4.3 Weak interactions (29)

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4.3.1 Electroweak symmetry breaking

Local symmetry group and associated gauge bosons:

- $SU(2)_L$: Weak isospin, coupling g. Fundamental doublets $\binom{u}{d}_L, \binom{\nu_e}{e}_L$ etc. Adjoint triplet W_μ^a.
- $U(1)_Y$: Weak hypercharge, coupling g'. Fundamental singlets u, d, ν_e, e etc. Adjoint singlet B_μ.
- (Electroweak) standard model (Glashow-Salam-Weinberg model): $SU(2)_L \otimes U(1)_Y \to U(1)_{em.}$

Higgs multiplet: (Complex) fundamental doublet H, hypercharge $Y = 1/2$.

Yang-Mills Lagrangian:

$$\boxed{\mathcal{L} = -\frac{1}{4}(W_{\mu\nu}^a)^2 - \frac{1}{4}(B_{\mu\nu})^2 + (D_\mu H)^\dagger (D^\mu H) + m^2 H^\dagger H - \lambda (H^\dagger H)^2}$$

- Covariant derivative: $\boxed{D_\mu H = \partial_\mu H - ig W_\mu^a \tau^a H - \frac{1}{2} i g' B_\mu H}$ Higgs hypercharge $\to$ factor $1/2$
- Higgs potential: $V(H) = -m^2 |H|^2 + \lambda |H|^4$. Normalization of $\lambda |H|^4$ is conventional.

Field expansion: Potential minimum (vacuum expectation value VEV) is at $v = \frac{m}{\sqrt{\lambda}}$

$$\boxed{H(x) = e^{2i\frac{\pi^a(x)\tau^a}{v}} \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}}$$

Factor $\frac{1}{\sqrt{2}}$ converts between canonical normalizations of complex $H(x)$ and real $h(x)$.

Higgs kinetic term: Unitary gauge: $\pi^a(x) = 0$, at the minimum $h(x) = 0$.

$$\boxed{\begin{aligned} |D_\mu H|^2 &= g^2 \frac{v^2}{8} (0\ 1) \begin{pmatrix} \frac{g'}{g} B_\mu + W_\mu^3 & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & \frac{g'}{g} B_\mu - W_\mu^3 \end{pmatrix} \begin{pmatrix} \frac{g'}{g} B_\mu + W_\mu^3 & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & \frac{g'}{g} B_\mu - W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= g^2 \frac{v^2}{8} \left[(W_\mu^1)^2 + (W_\mu^2)^2 + \left(\frac{g'}{g} B_\mu - W_\mu^3\right)^2 \right] \end{aligned}}$$

- Three gauge boson mass terms, created by electroweak symmetry breaking (EWSB)
- Diagonalization by rotation (no rescaling, since B_μ and W_μ^a canonically normalised)

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \quad \Leftrightarrow \quad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

$$A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu \quad \Leftrightarrow \quad W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$$

- Weak mixing angle $\boxed{\tan\theta_W = \frac{g'}{g}}$

Kinetic terms of Lagrangian for neutral gauge bosons:

$$\boxed{\mathcal{L}_{kin.} = -\frac{1}{4}(A_{\mu\nu})^2 - \frac{1}{4}(Z_{\mu\nu})^2 + \frac{1}{2} m_Z^2 Z_\mu Z^\mu}$$

with Z-boson mass $m_Z = \frac{gv}{2\cos\theta_W}$

Photon interactions:

$$g[A_\mu, W_\nu^a \tau^a] = g \sin\theta_W W_\mu^3 W_\nu^a [\tau^3, \tau^a]$$

No interactions with neutral W_μ^3.

Interactions with $W_\mu^{1,2}$ have coupling $e := g \cdot \sin\theta_W = g' \cdot \cos\theta_W$

Ladder operators: $\tau^\pm = \frac{1}{\sqrt{2}}(\tau^1 \pm i\tau^2)$. Associated bosons: $W_\mu^\pm := \frac{1}{\sqrt{2}}(W_\mu^1 \pm i W_\mu^2)$, since

$W^a \tau^a = W^+ \tau^+ + W^- \tau^- + W^3 \tau^3$

Since $[\tau^3, \tau^\pm] = \pm \tau^\pm$, the $W^\pm$ bosons coupling to $\tau^\pm$ have charge ± 1

(up to now just a convention!).

