Introduction to QFT
Assignment 9

Due on 27.01.16

1. (30 %) Express the following quantities in terms of creation and annihilation operators:
   a) charge: $Q = -e \int d^3x : \psi^\dagger \psi :$
   b) energy: $H = \int d^3x : \bar{\psi}(-i\gamma^j \partial_j + m)\psi := i \int d^3x : \psi^\dagger \partial_0 \psi :$
   c) momentum: $\boldsymbol{P} = -i \int d^3x : \psi^\dagger \nabla \psi :$

2. (10 %) Show that $i[H, \psi(x)] = \partial_t \psi(x)$

3. (40 %) Let $|\boldsymbol{p}_1, r_1; \boldsymbol{p}_2, r_2\rangle = a^\dagger_{r_1}(\boldsymbol{p}_1) a^\dagger_{r_2}(\boldsymbol{p}_2)|0\rangle$ be a two-particle state. Find the energy, charge and helicity of this state. Here, $r_{1,2}$ are the helicities of the single particle states. *Hint:* The helicity operator for the Dirac field is given by
$$S = \frac{1}{2} \sum_r \int d^3p (-1)^{r+1} [a^\dagger_r(\boldsymbol{p}) a_r(\boldsymbol{p}) + b^\dagger_r(\boldsymbol{p}) b_r(\boldsymbol{p})] \tag{1}$$

4. (20 %) Prove the following identities without using any particular representation for the Dirac matrices:
   - $\not{p}^2 = p^2$
   - $\gamma_\mu \gamma^\alpha \gamma^\beta \gamma^\mu = 4g^{\alpha\beta}$
   - $\gamma_\mu \gamma^\alpha \gamma^\beta \gamma^\eta \gamma^\mu = -2\gamma^\eta \gamma^\beta \gamma^\alpha$
   - $\mathrm{Tr}\, \gamma^5 = 0$
   - $\mathrm{Tr}\, (\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}$
   - $\mathrm{Tr}\, (\gamma^5 \gamma^\mu) = 0$
   - $\mathrm{Tr}\, (\gamma^5 \gamma^\mu \gamma^\nu) = 0$