Introduction to QFT
Assignment 2

Due on 10.11.17

This assignment has to be handed in **not later than at noon 09.11.17**.

1. (35%) Without using any particular representation of the Dirac matrices α_i and β, evaluate the commutators of the Dirac Hamiltonian $H_D = \boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta m$ with the following operators:

   (a) $\boldsymbol{p} = -i\nabla$

   (b) $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$

   (c) $\boldsymbol{L}^2$

   (d) $\boldsymbol{S} = \frac{1}{2}\boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma} = \frac{i}{2}\boldsymbol{\gamma} \times \boldsymbol{\gamma}$ and $\boldsymbol{\gamma} = \beta\boldsymbol{\alpha}$

   (e) $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$

   (f) $\boldsymbol{J}^2$

   (g) $\boldsymbol{\Sigma} \cdot \frac{\boldsymbol{p}}{|p|}$

2. (45%) Show that

$$\sum_{s=1,2} u^s(p)\bar{u}^s(p) = \not{p} + m,$$
$$\sum_{s=1,2} v^s(p)\bar{v}^s(p) = \not{p} - m,$$

   and that

$$\bar{u}^i(p)u^j(p) = 2m\,\delta_{ij},$$
$$\bar{v}^i(p)v^j(p) = -2m\,\delta_{ij},$$

   where $u^s(p)$ and $v^s(p)$ are solutions of the Dirac equation.

3. (20%) Without using any particular representation of the Dirac spinors, prove the following identities ($\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$):

$$2m\bar{u}(\boldsymbol{p}_1)\gamma_\mu u(\boldsymbol{p}_2) = \bar{u}(\boldsymbol{p}_1)[(p_1 + p_2)_\mu + i\sigma_{\mu\nu}(p_1 - p_2)^\nu]u(\boldsymbol{p}_2)\ ,$$
$$\bar{u}(\boldsymbol{p}_1)\sigma_{\mu\nu}(p_1 + p_2)^\nu u(\boldsymbol{p}_2) = i\bar{u}(\boldsymbol{p}_1)(p_1 - p_2)_\mu u(\boldsymbol{p}_2)\ .$$