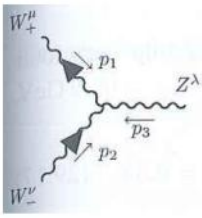
Yang-Mills-Lagrangian after electroweak symmetry breaking:

$$\boxed{\begin{aligned}
\mathcal{L}_{\text{gauge}} = &-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}Z_{\mu\nu}^2 + \frac{1}{2}m_Z^2 Z^\mu Z_\mu - \frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-)\\
&+ m_W^2 W_\mu^+ W_\mu^- - ie\cot\theta_w \Big[\partial_\mu Z_\nu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-)\\
&\qquad + Z_\nu(-W_\mu^+ \partial_\nu W_\mu^- + W_\mu^- \partial_\nu W_\mu^+ + W_\mu^+ \partial_\mu W_\nu^- - W_\mu^- \partial_\mu W_\nu^+)\Big]\\
&- ie\Big[\partial_\mu A_\nu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-)\\
&\qquad + A_\nu(-W_\mu^+ \partial_\nu W_\mu^- + W_\mu^- \partial_\nu W_\mu^+ + W_\mu^+ \partial_\mu W_\nu^- - W_\mu^- \partial_\mu W_\nu^+)\Big]\\
&- \frac{1}{2}\frac{e^2}{\sin^2\theta_w} W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}\frac{e^2}{\sin^2\theta_w} W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-\\
&- e^2\cot^2\theta_w (Z_\mu W_\mu^+ Z_\nu W_\nu^- - Z_\mu Z_\mu W_\nu^+ W_\nu^-) + e^2(A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\mu W_\nu^+ W_\nu^-)\\
&+ e^2\cot\theta_w \Big[A_\mu W_\mu^+ W_\nu^- Z_\nu + A_\mu W_\mu^- Z_\nu W_\nu^+ - W_\mu^+ W_\mu^- A_\nu Z_\nu\Big], \qquad (29.9
\end{aligned}}$$

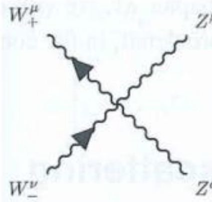
- Weak gauge boson masses: $m_W = \frac{vg}{2}$, $m_Z = \frac{gv}{2\cos\theta_W} = \frac{v}{2}\sqrt{g^2 + g'^2} = \frac{m_W}{\cos\theta_W}$

- Theoretical prediction: $m_W < m_Z$

Feynman rules for gauge boson vertices:



$$= -ie\cot\theta_w \left[g^{\mu\nu}(p_1 - p_2)^\lambda + g^{\nu\lambda}(p_2 - p_3)^\mu + g^{\lambda\mu}(p_3 - p_1)^\nu\right]$$



$$= ie^2\cot^2\theta_w \left[g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu} - 2g^{\alpha\beta}g^{\mu\nu}\right]$$

Expanded Higgs Lagrangian:

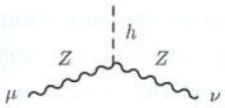
$$\boxed{\mathcal{L} = -\frac{1}{2}h(\Box + m_h^2)h - g\frac{m_h^2}{4m_W}h^3 - \frac{g^2}{32}\frac{m_h^2}{m_W^2}h^4 + 2\frac{h}{v}\left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu^2\right) + \left(\frac{h}{v}\right)^2\left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu\right)}$$

with $m_h = \sqrt{2}\, m$ and $v = 2 m_W \sin\theta_W / e$.

No interactions with derivatives on h, no $hh\partial_\mu Z^\mu$ term.

Feynman rules for Higgs vertices:

$$= i\frac{e}{\sin\theta_w} m_W g^{\mu\nu}$$



$$= i\frac{e}{\sin\theta_w}\frac{m_Z^2}{m_W} g^{\mu\nu} = i\frac{e}{\sin\theta_w \cos^2\theta_w} m_W g^{\mu\nu},$$

Four free parameters:

- Couplings: $e = 0.303$, $\sin^2\theta_w = 0.223$ $\Leftrightarrow$ $g = \frac{e}{\sin\theta_w} = 0.64$, $g' = \frac{e}{\sin\theta_w} = 0.34$
- Masses: $v = \frac{2m_W}{g} = 247$ GeV (from G_F), λ $\Leftrightarrow$ $m_W = 80.4$ GeV, $m_h = 125$ GeV