Introduction to QFT
Assignment 3

Due on 18.11.16

1. (33%) Show for every representation of gamma matrices that $S^{\mu\nu} = \frac{1}{2}\sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ are a representation of the generators of the Lorentz group, i. e.

$$[S^{\mu\nu}, S^{\rho\sigma}] = i(g^{\nu\rho}S^{\mu\sigma} - g^{\mu\rho}S^{\nu\sigma} - g^{\nu\sigma}S^{\mu\rho} + g^{\mu\sigma}S^{\nu\rho}) \ .$$

2. (33%) Show that the Lagrangian density

$$\mathcal{L} = -\frac{1}{2}\left[\partial_\alpha \phi_\beta(x)\right]\left[\partial^\alpha \phi^\beta(x)\right] + \frac{1}{2}\left[\partial_\alpha \phi^\alpha(x)\right]\left[\partial_\beta \phi^\beta(x)\right] + \frac{\mu^2}{2}\phi_\alpha(x)\,\phi^\alpha(x) \ ,$$

for the real vector field $\phi^\alpha(x)$ leads to the field equations

$$\left[g_{\alpha\beta}\left(\Box + \mu^2\right) - \partial_\alpha \partial_\beta\right]\phi^\beta(x) = 0 \ ,$$

and that the field $\phi^\alpha(x)$ satisfies the Lorentz condition: $\partial_\alpha \phi^\alpha(x) = 0$.

3. (33%) Find the Euler-Lagrange equations for the following Lagrangian densities:

$$a)\ \ \mathcal{L} = (\partial_\mu \phi - ieA_\mu \phi)(\partial^\mu \phi^* + ieA^\mu \phi^*) - m^2\phi^*\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},$$

$$b)\ \ \mathcal{L} = \bar{\psi}\left(i\gamma_\mu \partial^\mu - m\right)\psi + \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 - ig\bar{\psi}\gamma_5\psi\phi \ ,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.