# Introduction to QFT
# Assignment 1

Due on 4.11.16

1. (40%) Find the energy spectrum for a scalar particle in a constant magnetic field $\boldsymbol{B} = B\boldsymbol{e}_z$.
   *Hint:* The interaction with the electromagnetic field is introduced via the replacement $\partial_\mu \to \partial_\mu + iqA_\mu$, where $q$ is the electric charge of the particle.

2. (20%) Prove the following properties of the matrices $\alpha_i$ and $\beta$:
   a) $\alpha_i$ and $\beta$ are Hermitian.
   b) $\mathrm{Tr}\,\alpha_i = \mathrm{Tr}\,\beta = 0$.
   c) The eigenvalues of $\alpha_i$ and $\beta$ are $\pm 1$.
   d) The dimension of $\alpha_i$ and $\beta$ is even.

3. (40 %) There are infinitely many different choices to represent the Dirac matrices $\gamma^\mu$. Two of the most often used representations are

   Dirac representation: $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$,

   Weyl representation: $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$.

   a) What is the explicit form of $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ in both representations?
   b) Show,without using an explicit representation, that the operators $P_R = \frac{1}{2}(1+\gamma^5)$ and $P_L = \frac{1}{2}(1-\gamma^5)$ are projectors.
   c) Check, for both Weyl and Dirac representation, that $\psi_R(x) = P_R\psi(x)$ and $\psi_L(x) = P_L\psi(x)$ are eigenstates of the chirality operator $\gamma^5$.

   We have now introduced the concept of chirality ("handedness") and we call $\psi_R$ the right-handed and $\psi_L$ the left-handed component of a Dirac fermion. The Weyl representation is particularly useful for the concept of chirality.

   d) Chirality is not a good quantum number for finite fermion mass $m \neq 0$. Explain why?

   A related concept is called helicity. It is defined as the projection of the particle's spin along the direction of its momentum. $h = \boldsymbol{\Sigma} \cdot \frac{\boldsymbol{p}}{|p|}$, with $\Sigma_i = \frac{i}{4}\epsilon^{ijk}[\gamma_j, \gamma_k]$.

   e) Show that for massless fermions the concepts of chirality and helicity are the same. *Hint:* You can do this by introducing the 4-spinor $\psi(x) = \begin{pmatrix} \tilde{\psi}_L(x) \\ \tilde{\psi}_R(x) \end{pmatrix}$ and plugging the 4-spinor into the massless Dirac equation in the Weyl representation.