fermion }\rangle \quad Q \mid \text { fermion }\rangle \sim \mid \text { boson }\rangle, \tag{32}
\end{equation*}
$$

and are thus the generators of SUSY. Mathematically multiple sets of these generators are possible and they are investigated in extended SUSY models but are not considered in this thesis. Note that due to Equation 31 the squared mass operator $P^{2}$ also commutes with the SUSY generator and consequently superpartners must have the same mass if SUSY is unbroken. Additionally, the generators of the SM gauge transformations commute with $Q$ and $Q^{\dagger}$ and therefore

## 2 Supersymmetry

superpartners also carry the same charges, i.e. the same colour, weak isospin and electric charge. The discussed group theory structure of the Poincaré group and the extension with SUSY generators mathematically motivate SUSY. Before introducing the new particle content we want to consider observations which motivate SUSY from a more phenomenological point of view.

### 2.2. Phenomenological Motivation

To begin our brief discussion about why a symmetry between bosons and fermions might be reasonable and which problems it could solve we want to first look at the quantum electrodynamics (QED) Lagrangian given by

$$
\begin{equation*}
\mathcal{L}=i \bar{\Psi} \gamma_{\mu} D^{\mu} \Psi-m_{f} \bar{\Psi} \Psi-\frac{1}{4} F^{\mu \nu} F_{\mu \nu} \tag{33}
\end{equation*}
$$

with a fermionic field $\Psi$. If not for the mass term, this Lagrangian would yield chiral symmetry. Although it is explicitly broken, the fact that chiral symmetry is respected in the limit $m_{f} \rightarrow 0$ still has consequences, in specific it ensures that the radiative correction to the fermion mass also vanishes in the zero mass limit. Following that argument a linearly diverging correction $\Delta m_{f} \propto \Lambda_{\mathrm{UV}}$ like it is indicated by naive dimensional analysis is not physical. Here, the loop integrals are regulated by $\Lambda_{U V}$, interpreted as the energy scale at which new physics gets important at the latest. The right correction given by $\Delta m_{f} \propto m_{f} \ln \left(\Lambda / m_{f}\right)$ is smaller and it is therefore said that fermion masses are protected from large divergences by chiral symmetry [32]. In contrast there is no such symmetry that protects the spin-0 Higgs particle. Considering the

(a) fermionic loop correction

(b) scalar loop correction

Figure 1: Shown are the Feynman diagrams of the one loop corrections, considering fermions and scalars, to the Higss mass.
coupling between Higgs field $H$ and fermion $f$ given by $-\lambda_{f} H \bar{f} f$ or assuming a coupling between Higgs field and a massive scalar $S$ like $-\lambda_{S}|H|^{2}|S|^{2}$ in the Lagrangian, loops as shown in fig. 1a and fig. 1b would yield the corrections

$$
\begin{align*}
\Delta m_{H}^{2} & =-\frac{\left|\lambda_{f}\right|^{2}}{8 \pi^{2}} \Lambda_{\mathrm{UV}}^{2}+\ldots  \tag{34}\\
\Delta m_{H}^{2} & =\frac{\lambda_{s}}{16 \pi^{2}}\left[\Lambda_{\mathrm{UV}}^{2}-2 m_{S}^{2} \ln \left(\Lambda_{\mathrm{UV}} / m_{S}\right)+\ldots\right] \tag{35}
\end{align*}
$$

respectively. Note the relative minus sign between fermion and boson loop corrections. As $\lambda_{f} \propto m_{f}$ the largest fermionic impact results from the top quark. In the bosonic case the $\Lambda_{\mathrm{UV}}^{2}$ term can be avoided by using dimensional regularization but one still keeps the sensitivity to the highest particle masses.

Knowing that it becomes necessary to consider quantum gravitational effects at the reduced Planck scale $M_{P}=2.4 \times 10^{18} \mathrm{GeV}$ sets an upper limit for the cut-off. But if $\Lambda_{\mathrm{UV}}$ is of order $M_{P}$ the corrections would exceed the required light Higgs mass by several orders of magnitude. It is desirable to avoid this problem without fine-tuning by canceling the fermionic with the bosonic divergences. Since we want this cancellation to happen not just at the one loop level but to all orders an accidental compensation is highly improbable. One would rather need a symmetry that relates bosonic and fermionic properties to each other, in other words SUSY, which requires that for every fermionic degree of freedom there is a bosonic one and vice versa. Next to the so far discussed hierarchy problem, SUSY might present a way out of other not yet solved problems in particle physics. One of which is dark matter. Gravitational observations like the rotation curves of dark galaxies imply that the known, visible matter only accounts for apprx. $4 \%$ of the universes matter content. The rest is $73 \%$ dark energy and $23 \%$ dark matter, which is matter that does not interact with photons or gluons. An explanation are weakly interacting massive particles (WIMPs) which are yet to be discovered. With the neutralino, many supersymmetric models yield a promising candidate for a WIMP [33].
Lastly, we want to mention the desired unification of gauge couplings. Following the hypothesis of a grand unified theory all gauge couplings are only different branches of one big gauge group realised in higher energy regions, so in the early universe. Examining of the running couplings by solving the renormalization group equations shows that the couplings do not meet if we just consider the SM case. As shown in fig. 2 unification can be achieved by including the particle content of the minimal supersymmetric standard model (MSSM), the SUSY model with the minimal possible additional particles, in the calculations.


Figure 2: Shown are the running couplings calculated at two loop level in the SM (dashed line) and for masses varying between 750 GeV and 2.5 TeV (solid lines) in theMSSM. Figure is taken from [29].

We can thus conclude, that SUSY is not just a mathematical construct that fits into the group

## 2 Supersymmetry

theoretical description of spacetime symmetries but is also well motivated by theoretical and phenomenological considerations. In the following, we will focus on the MSSM rather than a general SUSY model.

### 2.3. Particle Content of the MSSM

Since the degrees of freedom of supersymmetric partners must equal, which can be derived via the (anti-)commutation relations above (see eqs. 29) to (31), Weyl fermions with two spin helicity degrees of freedom need to be assigned to two real scalars with one degree of freedom each. The scalars are put together in one complex field and the combination of this complex field with the Weyl fermion is called chiral supermultiplet. Therefore, the MSSM contains chiral supermultiplets with the known Weyl fermions like left- and right-handed quarks and leptons combined with their superpartners the squarks and sleptons respectively. Since the latter are spin-0 particles they do not have a helicity but are still marked with indices $L$ and $R$ to indicate which superpartner they are assigned to. Furthermore, all partner particles to known SM particles are denoted with a tilde.
The spin-0 SM Higgs boson is also part of a chiral supermultiplet, but adding just one fermionic partner would result in gauge anomalies. This is resolved if there are two Higgs supermultiplets which transform as doublets under $S U(2)_{L}$ called $H_{u}$ and $H_{d}$ with one electrically charged and one neutral component each for the spin-0 Higgs particles and the spin- $1 / 2$ higgsinos. The neutral SM Higgs is a linear combination of the two neutral components.

Additionally, the combination of a massless (before spontaneous symmetry breaking) spin-1 vector boson with a massless spin- $1 / 2$ Weyl fermion is called gauge supermultiplet. If one uses spin-3/2 fermions instead, the theory would not be renormalizable any more. In the MSSM the superpartners for the gauge bosons before electroweak symmetry breaking gluon, $W$-boson and $B$-boson are the gluino, wino and bino. The particle content of the chiral and gauge multiplets in the MSSM is summarised in table 1 .
None of the superpartners to any SM particle is found so far, which might be surprising since they should be of the same mass. Therefore, especially light particles like the selectron should have been detected already. Their absence in the low mass regime indicates that if SUSY exists it must be a broken symmetry, which is expected to be broken spontaneously. Since it is unknown how exactly SUSY is broken one just simplifies this problem by introducing terms that explicitly break the symmetry:

$$
\begin{equation*}
\mathcal{L}=\mathcal{L}_{\mathrm{SUSY}}+\mathcal{L}_{\mathrm{soft}} . \tag{36}
\end{equation*}
$$

The first term containing all gauge and Yukawa particles is preserved by SUSY, whereas the second one containing masses and couplings with positive mass dimension (soft) breaks SUSY. Therefore, the dimensionless couplings $\lambda_{f}$ and $\lambda_{S}$ from eq. (34) and eq. (35) do not change due to symmetry breaking and following that, although SUSY is broken it still yields a solution for the hierarchy problem.
The most general MSSMLagrangian introduces a total of 105 new parameters in form of masses,

|  | naming |  | spin-0 | spin-1/2 | spin-1 | SM |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| chiral | (s-)quarks | $Q$ | $\left(\tilde{\mathrm{u}}_{L} \tilde{\mathrm{~d}}_{L}\right)$ | $\left(u_{L} d_{L}\right)$ | - | $\left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right)$ |
| multiplet |  | $\bar{u}$ | $\tilde{\mathrm{u}}_{R}^{*}$ | $u_{R}^{\dagger}$ | - | $\left(\overline{\mathbf{3}}, \mathbf{1},-\frac{2}{3}\right)$ |
|  |  | $\bar{d}$ | $\tilde{\mathrm{~d}}_{R}^{*}$ | $d_{R}^{\dagger}$ | - | $\left(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}\right)$ |
|  | $(\mathrm{s}-)$ leptons | $L$ | $\left(\widetilde{\nu} \tilde{\mathrm{e}}_{L}\right)$ | $\left(\nu e_{L}\right)$ | - | $\left(\mathbf{1}, \mathbf{2},-\frac{1}{2}\right)$ |
|  |  | $\bar{e}$ | $\tilde{\mathrm{e}}_{R}^{*}$ | $e_{R}^{\dagger}$ | - | $(\mathbf{1}, \mathbf{1}, 1)$ |
|  | Higgs(-inos) | $H_{u}$ | $\left(H_{u}^{+} H_{u}^{0}\right)$ | $\left(\widetilde{H}_{u}^{+} \widetilde{H}_{u}^{0}\right)$ | - | $\left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right)$ |
| gauge | gluino, gluon | $H_{d}$ | $\left(H_{d}^{0} H_{d}^{-}\right)$ | $\left(\widetilde{H}_{d}^{0} \widetilde{H}_{d}^{-}\right)$ | - | $\left(\mathbf{1}, \mathbf{2},-\frac{1}{2}\right)$ |
| multiplet | winos, W bosons |  | - | $\widetilde{g}^{\prime}$ | $g$ | $(\mathbf{8}, \mathbf{1}, 0)$ |
|  | bino, B boson |  | - | $\left(\widetilde{W}^{ \pm} \widetilde{W}^{0}\right)$ | $\left(W^{ \pm} W^{0}\right)$ | $(\mathbf{1}, \mathbf{3}, 0)$ |
|  |  | - | $\widetilde{B}^{0}$ | $B^{0}$ | $(\mathbf{1}, \mathbf{1}, 0)$ |  |

Table 1: Shown is the particle content of the MSSM separated in chiral and gauge supermultiplets. The spin- 0 fields are complex scalars, the spin- $1 / 2$ fields are Weyl fermions and the spin-1 fields are gauge bosons. Note that for the (s-)quarks and (s-)leptons we show only one of the three families. In the last column the transformation behaviour under the $S M$ gauge groups, $S U(3)_{C}, S U(2)_{L}, U(1)_{Y}$, is given in the from that left handed quarks transform as triplets under $S U(3)$, as doublets under $S U(2)$ and carry the hypercharge $1 / 6$. This table is inspired by [29].
phases, mixing angles and couplings. This includes for example the masses $M_{1}, M_{2}$ and $M_{3}$ of the bino, wino and gluino, and the Higgs mass term $\mu$. However, some of the free parameters are not independent or bound due to experimental observations, reducing the arbitrariness.
The last missing piece is $R$-parity. If it were just for the matter content of table 1 baryon number $B$ and lepton number $L$ could generally be violated. This could lead to a rapid proton decay with for example squark mediators resulting in $p^{+} \rightarrow \pi^{0} e^{+}$. This is not favourable since postulating lepton and baryon number conservation would be a step back from the SM where this is realised as an accidental symmetry which is eventually broken by the weak interaction. Thus, a new discrete $\mathbb{Z} 2$ symmetry called $R$-parity is introduced, defined by

$$
\begin{equation*}
R_{P}=(-1)^{3(B-L)+2 s} \tag{37}
\end{equation*}
$$

with spin $s$. This yields an even $R$-parity of $R_{P}=+1$ for all particles (including additional Higgs) and an odd $R$-parity of $R_{P}=-1$ for all additional MSSM particles and if $R$-parity is exactly conserved, which we will assume from now on, no mixing of these two types of particles is possible. Further consequences are that every interaction vertex can only include an even number of sparticles, the lightest supersymmetric particle is stable and thus called lightest stable particle (LSP) and every other sparticle will eventually decay into a final state with an odd number of LSPs.

### 2.4. Mass Spectrum of the MSSM

In order to be in accordance with observations the MSSM must also break the electroweak symmetry spontaneously. Compared to the SM this is more complicated since there are two

## 2 Supersymmetry

scalar complex Higgs doublets $H_{u}$ and $H_{d}$ now. Using gauge transformations one can rotate away possible vacuum expectation values (VEVs) of the charged elements $H_{u}^{+}=H_{d}^{-}=0$ leaving only

$$
\begin{equation*}
\left\langle H_{u}^{0}\right\rangle=v_{u} \quad \text { and } \quad\left\langle H_{d}^{0}\right\rangle=v_{d} \tag{38}
\end{equation*}
$$

These are related to the SM Higgs VEV and thus to the mass of the $Z^{0}$ boson following

$$
\begin{equation*}
v_{u}^{2}+v_{d}^{2}=v^{2}=\frac{2 m_{Z}^{2}}{g^{2}+g^{\prime 2}} \approx(174 \mathrm{GeV})^{2} \tag{39}
\end{equation*}
$$

and thus yield only one new parameter, traditionally parameterised through their ratio

$$
\begin{equation*}
\tan \beta \equiv \frac{v_{u}}{v_{d}} \tag{40}
\end{equation*}
$$

This value is not fixed through experiments but depends on the parameters chosen in the MSSM Lagrangian. Usually tools like for example SPheno 34 are used to compute values of the supersymmetric particle spectrum.

Similar to the mixing of the B- and W-bosons after electroweak symmetry breaking in the SM, the neutral wino, bino and higgsinos as well as the charged winos and higgsinos mix following the electroweak symmetry breaking in the MSSM. They form the neutral majorana particles, neutralinos, and the charged particles, charginos, jointly called electroweakinos. The relevant part of the Larangian in the gauge-eigenstate basis with $\psi^{0}=\left(\widetilde{B}, \widetilde{W} 0, \widetilde{H}_{d}^{0}, \widetilde{H}_{u}^{0}\right)$ and $\psi^{ \pm}=\left(\widetilde{W}^{+}, \widetilde{H}_{u}^{+}, \widetilde{W}^{-}, \widetilde{H}_{d}^{-}\right) \mathrm{read}$

$$
\begin{equation*}
\mathcal{L} \subseteq-\frac{1}{2}\left(\psi^{0}\right)^{T} \mathbf{M}_{n} \psi^{0}-\frac{1}{2}\left(\psi^{ \pm}\right)^{T} \mathbf{M}_{c} \psi^{ \pm}+\text {h.c. } \tag{41}
\end{equation*}
$$

including the generally non-diagonal mass matrices for neutralinos and charginos $\mathbf{M}_{n}$ and $\mathbf{M}_{c}$. After electroweak symmetry breaking, the Higgs scalars are replaced by their VEVs, parameterised with $\beta$ (see eq. 40), yielding

$$
\mathbf{M}_{n}=\left(\begin{array}{cccc}
M_{1} & 0 & -c_{\beta} s_{W} m_{Z} & s_{\beta} s_{W} m_{Z}  \tag{42}\\
0 & M_{2} & c_{\beta} c_{W} m_{Z} & -s_{\beta} c_{W} m_{Z} \\
-c_{\beta} s_{W} m_{Z} & c_{\beta} c_{W} m_{Z} & 0 & -\mu \\
s_{\beta} s_{W} m_{Z} & -s_{\beta} c_{W} m_{Z} & -\mu & 0
\end{array}\right)
$$

which can be diagonalised to form the mass eigenstates

$$
\begin{equation*}
\tilde{\chi}_{i}^{0}=\mathbf{N}_{i j} \psi_{j}^{0} \quad \text { with } \quad \mathbf{N}^{*} \mathbf{M}_{n} \mathbf{N}^{-1}=\operatorname{diag}\left(m_{\tilde{\chi}_{1}^{0}}, m_{\tilde{\chi}_{2}^{0}}, m_{\tilde{\chi}_{3}^{0}}, m_{\tilde{\chi}_{4}^{0}}\right) \tag{43}
\end{equation*}
$$

Here, the sine and cosine of the weak mixing angle and $\beta$ are abbreviated such that $\cos \theta_{W}=c_{W}$ and $\sin \beta=s_{\beta}$. The diagonal elements, giving the masses of the neutralinos, are the eigenvalues of $\mathbf{M}_{n}$. Both, the masses and the mixing matrix $\mathbf{N}$ thus depend on the MSSM parameters $M_{1}$, $M_{2}, \tan \beta$ and $\mu$, however, their explicit values show no real simplification and are of no further
interest here. Usually the neutralinos $\tilde{\chi}_{i}^{0}$ are sorted by mass, such that $m_{\tilde{\chi}_{1}^{0}}<m_{\tilde{\chi}_{2}^{0}}<m_{\tilde{\chi}_{3}^{0}}<m_{\tilde{\chi}_{4}^{0}}$ with $\tilde{\chi}_{1}^{0}$ being the LSP because it is the favourable dark matter particle within the MSSM. Similarly the charged mass matrix, given in a $2 \times 2$ block form as

$$
\mathbf{M}_{c}=\left(\begin{array}{cc}
\mathbf{0} & \mathbf{X}^{T}  \tag{44}\\
\mathbf{X} & \mathbf{0}
\end{array}\right) \quad \text { with } \quad \mathbf{X}=\left(\begin{array}{cc}
M_{2} & \sqrt{2} s_{\beta} m_{W} \\
\sqrt{2} c_{\beta} m_{W} & \mu
\end{array}\right)
$$

after electroweak symmetry breaking can be examined. The mass eigenstates forming after diagonalisation are easier to calculate due to the block structure, but are again of no further interest. The mixing matrices $\mathbf{V}$ and $\mathbf{U}$ form the positively and negatively charged charginos as:

$$
\begin{equation*}
\binom{\tilde{\chi}_{1}^{+}}{\tilde{\chi}_{2}^{+}}=\mathbf{V}\binom{\widetilde{W}^{+}}{\widetilde{H}_{u}^{+}} \quad \text { and } \quad\binom{\tilde{\chi}_{1}^{-}}{\tilde{\chi}_{2}^{-}}=\mathbf{U}\binom{\widetilde{W}^{-}}{\widetilde{H}_{d}^{-}} \quad \text { with } \quad \mathbf{U}^{*} \mathbf{X} \mathbf{V}^{-1}=\operatorname{diag}\left(\tilde{\chi}_{1}^{ \pm}, \tilde{\chi}_{2}^{ \pm}\right) . \tag{45}
\end{equation*}
$$

In general scalars carrying the same charges and $R$-parity can mix. Therefore, the diagonalisation of the $6 \times 6$ mass matrices of up-type sqaurks ( $\tilde{\mathrm{u}}_{L}, \tilde{\mathrm{c}}_{L}, \tilde{\mathrm{t}}_{L}, \tilde{\mathrm{u}}_{R}, \tilde{\mathrm{c}}_{R}, \tilde{\mathrm{t}}_{R}$ ), down-type squarks $\left(\tilde{\mathrm{d}}_{L}, \tilde{\mathrm{~s}}_{L}, \tilde{\mathrm{~b}}_{L}, \tilde{\mathrm{~d}}_{R}, \tilde{\mathrm{~s}}_{R}, \tilde{\mathrm{~b}}_{R}\right)$, charged sleptons ( $\tilde{\mathrm{e}}_{L}, \tilde{\mu}_{L}, \tilde{\tau}_{L}, \tilde{\mathrm{e}}_{R}, \tilde{\mu}_{R}, \tilde{\tau}_{R}$ ) and the diagonalisation of the $3 \times 3$ matrix of the sneutrinos ( $\tilde{\nu}_{e}, \tilde{\nu}_{\mu}, \tilde{\nu}_{\tau}$ ) would yield the mass eigenstates. Fortunately, in most models the mixing angles of the lighter two families are small and can be neglected, such that the gauge eigenstates $\tilde{\mathrm{u}}_{L}, \tilde{\mathrm{~d}}_{L}, \tilde{\mathrm{e}}_{L}$, etc. are already the mass eigenstates. Only for the third family, the mixing can not be neglected, such that the superpartners of the left and right handed fermions mix and yield the states $\tilde{\mathrm{t}}_{1}, \tilde{\mathrm{t}}_{2}, \tilde{\mathrm{~b}}_{1}, \tilde{\mathrm{~b}}_{2}, \tilde{\tau}_{1}, \tilde{\tau}_{2}$. Since there is only the left handed $\nu_{\tau}$, there is no right handed counterpart and $\tilde{\nu}_{\tau}$ is also the mass eigenstate.
Next to the gluino there is no further colour octet fermion, so there is no gluino mixing possible. In most models the gluino is assumed to be much heavier than the lightest eweakinos.

## 3. Motivation and Setup

Within this thesis we aim to show how the combination of uncorrelated SRS of different analyses improve the exclusion limits on the masses of BSM particles. We choose a simplified SUSYmodel and employ three different processes to showcase the relevant steps and techniques. We start off by introducing the relevant scenario, the considered processes and the used toolchain.

### 3.1. Simplified Scenario

To showcase the enhanced exclusion limits achieved by combining uncorrelated SRs, we consider a simplified 1-flavour MSSM model in which all SUSY particles but $\tilde{\mathrm{u}}_{r}$ and $\tilde{\chi}_{1}^{0}$ are as heavy as 30 TeV , and therefore decoupled. The masses of $\tilde{u}_{r}$ and $\tilde{\chi}_{1}^{0}$ only obey a hierarchical mass spectrum $m_{\tilde{\mathrm{u}}_{r}}>m_{\tilde{\chi}_{1}^{0}}$ but otherwise vary freely. Since R-parity conservation is imposed, the neutralino $\tilde{\chi}_{1}^{0}$ cannot decay and is thus the LSP. The neutralino mixing matrix is taken to be diagnoal, hence $\tilde{\chi}_{1}^{0}$ is effectively a bino $\widetilde{B}^{0}$. In the following, the bino and up-squark will be denoted as $\tilde{\chi}_{1}^{0}$ and $\tilde{\mathrm{q}}$ rather than $\widetilde{B}^{0}$ and $\tilde{\mathrm{u}}_{r}$ since the used neutralino mixing and squark flavour represent a simplified model choice, however, the method could be demonstrated with other scenarios as well. We investigate in three different processes: squark pair production, neutralino squark (and neutralino antisquark) production and neutralino pair production.

(a)
(c)


(b)
(d)


(e)

Figure 3: Shown are the leading order (LO) Feynman diagrams generating a squark pair final state.

The LO Feynman diagrams, created with [35], of the squark pair final state are shown in fig. 3 . First of we notice that both t-channel diagrams are suppressed, the first due to a phase space suppression since the gluino is taken to be really massive, and the second because of the weak coupling of the neutralino. Therefore, only the two s-channel and the 4 -point vertex diagram
contribute significantly, indicating that the squark pair production is purely strong. For later reference, it is also important to notice, that the final state squark flavour does not depend on the initial state quark flavour any more, which reduces the parton distribution function (PDF) dependency.
In the associated neutralino squark production channel, with LO diagrams shown in fig. 4 , we see in the s-channel that following the quark-squark-neutralino vertex we obtain a squark flavour which is connected to the initial quark flavour. Due to the much higher abundance of up quarks compared to anti up quarks in the proton, the neutralino antisquark process is suppressed. However, for completeness, we include both processes and always refer to both when addressing the associated squark neutralino production. Finally, the two neutralinos are

(a)

(b)

Figure 4: Shown are the LO Feynman diagrams generating a squark neutralino final state.
produced in a purely weak channel, as can be seen in fig. 5. Opposed to the unstable squarks, decaying promptly via

$$
\begin{equation*}
\tilde{\mathrm{q}} \rightarrow \tilde{\chi}_{1}^{0} q, \quad \overline{\tilde{\mathrm{q}}} \rightarrow \tilde{\chi}_{1}^{0} \bar{q}, \tag{46}
\end{equation*}
$$

the LSP is obviously stable, i.e. showing no signature in the form of jets in particle detectors. Hence, the only noticeable effect of the neutralinos is missing transverse momentum, which is only quantified by detecting other momenta, like jet momenta, first. For this reason, and of course because radiation is always present in colored processes, it is inevitable to also include additional jets in the final state if one seeks to increase the precision of the calculations.

