Introduction to QFT
Assignment 6

Will be discussed on 08.12.17

This assignment has to be handed in **not later than at noon 07.12.17**.

1. (10%) There are 4 primary SI units: three kinematic (meter, second, kilogram) and one electrical (Ampere). Now, since we know from relativity that space and time are just different components of a unified four dimensional space-time, let us use the same unit for measuring lengths and times, i.e. $c = 1$. Let us further set $\hbar = 1$. We have imposed two constraints on our three kinematic units, leaving us one free choice for one of the three units. We will choose to fix the unit of energy and use eV to measure it. This way we have defined a new system of units that we call natural units.
   a) Find what are the units of length, time, mass, momentum and angular momentum in this new system?
   b) Convert back to SI: How many meters is a length of $1(\text{GeV})^{-1}$?
   c) What is the unit of the electric charge?
   d) What is the unit and value for the electric fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$ in SI and natural units?

2. (45%) Evaluate the function

$$\langle 0|\phi(x)\phi(y)|0\rangle = D(x-y) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}e^{-ip(x-y)} \,,$$

   for $(x-y)$ spacelike so that $(x-y)^2 = -r^2$, explicitly in terms of Bessel functions. *Hint:* Use the relation

$$\int_0^\infty dx \frac{\cos(ax)}{\sqrt{\beta^2+x^2}} = K_0(a\beta) \,,$$

   where K_0 is a modified Bessel function of the second kind. Further you will have to find a relation between $K_0(x)$ and $K_1(x)$.

3. (45%)
   a) Prove the relation

$$(\partial_x^2 + m^2)\langle 0|T\phi(x)\phi(y)|0\rangle = -i\delta^{(4)}(x-y) \,,$$

      where T is the time-ordering operator.
   b) Using the integral representation of the functions D_F and D_R show that $D_F - D_R$ is a solution to the homogeneous Klein-Gordon equation.