Introduction to QFT
Assignment 4

Will be discussed on 25.11.17

This assignment has to be handed in **not later than at noon 24.11.17**.

1. (30%) Show that the Lagrangian density (Proca action for real vectors),

$$\mathcal{L} = -\frac{1}{2}\left[\partial_\alpha V_\beta(x)\right]\left[\partial^\alpha V^\beta(x)\right] + \frac{1}{2}\left[\partial_\alpha V^\alpha(x)\right]\left[\partial_\beta V^\beta(x)\right] + \frac{\mu^2}{2}V_\alpha(x)V^\alpha(x)$$

for the real vector field $V^\alpha(x)$ leads to the field equations

$$\left[g_{\alpha\beta}\left(\Box + \mu^2\right) - \partial_\alpha\partial_\beta\right]V^\beta(x) = 0,$$

and that the field $V^\alpha(x)$ satisfies the Lorentz condition: $\partial_\alpha V^\alpha(x) = 0$.

2. (30%) Find the Euler-Lagrange equations for the following Lagrangian densities:

   a) $\mathcal{L} = \left(\partial_\mu\phi - ieA_\mu\phi\right)\left(\partial^\mu\phi^* + ieA^\mu\phi^*\right) - m^2\phi^*\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$,

   b) $\mathcal{L} = \bar{\psi}\left(i\gamma_\mu\partial^\mu - m\right)\psi + \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 - ig\bar{\psi}\gamma_5\psi\phi$,

   c) $\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4$,

   where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

3. (20%) Show that the Langrangian density

$$\mathcal{L} = \frac{1}{2}[(\partial\phi_1)^2 + (\partial\phi_2)^2] - \frac{m}{2}(\phi_1^2 + \phi_2^2) - \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2 \ ,$$

is invariant under the transformation

$$\phi_1 \to \phi_1' = \phi_1\cos\theta - \phi_2\sin\theta \ ,$$
$$\phi_2 \to \phi_2' = \phi_1\sin\theta + \phi_2\cos\theta \ .$$

Find the corresponding Noether current and charge.

4. (20%) Study the invariance properties of the Dirac Lagrangian, $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$, under chiral transformations

$$\psi(x) \to \psi'(x) = e^{i\alpha\gamma_5}\psi(x) \ ,$$

where α is a constant. Find the Noether current and its four-divergence.