To be consistent, we therefore generate further hard radiation, in the form of up to two additional jets in each process, such that they are described by

$$
\begin{align*}
& p p \rightarrow \tilde{\mathrm{q}} \overline{\tilde{q}}^{(+j j)}  \tag{47}\\
& p p \rightarrow \tilde{\mathrm{q}} \tilde{\chi}_{1}^{0}(+j j) \text { and }  \tag{48}\\
& p p \rightarrow \tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0}(+j j) \tag{49}
\end{align*}
$$

Finally, all three processes yield the same signature of jets plus missing energy in a particle detector, justifying the consideration of their combination in the recasting procedure.

(a)

(b)

Figure 5: Shown are the Feynman diagrams generating a neutralino pair final state.

### 3.2. Toolchain

In order to create exclusion limits, several high-energy physics tools are used. The whole toolchain executed for every mass point of every process individually, is summarised in the flowchart diagram in fig. 6. To outline the general structure of this work, the steps are briefly described in here, however, a more thorough explanation and discussion of the individual stages is provided in the corresponding sections below.
We start by generating 150 k events with the general purpose Monte Carlo event generator MadGraph5_aMC@NLO 3.5.1 [36] interfaced with LHAPDF [37] to include PDF sets. Furthermore, Pythia $8.306[38]$ is executed within MadGraph, adding parton showers and carrying out the jet merging.
The detector simulation is then carried out with MadAnalysis5 1.10.5 39], employing the relevant LO merged cross sections from MadGraph, which are scaled to higher order using Resummino $3.1 .2[13]$ or NNLL-fast 1.1 [14] in combination with NLL-fast 3.140 for higher accuracy. For the determination of the correlation between SRS, MadAnalysis also generates a binary acceptance matrix between events and SRS. If such a combination is pursued, these acceptance matrices are passed to TACO [26], which then calculates their overlap summarised in a correlation matrix. The best combination of uncorrelated SRs is found via a pathfinding algorithm, originally introduced in TACO, but outsourced and generalised in PathFinder The statistics tool Spey 41 offers a likelihood based method to derive the confidence level (CL) of both a single SR and a combination of multiple SRs.
Since the number of events surviving the cuts of the detector simulation vary, there might be mass points with low statistics where the uncertainties of the detector simulation affect the CL significantly. Therefore, if the statistics of the SR with the highest exclusion power is too low, we increase the number of events to 300 k in the first iteration and to 500 k in the second iteration. Finally, the calculated CL allows to decide if the particular mass point of one process is excluded based on a $95 \%$ confidence level.
Lastly, we interpolate between the CLS in the mass plane to find the $95 \%$ confidence level of exclusion.

[^0]

Figure 6: Shown is the used toolchain visualised in a flowchart diagram.

## 4. Event Generation and Parton Shower

We now want to turn to the event generation, parton shower and jet merging, executed by MadGraph which is interfaced with Pythia. The basics of Monte Carlo generators are not discussed here, however, a short explanation of the working principle is provided in appendix A.

### 4.1. Matrix Element Calculations

When generating the events for the three final states (see eqs. (47) to (49)) with MadGraph one needs to prevent taking the same processes into account multiple times. This problem of double counting becomes evident when the intermediate signature of a process equals the final state signature of another. Consider as an example the production of a neutralino pair with one additional hard jet, following fig. 7a. Naturally, all final state particles need to be on-shell, but since the intermediate squark can take any momentum, it can also go on-shell. In this case, the same signature as in the associated production following fig. 7b would be generated. Of course, the on-shell squark will eventually decay either way and thus the particle generation is the exact same but is executed in two different production channels. However, double counting

(a) $p p \rightarrow \tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0} j$

(b) $p p \rightarrow \tilde{\mathrm{q}} \tilde{\chi}_{1}^{0}$

Figure 7: Shown are two Feynman diagrams of processes with different final states but the same intermediate signature as an example for double counting.
can be prevented in this case by allowing only off-shell intermediate squarks. On the Monte Carlo generator level, a particle achieves on-shell status when its momentum is situated close to its pole mass. The designated range is based on the width $\Gamma$ of the Breit Wigner peak and typically one considers particles on-shell if they are no further than $0.15 \cdot \Gamma$ away from the pole. MadGraph simply enforces a cut to make sure final state particles are on- and intermediate squarks are off-shell.
The event generation of the hard process bases on matrix elements, i.e. analytic process calculation. However, the processes we considered so far lack applicability because the final states are inclusive (at least a given number of jets). Since coloured particles radiate gluons, which
themselves are coloured and radiate further, before hadronization ${ }^{2}$ it is likely that further jets are emitted after reaching the final state. Thus, just adding the different jet multiplicity final states might lead to double counting and one needs to make the final states exclusive (exact amount of jets). However, then we restrict ourselves to at most two additional jets, which is hardly enough, but adding further jets in the matrix element calculation is computationally expensive and therefore unattractive.
Another disadvantage arises when examining the soft and collinear regime, as illustrated by the example of $e^{+} e^{-} \rightarrow q \bar{q}$ at LO. The total cross section $\sigma_{q \bar{q}}$ is finite but adding a gluon in the final state, so $e^{+} e^{-} \rightarrow q \bar{q} g$ results in

$$
\begin{equation*}
\frac{\mathrm{d} \sigma_{q \bar{q} g}}{\mathrm{~d} \cos \theta d z_{g}} \approx \sigma_{q \bar{q}} C_{F} \frac{\alpha_{S}}{2 \pi} \frac{2}{\sin ^{2} \theta} \frac{1+\left(1-z_{g}\right)^{2}}{z_{g}} \tag{50}
\end{equation*}
$$

where $\theta$ and $z_{g}$ encode the opening angle and energy fraction, respectively, of the emitted gluon in relation to its mother particle. Clearly, the cross section diverges in both the soft ( $z_{g} \rightarrow 0$ ) and collinear $(\theta \rightarrow 0, \pi)$ limit [42] and another approach is needed. Although this is illustrated in a specific example, the divergence of the cross section is no unique characteristic of this process but is a general problem.

### 4.2. Parton Shower

Parton showers are effectively used to describe the momenta of outgoing jets following an evolution from high momentum transfer, associated to the hard process, down to $\mathcal{O}(1 \mathrm{GeV})$, associated with the hadronization scale. The following discussion reflects the central idea of parton showers based on 42, 43.
As an instructive example, we use the quark pair production with an additional gluon emission, introduced in the previous section. We start our discussion of the collinear divergence of the differential cross section (see eq. (50)) by realising that the relevant part can be separated following

$$
\begin{equation*}
\frac{2}{\sin ^{2} \theta}=\frac{1}{1-\cos \theta}+\frac{1}{1+\cos \theta} \approx \frac{1}{1-\cos \theta}+\frac{1}{1-\cos \hat{\theta}} . \tag{51}
\end{equation*}
$$

In the first step, the divergence resulting from actual collinear jets $(\theta \rightarrow 0)$ and from jets that are emitted back-to-back from a parton $(\theta \rightarrow \pi)$, got split. Both cases were addressed as collinear divergences before, since the jet that is emitted back-to-back from the quark is just collinear to the antiquark and vice versa. This is made explicit in the second step by introducing the angle $\hat{\theta}$ between the gluon and the non-emitting parton. We have now transformed the distribution as the sum of the emission close to the direction of the quark and the emission close to the antiquark direction. By approximating the cosine for small angles and explicitly introducing

[^1]
## 4 Event Generation and Parton Shower

the sum, eq. (47) is transformed to

$$
\begin{equation*}
d \sigma_{q \bar{q} g} \approx \sigma_{q \hat{q}} \sum_{\text {partons }} C_{F} \frac{\alpha_{S}}{2 \pi} \frac{\mathrm{~d} \theta^{2}}{\theta^{2}} \mathrm{~d} z_{g} \frac{1+\left(1-z_{g}\right)^{2}}{z_{g}} . \tag{52}
\end{equation*}
$$

By positioning the total quark pair cross section at the forefront, we aim to emphasise the interpretability of the entire expression as a factorization of the $q \bar{q}$ production and a term describing the additional gluon emission. The dependency on the angle $\theta$ is no requirement to get a form like this, an identical expression can be derived if one chooses the transverse momentum $k_{T}$ of the gluon with respect to the emitting quark or the virtuality $q$ of the off-shell quark propagator in the parametrization of eq. (47), since

$$
\begin{align*}
k_{T}^{2}=z_{g}^{2}\left(1-z_{g}\right)^{2} \theta^{2} E^{2} & \Longrightarrow \frac{\mathrm{~d} \theta^{2}}{\theta^{2}}=\frac{\mathrm{d} k_{T}^{2}}{k_{T}^{2}} \text { and }  \tag{53}\\
q^{2}=z_{g}\left(1-z_{g}\right) \theta^{2} E^{2} & \Longrightarrow \frac{\mathrm{~d} \theta^{2}}{\theta^{2}}=\frac{\mathrm{d} q^{2}}{q^{2}} . \tag{54}
\end{align*}
$$

In the following, we will stick to $q$ but if not explicitly stated otherwise, this can always be interchanged with $\theta$ or $k_{T}$ and thus is just a placeholder for any of these scales. Because the derivation followed the explicit quark pair production example, eq. (52) needs to be generalised. For the production of partons with cross section $\sigma_{0}$, the emission of a further parton $j$ from parton $i$ with energy fraction $z$ is described by

$$
\begin{equation*}
d \sigma \approx \sigma_{0} \sum_{\text {partons }} \frac{\alpha_{S}}{2 \pi} \frac{\mathrm{~d} q^{2}}{q^{2}} \mathrm{~d} z P_{j i}(z, \phi) d \phi \tag{55}
\end{equation*}
$$

with the splitting functions $P_{i j}$ depending on the momentum fraction and the azimuthal angle of $j$ around the $i$ axis. Including a spin average and integrating out the $\phi$ dependence they are given as

$$
\begin{array}{ll}
P_{q q}(z)=C_{F} \frac{1+z^{2}}{1-z}, & P_{g q}(z)=C_{F} \frac{1+(1-z)^{2}}{z} \\
P_{g g}(z)=C_{A} \frac{z^{4}+1+(1-z)^{4}}{z(1-z)}, & P_{q g}(z)=T_{R}\left(z^{2}+(1-z)^{2}\right) \tag{57}
\end{array}
$$

describing the splitting of $q \rightarrow q g, q \rightarrow g q, g \rightarrow g g$ and $g \rightarrow q \bar{q}$, respectively ${ }^{3}$ Still, the divergence problem is not solved so far. However, we can interpret its origin, since an emitter and a completely collinear emitted particle with a combined momentum are together indistinguishable from just the emitter particle with this exact momentum. Therefore, one introduces a cutoff $Q_{0}$, which again could be an angle, transverse momentum or virtuality, at which two partons are not resolvable anymore. The divergence below this cutoff would cancel out with loop corrections of the hard process to a finite result, but these calculations are tedious. The probability of a further emission and the probability of no further emission add up to one, hence, the behaviour

[^2]below the cutoff scale does not change the inclusive cross section, but just the event shape, which is experimentally not resolvable anyway.
Ignoring the unresolvable part of the phase space solves the divergence, so generating splittings from a parton following eq. (55) with the probability of emitting a parton within ( $Q_{0}, Q_{\mathrm{MS}}$ ) and $\left(z_{\text {min }}, z_{\text {max }}\right)$ is given as
\[

$$
\begin{equation*}
\mathcal{P}_{\text {incl }}=\int_{Q_{0}}^{Q_{\mathrm{MS}}} \frac{\mathrm{~d} \tilde{q}^{2}}{\tilde{q}^{2}} \frac{\alpha_{S}}{2 \pi} \int_{z_{\min }}^{z_{\max }} \mathrm{d} z P_{j i}(z) . \tag{58}
\end{equation*}
$$

\]

$Q_{\text {MS }}$ represents the upper cut-off scale at which the parton shower loses accuracy due to the approximation done before. As indicated by the subscript, this is the inclusive probability distribution, the probability that at least one particle is emitted within these limits. Similarly to before, the unitarity argument that the probability of emitting any number of particles added with the probability of no further emission is one holds. The multiplicative no emission probability can be approximated following

$$
\begin{align*}
\mathcal{P}_{\text {no-em }}\left(q_{\max }>q>q_{\text {min }}\right) & =\lim _{n \rightarrow \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text {no-em }}\left(q_{i}>q>q_{i+1}\right) \\
& =\lim _{n \rightarrow \infty} \prod_{i=0}^{n-1}\left(1-\mathcal{P}_{\text {incl }}\left(q_{i}>q>q_{i+1}\right)\right) \\
& \approx \lim _{n \rightarrow \infty} \exp \left(-\sum_{i=0}^{n-1} \mathcal{P}_{\text {incl }}\left(q_{i}>q_{\max }>q_{i+1}\right)\right) \\
& =\exp \left(-\int_{q_{\min }}^{q_{\max }} \mathrm{d} \mathcal{P}_{\text {incl }}(q)\right), \tag{59}
\end{align*}
$$

with $q_{0}=q_{\text {max }}$ and $q_{n}=q_{\min }$, and we used that the inclusive emission probability within a shrinking emission scale interval goes to zero, to approximate an exponential. We can thus write the no emission probability between two scales as

$$
\begin{equation*}
\Delta_{i}\left(q_{1}, q_{2}\right):=\mathcal{P}_{\text {no-em }}\left(q_{1}>q>q_{2}\right)=\exp \left\{-\int_{q_{2}}^{q_{1}} \frac{\mathrm{~d} q^{2}}{q^{2}} \int_{z_{\min }}^{z_{\max }} \frac{\alpha_{S}}{2 \pi} \mathrm{~d} z P_{j i}(z)\right\} \tag{60}
\end{equation*}
$$

which is called Sudakov form factor. The emission scale $q$ also acts as an ordering scale, such that if there is no emission between $q_{1}$ and $q_{2}$ with $q_{1}>q_{2}$, any further emission can only be at scale $q$ with $q_{2}>q$. Although this seems to be an obvious statement, this is a notable characteristic of the parton shower. Depending on the scale one then says that a parton shower is angular or $k_{T}$ ordered for example.
For the Monte Carlo implementation of the parton shower, a random number $\rho_{1}$ within $(0,1)$ is generated [44. Then the emission scale at which the no emission probability $\rho$ is reached is calculated, this means $\Delta_{i}\left(Q_{\mathrm{MS}}, q_{1}\right)=\rho$ is solved for $q_{1}$. Therefore, the probability that there is no emission at a scale larger than $q_{1}$ is $\rho$ and if $q_{1}>Q_{0}$ we generate a resolvable branching at scale $q_{1}$. In order to find the fitting momentum fraction $z$ a second random number $\rho_{1}^{\prime}$ is used

## 4 Event Generation and Parton Shower

to solve

$$
\begin{equation*}
\int_{z_{\min }}^{z} \frac{\alpha_{S}}{2 \pi} \mathrm{~d} \tilde{z} P(\tilde{z})=\rho_{1}^{\prime} \int_{z_{\min }}^{z_{\max }} \frac{\alpha_{S}}{2 \pi} \mathrm{~d} \tilde{z} P(\tilde{z}) \tag{61}
\end{equation*}
$$

with the appropriate splitting function $P(z)$. Afterwards, the procedure is repeated for smaller scales $q$, i.e. for another random number $\rho_{2}$ solve $\Delta_{i}\left(Q_{\mathrm{MS}}, q_{2}\right)=\rho$ and if $q_{2}>Q_{0}$ and $q_{1}>q_{2}$ a second emission at $q_{2}$ is radiated and the momentum fraction is calculated analogous to the above. Eventually, one of the scaling conditions for the emission scale is not met and the parton shower along the considered branch stops. The described procedure must be executed for all partons in the final state, as well as for all particles emitted during the parton shower, which eventually creates events as structurally depicted in fig. 8. It goes without saying that colour charge, momentum and energy are conserved in any particle emission along the way. Naturally,


Figure 8: Structural sketch of a parton shower taken from [44].
other parton shower algorithms like the Sudakov veto algorithm [45] exist, but will not be discussed in this thesis.
The sketched derivation is only valid for collinear emissions due to the small angle approximation executed in eqs. (51) and (52). However, the soft gluon emission problem, i.e. the divergence for $z \rightarrow 0$, is avoided but not solved by introducing cuts in terms of $z_{\min }$ and $z_{\max }$. Without delving into further detail, we claim that the occurrence of a soft emission subsequent to collinear radiation can be understood at the amplitude level as if the soft emission occurred before the collinear one. This motivates the rather simple solution, that soft emission effects are correctly accounted for in a collinear parton shower ordered for the opening angle $\theta$.

### 4.3. Jet Merging

Now we can conclude that the matrix element calculation precisely describes hard jets, but it is computationally expensive, it generates inclusive final states and it diverges in the soft- and collinear limit. On the other hand, the parton shower produces exclusive final states and models soft- and collinear emissions well but is also only valid in this regime. To get the best of both worlds, one aims to combine the event generator final states with a parton shower for accurate
description of the jets and a high final state jet multiplicity. But generating, for example, one additional hard jet on matrix element level and dressing it with a parton shower could result in a wide angle emission. This wide angle emission might also be described by the matrix element calculation with two additional jets as sketched in fig. 9 and thus it needs to be decided which event to trust or else counting the same jet twice. To prevent both double counting and the opposite, dead regions, so jets which are neither described by the matrix element or the parton shower, one needs to carefully split the phase space between these two. This is done by correctly choosing the upper scale limit for the parton shower and a lower scale for the matrix element, which was already introduced and anticipatory called $Q_{\mathrm{MS}}$ to denote the merging scale, in the previous section. Additionally to the correct merging scale, we need to take care of the inclu-


Figure 9: Double counting jets schematically visualised in case of two additional dressed hard jets compared to just one dressed hard jet.
siveness of the matrix element generator. Merging together events with one additional jet and events with two additional jets is only meaningful if the events are exclusive, else there would be double counting again. Within the parton shower, the Sudakov form factor secures exclusiveness depending on the emission scale. On the matrix element level, however, the emissions are not ordered and thus the notion of emitting one parton before another (in terms of the emission scale) is not well defined and thus, the scales need to be reconstructed. The whole procedure of adding a parton shower to the hard process, taking the described problems into account, is combined in merging algorithms. There are multiple algorithms available which all aim to split the phase space and make the matrix elements exclusive while minimizing the dependency on the artificial merging scale. Within this project, the CKKW-L algorithm [46], implemented in Pythia [47] and extending the CKKW algorithm [48], is used and thus explained in the following. For the full algorithm, please refer to the original publication and for general merging procedures to the summarising publications [49, 50].
Of course any merging algorithm is based on matrix element calculations of events with a usually fixed strong coupling $\alpha_{S}$, a specified maximum parton multiplicity $N$ and a lower cutoff $Q_{\text {MS }}$. Then the algorithm selects one parton multiplicity $n$ with a probability scaling with its total cross section and the full parton shower history, so all intermediate states in the cascade are reconstructed, yielding different paths. In summary, this entails reconstructing all possible parton shower cascades capable of producing the identical final state as provided by the matrix element. One of these histories is then picked with a probability proportional to the product

## 4 Event Generation and Parton Shower

of the branching probabilities, and only if there are no ordered parton shower paths unordered ones are considered. The resulting path is characterised by its intermediate states ( $S_{2}, S_{3}, \ldots, S_{n}$ ) and emission scales $\left(q_{2}, q_{3}, \ldots, q_{n}\right)$, with $q_{i}$ denoting the scale at which an emission from $S_{i-1}$ is generated to produce $S_{i}$. In case the matrix element is generated with a fixed strong coupling $\alpha_{S}(\mu)$, one obtains the running coupling by multiplying with $\prod_{i=2}^{n-1} \alpha_{s}\left(q_{i}\right) / \alpha_{S}(\mu)$.
The actual transformation to exclusive final states is executed by generating a test emission at each intermediate state $S_{i}$. If the scale $q$ of the test emission succeeds the scale of the originally subsequent emission in the reconstructed history, the event is rejected. Else, the test emission is kept, and the algorithm continues with the next intermediate state. For the final reconstructed state $S_{n}$ the test emission is rejected if it is above the merging scale $Q_{\mathrm{MS}}$, else the parton shower cascade is launched. With this, all remaining final states are exclusive, however, the final state of the highest jet multiplicity should be inclusive. This secures that, although we did not generate higher jet multiplicities, there could be more and they are accounted for, while keeping the lower jet multiplicity final states exclusive, such that we can merge them together. Therefore, in case of $n=N$, any test emission from $S_{n}$ is accepted.
Accepting or rejecting test emissions as described above correctly takes the Sudakov form factor into account without actually multiplying with it. This is analogous to the unweighting in a general Monte Carlo framework, as described in appendix A.

We choose the merging scale to be transverse momentum and set it to one quarter of the SUSY hard scale:

$$
Q_{\mathrm{MS}}=p_{\mathrm{T}}^{j}=\left\{\begin{array}{ll}
\frac{1}{4} m_{\tilde{\chi}_{1}^{0}} & \text { for } p p \rightarrow \tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0}  \tag{62}\\
\frac{1}{8}\left(m_{\tilde{\mathrm{q}}}+m_{\tilde{\chi}_{1}^{0}}\right) & \text { for } p p \rightarrow \tilde{\chi}_{1}^{0} \tilde{\mathrm{q}} \\
\frac{1}{4} m_{\tilde{\mathrm{q}}} & \text { for } p p \rightarrow \tilde{\mathrm{q}} \tilde{\tilde{\mathrm{q}}}
\end{array} .\right.
$$

To verify the success of the merging procedure, one examines the differential jet rate (DJR) plots, which illustrate the transition between $n-1$ and $n$ additional final-state hard jets in case of a DJR plot [51. Plotted is the cross section of every jet multiplicity state on the y-axis and $\log \sqrt{d_{i j}}$ on the x-axis with $d_{i j}$ being the minimised distance measure between two particles like the Durham $k_{T}$ [52]. If a smooth curve in the shape of an inverted parabola on a log scale describes the sum of all jet multiplicities, the merging process worked successfully. However, with the default settings peaks as exemplary shown in the DJR1 plot in fig. 10a, appeared at some mass points, preventing a smooth transition. By increasing the Breit Wigner cut, introduced in section 4.1, to $35 \%$ the all jets curve (green) smoothens out as the comparison with fig. 10 b shows. Note, this does not lead to neglecting some squark momenta since this only shuffles around which momenta are accounted for in which process. The final state squarks are considered on-shell in a wider area around the pole, which is compensated by cutting out the on-shell resonance of intermediate squarks more broadly. In the example of fig. 7 , the squark momenta outside of the $35 \%$ area around the pole are now considered in the neutralino pair production and all squark momenta within this area are accounted for in the associated neutralino squark process. We validate the merging for different mass combinations, but for


Figure 10: Shown are the DJR1 and DJR2 plot of the neutralino pair production with masses set at $m_{\tilde{q}}=1000 \mathrm{GeV}$ and $m_{\tilde{\chi}_{1}^{0}}=400 \mathrm{GeV}$. The upper and lower plots differ only by the chosen Breit Wigner cut.
brevity only show the DJR plots at one further representative mass point for all processes in fig. 11. The plots of the different processes show slightly different behaviour, but the previously listed features of a successful merging are present. The sum of the jet multiplicities (green) exhibits an inverted parabola shape without peaks and bumps. Especially, the transitions between the jet multiplicities are smooth.
This finalises the event generation. To ensure completeness and reproducibility, the relevant MadGraph commands are provided in appendix B


Figure 11: Shown are the $D J R 1$ and $D J R 2$ plot of the three production channels with masses set at $m_{\tilde{q}}=1000 \mathrm{GeV}$ and $m_{\tilde{\chi}_{1}^{0}}=300 \mathrm{GeV}$ using a Breit Wigner cut of $35 \%$.

## 5. Recasting analyses

We have now completed the event generation of the three processes, added a parton shower and took care of the merging via the CKKW-L scheme. Following the toolchain outlined in fig. 6 the next step is to actually recast analyses to generate an exclusion limit in the $m_{\tilde{q}}-m_{\tilde{\chi}_{1}^{0}}$-plane. However, to gain the most precise exclusion limits possible, it is preferable to use higher order cross sections instead of the merged LO cross sections returned by the previous step. Therefore, section 5.1 starts with discussing the cross sections in detail before we elaborate on the different analyses and the detector simulation in section 5.2 . Afterwards, section 5.3 focuses on how Spey uses a likelihood description to calculate the CL for each mass point. Finally, we provide an explanation for the combination of SRs with MadAnalysis, TACO, PathFinder and Spey in section 5.4. This concludes the toolchain presented in section 3.2 .

