Einführung in das Standardmodell der Teilchenphysik

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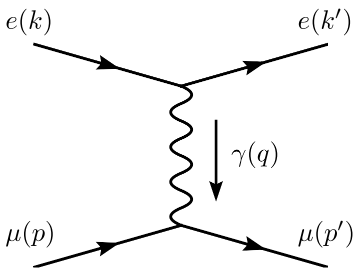
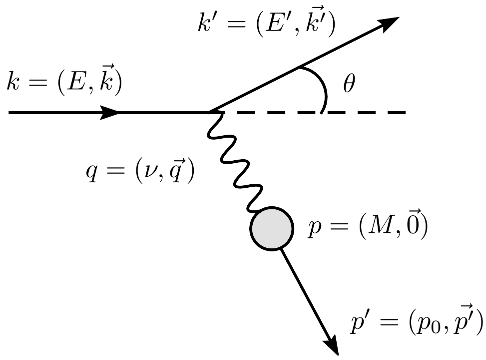
Sheet 8

Hand-in: Wed 28th June 2017 (12am)
Postfach von S. Schmiemann oder P. Scior KP306

Problem 1: e$^-$-μ-scattering. **[12 Points]**





(a) (1 Point) Give the corresponding matrix element $\mathcal{M}$.

(b) (3 Points) The squared spin-average matrix element can be written as

$$\overline{|\mathcal{M}|^2} = \frac{e^4}{q^4} L^{\mu\nu}_{(e)} L^{(muon)}_{\mu\nu} \,,$$

where the lepton tensors $L^{\mu\nu}$ contain the spin sums and the average factors. Calculate the lepton tensors $L^{\mu\nu}$ for the electron and muon. In the end you should receive

$$\overline{|\mathcal{M}|^2} \;=\; \frac{8e^4}{q^4}\Big[(k.p)(k'.p') + (k'.p)(k.p') - M^2(k.k')\Big] \,,$$

(c) (2 Points) In the following we neglect the electron mass m. Use the relation $q^2 = (k - k')^2$ to show that the squared and spin averaged matrix element is equal to

$$\overline{|\mathcal{M}|^2} \;=\; \frac{8e^4}{q^4}\Big[-\frac{1}{2}q^2(k.p - k'.p) + 2(k'.p)(k.p) + \frac{1}{2}M^2q^2\Big] \,.$$

(d) (3 Points) By using energies and scattering angles of the laboratory frame rewrite the result from (c) to

$$\overline{|\mathcal{M}|^2} \;=\; \frac{8e^4}{q^4}\Big[-\frac{1}{2}q^2M(E - E') + 2M^2EE' + \frac{1}{2}M^2q^2\Big] \,.$$

Now using the fact that the scattered particle stays in tact ($p' = p + q$ and $p'^2 = p^2 = M^2$) derive

$$\overline{|\mathcal{M}|^2} \;=\; \frac{8e^4}{q^4}\,2M^2EE'\Big[\cos^2\frac{\theta}{2} + \frac{(E - E')}{M}\sin^2\frac{\theta}{2}\Big] \,.$$

(e) (2 Points) The cross-section formula in the laboratory system is given by

$$d\sigma \;=\; \frac{1}{4ME}\,\overline{|\mathcal{M}|^2}\,\frac{1}{(2\pi)^2}\frac{E'}{4M}\,dE'd\Omega\;\delta(\nu+\frac{q^2}{2M})\,.$$

Replace $\overline{|\mathcal{M}|^2}$ with the result from (d) and perform the integration over E' to receive:

$$\left(\frac{d\sigma}{d\Omega}\right)=\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{1}{1+\frac{2E}{M}\sin^2\frac{\theta}{2}}\left[\cos^2\frac{\theta}{2}+\frac{\frac{2E^2}{M^2}\sin^2\frac{\theta}{2}}{1+\frac{2E}{M}\sin^2\frac{\theta}{2}}\sin^2\frac{\theta}{2}\right].$$

with $\alpha=\frac{e^2}{4\pi}$.

(f) (1 Point) Use $E'=\frac{E}{1+\frac{2E}{M}\sin^2\frac{\theta}{2}}$ and $q^2=-2M(E-E')$ and compare the result with the Mott scattering formula for protons (Z=1)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}=\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\cos^2\frac{\theta}{2}\,.$$

Problem 2: Inelastic scattering. [10 Points]

Up to now we have treated the case where the target particle stayed in tact after the collision. This is of course natural in a case of a muon as it is a point-like fundamental particle but in case of a proton one can imagine different outcomes of the scattering. When large energies are transferred from the electron to the proton, the proton can either transform into a different hadron (e.g. an excited state of the proton such as a Δ-resonance) or break up completely giving rise to a lot of different hadrons (see Fig. **??**).

