Einführung in das Standardmodell der Teilchenphysik

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Exercises: Dr. K. Kovařík, Dr. P. Scior, S. Schmiemann

Sheet 9

Hand-in: Wed 5th Juli 2017 (12am)
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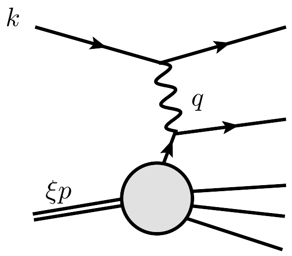
Problem 1: Deep inelastic scattering **[17 Points]**



In the last problem set we have analyzed the inelastic scattering of electrons on protons. Now we will carry on taking a look at the deep inelastic scattering with $Q^2 = -q^2 \gg m_p^2$.
However we will view the proton-electron scattering differently. Let us assume that the proton is made up of some spin$\frac{1}{2}$ constituents that we name *partons*. Furthermore we assume that the scattering of the electron with with the proton only involves scattering of the electron with one of those partons, carrying only a fraction ξp of the proton's momentum (see figure above). This time we are able to compute the hadronic tensor $W_{\mu\nu}$ (or strictly speaking the quark tensor) explicitly.

(a) (5 Points) Show that $W_{\mu\nu}$ is given by

$$W_{\mu\nu} = \frac{1}{4\pi} e_q^2 \int \frac{d^3p'}{(2\pi)^3} \frac{1}{2\xi E_{p'}} H_{\mu\nu} (2\pi)^4 \delta^{(4)}(\xi p + k - k' - p')\,,$$

with

$$h_{\mu\nu} = 2\left(\xi p^\mu p'^\nu + \xi p'^\mu p^\nu - (\xi p'.p - m^2) g^{\mu\nu}\right)\,,$$

and e_q the electric charge of the parton in units of e.

(b) (3 Points) Since we consider an elastic process you can perform all the momentum integrals in the final state. Show the following relations:

$$\int \frac{d^3p'}{(2\pi)^3} \frac{1}{2E_{p'}} = \int \frac{d^4p'}{(2\pi)^4} 2\pi\delta((\xi p + q - p')^2) = \int \frac{d^4p'}{(2\pi)^4} \frac{2\pi}{2p.q} \delta(\xi + \frac{q^2}{2p.q})\,.$$

(c) (5 Points) Show that $W_{\mu\nu}$ is given by

$$W_{\mu\nu} = \frac{e_q^2}{2\xi p.q} \left[2\xi^2 p_\mu p_\nu - g_{\mu\nu} \xi p.q\right] \delta(\xi - x)\,,$$

where x is the Bjorken-x variable. To derive this relation you had to drop terms proportional to q^μ and q^ν. Why is it allowed to drop these terms?

HINT: Futher also consider $m^2 \ll p.q$.

(d) (4 Points) Argue (use the results from sheet 8 Problem 1) that the differential cross section for the elastic scattering of an electron on a single parton is given by

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{2F_1(x,Q)}{m} \sin^2 \frac{\theta}{2} + \frac{F_2(x,Q)}{E-E'} \cos^2 \frac{\theta}{2} \right) ,$$

with $F_1(x,Q) = \frac{me_q^2}{2}\delta(x-\xi)$ and $F_2(x,Q) = me_q^2\delta(x-\xi)$.

This is an amazing and highly non-trivial result. We have just shown that in the parton model, the structure function F_1,F_2 are independent of Q at leading order. This was experimentally observed in several experiments in the 1950s.