Introduction to QFT
Assignment 4

Due on 25.11.16

1. (33%) Show that the Langrangian density

$$\mathcal{L} = \frac{1}{2}[(\partial\phi_1)^2 + (\partial\phi_2)^2] - \frac{m}{2}(\phi_1^2 + \phi_2^2) - \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2 ,$$

   is invariant under the transformation

$$\phi_1 \to \phi_1' = \phi_1 \cos\theta - \phi_2 \sin\theta ,$$
$$\phi_2 \to \phi_2' = \phi_1 \sin\theta + \phi_2 \cos\theta .$$

   Find the corresponding Noether current and charge.

2. (33%) Study the invariance properties of the Dirac Lagrangian, $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$, under chiral transformations

$$\psi(x) \to \psi'(x) = e^{i\alpha\gamma_5}\psi(x) ,$$

   where α is a constant. Find the Noether current and its four-divergence.

3. (33%) Let

$$f(x) = \int \frac{d^3(p)}{(2\pi)^3\sqrt{2\omega_p}} \tilde{f}(p) e^{-ipx} ,$$

   be a classical field that solves the Klein-Gordon equation. Define the operators

$$a = C\int \frac{d^3(p)}{(2\pi)^3\sqrt{2\omega_p}} \tilde{f}^*(p) a(\mathbf{p}) , \quad a^\dagger = C\int \frac{d^3(p)}{(2\pi)^3\sqrt{2\omega_p}} \tilde{f}^(p) a^\dagger(\mathbf{p}) , \tag{1}$$

   where $a(\mathbf{p})$ and $a^\dagger(\mathbf{p})$ are the annihilation and creation operators for the Klein-Gordon field, and C is a normalization constant.

   a) Calculate the following commutators

$$[a(\mathbf{p}), a^\dagger] , \quad [a(\mathbf{p}), a] .$$

   b) Prove that

$$[a(\mathbf{p}), (a^\dagger)^n] = n\frac{C\tilde{f}(p)}{\sqrt{2\omega_p}}(a^\dagger)^{n-1} .$$

   c) Show the the state $|z\rangle = e^{-|z|^2/2}e^{za^\dagger}|0\rangle$ is an eigenstate of the operator $a(\mathbf{p})$.