### 5.1. Cross Sections and K-Factors

As stated above, the accuracy of the exclusion limit calculation is increased by using higher order cross section instead of the merged LO cross sections provided by MadGraph. For the weak and associated channel Resummino yields resummed cross sections at aNNLO+NNLL next to next to leading order (NNLO) quantum chromo dynamics (QCD) and next to leading order (NLO) SUSY corrections) [53 56] and NLO+NLL 57] respectively. The chosen PDF set for these calculations are MSHT201o_as130 for LO and MSHT20nlo_as118 for all higher orders such that we stay consistent with the PDF used in MadGraph. Within this project Resummino is executed via HEPi [58], a Python interface which simplifies the use of several high-energy physics tools.
For the strong production, we rely on the resummed cross sections from NNLL-fast at aNNLO+NNLL [59 65. NNLL-fast as well as its predecessor NLL-fast bases on grid files yielding cross sections for specific scenarios at fixed mass points. Predictions of different masses are provided following an interpolation scheme. This method has the disadvantage that only a finite set of scenarios calculated with only a few specific PDF sets are available. The one closest to our simplified scenario is the squark production in the heavy gluino limit denoted by sdcpl with PDF4LHC21_40_pdfas [66]. However, the differences are that it uses a 10-flavour mass degeneracy, so ten instead of one light squark flavour and it does not incorporate MSHT20 as a PDF set. We can tackle the later problem by introducing a factorisation method (K-factor) 40, 55] instead of using the NNLL-fast cross sections directly. This means, we take the ratio of higher order against LO NNLL-fast cross sections, resulting in an approximately PDF independent scale factor, which is then multiplied with the merged LO cross section given by MadGraph. Fortunately, this also resolves the scenario discrepancy problem. Since the gluino is decoupled, the t-channel gluino process is negligible compared to the other diagrams given in fig. 3. Therefore, the initial state quarks do not have an impact on how likely a specific squark flavour is in the final state, so no PDF induced differences for different squark flavours emerge. Additionally, the squark mass degeneracy ensures that the partonic cross section for all final state squark flavours are equal, so no mass induced cross section differences emerge either. The in-

## 5 Recasting analyses

dependence of the cross section from the final state squark flavour ensures that the ratio of two 10-flavour mass degenerate scenario cross sections yield the same scaling factor as two 1-flavour mass degenerate scenario cross sections would.

In order to stay consistent between the different processes, we also use the K-factor approach for the weak and associated production. The higher order cross sections are calculated following

$$
\begin{align*}
p p \rightarrow \tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0}: & \sigma=\sigma(\mathrm{LO})_{\mathrm{LO}}^{\mathrm{MG}} \frac{\sigma(\mathrm{aNNLO}+\mathrm{NNLL})_{\mathrm{NLO}}^{\mathrm{RS}}}{\sigma(\mathrm{LO})_{\mathrm{LO}}^{\mathrm{RS}}},  \tag{63}\\
p p \rightarrow \tilde{\chi}_{1}^{0} \tilde{\mathrm{q}}: & \sigma=\sigma(\mathrm{LO})_{\mathrm{LO}}^{\mathrm{MG}} \frac{\sigma(\mathrm{NLO}+\mathrm{NLL})_{\mathrm{NLO}}^{\mathrm{RS}}}{\sigma(\mathrm{LO})_{\mathrm{LO}}^{\mathrm{RS}}},  \tag{64}\\
p p \rightarrow \tilde{\mathrm{q}} \overline{\mathrm{q}}: & \sigma=\sigma(\mathrm{LO})_{\mathrm{LO}}^{\mathrm{MG}} \frac{\sigma(\mathrm{NLO})_{\mathrm{NLO}}^{\text {nllfast }}}{\sigma(\mathrm{LO})_{\mathrm{LO}}^{\text {nllfast }}} \frac{\sigma(\mathrm{aNNLO}+\mathrm{NNLL})_{\mathrm{NNLO}}^{\mathrm{nnllfast}}}{\sigma(\mathrm{NLO})_{\mathrm{NLO}}^{\mathrm{nnnllfast}}} . \tag{65}
\end{align*}
$$

Here, the superscript denotes the software used to calculate the cross section, the subscript denotes the order of the PDF and the order of the cross section itself is given in the brackets. The reason for inserting a second K-factor based on NLL-fast in the strong channel (see eq. 65) is that the lowest order given by its successor is NLO but we aim for a LO input in the denominator. The NLL-fast scenario is the same as used in NNLL-Fast but with MSTW2008LO and MSTW2008NLO 67].
In fig. 12 the LO and the scaled cross sections from eqs. 63) to 65 are plotted over the squark mass with an additional panel to show the K-factor. Here, the neutralino mass is fixed at $m_{\tilde{\chi}_{1}^{0}}=250 \mathrm{GeV}$ on the left and at $m_{\tilde{\chi}_{1}^{0}}=400 \mathrm{GeV}$ on the right-hand side. In both cases, the purely strong channel $\left(\propto \alpha_{S}^{2}\right)$ yields the highest scaled cross section for low squark masses, followed by the associated channel ( $\propto \alpha_{S} \alpha$ ) about one order of magnitude lower and finally the weak channel $\left(\propto \alpha^{2}\right)$ another two orders of magnitude lower. This behaviour is anticipated since the coupling constant $\alpha_{S}$ of the strong interaction is much larger than the weak $\alpha$, which justifies their naming. However, since two squarks are produced in the strong channel, the corresponding cross section decreases faster with increasing squark mass compared to the associated and weak channel with the production of just one or no squark due to an increased phase space suppression. Therefore, the weakness of the weak coupling is compensated in the high squark mass regime, such that eventually the associated cross section exceeds the strong one at approximately $m_{\tilde{q}}=850 \mathrm{GeV}$ and $m_{\tilde{\mathrm{q}}}=950 \mathrm{GeV}$ for left and right, respectively. The neutralino pair cross section approaches the production cross sections of squark pairs, reaching a separation of roughly one order of magnitude. This gap would diminish with rising squark mass, eventually reaching a point where even the weak channel surpasses the strong one. This underlines the importance of considering the weak production in order to get accurate exclusion limits, as the squark exclusion limit is already pushed into the TeV-regime in many scenarios. A tendency of increasing and large K-factors in the weak process towards low squark masses is noted. This behaviour is traced back to t-channel bubble diagrams including the gluino and was cross checked with Prospino [68]. A similar behaviour is also outlined in 69].
By comparing the two plots, we see that for the same squark mass only the cross section of the weak and the associated production decrease notably when increasing the neutralino mass. This


Figure 12: Shown are the LO (dashed lines) and scaled higher order (solid lines) cross sections for $m_{\tilde{\chi}_{1}^{0}}=250 \mathrm{GeV}$ on the left and $m_{\tilde{\chi}_{1}^{0}}=400 \mathrm{GeV}$ on the right. Additionally, the resulting K-factors are shown in the lower panel.
behaviour is more apparent in the mass plane shown in fig. 13 for the three processes. Here, the red lines indicate the cut of the mass plane that is shown in fig. 12. Based on the inclination of the contour lines, we see the weak and the semi-weak production depend on both the squark and the neutralino mass, while the strong production nearly exclusively depends on the squark mass. Comparing this with the LO diagrams given in figs. 3 to 5 show that the weak production cross section depends on the squark mass due to the propagator in the t -channel. However, the total strong production cross section would only be minimally dependent on the neutralino mass following the t-channel neutralino propagator since the other diagrams dominate. Naturally, if the final state incorporates a specific particle, the cross section depends on its mass, i.e. the neutralino pair production cross section always depend on the neutralino mass, the squark pair production cross section on the squark mass and the associated production cross section on both.

### 5.2. Experimental Analyses and Detector Simulation

MadAnalysis5 allows to reinterpret LHC results, simply by feeding in the Monte Carlo events and selecting the relevant analyses available in the public analysis database (PAD) [70]. Depending on the analysis, the detector simulation is executed either via a FastJet [52] or DELPHES3 [7] interface, or alternatively by using the simplified detector simulation [71] implemented directly in MadAnalysis5.

## 5 Recasting analyses



Figure 13: Shown are the scaled higher order cross sections of the three processes in the mass plane. The red lines indicate the $m_{\tilde{\chi}_{1}^{0}}=250 \mathrm{GeV}$ and $m_{\tilde{\chi}_{1}^{0}}=400 \mathrm{GeV}$ levels.

For the simplified scenario at hand, four analyses, two by ATLAS and two by CMS, validated in MadAnalysis5 $72 \sqrt{75}$, proved to be the most sensitive. They are originally named ATLAS-EXOT-2018-06 [76], ATLAS-CONF-2019-040 [77], CMS-SUS-19-006 [78] and CMS-EXO-20004 [79, but the shorthands ATLAS-EXOT, ATLAS-CONF, CMS-SUS, and CMS-EXO excluding the numerical identifiers will be used for brevity. The data is taken in LHC Run-2 at $\sqrt{s}=13 \mathrm{TeV}$ with an integrated luminosity of $L=139 \mathrm{fb}^{-1}$ and $L=137 \mathrm{fb}^{-1}$ for the ATLAS and CMS analyses respectively and is evaluated in mono- and multijet-searches plus missing transverse momentum. The preselection criteriums are summarised in table 2 Since the BSM

| cuts | ATLAS-EXOT | ATLAS-CONF | CMS-SUS | CMS-EXO |
| :--- | :--- | :--- | :--- | :--- |
| veto | $e, \mu, \tau, \gamma$ | $e, \mu$ | $e, \mu, \gamma$ | $e, \mu, \tau, \gamma, b$-jet |
| $N_{j}$ | $\geq 1$ | $\geq 2$ | $\geq 2$ | $\geq 1$ |
| $E_{T}^{\text {miss }}$ | $>200 \mathrm{GeV}$ | $>300 \mathrm{GeV}$ | - | $>250 \mathrm{GeV}$ |
| $\|\eta\|$ | $<2.4$ | - | $<2.4$ | $<2.4$ |
| $p_{T}\left(j_{1}\right)$ | $>150 \mathrm{GeV}$ | $>200 \mathrm{GeV}$ | - | $>100 \mathrm{GeV}$ |
| $p_{T}\left(j_{2}, \ldots, j_{N_{j}}\right)$ | $>30 \mathrm{GeV}$ | $>50 \mathrm{GeV}$ | - | - |
| $\Delta \Phi\left(\mathrm{jet}, \mathbf{p}_{T}^{\text {miss }}\right)$ | $>0.4$ | $>0.2$ | $>0.5$ | $>0.5$ |
| $m_{\mathrm{eff}}$ | - | $>800 \mathrm{GeV}$ | - | - |
| $H_{T}$ | - | - | $>300 \mathrm{GeV}$ | - |
| $\left\|\vec{H}_{T}^{\text {miss }}\right\|$ | - | - | $>300 \mathrm{GeV}$ | - |

Table 2: Summary of the event selection cuts of the four analyses ATLAS-EXOT-2018-06, ATLAS-CONF-2019-040, CMS-SUS-19-006 and CMS-EXO-20-004.
signal from heavy particles will result in energetic jets and high missing momenta and energy, it is crucial to sort out SM signals in this region. Also, signals from leptons like electrons and muons and in some cases additionally for taus and photons need to be vetoed. Naturally, the selection of the transverse momentum and the rapidity of the vetoed particles vary, but this is of no greater importance here.
The ATLAS-EXOT preselection criteria necessitate the presence of at least one jet and a minimum missing transverse energy $E_{T}^{\text {miss }}$ exceeding 200 GeV . Additionally, selected events impose
conditions, including a primary jet with a transverse momentum $p_{T}$ surpassing 150 GeV and a rapidity $\eta$ within the range of $\pm 2.4$. Subsequent jets are subject to a lower $p_{T}$ threshold of 30 GeV . An azimuthal angle separation of $\Delta \Phi\left(\mathrm{jet}, \mathbf{p}_{T}^{\mathrm{miss}}\right)>0.4$ is mandated between each jet and the direction of the missing transverse momentum. Cuts on the effective mass and the sum of the absolute and vector missing transverse momenta defined as

$$
\begin{align*}
& m_{\mathrm{eff}}=E_{T}^{\mathrm{miss}}+\sum_{p_{T}>50 \mathrm{GeV}} p_{T}(j),  \tag{66}\\
& H_{T}=\sum_{|\eta|<2.4} p_{T}(j),  \tag{67}\\
& \vec{H}_{T}^{\mathrm{miss}}=\sum_{|\eta|<5} \vec{p}_{T}(j) . \tag{68}
\end{align*}
$$

are not imposed. The classification into SRs for the ATLAS-EXOT analysis is based on varying $E_{T}^{\mathrm{miss}}$.
Compared to the previous analysis, ATLAS-CONF demands a higher jet multiplicity of at least two and slightly higher limits on the transverse missing energy and the transverse momenta of the primary and subsequent jets. Additionally, there is a cut dependent on the effective mass of at least 800 GeV which is considered an effective discriminant between SM background and a signal from heavy BSM particles. The SRs differ by combining differently modified cuts obeying the discussed preselection criteria.

CMS-SUS applies a higher cut on the angle between the two highest $p_{T}$ jets and the missing momentum direction. Otherwise it bases on $H_{T}$ the scalar sum of jet transverse momenta and $\left|\vec{H}_{T}^{\text {miss }}\right|$ the vector sum of jet transverse momenta with a more inclusive rapidity interval. Furthermore, only events with $\left|\vec{H}_{T}^{\text {miss }}\right|>H_{T}$ are considered preventing mismeasurements. The analysis is split in SRs each representing an area in the two-component phase space spanned by $H_{T}$ and $\left|\vec{H}_{T}^{\text {miss }}\right|$.
The SR with the lowest transverse momentum cut on the leading jet in CMS-EXO undercuts the previous ones with 100 GeV . However, several of the SRs impose a significantly higher cut in this regime.
In general, we can say that the differences seem minor on the first look, however, their slightly varying cuts imply different sensitivity in different regions of the phase space. Note, that for example CMS-SUS is triggered if the sum of the jets surpasses a threshold, while for the others, one leading jet carrying a minimal energy is required. Another difference, promising a spread and an in combination well covered phase space sensitivity is that we employ analyses that select events with only one jet, while others start with at least two.
Feeding the BSM events and the corresponding cross section $\sigma$ to MadAnalysis5 for the detector simulation results in the expected signal events $n_{s}$ for each SR of the four analyses. The PAD 70 also yields some analysis specific information. In specific, for the purpose of getting exclusion limits with a $95 \%$ CL the number of experimentally observed events $n_{\text {obs }}$, as well as the expected number of SM background events $n_{\mathrm{b}}$ and the corresponding uncertainty $\Delta n_{\mathrm{b}}$ is needed.

## 5 Recasting analyses

### 5.3. Confidence Level Calculation

At this stage the number of expected signal events $n_{s}$ following our SUSY model is known from the MadAnalysis5 run calculated following the description in section 5.2 Keep in mind, this value already incorporates the cross section, as well as the integrated luminosity. Furthermore, the PAD yields the number of events $n_{\text {obs }}$ that were actually observed in the experiment, as well as the expected number of SM background events $n_{\mathrm{b}}$ and the corresponding uncertainty $\Delta n_{\mathrm{b}}$.
From these four values, the software Spey [41] calculates the exclusion limit or equivalently the $p$-value based on the corresponding likelihood functions. A basic understanding of statistics is assumed to understand the following sections. For the reader who is not well-acquainted with this concept or simply wants to review his knowledge, a more instructive introduction to statistical data analysis is given in appendix C.

### 5.3.1. Likelihood Description

Spey serves as a statistics tool that is built around a plug-in system, designed to treat a diverse set of likelihood based approaches. Here, we will focus on the likelihood description that leads to calculating the CL of exclusion.
Consider a general binned counting experiment where in each bin $i$ we find $x_{i}$ counts. The likelihood function yields the probability of finding this distribution of counts depending on the general parameters $\mu$, the parameter of interest or the signal strength, and $\theta$, the nuisance parameters. Therefore, the general composite likelihood is described by

$$
\begin{equation*}
\mathcal{L}(\mu, \boldsymbol{\theta})=\prod_{i \in \text { bins }} \mathcal{M}\left(x_{i} \mid \lambda_{i}(\mu, \boldsymbol{\theta})\right) \cdot \prod_{j \in \text { nui }} \mathcal{C}\left(\theta_{j}\right), \tag{69}
\end{equation*}
$$

with the main $\mathcal{M}$ representing the probability distribution in each bin and $\mathcal{C}$ representing the probability distribution of the constraint terms which are associated to the uncertainties in the measurement. $\lambda_{i}$ is a function relating the strength and nuisance parameter to the corresponding bin.
In the specific case at hand, there are no bins but just the already mentioned values $n_{s}, n_{\mathrm{obs}}, n_{\mathrm{b}}$ and $\Delta n_{\mathrm{b}}$. Furthermore, like in most LHC counting experiments, one assumes a Poisson distribution for the counted events while the expected background events are normally distributed. This yields:

$$
\begin{align*}
\mathcal{L}(\mu, \theta) & =\operatorname{Poiss}\left(n_{\text {obs }} \mid \mu n_{s}+n_{b}+\theta \cdot \Delta n_{b}\right) \text { Gauss }\left(n_{b}+\theta \cdot \Delta n_{b} \mid n_{b}, \Delta n_{b}\right)  \tag{70}\\
& =\frac{\left(\mu n_{s}+n_{b}+\theta \cdot \Delta n_{b}\right)^{n_{\text {obs }}}}{n_{\text {obs }}!} e^{-\left(\mu n_{s}+n_{b}+\theta \cdot \Delta n_{b}\right)} \cdot \frac{1}{\sqrt{2 \pi}} e^{-\frac{1}{2}\left(\frac{\left(n_{b}+\theta \cdot \Delta n_{b}\right)-n_{b}}{\Delta n_{b}}\right)^{2}} . \tag{71}
\end{align*}
$$

We see that the Poisson distribution describes the probability of observing $n_{\text {obs }}$ events given an expected value. The expected value is taken to be the sum of the background events $n_{b}$, its uncertainty regulated by the only nuisance parameter $\theta$ which can take any value and some
ratio of the actual signal events $n_{s}$ tuned by the strength parameter $\mu$. In order to get physical results, the strength parameter has to be between zero and one. The Gauss distribution yields the probability of getting a deviation from the actual background events regulated by the nuisance parameter. The mean and the standard deviation are taken to be the number of background events $n_{b}$ and its uncertainty $\Delta n_{b}$.

The strength parameter $\mu$ is defined by the hypothesis, with $\mu=0$ for the null hypothesis, i.e. there is no signal and $\mu=1$ for the alternative hypothesis, i.e. there is a signal. The nuisance parameter is unknown but following the maximum likelihood approach it is estimated to be $\hat{\hat{\theta}}(\mu)$, so the value of $\theta$ given a specific $\mu$, such that the likelihood maximises. Due to the estimator dependency on $\mu$ we call $\mathcal{L}(\mu, \hat{\hat{\theta}}(\mu))$ the conditional likelihood.
Typically, one distinguishes two different types of exclusion limit, the observed and expected one. So far, we assumed that the observed number of events are the true number of events and we are therefore in the observed regime. Contrary, for the expected exclusion limit, one assumes the expected number of SM events $n_{b}$ to be the truth and therefore the approach changes by replacing $n_{\text {obs }}$ with $n_{b}$ in the calculation.

Consider as an artificial example the values:

$$
\begin{equation*}
n_{s}=16, \quad n_{\mathrm{obs}}=56, \quad n_{\mathrm{b}}=48 \quad \text { and } \quad \Delta n_{\mathrm{b}}=20 \tag{72}
\end{equation*}
$$

On the left-hand side in fig. 14 the likelihood functions based on eq. 71 for these values with varying strength parameter are plotted. Note, $\mu$ varies in a broader interval than just between zero and one in the figure. This is to clarify the form of the likelihood in the plot, but still a strength parameter smaller than zero or bigger than one is unphysical. It is easy to see that the likelihood of the observed calculation peaks at a value that is greater than zero because more counts are observed than predicted by the SM. The expected likelihood peaks at zero since the background events are assumed to be the truth.
We define the profile likelihood ratio as

$$
\begin{equation*}
\lambda(\mu)=\frac{\mathcal{L}(\mu, \hat{\hat{\theta}}(\mu))}{\mathcal{L}(\hat{\mu}, \hat{\theta})} . \tag{73}
\end{equation*}
$$

In contrast to the conditional likelihood $\mathcal{L}(\mu, \hat{\hat{\theta}}(\mu))$, where the strength parameter $\mu$ is kept free and only the estimator for $\theta$ is calculated, $\mathcal{L}(\hat{\mu}, \hat{\theta})$ represents the maximised unconditional likelihood, using the estimators for both $\mu$ and $\theta$. Thus the unconditional likelihood is the best fit for the data and therefore, $\hat{\mu}$ is said to describe the data of the experiment, while $\mu$ describes the hypothesis, either null or alternative hypothesis, in the following.
The profile likelihood ratio represents a normalization of the likelihood, since $0 \leq \lambda \leq 1$ and its maximum is at $\lambda(\hat{\mu})=1$. Also, a profile likelihood ratio close to one implies that the hypothesised strength parameter $\mu$ is close to the maximum likelihood estimator $\hat{\mu}$, i.e. the hypothesis is in good agreement with the data.
On the right-hand side of fig. 14 the negative logarithm of the profile likelihood ratio is shown.

## 5 Recasting analyses

The impact of the logarithm is that the peak value is now zero following $\log \lambda(\hat{\mu})=0$ and the form turned upside down due to the extra sign. Naturally, the extrema are still at the same values of $\mu$. Consider now the null hypothesis $\mu=0$ and the alternative hypothesis $\mu=1$. For the former, the expected likelihood yields a lower negative logarithmic likelihood and is thus in better agreement with the data given the null hypothesis than the observed likelihood. Conversely, the observed likelihood yields a lower value for $\mu=1$ and is thus in better agreement with the data given the alternative hypothesis than the expected likelihood. As long as $n_{\mathrm{obs}}>n_{b}$ this is always the case.
Of course, the outlined differences in the negative logarithmic likelihood (see fig. 14 right) are already present at the likelihood level (see fig. 14 left). Nevertheless, it is advantageous to make clear if high or low values describe good or bad agreement with the data on both levels, in order to relate this to the CLlater.
Note that it can occur that the maximum likelihood estimator yields $\hat{\mu}<0$. However, we are only interested in $\mu \geq 0$ and since the best agreement between the data and a physical value $\mu$ is $\mu=0$ we tune the profile likelihood to be

$$
\tilde{\lambda}(\mu)=\left\{\begin{array}{ll}
\frac{\mathcal{L}(\mu, \hat{\hat{\theta}}(\mu))}{\mathcal{L}(\hat{\mu}, \hat{\theta})} & \text { if } \hat{\mu} \geq 0  \tag{74}\\
\frac{\mathcal{L}(\mu, \hat{\hat{\hat{N}}}(\mu))}{\mathcal{L}(0, \hat{\hat{\theta}}(0))} & \text { otherwise }
\end{array},\right.
$$

where the denominator turned into a conditional likelihood in case of $\hat{\mu}<0$.


Figure 14: In this plot we compare the conditional likelihood function of the observed and the expected calculation for the artificial example values given in eq. (72). On the left hand side the likelihood function is plotted and on the right we plot the negative logarithm of the profile likelihood ratio. The extrema are denoted by a dot.

