Introduction to QFT
Assignment 6

Due on 9.12.16

1. (25%) The complex Klein-Gordon fields $\phi(x)$ and $\phi^\dagger(x)$ can be expressed in terms of two independent real Klein-Gordon fields, $\phi_1(x)$, $\phi_2(x)$ as

$$\phi(x) = \frac{\phi_1(x) + i\phi_2(x)}{\sqrt{2}} \ , \quad \phi(x) = \frac{\phi_1(x) - i\phi_2(x)}{\sqrt{2}} \ .$$

   a) Use the commutation relations between fields and their conjugate momenta for the real fields to find the respective relations for the complex fields.

   b) Write the creation and annihilation operators of the complex fields in terms of those of the real fields. Use the commutation relations between creation and annihilation operators for the real fields to find the respective relations for the complex fields.

2. (45 %) Show that $\langle 0|\phi(t,\mathbf{x})\phi(t,\mathbf{x})|0\rangle$ is a diverging quantity. Let us instead define the smeared field

$$\phi_f(t,\mathbf{x}) = \int d^3y \phi(t,\mathbf{y}) f(\mathbf{x}-\mathbf{y}) \ ,$$

   with

$$f(\mathbf{x}-\mathbf{y}) = \frac{1}{(a^2\pi)^{3/2}} e^{-\frac{(\mathbf{x}-\mathbf{y})^2}{a^2}} \ .$$

   Calculate $\langle 0|\phi_f(t,\mathbf{x})\phi_f(t,\mathbf{x})|0\rangle$ and evaluate it for vanishing mass.

3. (20%) Prove the relation

$$(\partial_x^2 + m^2)\langle 0|T\phi(x)\phi(y)|0\rangle = -i\delta^{(4)}(x-y) \ ,$$

   where T is the time-ordering operator.

4. (10%)If Δ_F is the Feynman propagator, and Δ_R is the retarded one, prove that $\Delta_f - \Delta_R$ is a solution to the homogeneous Klein-Gordon equation.