Introduction to QFT
Assignment 10

Due on 03.02.16

1. (30%) Evaluate explicitly (that is, with no integrals left) the commutator

$$iD^{\mu\nu}\left(x-y\right)=\left[A^{\mu}\left(x\right),A^{\nu}\left(y\right)\right], \tag{1}$$

in the Lorenz gauge.

2. (30 %)The arbitrary state not containing any transversal photons has the form

$$|\Phi\rangle=\sum_{n}C_{n}|\Phi_{n}\rangle, \tag{2}$$

where C_n are constants and

$$|\Phi_{n}\rangle=\int d^{3}k_{1}...d^{3}k_{n}\,f\left(\mathbf{k}_{1}...\mathbf{k}_{n}\right)\prod_{i=1}^{n}\left(a_{0}^{\dagger}\left(\mathbf{k}_{i}\right)-a_{3}^{\dagger}\left(\mathbf{k}_{i}\right)\right)|0\rangle, \tag{3}$$

where $f\left(\mathbf{k}_1...\mathbf{k}_n\right)$ are arbitrary functions. The vacuum is $|\Phi_0\rangle=|0\rangle$

a) Prove that $\langle\Phi_n|\Phi_n\rangle=\delta_{n,0}$.

b) Show that for these states $\langle A^{\mu}\rangle$ is a pure gauge, i.e. $\langle\Phi|A^{\mu}\left(x\right)|\Phi\rangle=\partial^{\mu}\Lambda$, where Λ is a function that you must find.

3. (40 %) If

$$i\mathcal{M}=\bar{u}\left(\mathbf{p},r\right)\gamma_{\mu}\left(1-\gamma^{5}\right)u\left(\mathbf{q},s\right)\epsilon^{\mu}\left(\mathbf{k},\lambda\right), \tag{4}$$

calculate the sum

$$\sum_{\lambda=1}^{2}\sum_{r,s=1}^{2}|\mathcal{M}|^{2} \tag{5}$$