### 5.3.2. Test Statistics

To construct a hypothesis test resulting in a $p$-value, one usually relies on test statistics. Spey employs three different kinds of test statistics for three different scenarios, based on the discussion in 80. First,

$$
q_{0}= \begin{cases}-2 \log \lambda(0)=-2 \log \frac{\mathcal{L}(0, \hat{\hat{\theta}}(0))}{\mathcal{L}(\hat{\mu}, \hat{\theta})} & \text { if } 0 \leq \hat{\mu}  \tag{75}\\ 0 & \text { if } \hat{\mu}<0\end{cases}
$$

is the test statistic to look for a discovery of a signal. The result of observed data fluctuating lower than the expected SM background is $\hat{\mu}<0$ which yields a test statistic of zero, indicating agreement between the null hypothesis and the data. Contrary, the result of the observed data increasing above the SM expectation is an increase in $q_{0}$ indicating a higher incompatibility between data and null hypothesis.
The test statistic to explore upper limits is given as

$$
q_{\mu}=\left\{\begin{array}{ll}
-2 \log \lambda(\mu)=-2 \log \frac{\mathcal{L}(\mu, \hat{\theta}(\mu))}{\mathcal{L}(\hat{\mu}, \hat{\theta})} & \text { if } \hat{\mu} \leq \mu  \tag{76}\\
0 & \text { if } \mu<\hat{\mu}
\end{array} .\right.
$$

One can see that this explores the compatibility between a hypothesis, usually the alternative hypothesis, and the data. Since one is interested in exclusion limits, data fluctuations above the alternative hypothesis don't matter and $q_{\mu}$ is set to zero. The more important scenarios are if the data fluctuates lower than the hypothesised $\mu$ yielding an increased $q_{\mu}$ and thus, an increasing incompatibility.
This however, does not account for unphysical estimators given as $\hat{\mu}<0$. Therefore, the alternative test statistic, given as,

$$
\tilde{q}_{\mu}=\left\{\begin{array}{ll}
-2 \log \tilde{\lambda}(\mu) & \text { if } \hat{\mu} \leq \mu  \tag{77}\\
0 & \text { if } \mu<\hat{\mu}
\end{array}= \begin{cases}-2 \log \frac{\mathcal{L}(\mu, \hat{\hat{\theta}}(\mu))}{\mathcal{L}(0, \hat{\hat{\theta}}(0))} & \text { if } \hat{\mu}<0 \\
-2 \log \frac{\mathcal{L}(\mu, \hat{\theta}(\mu))}{\mathcal{L}(\hat{\mu}, \hat{\theta})} & \text { if } 0 \leq \hat{\mu} \leq \mu \\
0 & \text { if } \mu<\hat{\mu}\end{cases}\right.
$$

distinguishes this case. Note that $\tilde{q}_{\mu}$ will be used in this work and we therefore focus on this test statistic from now on. However, the difference between $\tilde{q}_{\mu}$ and $q_{\mu}$ is usually negligible.

### 5.3.3. Approximations and the Asimov Data Set

In order to find the $p$-value, that is the probability of finding data which is as extreme or extremer than the measured one, we have to calculate

$$
\begin{equation*}
p_{\mu}=\int_{\tilde{q}_{\mu}}^{\infty} f\left(\tilde{q}_{\mu} \mid \mu\right) d \tilde{q}_{\mu} \tag{78}
\end{equation*}
$$

Here, we denote the probability density function (pdf) to find $\tilde{q}_{\mu}$ given the hypothesis $\mu$ as $f\left(\tilde{q}_{\mu} \mid \mu\right)$. The first step to find the general distribution $f\left(\tilde{q}_{\mu} \mid \mu^{\prime}\right)$ is to approximate the profile likelihood. Wald [81] showed that

$$
\begin{equation*}
-2 \log \lambda(\mu)=\frac{(\mu-\hat{\mu})^{2}}{\sigma^{2}}+\mathcal{O}(1 / \sqrt{N}) \tag{79}
\end{equation*}
$$

with $N$ being the sample size and $\hat{\mu}$ following a normal distribution with mean $\mu^{\prime}$ and standard deviation $\sigma$. If the higher order terms are negligible, one can show that $t_{\mu}:=-2 \log \lambda(\mu)$ follows
a non-central chi-square distribution

$$
\begin{equation*}
f\left(t_{\mu} ; \Lambda\right)=\frac{1}{2 \sqrt{2 \pi}} \frac{1}{\sqrt{t_{\mu}}}\left[\exp -\frac{1}{2}\left(\sqrt{t_{\mu}}+\sqrt{\Lambda}\right)^{2}+\exp -\frac{1}{2}\left(\sqrt{t_{\mu}}-\sqrt{\Lambda}\right)^{2}\right] \tag{80}
\end{equation*}
$$

with the non-centrality parameter

$$
\begin{equation*}
\Lambda=\frac{\left(\mu-\mu^{\prime}\right)}{\sigma^{2}} \tag{81}
\end{equation*}
$$

Wilks 82 showed this earlier for the special case of $\mu^{\prime}=\mu$ which implies $\Lambda=0$.
One way of calculating the standard derivation $\sigma$ is by employing the so called Asimov data set. The Asimov data set is defined such that when it is used in the maximum likelihood approach, one finds the true parameters as estimators. In the case at hand that means the estimator of $\mu$ is equal to the true value $\mu^{\prime}$, i.e. $\hat{\mu}_{A}=\mu^{\prime}$, with $\hat{\mu}_{A}$ being the estimator of the Asimov data set. Plugging this equality into the Wald approximation eq. (79), yields that for negligible higher orders

$$
\begin{equation*}
-2 \log \lambda_{A}(\mu) \approx \frac{\left(\mu-\mu^{\prime}\right)}{\sigma_{A}^{2}}=\Lambda \tag{82}
\end{equation*}
$$

and therefore one finds an estimate for the non-centrality parameter. In the special case, where one looks for the exclusion significance of a hypothesis $\mu$ assuming there is no signal $\mu^{\prime}=0$ this simplifies to

$$
\begin{equation*}
q_{\mu, A}=\frac{\mu^{2}}{\sigma_{A}^{2}} \Longrightarrow \sigma_{A}^{2}=\frac{\mu^{2}}{q_{\mu, A}} \tag{83}
\end{equation*}
$$

with $q_{\mu, A}=-2 \log \lambda_{A}(\mu)$. With this last result, we now have a way to model the pdf using a test statistic (see. eq. 80 ) for which we need the standard deviation of the strength parameter (see eq. (83)).

### 5.3.4. Distribution of $\tilde{q}_{\mu}$

With the Wald approximation of the profile likelihood eq. 79, one can simplify the test statistic

$$
\tilde{q}_{\mu}= \begin{cases}\frac{\mu^{2}}{\sigma^{2}}-\frac{2 \mu \hat{\mu}}{\sigma^{2}} & \text { if } \hat{\mu}<0  \tag{84}\\ \frac{(\mu-\hat{\mu})^{2}}{\sigma^{2}} & \text { if } 0 \leq \hat{\mu} \leq \mu \\ 0 & \text { if } \mu<\hat{\mu}\end{cases}
$$

But more importantly, we also get an approximation for the pdf which is

$$
f\left(\tilde{q}_{\mu}, \mu\right)=\frac{1}{2} \delta\left(\tilde{q}_{\mu}\right)+ \begin{cases}\frac{1}{2 \sqrt{2 \pi}} \frac{1}{\sqrt{\tilde{q}_{\mu}}} e^{-\tilde{q}_{\mu} / 2} & \text { if } 0 \leq \tilde{q}_{\mu} \leq \frac{\mu^{2}}{\sigma^{2}}  \tag{85}\\ \frac{1}{\sqrt{2 \pi}(2 \mu / \sigma))} \exp -\frac{1}{2} \frac{\left(\tilde{q}_{\mu}+\mu^{2} / \sigma^{2}\right)^{2}}{(2 \mu / \sigma)^{2}} & \text { if } \frac{\mu^{2}}{\sigma^{2}}<\tilde{q}_{\mu}\end{cases}
$$

for the special case of $\mu^{\prime}=\mu$. The $p$-value is then calculated as

$$
\begin{equation*}
p_{\tilde{q}_{\mu}}=1-F\left(\tilde{q}_{\mu} \mid \mu\right) \tag{86}
\end{equation*}
$$

with the cumulative distribution

$$
F\left(\tilde{q}_{\mu} \mid \mu\right)= \begin{cases}\Phi\left(\sqrt{\tilde{q}_{\mu}}\right) & \text { if } 0<\tilde{q}_{\mu} \leq \frac{\mu^{2}}{\sigma^{2}}  \tag{87}\\ \Phi\left(\frac{\tilde{q}_{\mu}+\mu^{2} / \sigma^{2}}{2 \mu / \sigma}\right) & \text { if } \frac{\mu^{2}}{\sigma^{2}}<\tilde{q}_{\mu}\end{cases}
$$

where $\Phi$ denotes the cumulative of the Gaussian distribution. Remember that the CD of exclusion of a particular mass point can now be easily calculated as $\mathrm{CL}=1-p_{\tilde{q}_{\mu}}=F\left(\tilde{q}_{\mu} \mid \mu\right)$.
However, the CLS given by SPEY use a combination of the null hypothesis $\mu=0$ and the alternative hypothesis $\mu=1$. The background only $\mathrm{CL}_{\mathrm{b}}(\mu=0)$ and the signal-plus-background $\mathrm{CL}_{\mathrm{s}+\mathrm{b}}(\mu=1)$ are calculated, and the final CL is obtained from the ratio $\mathrm{CL}_{\mathrm{s}}=\mathrm{CL}_{\mathrm{s}+\mathrm{b}} / \mathrm{CL}_{\mathrm{b}}$. This works for both the expected and the observed likelihood.
We have now traced the important steps that get us from the likelihood to the CL We see that an increasing test statistic value $\tilde{q}_{\mu}$ indicates an increasing incompatibility between the hypothesis $\mu$ and the data $\hat{\mu}$. A higher $\tilde{q}_{\mu}$ ensures a higher $F\left(\tilde{q}_{\mu} \mid \mu\right)$ and therefore a higher CL Considering the ratio $\mathrm{CL}_{\mathrm{s}}=\mathrm{CL}_{\mathrm{s}+\mathrm{b}} / \mathrm{CL}_{\mathrm{b}}$, this means, to exclude a point in the phase space, we require a low $\mathrm{CL}_{\mathrm{b}}$ and a high $\mathrm{CL}_{\mathrm{s}+\mathrm{b}}$, so we require low test statistics for the null hypothesis and high test statistics for the alternative hypothesis.

### 5.4. Combination

There are two types of combinations, first, the combination of different uncorrelated SRs for one process, and second, the combination of different processes with the same signature. In the following, we start by focussing on the SR first before discussing the combination of processes. For the sake of calculating the overlap of SRs MA5 also generates a binary acceptance matrix which is an event $\times S R$-matrix denoting if the event passed the cuts of the SR. An exemplary acceptance matrix with $N$ events of process $p$ evaluated in the analysis $a$ which incorporates $n$ different SRs would look like

Here, the first event, for example, passed the cuts of the first and $n$-th SR but not the cuts of the second SR To also calculate the correlation of SRS of the previously discussed four different analyses, a combined acceptance matrix is needed. Therefore, we tag the events, such that we can allocate them in the four individual acceptance matrices and then merge the

## 5 Recasting analyses

corresponding event rows in the acceptance matrices horizontally, i.e. along the SR-axis. This can be structurally outlined as
with abbreviations to denote the SR columns and the event rows. The combined acceptance matrix now includes all SRs, so SR1 to SR $n$ of each analysis.
The resulting matrix is passed to TACO which determines if the minimal number of events to estimate the overlap of two SRS is met. Furthermore, it calculates their Pearson correlations and summarises them in one symmetric correlation matrix, i.e. a $S R \times S R$ matrix containing the SRS from all four analyses, structurally represented by

$$
\begin{array}{cccccc} 
& \mathrm{SR} 1_{1} & \mathrm{SR} 2_{1} & \cdots & \mathrm{SR}\left(n_{4}-1\right) & \mathrm{SR} n_{4}  \tag{90}\\
\mathrm{SR} 1_{1} \\
\mathrm{SR} 2_{1} \\
\vdots & \begin{array}{ccccc}
1 & 0.73 & \cdots & 0.002 & 0.43 \\
0.73 & 1 & \cdots & 0.6 & 0.005 \\
\mathrm{SR}\left(n_{4}-1\right) \\
\mathrm{SR} n_{4}
\end{array}\left(\begin{array}{ccccc} 
& \vdots & \ddots & \vdots & \vdots \\
0.002 & 0.6 & \cdots & 1 & 0.82 \\
0.43 & 0.005 & \cdots & 0.82 & 1
\end{array}\right) .
\end{array}
$$

If the correlation of a $\operatorname{SR}$ pair is below a chosen threshold of 0.01 as proposed in TACO, they are considered uncorrelated, else correlated. For the example given in eq. 90 we can therefore deduce from the first column that $\mathrm{SR} 1_{1}$ and $\mathrm{SR}\left(n_{4}-1\right)$ are uncorrelated while $\mathrm{SR} 2_{1}$ and $\mathrm{SR} n_{4}$ correlate with $\mathrm{SR}_{1}$.
Considering that the same proton-proton collision cannot be detected by both ATLAS and CMS detectors, we regard any ATLASSR to be uncorrelated to an CMS SR and set the corresponding values in the correlation matrix manually to zero. However, this is just an approximation because the experiments might share uncertainties and since any correlation would reduce the exclusion limit, the derived one should be taken as the maximal possible one 41.
This final correlation matrix is then passed to the PathFinder which finds the combination of uncorrelated SRs with the highest exclusion power based on the weighted hereditary depth-first search (WHDFS) proposed in TACO. Hence, a weight needs to be assigned to each SR which gives a measure for the corresponding exclusion power. It is important to stress at this point that in order to calculate the combined CL of exclusion later, we will combine the likelihood of different SRS. If the individual likelihoods are independent (uncorrelated SRs), they combine
multiplicatively, i.e.

$$
\begin{equation*}
\mathcal{L}_{\text {combined }}(\mu, \boldsymbol{\theta})=\prod_{i \in \mathrm{SRs}} \mathcal{L}_{i}\left(\mu, \theta_{i}\right) \tag{91}
\end{equation*}
$$

Therefore, the logarithmic likelihood and consequently the logarithmic likelihood ratio combines additively. Combining this with the previously discussed connection of an increasing test statistic implying an increasing CL justifies assigning weights in the form of

$$
\begin{equation*}
\mathcal{W}=-\log \left(\frac{\mathcal{L}(1, \hat{\hat{\theta}}(1))}{\mathcal{L}(\hat{\mu}, \hat{\theta})}\right) \tag{92}
\end{equation*}
$$

to each SR. Because the weights, which represent an approximation for test statistic, combine additively, the PathFinder is just looking to maximise the sum of weights of uncorrelated SRs. To reduce the runtime, the SRs in the correlation matrix is sorted for the weight in descending order before running the PathFinder.
In fig. 15 the correlation matrix $\rho$, of the squark pair production for $m_{\tilde{\chi}_{1}^{0}}=350 \mathrm{GeV}$ and


Figure 15: Visualised is the correlation matrix $\rho$ of the four discussed analyses in case of squark pair production at $m_{\tilde{\chi}_{1}^{0}}=350 \mathrm{GeV}$ and $m_{\tilde{q}}=850 \mathrm{GeV}$. Black dots encode a correlated pair of SR and white blocks an uncorrelated pair. The threshold is set to $T=0.01$. In the left panel, the unsorted correlation matrix and in the right the correlation matrix sorted for the likelihood ratio is shown. Note, for visual clarity, only 125 of the original 281 SRs are shown.
$m_{\tilde{q}}=1000 \mathrm{GeV}$ is shown as an example. Eventually, the PathFinder runs over the sorted correlation matrix of each mass point of each process. The diagonal elements, shaded in dark grey, correspond to the correlation of a SR with itself and are therefore of no interest for the combining and pathfinding procedure. Furthermore, the correlation matrix is symmetric, thus, the PathFinder only runs over the elements below the diagonal, forming the triangular shape

## 5 Recasting analyses

which is colour coded such that black and white blocks correspond to correlated and uncorrelated SRs, respectively. For the unsorted correlation matrix shown in fig. 15a, one finds block shaped white areas where ATLAS and CMS analyses meet. For clarity, we only give shorthands for the SRs and the corresponding analysis as axis descriptions in the zoom-in on the upper right. After sorting they get mixed up and only the white line like areas remain as shown in fig. 15b. In the zoom-in, it is clear to see that combinations of ATLAS and CMS analyses are always uncorrelated, while the majority of combinations within one analysis are correlated. However, there are exceptions like the intersection of cms_sus::SR6 and cms_sus::AGGSR3. The coloured lines indicate the top four best paths, i.e. combinations with the highest exclusion power found by the pathfinding algorithm. Following the red line, the best path, we see that all three intersection points between cms_sus::SR6, cms_sus::AGGSR3 and atlas_conf::SR4j are white in the plot and therefore they are all uncorrletaed. Furthermore, one notices that even though the first SR yields the highest exclusion power individually, it is not included in the best path, since a combination of other SRs achieves a higher exclusion power than combinations with the first one.
Ultimately, we feed the SRs of the best path into Spey which calculates the combined likelihood


Figure 16: Shown is the negative logarithmic likelihood ratio plotted over the strength parameter $\mu$. This compares the most sensitive SR and the combined SRs for the squark production process at $m_{\tilde{\chi}_{1}^{0}}=200 \mathrm{GeV}$ and $m_{\tilde{q}}=900 \mathrm{GeV}$.
following eq. (91) with the individual likelihoods as given in eq. 70). The CL is then calculated following the steps described in the previous section. In fig. 16 the impact of combining SRs is shown on the likelihood level. The course of the curve shows that the minimum, which denotes the best data agreement $\hat{\mu}$, shifts to a lower value of the strength parameter $\mu$ in the combined process. Combining SRs therefore ensures that the recorded data is more consistent with the SM, i.e. the null hypothesis, which again indicates that it is more likely to exclude the alternative hypothesis. For the alternative hypothesis that there is a BSM signal indicated by $\mu=1$, there is a higher value for the combined likelihood compared with the likelihood of the single most sensitive SR. This means the discrepancy between data and alternative hypothesis is higher for the combination and thus this mass point is more likely to be excluded by the combination and therefore we find a higher CL. If these relations are still unclear, refer back to the very end of section 5.3 .

Lastly, the combination of different processes need to be discussed. For the single SR evaluation, it is sufficient to add the number of expected signal events $n_{s}$ and the cross sections $\sigma$ of the different processes together, following

$$
\begin{equation*}
n_{s}=n_{s}^{\tilde{\mathrm{q}} \tilde{\tilde{q}}}+n_{s}^{\tilde{\mathrm{q}} \tilde{\chi}_{1}^{0}}+n_{s}^{\tilde{\chi}_{s}^{0} \tilde{\chi}_{1}^{0}} \text { and } \sigma=\sigma^{\tilde{\mathrm{q}} \tilde{\tilde{\mathrm{q}}}}+\sigma^{\tilde{\mathrm{q}} \tilde{\chi}_{1}^{0}}+\sigma^{\tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0}} \tag{93}
\end{equation*}
$$

The other necessary values, the number of observed events $n_{\text {obs }}$, the expected number of SM background events $n_{\mathrm{b}}$ and the corresponding uncertainty $\Delta n_{\mathrm{b}}$ are not process dependent and are therefore unchanged. Passing these values to SPEY results in CL values for the combined process evaluated in individual SRs.
In order to also combine multiple uncorrelated SRs for the combined process, an acceptance matrix is needed. All three processes yield the same final state signature, therefore, the correlation needs to be calculated taking into account the events of all the three processes. Hence, the acceptance matrix of the three processes is combined by merging them vertically, i.e.
where the event axis subscripts $1,2,3$ encode the first, second and third process. Also, the horizontal SR-axis subscripts clarify that the already horizontally merged acceptance matrices (see eq. 89 ) are now merged vertically. Thus, we accomplished to build one acceptance matrix for allSRs and all processes. This is then passed to TACO, PathFinder and SPEY to calculate the correlation matrix, find the best path and compute the resulting CL of exclusion, as described previously.

## 6 Results

## 6. Results

In the following we present the results from the single analysis considerations and from the combination of all four analyses. In both cases we take the single and combined processes into account. Numerical values supplementing the exclusion plots are given in the tables in appendix D.

### 6.1. Single Analysis

The results of the individual analyses are displayed in fig. 17. The individual analysis exclusion limits are derived by using the most constraining SR from the respective analysis at each mass point in the phase space to calculate the CL of exclusion. However, if the number of accepted events in the best SR is too low ( $\lesssim 50$ events) statistical uncertainties might have a high impact and therefore we increase the number of events to 300 k in the first iteration and to 500 k in the second iteration. Eventually, the exclusion limits shown result from interpolating between the mass points to find the $95 \%$ CL of exclusion. Every mass point to the lower left of the lines, so points with lower masses, are excluded, while everything to the upper right, so higher mass points are still possible. The shown diagonal restricting the mass plane originates from the fact that the neutralino is the LSP and thus $m_{\tilde{\chi}_{1}^{0}}<m_{\tilde{q}}$ always holds as introduced in section 3.1.
For all four analyses we show the expected (solid line) and observed (dashed line) exclusion limits in figs. 17 a to 17 d . They are derived by using the squark pair production only (blue line), adding the associated squark neutralino production (orange line) and finally additionally combining the neutralino pair production (red line). However, the orange line is not visible in all four cases, because it is hidden behind the red line, indicating that the neutralino pair production contributes only minimally.

First, we want to focus on one of the analyses, namely ATLAS-EXOT. Figure 17a shows, that in the area where the exclusion lines approach the diagonal, so for greater neutralino masses, the limits from the squark pair production and from the combination are close to each other, meaning mainly the squark-squark channel is sensitive in this region. For smaller neutralino masses the limits diverge from the diagonal and from each other. Eventually at $m_{\tilde{\chi}_{1}^{0}}=200 \mathrm{GeV}$, the expected combined limit reaches a surplus of nearly 200 GeV in the squark mass respective to the expected squark pair production limit. For the observed limits, this difference is with about 50 GeV a little smaller but still present. The described behaviour is in good agreement with the fact that the cross section of the associated channel decreases slower with increasing squark mass than the strong channel cross section (see fig. 12). Therefore, the significance of the squark neutralino production for the exclusion limit grows compared to the squark pair production with decreasing neutralino and increasing squark mass. The comparison of the observed and expected limit yield a similar trend of becoming close to each other for high neutralino and small squark masses and growing apart at the other end of the spectrum. In this case the observed limits are generally less constraining.
The effect of the squark pair production limit and the combined processes limit distancing from each other for smaller neutralino masses is present in the other analyses as well. The comparison


Figure 17: Shown are the expected (solid lines) and observed (dashed lines) exclusion limits with $95 \%$ confidence for one individual analysis computed with higher order cross sections.
of the observed and expected limit yield no general behaviour since for ATLAS-CONF and CMSSUS in figs. 17 b and 17 c the observed is more constraining while for CMS-EXO in fig. 17d both give similar exclusion limits. Still, both limits diverge from each other in the small neutralino mass and high squark mass regime.
The comparison of the analyses with each other shows that CMS-SUS yields the largest limits over the whole mass plane. A reason for that might be that CMS-SUS is triggered by the sum of jet momenta surpassing a specific limit as opposed to the other analyses where always at least one high energetic jet is needed as pointed out in section 5.2. Since the dominant channel, the squark pair production, always yields at least two jets from the squark decay, possibly two additional hard jets and even more soft jets (see matrix element and parton shower jet production in section (4), the jet multiplicity can generally be high. Furthermore, the decaying squarks result in two similar energetic jets rather than one energetic and further less energetic

## 6 Results

ones.
Next to the strength over the whole mass plane we notice that ATLAS-EXOT and CMS-EXO show an increased sensitivity along the diagonal. The common feature of these analyses outlined in section 5.2 is that they can be triggered by one jet while the other two analyses always need at least two. Close to the diagonal, the squark and neutralino are equally heavy, therefore, following the squark decay, the majority of the squark energy is transferred to the neutralino rather than the quark. Thus, possible high energetic jets triggering the analyses are less abundant and only originating from the additionally generated hard jets. This reasons that the probability to find multiple energetic jets is decreasing as squark and neutralino masses come closer and hence one jet analyses are more likely to be triggered. However, the justification for the different sensitivity regions are just assumptions derived from comparing the analyses cuts with the final exclusion plots. This means we can not be certain whether or not these are the actual reasons.

