# Introduction to QFT
# Assignment 11

Will be discussed on 26.01.18

This assignment has to be handed in **not later than at noon 25.01.18**.

1. (50%) Z-boson decay.

   Consider a tree level process $Z \to f\bar{f}$ (Z-boson decay into a pair fermion - antifermion).

   The $Zf\bar{f}$ vertex is

$$\frac{ig}{\cos\theta_W}\gamma^\mu\left(C_L\frac{1+\gamma_5}{2}+C_R\frac{1-\gamma_5}{2}\right), \tag{1}$$

   where coefficients $C_L$ and $C_R$ depend on fermion $f$.

   Let's denote:

   - $Z$-boson has: momentum P, polarization $\varepsilon^\kappa_\mu(P)$, mass $M_Z$
   - $f$ has: momentum p, mass m
   - $\bar{f}$ has: momentum q, mass m

   a) Write down the amplitude for the process $Z \to f\bar{f}$.

   b) Find the matrix element squared. Take the sum over fermion's polarizations. Write down the corresponding trace.

   The polarization sum for the massive spin-one particle is a gauge dependent quantity. In this exercise use

$$\sum_{\kappa=1}^{3}\varepsilon^\kappa_\mu(k)\varepsilon^\kappa_\nu(k) = -\left(g_{\mu\nu}-\frac{k_\mu k_\mu}{M_Z^2}\right). \tag{2}$$

   c) Assuming the polarization vectors $\varepsilon^\kappa_\mu$ to be real, take the average over $Z$-boson polarization of the matrix element squared from the previous question.

   [*Hint*: A massive spin-one particle has three polarizations.]

   d) Give the matrix element squared averaged over fermion's polarizations and Z-boson's polarizations. Show that the answer is a Lorentz scalar.

   [*Hint*: Express momenta in terms of masses .]

   The decay width $\Gamma_{f\bar{f}}$ is given by

$$\Gamma_{f\bar{f}} = \frac{1}{16\pi M_Z^3}\sqrt{\Delta\left(M_z^2, m^2, m^2\right)}\,4m^2\,\frac{1}{3}\sum_{\alpha,\beta,\kappa}|M|^2,$$

   where

$$\Delta\left(x,y,z\right) = x^2+y^2+z^2-2xy-2xz-2yz,$$

   and the sum runs over all possible polarizations.

   e) Assuming that $M_Z^2 \gg m^2$ give the answer for the decay width $\Gamma_{f\bar{f}}$.

2. (50%) Bhabha scattering.

   Consider a process $e^+\,e^- \to e^+\,e^-$. In this exercise we consider all electrons as being massless.

   a) Draw the corresponding diagrams. How many channels do you have?

b) Find a matrix element squared (Consider contribution of each diagram and the interference term separately).

c) Assuming that the initial state is not polarized and that the spin of the final state is not measured take the average over the spins in initial state and the sum over all spins in the final state. Express these sums as traces. Write down the contribution of each diagram and the interference term.

d) Take the traces and express the answer in terms of Mandelstam variables $s$, $t$, $u$.

[*Hint*: For massless particles $s+u+t=0$ holds.]

e) Find the differential cross section $\frac{d\sigma}{d\cos\theta}$ as a function of $s$, $t$, $u$.

3. (**BONUS** if you need more points!) Furry's Theorem

In this problem you can use the simplified Feynman rules without explicitly writing down the LSZ-reduction formula. This is the more practical way of evaluating the Feynman diagrams.

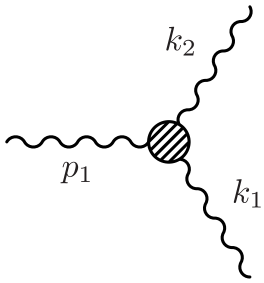


Figure 1: The process to compute at lowest order, which is $\mathcal{O}\left(e^3\right)$.

**(a)** Draw the two contributing loop diagrams to the process shown in **Fig. 1**?

In order to compute this process, we recommend a strategy wherein one uses the properties of the following matrix:

**(b)** Verify that for $C = \mathrm{i}\gamma^0\gamma^2$ the following properties hold:

- $\mathcal{C}^\dagger\mathcal{C} = \mathbb{1}$
- $\mathcal{C}\gamma^{\mu\mathrm{T}}\mathcal{C}^{-1} = -\gamma^\mu$
- $\mathcal{C}\gamma^5\mathcal{C}^{-1} = \gamma^5$

The matrix $\mathcal{C}$ with the properties that you were asked to verify is the explicit representation of charge conjugation matrix in the case that the $\gamma$-matrices are given in the chiral basis:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \quad \text{where } \sigma^\mu = (\mathbb{1}, \sigma) \quad , \quad \overline{\sigma}^\mu = (\mathbb{1}, -\sigma) \tag{3}$$

**(c)** Apply the QED Feynman rules to the diagrams that you drew in problem (a) and give mathematical expressions for both diagrams. Do not further simplify the spinor structures of these expressions.

**(d)** By using the charge conjugation matrix it is possible to relate the two expressions to each other and evaluate their sum. Show this.