Einführung in das Standardmodell der Teilchenphysik

Lectures: Prof. M. Klasen & Prof. D. Frekers

Exercises: Dr. K. Kovařík, Dr. P. Scior, S. Schmiemann

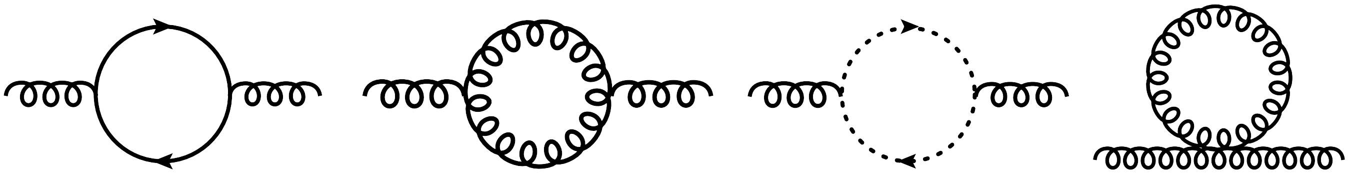
Sheet 7

Hand-in: Wed 21th June 2017 (12am)

Postfach von S. Schmiemann oder P. Scior KP306

Problem 1: Gluon vacuum polarization and its renormalization. **[22 Points]**

The gluon 2-point function at one loop consists of the following 4 diagrams



The amplitude which corresponds to these diagrams can be written as $\mathcal{M}^{\mu\nu,ab}$.

(a) (2 Points) Show that if the amplitude satisfies the Ward identity as it should, it has to have the following form

$$\mathcal{M}^{\mu\nu,ab}(p) = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi^{ab}(p^2)$$

(b) (4 Points) The amplitude for the contribution with the loop involving massless fermions is

$$\mathcal{M}^{\mu\nu,ab} = -g_s^2\,\mathrm{Tr}\big(T^a T^b\big)\,\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{1}{(q^2+i\varepsilon)((q-p)^2+i\varepsilon)}\mathrm{Tr}\Big[\gamma^\mu(\not{q}-\not{p})\gamma^\nu\not{q}\Big]$$

Show that this contribution gives

$$\mathcal{M}^{\mu\nu,ab} = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\delta^{ab}\,T_F\frac{ig_s^2}{16\pi^2}\,p^2\left(-\frac{4}{3}\right)\left[\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}+\frac{5}{3}\right]$$

(c) (2 Points) Show that the contribution involving the 4-gluon coupling vanishes.

(d) (4 Points) The amplitude for the contribution with the loop involving gluons is

$$\mathcal{M}^{\mu\nu,ab} = -\frac{g_s^2}{2}f^{acd}f^{bcd}\,\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{N^{\mu\nu}}{(q^2+i\varepsilon)((q-p)^2+i\varepsilon)}$$

where

$$N^{\mu\nu} = \big[g^{\mu\rho}(2p-q)^\sigma + g^{\rho\sigma}(2q-p)^\mu + g^{\sigma\mu}(-p-q)^\rho\big]\times\big[g^{\nu\rho}(q-2p)^\sigma + g^{\rho\sigma}(p-2q)^\nu + g^{\sigma\nu}(p+q)^\rho\big]$$

Show that this contribution gives

$$\mathcal{M}^{\mu\nu,ab} = -\frac{ig_s^2}{2}\delta^{ab}C_A\frac{\mu^{4-D}}{(4\pi)^{D/2}}\int_0^1 dx\left(\frac{1}{\Delta}\right)^{2-D/2}\Big\{g^{\mu\nu}3(d-1)\Delta\,\Gamma\Big(1-\frac{D}{2}\Big)$$
$$+p^\mu p^\nu\big[6(x^2-x+1)-d(1-2x)^2\big]\Gamma\Big(2-\frac{D}{2}\Big)+g^{\mu\nu}p^2(-2x^2+2x-5)\Gamma\Big(2-\frac{D}{2}\Big)\Big\}$$

where $\Delta = p^2x(x-1)$.

(e) (3 Points) The amplitude for the contribution with the loop involving ghosts is

$$\mathcal{M}^{\mu\nu,ab} = -g_s^2 f^{acd} f^{bcd} \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{q^\mu (q-p)^\nu}{(q^2+i\varepsilon)((q-p)^2+i\varepsilon)}$$

$$\mathcal{M}^{\mu\nu,ab} = i g_s^2 \delta^{ab} C_A \frac{\mu^{4-D}}{(4\pi)^{D/2}} \int_0^1 dx \left(\frac{1}{\Delta}\right)^{2-D/2} \left\{ g^{\mu\nu} \frac{\Delta}{2} \Gamma\left(1-\frac{D}{2}\right) + p^\mu p^\nu x(1-x) \Gamma\left(2-\frac{D}{2}\right) \right\}$$

where again $\Delta = p^2 x(x-1)$.

(f) (4 Points) Show that the sum of the diagrams where gluons and ghosts are running in the loop expanded in $\varepsilon = (4-D)/2$ gives

$$\mathcal{M}^{\mu\nu,ab} = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) \delta^{ab}\, C_A \frac{i g_s^2}{16\pi^2} p^2 \left[\frac{5}{3\,\varepsilon} + \frac{5}{3} \ln \frac{\mu^2}{-p^2} + \frac{31}{9}\right]$$

(g) (3 Points) In order to make the gluon vacuum polarization finite, we absorb the divergence into the gluon wave-function renormalization constant $Z_3 = 1 + \delta_3$. Show that

$$\delta_3 = \frac{1}{\varepsilon} \frac{g_s^2}{16\pi^2} \left[\frac{5}{3} C_A - \frac{4}{3} n_f T_F\right]$$

where we have included the fermion loop for n_f fermions.