### 6.2. Combined Analyses

The combination of the most constraining uncorrelated SRs of the four different analyses yield the exclusions as shown in fig. 18 including both the expected (left) and observed (right) limits. Additionally, to the combined analyses limits (blue, orange and red line) we show for comparison the most constraining limit from the single analysis considerations, namely the combined processes limit from CMS-SUS (dashed black line). As before, we note that the squark pair


Figure 18: Shown are the expected (left) and observed (right) exclusion limits with $95 \%$ confidence for the best combination of uncorrelated SRs computed with higher order cross sections. Additionally, the limit for the combined processes with only CMS-SUS is visualised (dashed black line).
production and the combined processes limits are very similar close to the diagonal but for small neutralino and high squark masses the combined one shows a greater exclusion. The observed limits exceed the expected limits, which is most apparent in the small neutralino and high squark mass regime again and is a feature that can be traced back to the most constraining


Figure 19: Shown are two different fractional distributions for the combined $\operatorname{SR}$ consideration of the combined process. On the left the number of combined SRs is noted as the fraction of all mass points and on the right the affiliation to one analysis is shown as the fraction of all considered SRs.
individual analysis, CMS-SUS.
The comparison with the CMS-SUS consideration demonstrates the significant gain in exclusion power achieved by the combination of uncorrelated SRs. For both the expected and observed limit a surplus of around 100 GeV to 200 GeV in squark mass direction over nearly the whole mass plane is obtained. Here, we want to stress that this combination does not just give the exclusion limit of one analysis in a region where it is the most constraining and the exclusion limit of a second analysis in another region of the phase space where this one is the most constraining. The combined analyses exclusion limit exceeds any exclusion limit of the individual analysis consideration as the comparison with fig. 17 shows. Furthermore, when comparing the combined process and single SR limit (i.e. the black line) with the single process and combined SRs limit (i.e. blue line) we see that the later is mostly more constraining. Thus, the combination of SRs yields better exclusion than the combination of processes in this case. Therefore, a joint evaluation of data from different analyses like this increases the information gained by the experiments and is thus needed in order to improve the exclusion limits.

The number of combined SRs at different mass points vary as fig. 19a shows. For the expected case, nearly $50 \%$ of the mass points include five and for the observed case nearly $50 \%$ of the mass points include four different uncorrelated SRs, but none include less than two or more than seven. It is questionable if including further analyses from ATLAS and CMS might increase this number. Another analysis would need to be both mostly uncorrelated with the considered analysis and sensitive to the BSM signal. Since we already picked the most constraining ATLAS and CMS analyses it is unlikely to find an analysis that combines these characteristics. However, including analyses from other experiments, which are thus uncorrelated, yield an attractive way to extend the number of SRs,
On the analysis level, we see in fig. 19b that over $40 \%$ of the used SRs originate from CMSSUS. This can be reasoned by the fact that CMS-SUS is the most constraining analysis and it

## 6 Results

is also the analysis which divides into the most SRs. However, the other analyses contribute substantially as well, showing that although their individual exclusion was less constraining than CMS-SUS, they impact the combination significantly. Since we assume CMS and ATLAS analyses to be uncorrelated, the combination of ATLAS SRs to CMS-SUS SRs is more likely than combinations of CMS-SUS and CMS-EXO SRs. Thus, CMS-EXO naturally contributes less compared to ATLAS-EXOT and ATLAS-CONF.

## 7. Conclusion

In this thesis, we considered a simplified MSSM scenario with all particles but one squark flavor and a bino like neutralino decoupled to showcase the exclusion power gained by combining uncorrelated SRs of different analyses. The examined processes include the purely strong squark pair production, the associated squark neutralino production, the purely weak neutralino pair production and their combination.
The general features of the Monte Carlo event generation, including its limitations, which lead to the concept of parton showers and consequently jet merging, were discussed and the event generation executed for the scenario at hand. We then scaled the resulting LO cross sections to higher orders using publicly available state-of-the-art precision calculations. Afterwards, we recast four different analyses with the generated events to get the number of expected signal events, which are then compared to the experimentally measured values. We traced the steps taking us from these values to the CL of exclusion for the individual processes and their combination. Also, the procedure of determining the correlation of SRS, finding the most constraining combination of uncorrelated SRS from the different analyses and eventually combining them, is explained. Finally, we gave the results in the form of exclusion plots, which showed that considering associated and strong production together significantly affects the mass limit, while the contribution from the weak production was insignificant. The combination of uncorrelated SRs substantially pushed the exclusion limit towards higher masses, compared to the most sensitive individual analysis.

With this we showed that by combining the data obtained from various already executed BSM searches conducted at the LHC one can still gain further information.

## A. Monte Carlo Method

Monte Carlo generators play a key role in high-energy physics and are employed in tools like MadGraph, Pythia, HERWIG and many more. Using them as black boxes is prone to errors and we therefore, aim to get at least a basic understanding of the procedure. In the following, a general description based on [83, 84] is given.

## A.1. Monte Carlo Integration

First of all, Monte Carlo methods serve as integration methods. Standard numerical integration techniques like the trapezium rule, Simpson's rule and Gaussian quadrature approximate the integrand with a polynomial. Alternatively, one can exploit that the value of an integral can be calculated by the average of the integrand:

$$
\begin{equation*}
I=\int_{t_{1}}^{t_{2}} f(t) \mathrm{d} t=\left(t_{2}-t_{1}\right)\langle f(t)\rangle \tag{95}
\end{equation*}
$$

The average can be estimated by taking $N$ values uniformly distributed in $\left(t_{1}, t_{2}\right)$, such that:

$$
\begin{equation*}
I \approx\left(t_{2}-t_{1}\right) \frac{1}{N} \sum_{i=0}^{N} f\left(t_{i}\right) . \tag{96}
\end{equation*}
$$

Since the order in the sum is of no importance, one can simply use random numbers $\rho_{i}$ within $(0,1)$, i.e.

$$
\begin{equation*}
t_{i}=t_{1}+\left(t_{2}-t_{1}\right) \rho_{i}, \tag{97}
\end{equation*}
$$

which concludes the method of Monte Carlo integration. For future purposes, we define the weight $W_{i}:=\left(t_{2}-t_{1}\right) f\left(t_{i}\right)$ and then describe the Monte Carlo integral, its variance and the standard deviation with

$$
\begin{align*}
& I \approx I_{N}=\frac{1}{N} \sum_{i=0}^{N} W_{i},  \tag{98}\\
& V_{N}=\frac{1}{N} \sum_{i=0}^{N} W_{i}^{2}-\left(\frac{1}{N} \sum_{i=0}^{N} W_{i}\right)^{2} \text { and }  \tag{99}\\
& \sigma_{N}=\sqrt{\frac{V_{N}}{N}} \tag{100}
\end{align*}
$$

respectively, resulting in

$$
\begin{equation*}
I=I_{N} \pm \sigma_{N} . \tag{101}
\end{equation*}
$$

Of course, this is also valid for multidimensional integration, where the random point $t_{i}$ becomes a multidimensional point $\mathbf{t}_{\mathbf{i}}=\left(t_{i_{1}}, t_{i_{2}}, t_{i_{3}}, \ldots\right)$. In fact, Monte Carlo integration reveals its true advantage in high dimensions, because its convergence in any dimension $d$ is $\propto 1 / \sqrt{N}$ which is in contrast to the other techniques (see table 3) which converge more slowly with
increasing dimension. However, in low dimensional integrals, Monte Carlo is slowly converging in

| technique | convergence |
| ---: | :---: |
| trapezium rule | $1 / N^{2 / d}$ |
| Simpson's rule | $1 / N^{4 / d}$ |
| $m$-th order Gaussian quadrature | $1 / N^{(2 m-1) / d}$ |
| Monte Carlo | $1 / \sqrt{N}$ |

Table 3: Given are the convergence rates of the different numerical integration techniques in $d$ dimensions. Table is taken from (84].
comparison. A closer look on the standard deviation $\sigma_{N}=\sqrt{V_{N} / N}$ reveals that the convergence decreases by reducing the variance in addition to simply increasing $N$. This is employed in importance sampling, which uses a transformation to flatten the integrand and hence results in a smaller variance. An example can be found in [83].
In particle physics, one often encounters many particle final states. The high dimensionality of the resulting phase space motivates using Monte Carlo methods.

## A.2. Monte Carlo Event Generator

In the language of Monte Carlo generators an event is understood to mean the multidimensional random point $t_{i}$ in combination with its weight $W_{i}$, which can then be used in experimental analyses, for example. However, carrying around the weight is cumbersome, inefficient and computationally expensive and therefore an unweighting procedure is needed. The hit-or-miss method exploits that the weight of an event is proportional to its occurence. Hence, by accepting an event with probability $f(t) / f_{\max }$ and else rejecting it, using the maximal value of the integrand $f_{\max }$, one accounts for the weight of the event in terms of its occurrence probability and one does not need to carry the explicit weight around. The algorithm steps are:

1. Generate $N_{1}$ random points $t_{i}$ within $\left(t_{1}, t_{2}\right)$ and find the maximal value $f_{\max }$ of the distribution $f(t)$.
2. Generate $N_{2}$ random points $t_{i}$ and keep them with a probability of $f(t) / f_{\max }$, such that one ends up with a set of accepted events.

Note, during the first step, the value of the integral, it's variance and standard deviation can already be computed if needed, following eqs. (98) to 100 .
The cross section of a $2 \rightarrow 2$ process with two colliding quarks in the initial state can often be expressed in an analytically solvable integral. However, quarks are bound in hadrons and such a quark collisions can only be described if they are asymptotically free, i.e. if they carry large momenta. Therefore, one needs to take the whole hadron hadron collision into account to get accurate predictions.


Figure 20: Structural visualisation of a hadron hadron collision. The collision of partons (red blob), originating from hadrons (dark green ovals), creates decaying particles (small red blobs). Hadronisation (light green blobs) takes place after this and also after secondary processes (purple blob) and the unstable hadrons decay. Additionally, photons (yellow) are emitted at any stage. The picture is taken from 85].

## A.3. QCD Factorization

The usual structure of a collision in a hadron collider includes more than just the hard collision of the partons. For the general case, this is visualised in fig. 20. The complexity of this is reduced since one can approximately factorise separate steps. First of all, unweighted events for the hard process (green ovals in fig. 20) are produced following the hit-or-miss method and consequently, the cross section can be calculated. The calculation bases on the assumption that the hadron physics and the short distance parton physics can be factorised like

$$
\begin{equation*}
\sigma=\sum_{a, b} \int_{0}^{1} \mathrm{~d} x_{a} \mathrm{~d} x_{b} f_{a, h_{1}}\left(x_{i}, \mu_{F}^{2}\right) f_{b, h_{2}}\left(x_{b}, \mu_{F}^{2}\right) \hat{\sigma}_{a b} . \tag{102}
\end{equation*}
$$

Here, the PDF for the parton $a$ in the hadron $h$ is denoted by $f_{a, h}, \mu_{F}$ is the relevant factorisation scale and $\hat{\sigma}$ is the partonic cross section (red blob), given by

$$
\begin{equation*}
\hat{\sigma}_{a b}=\int\left[\prod_{i=1}^{N} \frac{\mathrm{~d}^{3} q_{i}}{(2 \pi)^{3} 2 E_{i}}\right] \delta^{4}\left(p_{1}+p_{2}-\sum_{i=1}^{N} q_{i}\right)\left|\mathcal{M}_{p_{1} p_{2} \rightarrow\{q\}}^{a b}\right|^{2} . \tag{103}
\end{equation*}
$$

The other, usually less energetic partons in the hadrons might collide (purple blob) which again be modelled using the hard process techniques.

Heavy resonances like the top quark decay before the parton shower. Of course, all particles carrying color radiate, quarks and gluons radiate gluons and gluons can also split into quark antiquark pairs, which is described by the matching splitting functions $\hat{P}$. Parton shower algorithms take this into account via the Sudakov form factor

$$
\begin{equation*}
\Delta_{b a}\left(t_{0}, t\right)=\exp \left(-\int_{t_{0}}^{t} \frac{\mathrm{~d} t^{\prime}}{t^{\prime}} \int \mathrm{d} z \frac{\alpha_{s}}{2 \pi} \hat{P}_{b a}(z)\right), \tag{104}
\end{equation*}
$$

which encodes the no emission probability, i.e. it yields the probability that parton $a$ is not emitting parton $b$ between the scales $t_{0}$ and $t$. More details regarding the parton shower are also given in section 4.2, here, it is just worth noticing that Monte Carlo methods are used to calculate the Sudakov form factor and create the resulting events.
After the parton shower, a set of partons will be present in the low momentum regime. This nonperturbative regime and thus the hadronisation of partons into hadrons, can only be described by phenomenological models.

## A.4. Monte Carlo Generator in $\mathrm{C}++$

At https://github.com/a-feike/MC_event_generator you can find a Monte Carlo project implemented in $\mathrm{C}++$. The motivation for programming this from scratch are solely to familiarise myself with the Monte Carlo method described above and to improve my programming skills. Therefore, only a very brief summary of the result is given here.
The cross section for muon pair production following electron positron is implemented, so far and the needed integral and exact cross section is taken from [83]. The event output is given in the Les Houches event (LHE) file format [50], for example:

```
<LesHoucheEvents>
<Header>
</Header>
<init>
    11-11 4.500000e+01 4.500000e+01 0 0 0 0 0 0
    1.061654e+03 2.564191 e+00 1.061654e+03 0
</init>
4 4 1 1.060650e+03 1.000000e+03 
    11, 0}
        -11-1 0
        -11 -1 
```



```
</event>
\(11-1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 4.500000 \mathrm{e}+01 \quad 0.000000 \mathrm{e}+0\)
</LesHoucheEvents>
```

The init block first yields the ID's of the incoming beams following the numbering of the Particle Data Group (PDG), here the electron (11) and its antiparticle (-11). Their respective energies in $\mathrm{GeV}(4.500000 \mathrm{e}+014.500000 \mathrm{e}+0)$, information for the $\mathrm{PDF}(0000)$, the weighting strategy ( 0 ) and the number of processes ( 0 ) is given in the same line. Below the cross section, its error and maximum ( $1.061654 \mathrm{e}+032.564191 \mathrm{e}+001.061654 \mathrm{e}+03)$ are given in pb and a process ID (0) is shown.
Each event block then carries number of particles (4), an ID (1), its weight $(1.061654 \mathrm{e}+03)$ the used scale (1.000000e+03) and the couplings $\alpha$ (7.546771e-03) and $\alpha_{S}$ (8.684367e-02). For each

## A Monte Carlo Method

particle we denote the PDGID (11), its status, here it is an incoming particle (-1), the mother particles ( 00 ) and the color ( 00 ), followed by the 4 -momentum $(4.500000 \mathrm{e}+010.000000 \mathrm{e}+00$ $0.000000 \mathrm{e}+004.500000 \mathrm{e}+01)$. Finally the mass $(0.000000 \mathrm{e}+00)$, proper lifetime $(0.000000 \mathrm{e}+00)$ and spin $(0.000000 \mathrm{e}+00)$ are noted.
For the presented case, we used $N=10000$ in the integration and found a cross section of (1060.650 $00 \pm 2.52977$ ) pb which agrees with the analytical value of 1060.94 pb .

## B. MadGraph Commands

In order to generate the process, the following commands are used for the neutralino pair production

```
define squa = ur ur~
generate p p > n1 n1 $squa @1
add process p p > n1 n1 j $squa @2
add process p p > n1 n1 j j $squa @3
output
```

for the associated production

```
define squa = ur ur~
generate p p > n1 ur $squa, ur > u n1 @1
add process p p > n1 ur~ $squa, ur~ > u~ n1 @1
add process p p > n1 ur j $squa, ur > u n1 @2
add process p p > n1 ur~ j $squa, ur~ > u~ n1 @2
add process p p > n1 ur j j $squa, ur > u n1 @3
add process p p > n1 ur~ j j $squa, ur~ > u~ n1 @3
output
```

and for the squark pair production production

```
define squa = ur ur~
generate p p > ur ur~ $squa, ur > u n1, ur~ > u~ n1 @1
add process p p > ur ur~ j $squa, ur > u n1, ur~ > u~ n1 @2
add process p p > ur ur~ j j $squa, ur > u n1, ur~ > u~ n1 @3
output
```

Here, the $\$$-notation denotes that only off-shell quarks are considered in intermediate states. Furthermore, the prompt decay of the squark is executed by implementing it in the process generation. After launching the process with

```
launch
shower = Pythia8
analysis = OFF
madspin = OFF
```

we declare, that only the Pythia plug-in is needed. Then

```
set pdlabel = lhapdf
set lhaid = 27000
set ebeam1 = 6500.0
set ebeam2 = 6500.0
set nevents = 500000
```

B MadGraph Commands

```
set bwcutoff = 35.0
set use_syst = False
```

sets the PDF set and its label, the energy for each beam in GeV , the number of events and the Breit Wigner cutoff in the run_card. Moreover, systematic studies are deactivated to omit scale variations in order to save runtime.

Then the masses are set in the param_card with

```
set mass mneu1 }30
set mass msu4 1000
set mass msd1 30000
set mass msd2 30000
set mass msd3 30000
set mass msd4 30000
set mass msd5 30000
set mass msd6 30000
set mass msu1 }3000
set mass msu2 30000
set mass msu3 30000
set mass msu5 30000
set mass msu6 30000
set mass mneu2 30000
set mass mneu3 30000
set mass mneu4 30000
set mass mch1 }3000
set mass mch2 30000
set mass mgo 30000
set mass msl1 30000
set mass msl2 30000
set mass msl3 30000
set mass msl4 30000
set mass msl5 30000
set mass msl6 30000
set mass msn1 30000
set mass msn2 30000
set mass msn3 30000
```

where the given configurations create the mass point at $m_{\tilde{q}}=1000 \mathrm{GeV}$ and $m_{\tilde{\chi}_{1}^{0}}=300 \mathrm{GeV}$. All other particles are decoupled by mass which is set to 30000 GeV . Additionally, the param_card includes the neutralino mixing matrix which is given as

```
set rnn1x1 1
set rnn1x2 0
```

```
set rnn1x3 0
set rnn1x4 0
set rnn2x1 0
set rnn2x2 1
set rnn2x3 0
set rnn2x4 0
set rnn3x1 0
set rnn3x2 0
set rnn3x3 0
set rnn3x4 0
set rnn4x1 0
set rnn4x2 0
set rnn4x3 0
set rnn4x4 0
```

and therefore being diagonal. The merging parameters

```
set ptlund = 250.0
set ickkw = 0
set xqcut = 0
set ktdurham = -1
set dparameter = 0.4
```

given in the run_card state that the CKKW-L algorithm with a $p_{T}$ scale is used while the MLM algorithm is not used. Combining the defined merging scale (see eq. (62)) with the used mass point, one can deduce that the given commands are chosen for the squark pair production. Finally, the pythia8_card is modified with

```
set Merging:TMS = -1
set Merging:nJetMax 2
set Merging:mayRemoveDecayProducts = on
set Merging:doPTLundMerging = on
set Merging:Process = pp>ur,ur~
```

which yield the number of jets, the process and some minor adjustments. For more information, please refer to the Pythia8 website [86].

## C. Statistical Introduction

For a correct interpretation of data from experiments in general, but especially in high-energy physics, knowledge of the underlying statistics is crucial. This field is often not covered in general particle physics lectures. Therefore, the following section briefly introduces the terms and concepts needed to understand this thesis and support this with examples.
For further reading and to get a deeper insight, we would like to recommend the book from G. Cowan 87 from which, if not explicitly stated otherwise, the information of this section is taken from. For more compressed and summarised information, there are also Cern lectures of G. Cowan available at 88.

## C.1. Basic Introduction

In the following section, the basic concepts of statistics such as conditional probabilities, Bayes' theorem and probability density functions are introduced.

## C.1.1. Primary Principles

In this framework, we consider a set of elements denoted as $S$ and associate any subset $A$ with a real number denoted as $P(A)$, referred to as the probability. It is defined using these three axioms:

1. The probability is larger than zero for each subset, i.e. $P(A)>0 \forall A \subseteq S$.
2. The probability of any two disjoint subsets is the sum of their individual probabilities, i.e. $\forall A, B \subseteq S$ with $A \cap B=\emptyset \Longrightarrow P(A \cup B)=P(A)+P(B)$.
3. The probability of the whole set is one, i.e $P(S)=1$.

Properties like $P(\bar{A})=1-P(A), P(A) \leq P(B)$ if $A \subseteq B$, etc. can be derived from these axioms only. They are also met by the conditional probability $P(A \mid B)$ which is the probability of $A$ given $B$ defined as

$$
\begin{equation*}
P(A \mid B)=\frac{P(A \cap B)}{P(B)} \tag{105}
\end{equation*}
$$

It is worth noting that the conventional probability $P(A)$ can be expressed as the probability of $A$ given the whole set $S, P(A)=P(A \mid S)$. Since $P(A \cap B)=P(B \cap A)$ one can combine $P(A \mid B)$ from eq. 105 with $P(B \mid A)$ to find

$$
\begin{align*}
P(A \cap B) & =P(A \mid B) P(B)=P(B \mid A) P(A)  \tag{106}\\
\Longrightarrow P(A \mid B) & =\frac{P(B \mid A) P(A)}{P(B)} \tag{107}
\end{align*}
$$

which is known as Bayes' theorem. This can be further transformed using the law of total probability. $S$ can be broken down to disjoint subsets $A_{i}$, such that $S=\cup_{i} A_{i}$. For any probability, one can then use:

$$
\begin{equation*}
P(B)=\sum_{i} P\left(B \cup A_{i}\right)=\sum_{i} P\left(B \mid A_{i}\right) P\left(A_{i}\right) \tag{108}
\end{equation*}
$$

Here, we used the definition of the conditional probability eq. 105 to transform the equation. This can now be inserted into Bayes' theorem eq. (107) and one gets:

$$
\begin{equation*}
\Longrightarrow P(A \mid B)=\frac{P(B \mid A) P(A)}{\sum_{i} P\left(B \mid A_{i}\right) P\left(A_{i}\right)} . \tag{109}
\end{equation*}
$$

This result can be used in a simple example calculation. Suppose there is a disease and there are tests to figure out if one is infected. We want to assume a prior probability of $1 \%$, so $1 \%$ of the population is sick and the rest is healthy, i.e. $P(s)=0.01$ and $P(h)=0.99$. Of course the test is not $100 \%$ accurate such that if a person is infected, it gives a positive test result with $97 \%$, i.e. $P(+\mid s)=0.97$ and $P(-\mid s)=0.03$. Also, given that a person is healthy, it falsely yields a positive test result with a probability of $4 \%$, i.e. $P(+\mid h)=0.04$ and $P(-\mid h)=0.96$. Now assume that somebody got a positive test result, how likely is it, that this person is actually sick? We just use eq. (109):

$$
\begin{equation*}
P(s \mid+)=\frac{P(+\mid s) P(s)}{P(+\mid s) P(s)+P(+\mid h) P(h)}=\frac{0.97 \cdot 0.01}{0.97 \cdot 0.01+0.04 \cdot 0.99} \approx 20 \% . \tag{110}
\end{equation*}
$$

So although one gets a positive test result, the posterior probability, so the probability to actually be infected, is only $20 \%$.

## C.1.2. Relative Frequency and Bayesian Probability

In high-energy particle physics, there are two dominant interpretations of probabilities, relative frequency and Bayesian probability (or subjective probability).
In relative frequency, one regards any subset $A$ as an outcome, so the occurrence of a specific event, in a repeatable experiment. Its probability is defined by the fraction of the number of times this event occurs if the experiment is repeated a sufficient amount of times:

$$
\begin{equation*}
P(A)=\lim _{n \rightarrow \infty} \frac{\text { number of outcome A }}{n} . \tag{111}
\end{equation*}
$$

In Bayesian statistics on the other hand, a subset $A$ is regarded as a hypothesis which can be true or false. The probability is then defined by how much one believes that this hypothesis is true, so:

$$
\begin{equation*}
P(A)=\text { degree of belief that } A \text { is true. } \tag{112}
\end{equation*}
$$

Consider doing an experiment to evaluate if a theory is correct. Thus, the hypothesis is given by the theory. The probability that the theory is true under the assumption of the experimental data

$$
\begin{equation*}
P(\text { theory } \mid \text { data }) \propto P(\text { data } \mid \text { theory }) P(\text { theory }) \tag{113}
\end{equation*}
$$

represents our degree of belief in the theory after the experiment, also called likelihood. Note that Bayesian statistics don't provide a prior probability for the theory but shows how the degree of belief changes if one has some data.