We will analyze the deep inelastic scattering which can be characterized by the energy transferred to the proton $Q^2 \gg M^2$ and by the fact that the invariant mass of the resulting hadrons is much bigger than the mass of the proton $W^2 \gg M^2$ which guarantees that the proton breaks up into many hadrons. The invariant squared mass W^2 can be written as

$$W^2=(p+q)^2=M^2+Q^2\,\frac{(1-x)}{x}\,, \tag{1}$$

where

$$Q^2=-q^2\,,\qquad x=\frac{Q^2}{2p.q}\,. \tag{2}$$

In the case of the inelastic scattering, the final state is not a single spin-1/2 particle but a collection of many different particles. Therefore we have to parameterize photon-proton-X interaction, where X is anything the proton can break up into. We begin at the level of a cross-section which we write as

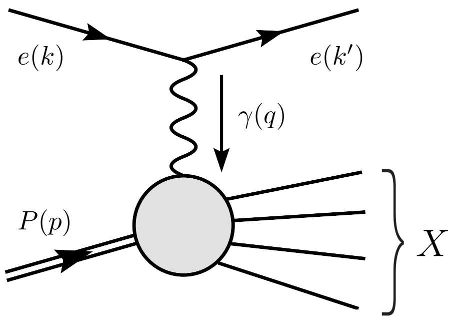
$$d\sigma=\frac{1}{4ME}\left(\frac{e^4}{q^4}\,L^{\mu\nu}_{(\mathrm{e})}\,H^X_{\mu\nu}\right)\,\frac{d^3k'}{(2\pi)^3}\frac{1}{2E'}\prod_{n=1}^{N}\frac{d^3p'_n}{(2\pi)^3}\frac{1}{2(p'_n)_0}\,(2\pi)^4\delta^{(4)}(p+k-k'-\sum p'_n)\,, \tag{3}$$

where $H^X_{\mu\nu}$ stands for the photon-proton-X interaction and there is an added phase-space integration for all N particles in the X final state. We are interested in all processes where the proton breaks up irrespective of N and so we rewrite the cross-section in the following form

$$d\sigma=\frac{4\pi M}{4ME}\left(\frac{e^4}{q^4}\,L^{\mu\nu}_{(\mathrm{e})}\,W_{\mu\nu}\right)\,\frac{d^3k'}{(2\pi)^3}\frac{1}{2E'}\,, \tag{4}$$

where $L^{\mu\nu}_{(\mathrm{e})}$ is the known leptonic tensor

$$L^{\mu\nu}_{(\mathrm{e})}=2\Big(k^\mu k'^\nu+k'^\mu k^\nu-(k'.k-m^2)g^{\mu\nu}\Big)\,, \tag{5}$$



$$W^2 = (p_1' + p_2' + \ldots + p_n')^2 \gg M^2$$

$$\Downarrow$$

'Inelastic'

$$Q^2 = -q^2 \gg M^2 \Rightarrow \text{'Deep'}$$

Figure 1: Inelastic scattering of electrons on protons with some useful kinematic variables.

and we have simply re-defined the hadronic tensor to include all the dependence on N as well

$$W_{\mu\nu} = \frac{1}{4\pi M} \prod_{n=1}^{N} \frac{d^3 p_n'}{(2\pi)^3} \frac{1}{2(p_n')_0} H_{\mu\nu}^X \, (2\pi)^4 \delta^{(4)}(p + k - k' - \sum p_n') \,. \tag{6}$$

The cross-section can be now written in terms of lab frame variables as

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{q^4} \frac{E'}{E} \, L^{\mu\nu}_{(\mathrm{e})} \, W_{\mu\nu} \,, \tag{7}$$

where unlike in the elastic scattering the energy E' of the outgoing electron is a free variable and not related to E and θ. This is a consequence of the fact that we study a case where the proton breaks up and so we have a multi-particle phase-space to integrate over. This time we cannot calculate the tensor as was the case with muons in the elastic scattering.

(a) (2 Points) Explain why the hadronic tensor $W^{\mu\nu}$ can be parametrized by the Lorentz structure

$$W^{\mu\nu} = -g^{\mu\nu} W_1 + \frac{p^\mu p^\nu}{M^2} W_2 + \frac{i}{2M^2} \varepsilon^{\mu\nu\rho\lambda} p_\rho q_\lambda \, W_3 + \frac{q^\mu q^\nu}{M^2} W_4 + \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} W_5 \,.$$

(b) (2 Points) We can now make use of the fact that the hadronic tensor couples to the photon an so due to the conservation of the charge current, we have

$$q^\mu W_{\mu\nu} = 0 \,.$$

Express the factors W_4 and W_5 in terms of W_1 and W_2.

(c) (2 Points) Moreover because we assume for simplicity that the scattering is mediated only by an exchange of a photon, the interaction conserves parity as an electromagnetic interaction should. What does it means for the term $\sim \epsilon^{\mu\nu\rho\lambda}$ in the hadronic tensor? Rewrite the hadronic tensor only in terms of W_1 and W_2

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \, \frac{1}{M^2} \left(p^\mu - \frac{p.q}{q^2} \, q^\mu \right) \left(p^\nu - \frac{p.q}{q^2} \, q^\nu \right) .$$

(d) (4 Points) Derive the differential cross-section

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{2F_1}{M} \, \sin^2 \frac{\theta}{2} + \frac{F_2}{E - E'} \, \cos^2 \frac{\theta}{2} \right) \tag{8}$$

with the structure functions $F_1 = M W_1$ and $F_2 = \frac{p \cdot q}{M} W_2$.