Introduction to QFT
Assignment 7

Will be discussed on 15.12.17

This assignment has to be handed in **not later than at noon 14.12.17**.

1. (20%) Show explicitly that for a field $f(x,t)$

$$\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2\, f(x_1,t_1)f(x_2,t_2) = \frac{1}{2}\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2\, \mathrm{T} f(x_1,t_1)f(x_2,t_2)\ , \tag{1}$$

   where T stands for time-ordering operator.

   [*Hint*: Think about the geometric interpretation of equation 1.]

2. (40%) Consider two different real massive scalar fields A, B with an interaction Lagrangian given by $\mathcal{L}_{\text{int}} = gA^2B$. Calculate the lowest-order non-trivial matrix element $\langle p_a k_A|S|q_A, r_A\rangle$ for the scattering process $AA \to AA$ and write down the corresponding diagrams.

   [*Hint*: Use LSZ reduction formula.]

3. (40%)
   a) Show that $\langle 0|\phi(t,\mathbf{x})\phi(t,\mathbf{x})|0\rangle$ is a diverging quantity.
   b) Let us instead define the smeared field

$$\phi_f(t,\mathbf{x}) = \int d^3y \phi(t,\mathbf{y}) f(\mathbf{x}-\mathbf{y})\ ,$$

   with

$$f(\mathbf{x}-\mathbf{y}) = \frac{1}{(a^2\pi)^{3/2}} e^{-\frac{(\mathbf{x}-\mathbf{y})^2}{a^2}}\ .$$

   Calculate $\langle 0|\phi_f(t,\mathbf{x})\phi_f(t,\mathbf{x})|0\rangle$ and evaluate it for vanishing mass.