## C.1.3. Probability Distribution Function

If the outcome $x$ of an experiment is not discrete, like the number of particles produced in a collision, but continuous, like the particles absolute momentum, one defines the probability to find $x$ in the interval $[x, x+d x]$ as $f(x) d x$, where $f(x)$ is referred to as the probability density function pdf). The pdf is normalised, such that the integral over the complete set yields one. The cumulative distribution $F(x)$ is defined as the probability of finding a value which is smaller or equal to $x$ :

$$
\begin{equation*}
F(x)=\int_{-\infty}^{x} f\left(x^{\prime}\right) d x^{\prime} \tag{114}
\end{equation*}
$$

Of course, the pdf concept is straightforwardly expandable to multiple variables, like measuring not the absolute value of the particles momentum but the momentum in $x$ - and $y$-direction. $f(x, y) d x d y$ describes the probability of finding $x$ in the interval $[x, x+d x]$ and $y$ in the interval $[y, y+d y]$. If one is interested in the pdf of $x$ for any $y$ or the other way around one gets

$$
\begin{equation*}
f_{x}(x)=\int_{-\infty}^{\infty} f(x, y) d y \text { and } f_{y}(y)=\int_{-\infty}^{\infty} f(x, y) d x \tag{115}
\end{equation*}
$$

respectively, called marginal pdf. The conditional pdf is then defined analogue to the conditional probability eq. (105) as

$$
\begin{equation*}
g(x \mid y)=\frac{f(x, y)}{f_{x}(x)} \text { and } \quad h(y \mid x)=\frac{f(x, y)}{f_{y}(y)} \quad \Longrightarrow g(x \mid y)=\frac{g(x \mid y) f_{x}(x)}{f_{y}(y)} . \tag{116}
\end{equation*}
$$

and one can express the marginal pdf as

$$
\begin{equation*}
f_{x}(x)=\int_{-\infty}^{\infty} g(x \mid y) f_{y}(y) d y \text { and } f_{y}(y)=\int_{-\infty}^{\infty} h(y \mid x) f_{x}(x) d x \tag{117}
\end{equation*}
$$

representing the law of total probability.

## C.2. Hypothesis Tests

In this section, we will introduce the concept of testing a hypothesis. One primary interest for this thesis is to distinguish if a given event is due to a signal or just background.

## C.2.1. General

One generally aims to test if the prediction, the so called null hypothesis $H_{0}$, is valid under consideration of the observed data and one compares this with alternative hypothesis $H_{1}$. Consider the data $\mathbf{x}=\left(x_{1}, \ldots, x_{n}\right)$ which could be a $n$-dimensional data point or data from $n$ repeated experiments as given. Furthermore, we define a function $\mathbf{t}(\mathbf{x})$ called test statistic which, for now, can be considered as a summary of the data to quantify the agreement between hypothesis and data. For simplicity, we will consider a scalar test statistic $\mathbf{t}(\mathbf{x})=t(x)$ if not stated
otherwise. The probability for the measured data given the null or the alternative hypothesis is described by the pdf $g\left(t \mid H_{0}\right)$ or $g\left(t \mid H_{1}\right)$ respectively.
We want to accept the null hypothesis $H_{0}$ if the test statistic value is below a certain threshold $t_{\text {cut }}$ and otherwise reject it. This is usually done by choosing a significance value $\alpha$ defined as:

$$
\begin{equation*}
\alpha=\int_{t_{\mathrm{cut}}}^{\infty} g\left(t \mid H_{0}\right) d t \tag{118}
\end{equation*}
$$

Thus, if the null hypothesis $H_{0}$ is true, we reject it with a probability of $\alpha$, which is therefore the error of the first kind. Consequently, there is also an error of the second kind given as the probability that we accept $H_{0}$ although $H_{1}$ is true:

$$
\begin{equation*}
\beta=\int_{-\infty}^{t_{\mathrm{cut}}} g\left(t \mid H_{1}\right) d t \tag{119}
\end{equation*}
$$

We call $1-\beta$ the power of the test. The above discussed concepts are visualised on the left-hand side and in the middle of fig. 21 .


Figure 21: Shown are exemplary conditional probabilities for a scalar test statistic given a null or an alternative hypothesis. On the left-hand side $\alpha$, in the middle $\beta$ and on the right-hand side the $p$-value are specifically shown.

## C.2.2. $p$-Value and Confidence Interval

If one is interested in how compatible measured data is with the null hypothesis, one usually gives the $p$-value. The $p$-value is defined as the probability to get data as extreme or extremer compared to the measured one. So in the example in fig. 21 we can see that the extreme end of the pdf is towards higher $t$. If we measure some data yielding $t_{\text {data }}=50$, the probability of getting a value being 50 or higher given the null hypothesis is the $p$-value (shown on the right-hand side of fig. 21. In contrast to the significance $\alpha$, the $p$-value is variable.
The confidence level (CL) is closely related to the significance value $\alpha$. Consider specifying an $\alpha$ and then taking data. As discussed before, a hypothesised value is rejected if it is in the critical region. If one inverts this, the confidence interval gives all the hypothesised values that
are not rejected. The $\mathbf{C L}$ is given as $1-\alpha$.
A typical physical example would be to decide whether some counts in a detector originate from background processes only (null hypothesis) or if there is actually a new physics signal (alternative hypothesis). We want to consider a total number of events given by the sum of the signal and background events $n=n_{s}+n_{b}$ with mean $\nu=\nu_{s}+\nu_{b}$. Following the Poisson distribution, the probability of observing $n$ events is

$$
\begin{equation*}
f\left(n ; \nu_{s}, \nu_{b}\right)=\frac{\left(\nu_{s}+\nu_{b}\right)^{n}}{n!} e^{-\left(\nu_{s}+\nu_{b}\right)} \tag{120}
\end{equation*}
$$

Now one observes $n_{\text {obs }}$ events and the question is, if this is compatible with there being no signal and just background. Since the extreme end of the distribution is here for a high number of events, the $p$-value is calculated as

$$
\begin{equation*}
P\left(n \geq n_{\mathrm{obs}}\right)=\sum_{n_{\mathrm{obs}}}^{\infty} f\left(n_{\mathrm{obs}} ; \nu_{s}=0, \nu_{b}\right)=1-\sum_{0}^{n_{\mathrm{obs}}-1} f\left(n_{\mathrm{obs}} ; \nu_{s}=0, \nu_{b}\right) \tag{121}
\end{equation*}
$$

Consider an expected background of $\nu_{b}=10$ events and the number of observed events $n_{\text {obs }}=$ 22 , the corresponding $p$-value is approximately $0.16 \%$. Thus, it is rather unlikely to observe these events if there was only background. We could have also applied a significance value of $\alpha=5 \%$ or equivalently a CL of $95 \%$. Then all observed events with $n_{\mathrm{obs}} \leq 17$ would be accepted by the null hypothesis and, therefore, be in the confidence interval. Following the Bayesian interpretation we can also formulate our degree of belief to be $95 \%$ that the events are actually background if they are in the confidence interval or the events are due to a signal if they exceed the highest number of events in the confidence interval.
Note that in this example, we only used the background only hypothesis to decide whether or not there is a discovery. Usually there are other factors to consider, like, for example the agreement with the alternative hypothesis.

## C.2.3. Neyman-Pearson Lemma

From the definition of the significance and power of a test, it is rather obvious that one aims to choose the perfect $\mathbf{t}_{\text {cut }}$ such that the former minimises and the latter maximises. If the test statistics get multidimensional, this is not straightforward. The Neyman-Pearson lemma states that one gets the highest power w.r.t. to the alternative hypothesis, for

$$
\begin{equation*}
\frac{g\left(\mathbf{t} \mid H_{1}\right)}{g\left(\mathbf{t} \mid H_{0}\right)} \geq c \tag{122}
\end{equation*}
$$

in the critical region. This ratio is called likelihood ratio. The constant $c$ yields the desired test size. Note that this is equivalent to choosing a scalar test statistic given as the likelihood ratio. However, this lemma is often impractical since one often doesn't know the conditional pdf.

## C.3. Likelihood

In the prior sections, we already mentioned the likelihood in the discussion of eq. 122) and the likelihood ratio in eq. 122). In this section, we revisit the basic concept of the likelihood or likelihood function and introduce the method of maximum likelihood.

## C.3.1. Basics

In a best case scenario a given hypothesis $H$ yields all values to define the pdf $P(\mathbf{x} \mid H)$. However, more often one deals with a composite hypothesis, meaning the kind of pdf is known but there are unknown parameters $\boldsymbol{\theta}$ denoted by $P(\mathbf{x} \mid \boldsymbol{\theta})$. One then defines the likelihood function as

$$
\begin{equation*}
P(\mathbf{x} \mid \boldsymbol{\theta})=\mathcal{L}(\boldsymbol{\theta})=\mathcal{L}(\boldsymbol{\theta} \mid \mathbf{x}) . \tag{123}
\end{equation*}
$$

It is important to note that unlike to the original pdf the data $\mathbf{x}$ is considered being given and therefore fixed and the unknown parameters are variable in the likelihood function. The unknown parameter could be any pdf characterising constant.
For example, consider having a radioactively decaying probe. The probability for $x$ decays per second is given in general by the Poisson distribution

$$
\begin{equation*}
P_{\lambda}(x)=\frac{\lambda^{x}}{x!} e^{-\lambda} \tag{124}
\end{equation*}
$$

with the decay constant $\lambda$. If one does know this constant, one can calculate the probability for any number of decays per second as shown left in fig. 22 . If one does not know the decay constant because the element of the probe is unknown but one counts the decays per second, one has data $x$ but an unknown parameter $\lambda$. In this case, we can plot the likelihood, so the probability of counting $x$ decays with any decay constant, shown right in fig. 22 for $x=12$.
For $n$ repeated measurements of a random variable, the data is independent and the joint pdf is given by the product of the individual ones. This reasoning also implies that the likelihood functions also combine multiplicatively:

$$
\begin{align*}
& P\left(x_{1}, \ldots, x_{n} \mid \theta\right)=\prod_{i=1}^{n} P\left(x_{i} \mid \theta\right)  \tag{125}\\
& \Longrightarrow \mathcal{L}\left(\theta \mid x_{1}, \ldots, x_{n}\right)=\prod_{i=1}^{n} \mathcal{L}\left(\theta \mid x_{i}\right) . \tag{126}
\end{align*}
$$

## C.3.2. Maximum Likelihood

By definition as unknown parameters, the true value $\boldsymbol{\theta}$ can usually not be determined. However, one can find estimators $\hat{\boldsymbol{\theta}}$ which are called consistent if they converge to the true values for an increasing number of observations, so, for increasing data. One way to find estimators based on a finite data sample is the method of maximum likelihood.
The idea of this technique is that one expects to get a high probability for parameter values


Figure 22: We show exemplary the difference between the pdf with variable $x$ and fixed $\lambda$ on the left and the likelihood function with variable $\lambda$ and fixed $x$ on the right.
close to the true values. Hence, to find estimators, one maximises the likelihood with

$$
\begin{equation*}
\frac{\partial \mathcal{L}}{\partial \theta_{i}}=0 \tag{127}
\end{equation*}
$$

assuming the likelihood function to be differentiable. If there is more than one maximum, the global one, so, the highest one is chosen.
Usually, the logarithm of the likelihood function eq. 126, the log-likelihood function is used for convenience. This simplifies $\mathcal{L}$ by converting products into sums and exponents into factors and since the logarithm is monotonic, the log-likelihood function maximises for the same values as the likelihood function.
Revisiting the radioactive decay example with a Poisson distribution (see eq. (124)) from the previous section and assuming that in $n$ observations one got the data $\mathbf{x}=\left(\begin{array}{lll}x_{1} & x_{2} & \ldots\end{array} x_{n}\right)$ yields the log-likelihood function

$$
\begin{equation*}
\log \mathcal{L}=\log \left(\prod_{i=1}^{n} \frac{\lambda_{i}^{x}}{x_{i}!} e^{-\lambda}\right)=\sum_{i=1}^{n}\left(x_{i} \log (\lambda)-\log \left(x_{i}!\right)-\lambda\right) \tag{128}
\end{equation*}
$$

and hence, the maximum is calculated as:

$$
\begin{equation*}
\frac{\partial}{\partial \lambda} \log \mathcal{L}=0 \Longrightarrow \hat{\lambda}=\frac{1}{n} \sum_{i=1}^{n} x_{i} \tag{129}
\end{equation*}
$$

It is no surprise to find the estimator $\hat{\lambda}$ being the arithmetic mean. The concept is visualised in fig. 23 on the left. Here, we generated $n=50$ random variables according to a Poisson distribution with $\lambda=19.4$. With this data and eq. 129 the maximum likelihood estimator is calculated as $\hat{\lambda}=19.52$. The distribution with the true parameter $\lambda$ (solid) and the distribution with the estimator $\hat{\lambda}$ (dashed) are nearly on top of each other.
We can treat the Gauss distribution

$$
\begin{equation*}
f(x)=\frac{1}{\sigma \sqrt{2 \pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}} \tag{130}
\end{equation*}
$$



Figure 23: On the left two Poisson distributions and on the right two Gauss distributions are shown. In both cases the blue (solid) line represents the distribution with the true parameters and the orange (dashed) line shows the distribution with the estimators calculated using the maximum likelihood approach. The random data is visualised as red dashes in the lower area of the plot.
in the same manner. For $n$ repeated experiments, we get the log-likelihood

$$
\begin{equation*}
\log \mathcal{L}=\sum_{i=1}^{n}\left(-\frac{1}{2} \log (2 \pi)-\frac{1}{2} \log \sigma^{2}-\frac{1}{2}\left(\frac{x_{i}-\mu}{\sigma}\right)^{2}\right) \tag{131}
\end{equation*}
$$

and by taking the derivatives, we get the estimators

$$
\begin{align*}
& \frac{\partial}{\partial \mu} \log \mathcal{L}=0 \Longrightarrow \hat{\mu}=\frac{1}{n} \sum_{i=1}^{n} x_{i} \quad \text { and }  \tag{132}\\
& \frac{\partial}{\partial \sigma^{2}} \log \mathcal{L}=0 \Longrightarrow \widehat{\sigma^{2}}=\frac{1}{n} \sum_{i=1}^{n}\left(x_{i}-\hat{\mu}\right)^{2} \tag{133}
\end{align*}
$$

This is visualised as an example in fig. 23 on the right. Here, $n=100$ random variables according to a Gauss distribution with $\mu=2$ and $\sigma=0.6$ were generated. The estimator calculation with eq. (132) and eq. (133) yields $\hat{\mu} \approx 2.029$ and $\hat{\sigma} \approx 0.582$. Naturally, there is still a statistical error on the estimators causing the slight discrepancy. Nevertheless, this example motivates the effectiveness of the maximum likelihood approach.
Note that depending on the likelihood function, it is not always possible to find the maximum likelihood estimators with a straightforward analytic approach. In these cases, the estimators are calculated numerically.
D. Tables

| $m_{\tilde{\chi}_{1}^{0}} / m_{\tilde{\mathrm{q}}}$ | $\begin{aligned} & \mathrm{qq} \\ & \mathrm{AC} \end{aligned}$ | AE | CS | CE | com. | $\frac{\mathrm{qq}}{\mathrm{Aq}} \mathrm{C}^{\mathrm{r}}$ | AE | CS | CE | com. | $\frac{\mathrm{qq}}{\mathrm{AC}}{ }^{-1}$ | $\begin{aligned} & \_\mathrm{nn} \\ & \mathrm{AE} \end{aligned}$ | CS | CE | com. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 200/900 | 0.88 | 0.68 | 0.92 | 0.61 | 0.99 | 0.95 | 0.86 | 0.99 | 0.79 | 1.0 | 0.95 | 0.86 | 0.99 | 0.79 | 1.0 |
| 200/950 | 0.85 | 0.67 | 0.88 | 0.57 | 0.98 | 0.93 | 0.83 | 0.93 | 0.74 | 0.99 | 0.93 | 0.84 | 0.94 | 0.74 | 0.99 |
| 200/970 | 0.8 | 0.58 | 0.81 | 0.53 | 0.97 | 0.93 | 0.81 | 0.92 | 0.93 | 1.0 | 0.93 | 0.81 | 0.92 | 0.93 | 1.0 |
| 200/1000 | 0.78 | 0.54 | 0.78 | 0.47 | 0.97 | 0.89 | 0.72 | 0.86 | 0.59 | 0.99 | 0.89 | 0.73 | 0.86 | 0.59 | 0.99 |
| 200/1050 | 0.68 | 0.48 | 0.68 | 0.35 | 0.87 | 0.8 | 0.68 | 0.81 | 0.55 | 0.96 | 0.8 | 0.69 | 0.81 | 0.55 | 0.96 |
| 200/1200 | 0.53 | 0.27 | 0.59 | 0.24 | 0.73 | 0.67 | 0.45 | 0.71 | 0.39 | 0.91 | 0.68 | 0.45 | 0.72 | 0.4 | 0.91 |
| 200/1300 | 0.46 | 0.26 | 0.5 | 0.25 | 0.66 | 0.59 | 0.42 | 0.66 | 0.43 | 0.84 | 0.59 | 0.42 | 0.66 | 0.43 | 0.85 |
| 250/750 | 0.9 | 0.85 | 0.99 | 0.8 | 1.0 | 0.94 | 0.95 | 1.0 | 0.91 | 1.0 | 0.94 | 0.95 | 1.0 | 0.91 | 1.0 |
| 250/850 | 0.9 | 0.74 | 0.98 | 0.79 | 1.0 | 0.95 | 0.9 | 0.99 | 0.9 | 1.0 | 0.95 | 0.9 | 0.99 | 0.9 | 1.0 |
| 250/940 | 0.82 | 0.56 | 0.96 | 0.57 | 0.99 | 0.91 | 0.77 | 0.96 | 0.73 | 1.0 | 0.91 | 0.78 | 0.96 | 0.73 | 1.0 |
| 250/950 | 0.79 | 0.58 | 0.75 | 0.49 | 0.95 | 0.88 | 0.75 | 0.88 | 0.66 | 0.98 | 0.88 | 0.75 | 0.88 | 0.67 | 0.98 |
| 250/1000 | 0.76 | 0.51 | 0.75 | 0.38 | 0.93 | 0.86 | 0.69 | 0.83 | 0.55 | 0.99 | 0.86 | 0.7 | 0.83 | 0.55 | 0.99 |
| 250/1050 | 0.68 | 0.48 | 0.82 | 0.5 | 0.92 | 0.79 | 0.67 | 0.83 | 0.61 | 0.96 | 0.79 | 0.67 | 0.83 | 0.61 | 0.96 |
| 250/1080 | 0.62 | 0.42 | 0.65 | 0.33 | 0.87 | 0.78 | 0.6 | 0.82 | 0.46 | 0.95 | 0.78 | 0.6 | 0.82 | 0.46 | 0.96 |
| 250/1100 | 0.72 | 0.39 | 0.62 | 0.3 | 0.87 | 0.75 | 0.55 | 0.74 | 0.45 | 0.92 | 0.75 | 0.55 | 0.75 | 0.45 | 0.92 |
| 300/750 | 0.84 | 0.79 | 0.99 | 0.73 | 1.0 | 0.93 | 0.9 | 1.0 | 0.86 | 1.0 | 0.93 | 0.9 | 1.0 | 0.87 | 1.0 |
| 300/850 | 0.84 | 0.68 | 0.94 | 0.7 | 1.0 | 0.87 | 0.83 | 0.98 | 0.81 | 1.0 | 0.87 | 0.83 | 0.98 | 0.82 | 1.0 |
| 300/900 | 0.83 | 0.61 | 0.96 | 0.72 | 0.99 | 0.91 | 0.78 | 0.99 | 0.8 | 1.0 | 0.91 | 0.78 | 0.99 | 0.8 | 1.0 |
| 300/1000 | 0.68 | 0.48 | 0.77 | 0.41 | 0.9 | 0.78 | 0.66 | 0.84 | 0.56 | 0.95 | 0.78 | 0.66 | 0.84 | 0.57 | 0.96 |
| 300/1200 | 0.46 | 0.27 | 0.53 | 0.25 | 0.66 | 0.58 | 0.44 | 0.65 | 0.37 | 0.79 | 0.58 | 0.44 | 0.65 | 0.37 | 0.79 |
| 350/650 | 0.87 | 0.68 | 1.0 | 0.67 | 1.0 | 0.95 | 0.81 | 1.0 | 0.77 | 1.0 | 0.95 | 0.81 | 1.0 | 0.77 | 1.0 |
| 350/700 | 0.75 | 0.7 | 0.99 | 0.7 | 1.0 | 0.86 | 0.84 | 1.0 | 0.82 | 1.0 | 0.86 | 0.84 | 1.0 | 0.83 | 1.0 |
| 350/800 | 0.77 | 0.59 | 0.93 | 0.64 | 0.99 | 0.84 | 0.75 | 1.0 | 0.78 | 1.0 | 0.84 | 0.75 | 1.0 | 0.78 | 1.0 |
| 350/850 | 0.66 | 0.58 | 0.89 | 0.56 | 0.95 | 0.78 | 0.73 | 0.95 | 0.67 | 0.99 | 0.78 | 0.73 | 0.95 | 0.68 | 0.99 |
| 350/900 | 0.69 | 0.49 | 0.82 | 0.48 | 0.93 | 0.81 | 0.69 | 0.92 | 0.63 | 0.99 | 0.81 | 0.69 | 0.92 | 0.63 | 0.99 |
| 350/1000 | 0.64 | 0.4 | 0.64 | 0.45 | 0.89 | 0.74 | 0.56 | 0.73 | 0.53 | 0.94 | 0.74 | 0.57 | 0.73 | 0.53 | 0.94 |
| 350/1100 | 0.5 | 0.31 | 0.6 | 0.45 | 0.82 | 0.59 | 0.42 | 0.65 | 0.51 | 0.84 | 0.6 | 0.43 | 0.65 | 0.51 | 0.84 |
| 400/600 | 0.75 | 0.45 | 1.0 | 0.64 | 1.0 | 0.87 | 0.57 | 1.0 | 0.83 | 1.0 | 0.87 | 0.57 | 1.0 | 0.83 | 1.0 |
| 400/750 | 0.83 | 0.5 | 0.97 | 0.68 | 0.99 | 0.87 | 0.67 | 0.99 | 0.74 | 1.0 | 0.87 | 0.68 | 0.99 | 0.74 | 1.0 |
| 400/800 | 0.61 | 0.49 | 0.9 | 0.6 | 0.97 | 0.71 | 0.66 | 0.95 | 0.69 | 0.99 | 0.71 | 0.66 | 0.95 | 0.69 | 0.99 |
| 400/850 | 0.59 | 0.49 | 0.85 | 0.45 | 0.92 | 0.68 | 0.63 | 0.9 | 0.57 | 0.96 | 0.68 | 0.63 | 0.9 | 0.58 | 0.96 |
| 400/870 | 0.52 | 0.48 | 0.77 | 0.42 | 0.92 | 0.65 | 0.64 | 0.87 | 0.59 | 0.94 | 0.65 | 0.64 | 0.87 | 0.59 | 0.94 |
| 400/920 | 0.59 | 0.41 | 0.8 | 0.41 | 0.93 | 0.72 | 0.57 | 0.89 | 0.58 | 0.95 | 0.72 | 0.57 | 0.89 | 0.59 | 0.95 |
| 400/950 | 0.59 | 0.4 | 0.81 | 0.5 | 0.89 | 0.71 | 0.56 | 0.82 | 0.59 | 0.95 | 0.71 | 0.56 | 0.82 | 0.59 | 0.95 |
| 400/1000 | 0.54 | 0.33 | 0.65 | 0.46 | 0.77 | 0.62 | 0.48 | 0.72 | 0.5 | 0.9 | 0.62 | 0.48 | 0.72 | 0.5 | 0.9 |
| 400/1200 | 0.38 | 0.27 | 0.42 | 0.26 | 0.57 | 0.52 | 0.38 | 0.56 | 0.35 | 0.76 | 0.52 | 0.38 | 0.56 | 0.35 | 0.77 |
| 450/550 | 0.17 | 0.29 | 1.0 | 0.52 | 1.0 | 0.17 | 0.51 | 1.0 | 0.87 | 1.0 | 0.17 | 0.51 | 1.0 | 0.87 | 1.0 |
| 450/600 | 0.17 | 0.27 | 1.0 | 0.49 | 1.0 | 0.17 | 0.39 | 1.0 | 0.55 | 1.0 | 0.17 | 0.4 | 1.0 | 0.55 | 1.0 |
| 450/650 | 0.6 | 0.25 | 1.0 | 0.42 | 1.0 | 0.71 | 0.36 | 1.0 | 0.49 | 1.0 | 0.71 | 0.36 | 1.0 | 0.5 | 1.0 |
| 450/820 | 0.17 | 0.38 | 0.8 | 0.42 | 0.89 | 0.17 | 0.51 | 0.89 | 0.5 | 0.96 | 0.17 | 0.52 | 0.89 | 0.5 | 0.96 |
| 450/850 | 0.17 | 0.37 | 0.84 | 0.42 | 0.9 | 0.17 | 0.5 | 0.85 | 0.49 | 0.93 | 0.17 | 0.5 | 0.85 | 0.49 | 0.93 |
| 450/950 | 0.17 | 0.33 | 0.72 | 0.31 | 0.87 | 0.17 | 0.45 | 0.72 | 0.42 | 0.95 | 0.17 | 0.46 | 0.72 | 0.42 | 0.95 |
| 500/520 | 0.79 | 0.69 | 1.0 | 0.94 | 1.0 | 0.84 | 0.77 | 1.0 | 0.95 | 1.0 | 0.84 | 0.77 | 1.0 | 0.95 | 1.0 |
| 500/600 | 0.17 | 0.3 | 0.99 | 0.5 | 1.0 | 0.17 | 0.38 | 0.99 | 0.89 | 1.0 | 0.17 | 0.39 | 0.99 | 0.89 | 1.0 |
| 500/650 | 0.17 | 0.17 | 0.79 | 0.61 | 0.92 | 0.17 | 0.26 | 0.89 | 0.64 | 0.98 | 0.17 | 0.26 | 0.9 | 0.64 | 0.98 |
| 500/700 | 0.17 | 0.22 | 0.84 | 0.22 | 0.93 | 0.17 | 0.3 | 0.85 | 0.32 | 0.94 | 0.17 | 0.3 | 0.85 | 0.32 | 0.94 |
| 500/750 | 0.17 | 0.23 | 0.86 | 0.35 | 0.6 | 0.17 | 0.3 | 0.87 | 0.41 | 0.67 | 0.17 | 0.3 | 0.87 | 0.41 | 0.67 |
| 500/800 | 0.17 | 0.24 | 0.74 | 0.26 | 0.82 | 0.17 | 0.35 | 0.84 | 0.42 | 0.93 | 0.17 | 0.35 | 0.84 | 0.42 | 0.93 |
| 500/850 | 0.48 | 0.28 | 0.68 | 0.29 | 0.79 | 0.52 | 0.38 | 0.75 | 0.36 | 0.9 | 0.52 | 0.38 | 0.75 | 0.36 | 0.9 |
| 500/950 | 0.39 | 0.25 | 0.57 | 0.25 | 0.72 | 0.44 | 0.35 | 0.66 | 0.33 | 0.79 | 0.44 | 0.36 | 0.66 | 0.33 | 0.79 |
| 550/700 | 0.41 | 0.14 | 0.64 | 0.33 | 0.72 | 0.46 | 0.23 | 0.71 | 0.37 | 0.87 | 0.46 | 0.23 | 0.71 | 0.37 | 0.87 |
| 550/750 | 0.41 | 0.11 | 0.54 | 0.3 | 0.69 | 0.47 | 0.18 | 0.61 | 0.33 | 0.75 | 0.48 | 0.18 | 0.61 | 0.33 | 0.75 |
| 550/800 | 0.5 | 0.17 | 0.72 | 0.19 | 0.81 | 0.53 | 0.24 | 0.78 | 0.29 | 0.89 | 0.53 | 0.24 | 0.78 | 0.29 | 0.89 |
| 550/850 | 0.43 | 0.17 | 0.92 | 0.22 | 0.94 | 0.47 | 0.25 | 0.92 | 0.26 | 0.95 | 0.47 | 0.25 | 0.92 | 0.26 | 0.95 |
| 550/900 | 0.31 | 0.2 | 0.56 | 0.18 | 0.65 | 0.34 | 0.28 | 0.59 | 0.24 | 0.76 | 0.34 | 0.28 | 0.59 | 0.24 | 0.76 |
| 600/650 | 0.56 | 0.2 | 0.89 | 0.32 | 0.94 | 0.6 | 0.28 | 0.89 | 0.36 | 0.93 | 0.6 | 0.28 | 0.89 | 0.36 | 0.93 |
| 600/750 | 0.37 | 0.09 | 0.48 | 0.15 | 0.6 | 0.4 | 0.16 | 0.56 | 0.22 | 0.72 | 0.4 | 0.16 | 0.56 | 0.22 | 0.72 |
| 600/850 | 0.29 | 0.11 | 0.5 | 0.24 | 0.56 | 0.35 | 0.17 | 0.55 | 0.27 | 0.7 | 0.35 | 0.17 | 0.55 | 0.27 | 0.7 |
| 600/1000 | 0.31 | 0.17 | 0.31 | 0.19 | 0.48 | 0.35 | 0.22 | 0.38 | 0.23 | 0.53 | 0.35 | 0.22 | 0.38 | 0.23 | 0.53 |
| 600/1200 | 0.2 | 0.12 | 0.32 | 0.13 | 0.41 | 0.25 | 0.18 | 0.32 | 0.18 | 0.45 | 0.25 | 0.18 | 0.32 | 0.18 | 0.45 |
| 625/750 | 0.35 | 0.06 | 0.55 | 0.14 | 0.62 | 0.39 | 0.14 | 0.6 | 0.18 | 0.7 | 0.39 | 0.14 | 0.6 | 0.18 | 0.7 |
| 650/800 | 0.29 | 0.07 | 0.36 | 0.16 | 0.46 | 0.34 | 0.12 | 0.42 | 0.19 | 0.61 | 0.34 | 0.12 | 0.42 | 0.19 | 0.61 |
| 650/850 | 0.28 | 0.06 | 0.5 | 0.38 | 0.56 | 0.31 | 0.11 | 0.51 | 0.4 | 0.66 | 0.31 | 0.11 | 0.51 | 0.4 | 0.66 |
| 700/705 | 0.24 | 0.51 | 0.61 | 0.5 | 0.74 | 0.32 | 0.56 | 0.7 | 0.53 | 0.85 | 0.32 | 0.56 | 0.7 | 0.53 | 0.85 |
| 700/720 | 0.31 | 0.27 | 0.57 | 0.62 | 0.67 | 0.47 | 0.41 | 0.57 | 0.68 | 0.77 | 0.47 | 0.41 | 0.57 | 0.68 | 0.77 |
| 800/805 | 0.1 | 0.19 | 0.22 | 0.24 | 0.32 | 0.14 | 0.3 | 0.33 | 0.48 | 0.57 | 0.14 | 0.3 | 0.33 | 0.48 | 0.57 |
| 800/850 | 0.08 | 0.11 | 0.34 | 0.18 | 0.37 | 0.13 | 0.13 | 0.35 | 0.2 | 0.47 | 0.13 | 0.13 | 0.35 | 0.2 | 0.47 |
| 800/1200 | 0.09 | 0.04 | 0.1 | 0.05 | 0.14 | 0.12 | 0.06 | 0.13 | 0.07 | 0.19 | 0.12 | 0.06 | 0.13 | 0.07 | 0.19 |

