Einführung in das Standardmodell der Teilchenphysik

Lectures: Prof. M. Klasen & Prof. D. Frekers

Exercises: Dr. K. Kovařík, Dr. P. Scior, S. Schmiemann

Sheet 11

Hand-in: Wed 19th Juli 2017 (12am)
Postfach von S. Schmiemann oder P. Scior KP306

Problem 1: n real scalar fields [14 Points]

Consider a theory with n real scalar fields and Lagrangian

$$\mathcal{L} = \frac{1}{2}\sum_n \partial_\mu\phi_n\partial^\mu\phi_n + \frac{1}{2}\mu^2\sum_n \phi_n\phi_n - \frac{\lambda}{4}\left(\sum_n \phi_n\phi_n\right)^2 .$$

(a) (2 Points) What are the global symmetries of this theory?

(b) (2 Points) What are all the possible vacua of this theory? Are all the vacua equivalent?

(c) (5 Points) Using simply group considerations, how many Goldstone bosons are there?

(d) (2 Points) Write down the Lagrangian for small excitations around one of the vacua. How many Goldstone bosons are there?

(e) (3 Points) If instead of real fields we had complex fields and the Lagrangian was characterized by $SU(N)$ symmetry, how many Goldstone bosons we should expect in this case?
Use only group considerations!

Problem 2: The scalar Mexican hat potential [6 Points]

Consider the case of one hermitian scalar field ϕ with scalar potential

$$V_0(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 .$$

Show that $V_0(\phi)$ has degenerate minimum at $\phi = \pm v$, with $v = \sqrt{\frac{\mu^2}{\lambda}}$.
Suppose now we add a cubic term to $V_0(\phi)$

$$\tilde{V}_0(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{2}{3}\xi\phi^3 + \frac{\lambda}{4}\phi^4 .$$

Show that the degeneracy in the minimum of $V_0(\phi)$ is now removed. Find the true minimum of $\tilde{V}_0(\phi)$.
Also, show that, as a function of the parameter ξ, the vacuum expectation value (VEV) $\langle\phi\rangle_0$ changes discontinuously from $\langle\phi\rangle_0 = -v$ to $\langle\phi\rangle_0 = +v$ as ξ changes from positive to negative values going through 0.
HINT: Do all the calculation in the hypothesis of very small ξ.