Introduction to QFT
Assignment 5

Due on 2.12.16

1. (10%) There are 4 primary SI units: three kinematic (meter, second, kilogram) and one electrical (Ampere). Now, since we know from relativity that space and time are just different components of a unified four dimensional space-time, let us use the same unit for measuring lengths and times, i.e. $c = 1$. Let us further set $\hbar = 1$. We have imposed two constraints on our three kinematic units, leaving us one free choice for one of the three units. We will choose to fix the unit of energy and use eV to measure it. This way we have defined a new system of units that we call natural units.
   a) In this new system what are the units of length, time, mass, momentum and angular momentum
   b) Convert back to SI: How many meters is a length of $1(\mathrm{GeV})^{-1}$?
   c) Are the two unit systems equivalent? If yes, where are the constraints in the SI system?

   One can fix the electrical unit by imposing $\epsilon_0 = 1$.
   d) What is the unit of the electric charge?
   e) What is the unit and value for the electric fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$ in SI and natural units?
   f) How would you define a constant of nature?

2. (20%) Show that the normalization of a single particle state in Fock space $|\vec{q}\rangle = \sqrt{2E_q}a^\dagger(\vec{q})|0\rangle$ is chosen such that

$$\langle\vec{p}|\vec{q}\rangle = 2E_p(2\pi)^3\delta^{(3)}(\vec{p}-\vec{q}) \,,$$

   is Lorentz-invariant.

3. (40%) Show that

$$G(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{1}{k^2-m^2} \,,$$

   is a Green's function of the Klein-Gordon equation, i.e. that it solves the inhomogeneous Klein-Gordon equation

$$(\partial^2 + m^2)G(x-y) = -\delta^{(4)}(x-y) \,. \tag{1}$$

   Evaluate the k^0 integral in $G(x-y)$ by shifting both poles into the upper half of the complex plane (i.e. to $-E_k + i\epsilon$ and $E_k + i\epsilon$) and taking $\epsilon \to 0$.

4. (30%) Evaluate the function

$$\langle 0|\phi(x)\phi(y)|0\rangle = D(x-y) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}e^{-ip(x-y)} \,,$$

   for $(x-y)$ spacelike so that $(x-y)^2 = -r^2$, explicitly in terms of Bessel functions. *Hint:* Use the relation

$$\int_0^\infty dx \frac{\cos(ax)}{\sqrt{\beta^2+x^2}} = K_0(a\beta) \,,$$

   where K_0 is a modified Bessel function of the second kind. Further you will have to find a relation between $K_0(x)$ and $K_1(x)$.