Table 4: Shown is a subset of the rounded expected CLs for squark pair production (qq), combined with neutralino squark production (qq_nq) and also all together with neutralino pair production (qq_nq_nn) obtained with ATLAS-CONF (AC), ATLASEXOT (AE), CMS-SUS (CS), CMS-EXO (CE) and the combined approach (comb.). The masses are given in GeV .

| $m_{\tilde{\chi}_{1}^{0}} / m_{\tilde{\mathrm{q}}}$ | $\begin{aligned} & \mathrm{qq} \\ & \mathrm{AC} \end{aligned}$ | AE | CS | CE | com. | $\begin{aligned} & \mathrm{qq} \\ & \mathrm{AC} \end{aligned}$ | AE | CS | CE | com. |  | $\begin{aligned} & \text { nn } \\ & \text { AE } \end{aligned}$ | CS | CE | com. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 200/900 | 0.96 | 0.63 | 1.0 | 0.61 | 1.0 | 0.99 | 0.83 | 1.0 | 0.76 | 1.0 | 0.99 | 0.83 | 1.0 | 0.76 | 1.0 |
| 200/950 | 0.94 | 0.64 | 0.97 | 0.57 | 1.0 | 0.98 | 0.8 | 1.0 | 0.69 | 1.0 | 0.98 | 0.8 | 0.99 | 0.69 | 1.0 |
| 200/970 | 0.92 | 0.64 | 0.96 | 0.65 | 0.99 | 0.98 | 0.8 | 0.99 | 0.91 | 1.0 | 0.98 | 0.81 | 0.99 | 0.91 | 1.0 |
| 200/1000 | 0.91 | 0.68 | 0.94 | 0.69 | 0.99 | 0.96 | 0.83 | 0.98 | 0.76 | 1.0 | 0.96 | 0.84 | 0.99 | 0.76 | 1.0 |
| 200/1050 | 0.85 | 0.63 | 0.89 | 0.7 | 0.97 | 0.92 | 0.8 | 0.97 | 0.81 | 1.0 | 0.92 | 0.81 | 0.97 | 0.81 | 1.0 |
| 200/1200 | 0.74 | 0.42 | 0.7 | 0.48 | 0.88 | 0.81 | 0.59 | 0.88 | 0.69 | 0.97 | 0.81 | 0.6 | 0.88 | 0.69 | 0.97 |
| 200/1300 | 0.69 | 0.43 | 1.0 | 0.38 | 0.82 | 0.8 | 0.6 | 1.0 | 0.7 | 0.95 | 0.8 | 0.61 | 1.0 | 0.7 | 0.95 |
| 250/750 | 0.94 | 0.54 | 1.0 | 0.84 | 1.0 | 0.98 | 0.81 | 1.0 | 0.94 | 1.0 | 0.98 | 0.81 | 1.0 | 0.94 | 1.0 |
| 250/850 | 0.97 | 0.54 | 0.99 | 0.74 | 1.0 | 0.99 | 0.72 | 1.0 | 0.86 | 1.0 | 0.99 | 0.72 | 1.0 | 0.86 | 1.0 |
| 250/940 | 0.93 | 0.55 | 0.99 | 0.59 | 1.0 | 0.97 | 0.76 | 0.99 | 0.77 | 1.0 | 0.97 | 0.76 | 0.99 | 0.77 | 1.0 |
| 250/950 | 0.91 | 0.59 | 0.96 | 0.65 | 0.99 | 0.96 | 0.75 | 0.99 | 0.73 | 1.0 | 0.96 | 0.75 | 0.99 | 0.73 | 1.0 |
| 250/1000 | 0.9 | 0.57 | 0.94 | 0.67 | 0.99 | 0.95 | 0.77 | 1.0 | 0.79 | 1.0 | 0.95 | 0.77 | 1.0 | 0.8 | 1.0 |
| 250/1050 | 0.85 | 0.63 | 0.89 | 0.7 | 0.99 | 0.91 | 0.79 | 0.96 | 0.8 | 1.0 | 0.91 | 0.79 | 0.96 | 0.8 | 1.0 |
| 250/1080 | 0.79 | 0.58 | 0.85 | 0.61 | 0.96 | 0.87 | 0.74 | 0.95 | 0.74 | 0.99 | 0.87 | 0.74 | 0.95 | 0.74 | 0.99 |
| 250/1100 | 0.76 | 0.54 | 0.83 | 0.62 | 0.95 | 0.85 | 0.7 | 0.94 | 0.72 | 0.98 | 0.85 | 0.7 | 0.94 | 0.72 | 0.98 |
| 300/750 | 0.88 | 0.48 | 1.0 | 0.83 | 1.0 | 0.94 | 0.71 | 1.0 | 0.9 | 1.0 | 0.94 | 0.72 | 1.0 | 0.9 | 1.0 |
| 300/850 | 0.91 | 0.44 | 0.99 | 0.7 | 1.0 | 0.95 | 0.64 | 1.0 | 0.82 | 1.0 | 0.96 | 0.64 | 1.0 | 0.82 | 1.0 |
| 300/900 | 0.93 | 0.51 | 0.98 | 0.57 | 1.0 | 0.97 | 0.67 | 1.0 | 0.71 | 1.0 | 0.97 | 0.67 | 1.0 | 0.72 | 1.0 |
| 300/1000 | 0.85 | 0.52 | 0.92 | 0.66 | 0.98 | 0.91 | 0.69 | 0.97 | 0.74 | 0.99 | 0.91 | 0.69 | 0.98 | 0.74 | 1.0 |
| 300/1200 | 0.64 | 0.38 | 0.69 | 0.5 | 0.72 | 0.72 | 0.54 | 0.86 | 0.61 | 0.95 | 0.72 | 0.54 | 0.86 | 0.61 | 0.95 |
| 350/650 | 0.88 | 0.32 | 1.0 | 0.86 | 1.0 | 0.95 | 0.55 | 1.0 | 0.92 | 1.0 | 0.95 | 0.55 | 1.0 | 0.92 | 1.0 |
| 350/700 | 0.76 | 0.45 | 0.99 | 0.83 | 1.0 | 0.87 | 0.62 | 1.0 | 0.89 | 1.0 | 0.87 | 0.62 | 1.0 | 0.89 | 1.0 |
| 350/800 | 0.82 | 0.38 | 0.98 | 0.75 | 1.0 | 0.93 | 0.59 | 1.0 | 0.85 | 1.0 | 0.93 | 0.59 | 1.0 | 0.85 | 1.0 |
| 350/850 | 0.84 | 0.33 | 0.98 | 0.64 | 1.0 | 0.91 | 0.53 | 0.99 | 0.74 | 1.0 | 0.91 | 0.53 | 0.99 | 0.74 | 1.0 |
| 350/900 | 0.86 | 0.42 | 0.96 | 0.55 | 0.98 | 0.92 | 0.57 | 0.99 | 0.68 | 1.0 | 0.92 | 0.58 | 0.99 | 0.68 | 1.0 |
| 350/1000 | 0.83 | 0.45 | 0.91 | 0.55 | 0.98 | 0.88 | 0.59 | 0.96 | 0.64 | 0.99 | 0.88 | 0.6 | 0.96 | 0.64 | 0.99 |
| 350/1100 | 0.72 | 0.45 | 0.81 | 0.59 | 0.92 | 0.79 | 0.57 | 0.9 | 0.64 | 0.97 | 0.8 | 0.58 | 0.9 | 0.64 | 0.97 |
| 400/600 | 0.73 | 0.32 | 1.0 | 0.88 | 1.0 | 0.92 | 0.52 | 1.0 | 0.91 | 1.0 | 0.92 | 0.52 | 1.0 | 0.91 | 1.0 |
| 400/750 | 0.84 | 0.24 | 0.97 | 0.62 | 0.99 | 0.88 | 0.4 | 0.99 | 0.72 | 1.0 | 0.88 | 0.4 | 0.99 | 0.72 | 1.0 |
| 400/800 | 0.64 | 0.31 | 0.94 | 0.67 | 0.99 | 0.77 | 0.48 | 0.98 | 0.74 | 0.99 | 0.77 | 0.49 | 0.98 | 0.74 | 0.99 |
| 400/850 | 0.76 | 0.3 | 0.95 | 0.52 | 0.98 | 0.85 | 0.48 | 0.98 | 0.63 | 1.0 | 0.85 | 0.49 | 0.98 | 0.63 | 1.0 |
| 400/870 | 0.74 | 0.28 | 0.95 | 0.58 | 0.98 | 0.83 | 0.44 | 0.98 | 0.72 | 1.0 | 0.83 | 0.45 | 0.98 | 0.72 | 1.0 |
| 400/920 | 0.78 | 0.3 | 0.94 | 0.53 | 0.98 | 0.85 | 0.44 | 0.97 | 0.62 | 0.99 | 0.85 | 0.44 | 0.97 | 0.63 | 0.99 |
| 400/950 | 0.79 | 0.35 | 0.92 | 0.45 | 0.98 | 0.87 | 0.54 | 0.97 | 0.58 | 0.99 | 0.87 | 0.54 | 0.97 | 0.59 | 0.99 |
| 400/1000 | 0.76 | 0.33 | 0.87 | 0.86 | 0.97 | 0.81 | 0.48 | 0.94 | 0.89 | 0.99 | 0.81 | 0.49 | 0.94 | 0.89 | 0.99 |
| 400/1200 | 0.56 | 0.36 | 0.69 | 0.41 | 0.81 | 0.67 | 0.49 | 0.83 | 0.52 | 0.93 | 0.67 | 0.49 | 0.83 | 0.52 | 0.93 |
| 450/550 | 0.38 | 0.38 | 1.0 | 0.54 | 1.0 | 0.38 | 0.6 | 1.0 | 0.78 | 1.0 | 0.38 | 0.6 | 1.0 | 0.78 | 1.0 |
| 450/600 | 0.38 | 0.23 | 1.0 | 0.56 | 1.0 | 0.38 | 0.35 | 1.0 | 0.61 | 1.0 | 0.38 | 0.36 | 1.0 | 0.61 | 1.0 |
| 450/650 | 0.59 | 0.22 | 1.0 | 0.48 | 1.0 | 0.7 | 0.37 | 1.0 | 0.56 | 1.0 | 0.7 | 0.37 | 1.0 | 0.57 | 1.0 |
| 450/820 | 0.38 | 0.18 | 0.86 | 0.48 | 0.93 | 0.38 | 0.38 | 0.93 | 0.56 | 0.99 | 0.38 | 0.38 | 0.93 | 0.56 | 1.0 |
| 450/850 | 0.38 | 0.36 | 0.9 | 0.52 | 0.96 | 0.38 | 0.47 | 0.95 | 0.57 | 0.98 | 0.38 | 0.47 | 0.95 | 0.57 | 0.98 |
| 450/950 | 0.38 | 0.26 | 0.9 | 0.46 | 0.95 | 0.38 | 0.39 | 0.94 | 0.57 | 0.99 | 0.38 | 0.39 | 0.94 | 0.57 | 0.99 |
| 500/520 | 0.83 | 0.44 | 1.0 | 0.92 | 1.0 | 0.87 | 0.59 | 1.0 | 0.94 | 1.0 | 0.87 | 0.59 | 1.0 | 0.94 | 1.0 |
| 500/600 | 0.38 | 0.18 | 1.0 | 0.49 | 1.0 | 0.38 | 0.33 | 1.0 | 0.64 | 1.0 | 0.38 | 0.34 | 1.0 | 0.64 | 1.0 |
| 500/650 | 0.38 | 0.31 | 0.82 | 0.43 | 0.96 | 0.38 | 0.43 | 0.91 | 0.5 | 0.97 | 0.38 | 0.43 | 0.91 | 0.5 | 0.97 |
| 500/700 | 0.38 | 0.14 | 0.88 | 0.41 | 0.94 | 0.38 | 0.34 | 0.89 | 0.47 | 0.95 | 0.38 | 0.35 | 0.89 | 0.47 | 0.95 |
| 500/750 | 0.38 | 0.18 | 0.8 | 0.51 | 0.88 | 0.38 | 0.33 | 0.87 | 0.56 | 0.9 | 0.38 | 0.33 | 0.87 | 0.56 | 0.9 |
| 500/800 | 0.38 | 0.19 | 0.74 | 0.58 | 0.74 | 0.38 | 0.34 | 0.85 | 0.63 | 0.93 | 0.38 | 0.34 | 0.85 | 0.63 | 0.94 |
| 500/850 | 0.5 | 0.15 | 0.8 | 0.48 | 0.93 | 0.58 | 0.3 | 0.88 | 0.54 | 0.92 | 0.58 | 0.3 | 0.88 | 0.54 | 0.92 |
| 500/950 | 0.58 | 0.22 | 0.84 | 0.4 | 0.93 | 0.66 | 0.32 | 0.9 | 0.48 | 0.96 | 0.66 | 0.32 | 0.9 | 0.48 | 0.96 |
| 550/700 | 0.4 | 0.17 | 0.69 | 0.34 | 0.77 | 0.44 | 0.27 | 0.81 | 0.41 | 0.87 | 0.44 | 0.28 | 0.81 | 0.41 | 0.87 |
| 550/750 | 0.39 | 0.17 | 0.69 | 0.43 | 0.79 | 0.47 | 0.3 | 0.79 | 0.46 | 0.89 | 0.47 | 0.3 | 0.79 | 0.46 | 0.89 |
| 550/800 | 0.54 | 0.12 | 0.79 | 0.36 | 0.87 | 0.62 | 0.38 | 0.82 | 0.42 | 0.93 | 0.62 | 0.39 | 0.82 | 0.42 | 0.93 |
| 550/850 | 0.47 | 0.13 | 0.85 | 0.39 | 0.88 | 0.52 | 0.24 | 0.85 | 0.45 | 0.85 | 0.52 | 0.24 | 0.85 | 0.45 | 0.85 |
| 550/900 | 0.4 | 0.12 | 0.73 | 0.32 | 0.83 | 0.49 | 0.21 | 0.82 | 0.39 | 0.84 | 0.49 | 0.21 | 0.82 | 0.39 | 0.84 |
| 600/650 | 0.54 | 0.34 | 0.97 | 0.37 | 0.98 | 0.58 | 0.42 | 0.98 | 0.42 | 0.98 | 0.58 | 0.42 | 0.98 | 0.42 | 0.98 |
| 600/750 | 0.35 | 0.16 | 0.64 | 0.31 | 0.63 | 0.38 | 0.29 | 0.76 | 0.38 | 0.83 | 0.38 | 0.3 | 0.76 | 0.38 | 0.83 |
| 600/850 | 0.32 | 0.21 | 0.62 | 0.45 | 0.68 | 0.41 | 0.29 | 0.73 | 0.48 | 0.84 | 0.41 | 0.29 | 0.73 | 0.48 | 0.84 |
| 600/1000 | 0.4 | 0.12 | 0.68 | 0.27 | 0.73 | 0.46 | 0.18 | 0.76 | 0.33 | 0.78 | 0.46 | 0.19 | 0.76 | 0.33 | 0.78 |
| 600/1200 | 0.44 | 0.21 | 0.61 | 0.26 | 0.72 | 0.49 | 0.28 | 0.71 | 0.33 | 0.8 | 0.49 | 0.28 | 0.72 | 0.33 | 0.8 |
| 625/750 | 0.33 | 0.17 | 0.62 | 0.22 | 0.75 | 0.37 | 0.28 | 0.73 | 0.27 | 0.79 | 0.37 | 0.29 | 0.73 | 0.27 | 0.79 |
| 650/800 | 0.27 | 0.1 | 0.57 | 0.22 | 0.63 | 0.32 | 0.2 | 0.67 | 0.26 | 0.73 | 0.32 | 0.2 | 0.67 | 0.26 | 0.73 |
| 650/850 | 0.28 | 0.1 | 0.75 | 0.37 | 0.84 | 0.33 | 0.17 | 0.75 | 0.4 | 0.84 | 0.33 | 0.17 | 0.75 | 0.4 | 0.84 |
| 700/705 | 0.48 | 0.37 | 0.84 | 0.42 | 0.89 | 0.5 | 0.52 | 0.88 | 0.45 | 0.91 | 0.5 | 0.52 | 0.88 | 0.45 | 0.91 |
| 700/720 | 0.39 | 0.26 | 0.72 | 0.64 | 0.77 | 0.55 | 0.4 | 0.82 | 0.7 | 0.88 | 0.55 | 0.4 | 0.82 | 0.7 | 0.88 |
| 800/805 | 0.22 | 0.27 | 0.62 | 0.23 | 0.58 | 0.28 | 0.48 | 0.68 | 0.35 | 0.8 | 0.28 | 0.48 | 0.68 | 0.35 | 0.8 |
| 800/850 | 0.25 | 0.19 | 0.58 | 0.33 | 0.61 | 0.27 | 0.26 | 0.64 | 0.34 | 0.68 | 0.27 | 0.26 | 0.64 | 0.34 | 0.68 |
| 800/1200 | 0.23 | 0.08 | 0.42 | 0.13 | 0.48 | 0.27 | 0.13 | 0.5 | 0.16 | 0.56 | 0.27 | 0.13 | 0.5 | 0.16 | 0.56 |

Table 5: Shown is a subset of the rounded observed CLs for squark pair production (qq), combined with neutralino squark production (qq_nq) and also all together with neutralino pair production (qq_nq_nn) obtained with ATLAS-CONF (AC), ATLASEXOT (AE), CMS-SUS (CS), CMS-EXO (CE) and the combined approach (comb.). The masses are given in GeV .

## Acronyms

BSM beyond standard model. 1, 7, 14, 30, 31, 40, 45, 47

CL confidence level. $16,27,31,32,34,37,42,47,59,60,64,65$

DJR differential jet rate. 24,26

LHC large hadron collider. 1, 29, 30, 47
LHE Les Houches event. 51

LO leading order. $14,16,19,27,29,47$
LSP lightest stable particle. $11,13,15,42$

MSSM minimal supersymmetric standard model. $9,14,47$

NLO next to leading order. 27, 28
NNLO next to next to leading order. 27

PAD public analysis database. 29, 31, 32
PDF parton distribution function. $15,16,27,28,50,51,54$
pdf probability density function. $35,36,58,62$
PDG Particle Data Group. 51, 52

QCD quantum chromo dynamics. 27

SM standard model. 1, 3, 7, 9, 12, 30, 33, 35, 40, 41
SR signal region. 1, 2, 14, 16, 27, 31, 37, 42, 44, 47
SUSY supersymmetry. 1, 3, 7, 10, 14, 24, 27

VEV vacuum expectation value. 12

WIMP weakly interacting massive particle. 9

## Bibliography

[1] Matthew D. Schwartz. Quantum Field Theory and the Standard Model. Cambridge University Press, Mar. 2014. ISBN: 978-1-107-03473-0, 978-1-107-03473-0.
[2] Michael E. Peskin and Daniel V. Schroeder. An Introduction to quantum field theory. Reading, USA: Addison-Wesley, 1995. ISBN: 978-0-201-50397-5.
[3] Jon Butterworth. "The Standard Model: How far can it go and how can we tell?" In: Phil. Trans. Roy. Soc. Lond. A 374.2075 (2016). Ed. by C. David Garner, p. 20150260. Doi: $10.1098 /$ rsta.2015.0260, arXiv: 1601.02759 [hep-ex].
[4] Peter W. Higgs. "Broken Symmetries and the Masses of Gauge Bosons". In: Phys. Rev. Lett. 13 (16 Oct. 1964), pp. 508-509. Doi: 10.1103/PhysRevLett. 13.508, URL: https : //link.aps.org/doi/10.1103/PhysRevLett.13.508.
[5] F. Englert and R. Brout. "Broken Symmetry and the Mass of Gauge Vector Mesons". In: Phys. Rev. Lett. 13 (9 Aug. 1964), pp. 321-323. Dor: 10.1103/PhysRevLett.13.321. URL: https://link.aps.org/doi/10.1103/PhysRevLett.13.321.
[6] Serguei Chatrchyan et al. "Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC". In: Phys. Lett. B 716 (2012), pp. 30-61. Doi: 10.1016/j. physletb.2012.08.021. arXiv: 1207.7235 [hep-ex].
[7] Y. Fukuda et al. "Evidence for oscillation of atmospheric neutrinos". In: Phys. Rev. Lett. 81 (1998), pp. 1562-1567. DOI: 10.1103/PhysRevLett.81.1562. arXiv: hep-ex/9807003.
[8] Q. R. Ahmad et al. "Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory". In: Phys. Rev. Lett. 89 (2002), p. 011301. DOI: 10.1103/PhysRevLett.89.011301, arXiv: nucl-ex/0204008.
[9] Sidney Coleman and Jeffrey Mandula. "All Possible Symmetries of the $S$ Matrix". In: Phys. Rev. 159 (5 July 1967), pp. 1251-1256. DOI: 10.1103/PhysRev.159.1251. URL: https://link.aps.org/doi/10.1103/PhysRev.159.1251.
[10] Rudolf Haag, Jan T. Lopuszanski, and Martin Sohnius. "All Possible Generators of Supersymmetries of the s Matrix". In: Nucl. Phys. B 88 (1975), p. 257. Dor: 10.1016/0550-3213(75)90279-5.
[11] J. Wess and B. Zumino. "A Lagrangian Model Invariant Under Supergauge Transformations". In: Phys. Lett. B 49 (1974), p. 52. DoI: 10.1016/0370-2693(74) 90578-4.
[12] J. Wess and B. Zumino. "Supergauge Transformations in Four-Dimensions". In: Nucl. Phys. B 70 (1974). Ed. by A. Salam and E. Sezgin, pp. 39-50. Dor: $10.1016 / 0550-$ 3213(74)90355-1.
[13] Juri Fiaschi et al. "Electroweak superpartner production at 13.6 Tev with Resummino". In: Eur. Phys. J. C 83.8 (2023), p. 707. DoI: $10.1140 / \mathrm{epjc} / \mathrm{s} 10052-023-11888-\mathrm{y}$. arXiv: 2304.11915 [hep-ph].
[14] Wim Beenakker et al. "NNLL-fast: predictions for coloured supersymmetric particle production at the LHC with threshold and Coulomb resummation". In: JHEP 12 (2016), p. 133. DOI: $10.1007 /$ JHEP12 (2016) 133, arXiv: 1607.07741 [hep-ph].
[15] Georges Aad et al. "Search for supersymmetry using final states with one lepton, jets, and missing transverse momentum with the ATLAS detector in $\sqrt{s}=7 \mathrm{TeV} p p$ ". In: Phys. Rev. Lett. 106 (2011), p. 131802. DOI: 10.1103/PhysRevLett.106.131802, arXiv: 1102.2357 [hep-ex]
[16] Vardan Khachatryan et al. "Search for Supersymmetry in pp Collisions at 7 TeV in Events with Jets and Missing Transverse Energy". In: Phys. Lett. B 698 (2011), pp. 196-218. Doi: 10.1016/j.physletb.2011.03.021. arXiv: 1101.1628 [hep-ex].
[17] Georges Aad et al. "Search for direct slepton and gaugino production in final states with two leptons and missing transverse momentum with the ATLAS detector in $p p$ collisions at $\sqrt{s}=7 \mathrm{TeV}$ ". In: Phys. Lett. B 718 (2013), pp. 879-901. Doi: $10.1016 / \mathrm{j}$. physletb. 2012.11.058, arXiv: 1208.2884 [hep-ex].
[18] Georges Aad et al. "Search for direct production of charginos and neutralinos in events with three leptons and missing transverse momentum in $\sqrt{s}=8 \mathrm{TeV} p p$ collisions with the ATLAS detector". In: JHEP 04 (2014), p. 169. DOI: 10.1007/JHEP04(2014)169, arXiv: 1402.7029 [hep-ex]
[19] Serguei Chatrchyan et al. "Search for electroweak production of charginos and neutralinos using leptonic final states in $p p$ collisions at $\sqrt{s}=7 \mathrm{TeV}$ ". In: JHEP 11 (2012), p. 147. DOI: 10.1007/JHEP11(2012)147, arXiv: 1209.6620 [hep-ex].
[20] "Search for direct EWK production of SUSY particles in multilepton modes with 8 TeV data". In: (2013).
[21] Georges Aad et al. "Search for R-parity-violating supersymmetry in a final state containing leptons and many jets with the ATLAS experiment using $\sqrt{s}=13 \mathrm{TeV}$ proton-proton collision data". In: Eur. Phys. J. C 81.11 (2021), p. 1023. Doi: $10.1140 / \mathrm{epjc} / \mathrm{s} 10052-$ 021-09761-x. arXiv: 2106.09609 [hep-ex].
[22] Georges Aad et al. "Search for pair production of squarks or gluinos decaying via sleptons or weak bosons in final states with two same-sign or three leptons with the ATLAS detector". In: (July 2023). arXiv: 2307.01094 [hep-ex].
[23] "Search for new physics in multijet events with at least one photon and large missing transverse momentum in proton-proton collisions at 13 TeV ". In: (2023).
[24] Armen Tumasyan et al. "Search for top squark pair production in a final state with at least one hadronically decaying tau lepton in proton-proton collisions at $\sqrt{s}=13 \mathrm{TeV}$ ". In: JHEP 07 (2023), p. 110. DOI: $10.1007 /$ JHEP07 (2023)110, arXiv: 2304.07174 [hep-ex].
[25] Iñaki Lara et al. "Monojet signatures from gluino and squark decays". In: JHEP 10 (2022), p. 150. DOI: 10.1007/JHEP10(2022)150, arXiv: 2208.01651 [hep-ph].
[26] Jack Y. Araz et al. "Strength in numbers: Optimal and scalable combination of LHC newphysics searches". In: SciPost Phys. 14.4 (2023), p. 077. DoI: $10.21468 /$ SciPostPhys. 14.4.077. arXiv: 2209.00025 [hep-ph].
[27] Alexander Feike et al. "Combination and Reinterpretation of LHC SUSY Searches". unpublished, preprint: MS-TP-23-49.
[28] Jakob Schwichtenberg. Physics from Symmetry. Undergraduate Lecture Notes in Physics. Cham: Springer International Publishing, 2018. ISBN: 978-3-319-66630-3, 978-3-319-666310. Dor: $10.1007 / 978-3-319-66631-0$.
[29] Stephen P. Martin. "A Supersymmetry Primer". In: Perspectives on Supersymmetry. World Scientific, July 1998, pp. 1-98. Doi: 10.1142/9789812839657_0001, URL: https: //doi.org/10.1142\\\%/2F9789812839657_0001.
[30] Sidney Coleman and Jeffrey Mandula. "All Possible Symmetries of the $S$ Matrix". In: Phys. Rev. 159 (5 July 1967), pp. 1251-1256. DoI: 10.1103/PhysRev.159.1251. URL: https://link.aps.org/doi/10.1103/PhysRev.159.1251.
[31] Rudolf Haag, Jan T. Lopuszanski, and Martin Sohnius. "All Possible Generators of Supersymmetries of the s Matrix". In: Nucl. Phys. B 88 (1975), p. 257. Dor: 10.1016/0550-3213(75)90279-5.
[32] Howard Baer and Xerxes Tata. Weak Scale Supersymmetry: From Superfields to Scattering Events. Cambridge University Press, 2006. Doi: $10.1017 /$ CB09780511617270.
[33] A. V. Gladyshev and D. I. Kazakov. "Supersymmetry and LHC". In: Physics of Atomic Nuclei 70.9 (Sept. 2007), pp. 1553-1567. Dor: $10.1134 /$ s1063778807090104. URL: https : //doi.org/10.1134\\\%2Fs1063778807090104.
[34] Werner Porod. "SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at e+ e- colliders". In: Comput. Phys. Commun. 153 (2003), pp. 275-315. DoI: $10.1016 /$ S0010-4655(03)00222-4. arXiv: hep-ph/0301101.
[35] Alexander Puck Neuwirth and Kevin Pedro. APN-Pucky/pyfeyn2: 2.3.10. Version 2.3.10. Nov. 2023. DoI: $10.5281 /$ zenodo.10067115, URL: https://doi.org/10.5281/zenodo. 10067115 .
[36] J. Alwall et al. "The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations". In: JHEP 07 (2014), p. 079. DOI: $10.1007 /$ JHEP07 (2014) 079. arXiv: 1405.0301 [hep-ph]
[37] Andy Buckley et al. "LHAPDF6: parton density access in the LHC precision era". In: Eur. Phys. J. C 75 (2015), p. 132. DoI: $10.1140 / \mathrm{epjc} /$ s10052-015-3318-8, arXiv: 1412.7420 [hep-ph].
[38] Christian Bierlich et al. "A comprehensive guide to the physics and usage of PYTHIA 8.3". In: (Mar. 2022). DoI: 10.21468/SciPostPhysCodeb.8, arXiv: 2203.11601 [hep-ph]
[39] Eric Conte, Benjamin Fuks, and Guillaume Serret. "MadAnalysis 5, A User-Friendly Framework for Collider Phenomenology". In: Comput. Phys. Commun. 184 (2013), pp. 222256. DOI: $10.1016 /$ j.cpc.2012.09.009. arXiv: 1206.1599 [hep-ph].
[40] Wim Beenakker et al. "NLO+NLL squark and gluino production cross-sections with threshold-improved parton distributions". In: Eur. Phys. J. C 76.2 (2016), p. 53. DOI: 10.1140/epjc/s10052-016-3892-4. arXiv: 1510.00375 [hep-ph].
[41] Jack Y. Araz. "Spey: smooth inference for reinterpretation studies". In: (July 2023). arXiv: 2307.06996 [hep-ph]
[42] Andy Buckley et al. "General-purpose event generators for LHC physics". In: Phys. Rept. 504 (2011), pp. 145-233. DOI: 10.1016 / j . physrep . 2011.03.005. arXiv: 1101.2599 [hep-ph].
[43] Leif Gellersen. "Basics of Parton Showers". In: 2019. URL: https://indico.cern.ch/ event/829653/contributions/3568527/.
[44] R. K. Ellis, W. J. Stirling, and B. R. Webber. QCD and Collider Physics. Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology. Cambridge University Press, 1996. DOI: $10.1017 / \mathrm{CBO9780511628788}$.
[45] Leif Lönnblad. "Fooling Around with the Sudakov Veto Algorithm". In: Eur. Phys. J. C 73.3 (2013), p. 2350. DOI: $10.1140 / \mathrm{epjc} / \mathrm{s} 10052-013-2350-9$, arXiv: 1211.7204 [hep-ph].
[46] Leif Lonnblad. "Correcting the color dipole cascade model with fixed order matrix elements". In: JHEP 05 (2002), p. 046. DOI: $10.1088 / 1126-6708 / 2002 / 05 / 046$ arXiv: hep-ph/0112284.
[47] Leif Lonnblad and Stefan Prestel. "Matching Tree-Level Matrix Elements with Interleaved Showers". In: JHEP 03 (2012), p. 019. DOI: 10.1007/JHEP03(2012)019. arXiv: 1109.4829 [hep-ph].
[48] Stefano Catani et al. "QCD Matrix Elements + Parton Showers". In: Journal of High Energy Physics 2001.11 (Nov. 2001), pp. 063-063. ISSN: 1029-8479. DOI: 10.1088/11266708/2001/11/063. URL: http://dx.doi.org/10.1088/1126-6708/2001/11/063
[49] Nils Lavesson and Leif Lönnblad. "Merging parton showers and matrix elements-back to basics". In: Journal of High Energy Physics 2008.04 (Apr. 2008), pp. 085-085. ISSN: 10298479. DOI: $10.1088 / 1126-6708 / 2008 / 04 / 085$. URL: http://dx.doi.org/10.1088/11266708/2008/04/085.
[50] J. Alwall et al. "Comparative study of various algorithms for the merging of parton showers and matrix elements in hadronic collisions". In: The European Physical Journal C 53.3 (Dec. 2007), pp. 473-500. ISSN: 1434-6052. DOI: $10.1140 / \mathrm{epjc} / \mathrm{s} 10052-007-0490-5$. URL: http://dx.doi.org/10.1140/epjc/s10052-007-0490-5.
[51] Benjamin Fuks. Supersymmetry - When Theory Inspires Experimental Searches. 2013. arXiv: 1401.6277 [hep-ph].
[52] Matteo Cacciari, Gavin P. Salam, and Gregory Soyez. "FastJet User Manual". In: Eur. Phys. J. C 72 (2012), p. 1896. Dor: $10.1140 / \mathrm{epjc} / \mathrm{s} 10052-012-1896-2$, arXiv: 1111.6097 [hep-ph].
[53] Jonathan Debove, Benjamin Fuks, and Michael Klasen. "Joint Resummation for Gaugino Pair Production at Hadron Colliders". In: Nucl. Phys. B 849 (2011), pp. 64-79. Doi: 10.1016/j.nuclphysb.2011.03.015, arXiv: 1102.4422 [hep-ph].
[54] Benjamin Fuks et al. "Gaugino production in proton-proton collisions at a center-of-mass energy of 8 TeV". In: JHEP 10 (2012), p. 081. DoI: 10.1007/JHEP10(2012) 081. arXiv: 1207.2159 [hep-ph]
[55] Juri Fiaschi and Michael Klasen. "Neutralino-chargino pair production at NLO+NLL with resummation-improved parton density functions for LHC Run II". In: Phys. Rev. D 98.5 (2018), p. 055014. Doi: 10.1103/PhysRevD.98.055014, arXiv: 1805.11322 [hep-ph].
[56] Juri Fiaschi and Michael Klasen. "Higgsino and gaugino pair production at the LHC with aNNLO+NNLL precision". In: Phys. Rev. D 102.9 (2020), p. 095021. Doi: $10.1103 /$ PhysRevD. 102.095021, arXiv: 2006.02294 [hep-ph].
[57] Juri Fiaschi et al. "Soft gluon resummation for associated squark-electroweakino production at the LHC". In: JHEP 06 (2022), p. 130. DOI: 10.1007/JHEP06(2022)130. arXiv: 2202.13416 [hep-ph].
[58] Alexander Puck Neuwirth. APN-Pucky/HEPi: Zenodo Release. Version 0.2.11. Oct. 2023. DOI: 10.5281/zenodo.8430837, uRL: https://doi.org/10.5281/zenodo.8430837.
[59] W. Beenakker et al. "Squark and gluino production at hadron colliders". In: Nucl. Phys. B 492 (1997), pp. 51-103. DOI: 10.1016/S0550-3213(97)80027-2 arXiv: hep-ph/9610490.
[60] A. Kulesza and L. Motyka. "Threshold resummation for squark-antisquark and gluinopair production at the LHC". In: Phys. Rev. Lett. 102 (2009), p. 111802. DoI: 10.1103/ PhysRevLett.102.111802, arXiv: 0807.2405 [hep-ph].
[61] A. Kulesza and L. Motyka. "Soft gluon resummation for the production of gluino-gluino and squark-antisquark pairs at the LHC". In: Phys. Rev. D 80 (2009), p. 095004. DoI: 10.1103/PhysRevD. 80.095004 arXiv: 0905.4749 [hep-ph].
[62] Wim Beenakker et al. "Soft-gluon resummation for squark and gluino hadroproduction". In: JHEP 12 (2009), p. 041. DOI: 10.1088/1126-6708/2009/12/041. arXiv: 0909.4418 [hep-ph].
[63] Wim Beenakker et al. "NNLL resummation for squark-antisquark pair production at the LHC". In: JHEP 01 (2012), p. 076. DOI: $10.1007 /$ JHEP01(2012) 076. arXiv: 1110.2446 [hep-ph].
[64] Wim Beenakker et al. "Towards NNLL resummation: hard matching coefficients for squark and gluino hadroproduction". In: JHEP 10 (2013), p. 120. DOI: 10.1007/JHEP10(2013) 120. arXiv: 1304.6354 [hep-ph].
[65] Wim Beenakker et al. "NNLL resummation for squark and gluino production at the LHC". In: JHEP 12 (2014), p. 023. DOI: $10.1007 /$ JHEP12(2014)023, arXiv: 1404.3134 [hepph].
[66] Richard D. Ball et al. "The PDF4LHC21 combination of global PDF fits for the LHC Run III". In: J. Phys. G 49.8 (2022), p. 080501. Doi: 10.1088/1361-6471/ac7216. arXiv: 2203.05506 [hep-ph].
[67] A. D. Martin et al. "Parton distributions for the LHC". In: Eur. Phys. J. C 63 (2009), pp. 189-285. DOI: $10.1140 / \mathrm{epjc} / \mathrm{s} 10052-009-1072-5$, arXiv: 0901.0002 [hep-ph].
[68] W. Beenakker, R. Hopker, and M. Spira. "PROSPINO: A Program for the production of supersymmetric particles in next-to-leading order QCD". In: (Nov. 1996). arXiv: hepph/9611232.
[69] Guang-ping Gao et al. "Loop effects and nondecoupling property of SUSY QCD in g b $\longrightarrow$ t H-". In: Phys. Rev. D 66 (2002), p. 015007. DOI: 10.1103/PhysRevD.66.015007. arXiv: hep-ph/0202016.
[70] B. Dumont et al. "Toward a public analysis database for LHC new physics searches using MADANALYSIS 5". In: Eur. Phys. J. C 75.2 (2015), p. 56. Doi: 10.1140/epjc/s10052-014-3242-3, arXiv: 1407.3278 [hep-ph].
[71] Jack Y. Araz, Benjamin Fuks, and Georgios Polykratis. "Simplified fast detector simulation in MADANALYSIS 5". In: Eur. Phys. J. C 81.4 (2021), p. 329. Doi: 10.1140/epjc/ s10052-021-09052-5, arXiv: 2006.09387 [hep-ph].
[72] Federico Ambrogi. Implementation of a search for squarks and gluinos in the multi-jet + missing energy channel (139 fb-1; 13 TeV; ATLAS-CONF-2019-040). Version V1. 2021. DOI: 10.14428/DVN/NW3NPG. URL: https://doi.org/10.14428/DVN/NW3NPG.
[73] Diyar Agin. Implementation of a search for new physics with jets and missing transverse energy (139/fb; 13 TeV; ATLAS-EXOT-2018-06). Version V1. 2023. DOI: 10.14428/DVN/ REPAMM, URL: https://doi.org/10.14428/DVN/REPAMM.
[74] Mrowietz Malte, Sam Bein, and Jory Sonneveld. Re-implementation of a search for supersymmetry in the HT/missing HT channel (137 fb-1; CMS-SUSY-19-006). Version V6. 2020. DOI: 10.14428/DVN/4DEJQM. URL: https://doi.org/10.14428/DVN/4DEJQM.
[75] Andreas Albert. Implementation of a search for new phenomena in events featuring energetic jets and missing transverse energy (137 fb-1; 13 TeV; CMS-EXO-20-004). Version V2. 2021. DOI: 10.14428/DVN/IRF7ZL. URL: https://doi.org/10.14428/DVN/IRF7ZL.
[76] Georges Aad et al. "Search for new phenomena in events with an energetic jet and missing transverse momentum in $p p$ collisions at $\sqrt{s}=13 \mathrm{TeV}$ with the ATLAS detector". In: Phys. Rev. D 103.11 (2021), p. 112006. DoI: $10.1103 /$ PhysRevD.103.112006, arXiv: 2102.10874 [hep-ex].
[77] Georges Aad et al. "Search for squarks and gluinos in final states with jets and missing transverse momentum using $139 \mathrm{fb}^{-1}$ of $\sqrt{s}=13 \mathrm{TeV} p p$ collision data with the ATLAS detector". In: JHEP 02 (2021), p. 143. DOI: $10.1007 /$ JHEP02(2021) 143, arXiv: 2010. 14293 [hep-ex].
[78] The Cms Collaboration et al. "Search for supersymmetry in proton-proton collisions at 13 TeV in final states with jets and missing transverse momentum". In: JHEP 10 (2019), p. 244. DOI: $10.1007 /$ JHEP10 (2019) 244 arXiv: 1908.04722 [hep-ex]
[79] Armen Tumasyan et al. "Search for new particles in events with energetic jets and large missing transverse momentum in proton-proton collisions at $\sqrt{s}=13 \mathrm{TeV}$ ". In: JHEP 11 (2021), p. 153. DOI: 10.1007/JHEP11(2021)153. arXiv: 2107.13021 [hep-ex].
[80] Glen Cowan et al. "Asymptotic formulae for likelihood-based tests of new physics". In: Eur. Phys. J. C 71 (2011). [Erratum: Eur.Phys.J.C 73, 2501 (2013)], p. 1554. Doi: 10. 1140/epjc/s10052-011-1554-0. arXiv: 1007.1727 [physics.data-an].
[81] Abraham Wald. "Tests of Statistical Hypotheses Concerning Several Parameters When the Number of Observations is Large". In: Transactions of the American Mathematical Society 54.3 (1943), pp. 426-482. ISSN: 00029947. URL: http://www.jstor.org/stable/1990256 (visited on 10/30/2023).
[82] S. S. Wilks. "The Large-Sample Distribution of the Likelihood Ratio for Testing Composite Hypotheses". In: The Annals of Mathematical Statistics 9.1 (1938), pp. 60-62. ISSN: 00034851. URL: http://www.jstor.org/stable/2957648 (visited on 10/30/2023).
[83] Andreas Papaefstathiou. "How-to: write a parton-level Monte Carlo particle physics event generator". In: Eur. Phys. J. Plus 135.6 (2020), p. 497. Doi: 10.1140/epjp/s13360-020-00499-1. arXiv: 1412.4677 [hep-ph]
[84] Andreas Papaefstathiou. "Phenomenological aspects of new physics at high energy hadron colliders". Other thesis. Aug. 2011. DoI: 10.17863/CAM.16577, arXiv: 1108.5599 [hepph].
[85] Frank Siegert. "Monte-Carlo event generation for the LHC". PhD thesis. Durham U., 2010.
[86] CKKW-L Merging. https://pythia.org/latest-manual/CKKWLMerging.html. Accessed: December 20, 2023.
[87] G. Cowan. Statistical data analysis. Oxford University Press, USA, 1998.
[88] Glen Cowan. Academic Training Lecture Regular Programme - Statistics for Particle Physics. https://indico.cern.ch/event/1040092/. 2021.

## Declaration of Academic Integrity

I hereby confirm that this thesis on Combination and Reinterpretation of LHC SUSY Searches is solely my own work and that I have used no sources or aids other than the ones stated. All passages in my thesis for which other sources, including electronic media, have been used, be it direct quotes or content references, have been acknowledged as such and the sources cited.

Münster, January 11, 2024
(signature of student)

I agree to have my thesis checked in order to rule out potential similarities with other works and to have my thesis stored in a database for this purpose.

Münster, January 11, 2024
(signature of student)


[^0]:    ${ }^{1}$ The generalised PathFinder can be found at https://github.com/J-Yellen/PathFinder.

[^1]:    ${ }^{2}$ Of course electrically charged particles radiate as well, see for example bremsstrahlung, but here we will restrict our discussion on coloured radiation.

[^2]:    ${ }^{3} \mathrm{~A}$ full derivation of the Altarelli-Parisi splitting functions can be found in 44 .

