4.2 Spontaneous symmetry breaking (28)

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Two possibilities to break a symmetry:

- Explicit: (Small) symmetry breaking term in the lagrangian (e.g. m_q break chiral symmetry)
- Spontaneous: lagrangian invariant, but ground state is not $\to$
  Phase transition at low $E(T)$.
  Effective Field Theory (e.g. Fermi model) can describe the broken phase very well.

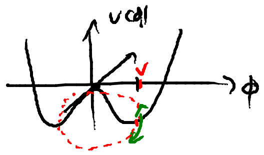
Example for spontaneous symmetry breaking: Ferromagnets (e.g. iron)

- Magnetic moment $\vec{M}(x)$ (related to local spin directions) rotationally symmetric at high T
- Below Curie temperature $T_C = 1032\,K$, $F = E - TS$ dominated by E from spin alignment.
  Aligned domains grow, long-range order emerges with correlation length ξ.
  Can expand $\vec{M}(x) = \vec{\mu} + \vec{s}(x)$ about the VEV $\vec{\mu}$ at $T=0$ $\to$
  Rotational symmetry is broken.
  $\vec{s}(x)$ are excitations $\to$ spin waves. Quanta are Goldstone bosons.

Particle physics:

- Continous global symmetry: Quanta are massless particles (e.g. pions in QCD, when $m_q = 0$)
- Continuous local (gauge) symmetry: Massless gauge bosons aquire mass $\to$ Higgs mechanism.

4.2.1 Discrete symmetries



Real scalar field ϕ with lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

Symmetric under $\phi \to -\phi$ $\Rightarrow$ Global $\mathbb{Z}_2$ symmetry

Describes ferromagnet near T_C (Ginzburg, Landau) with $m^2(T)$ and $\lambda(T)$ microscopically calculable

Expand $m^2(T) = c(T - T_c)$ $\Rightarrow$ $m^2 < 0$, when $T < T_c$. $\Rightarrow$

$V = -\mathcal{L}_{int}$ has a local maximum at $\phi = 0$.

Corresponds to spacelike momentum $\Rightarrow$ Replace $m^2 \to -m^2$.

V then has minima $v = \pm\sqrt{\frac{6m^2}{\lambda}}$.

Expand about positive solution $\phi = v + \tilde{\phi}$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \tilde{\phi})^2 + \frac{3m^4}{2\lambda} - m^2\tilde{\phi}^2 - \sqrt{\frac{\lambda}{6}}\, m\tilde{\phi}^3 - \frac{\lambda}{4!}\tilde{\phi}^4$$

Excitation $\tilde{\phi}$ has positive m^2 $\to$ Tachyon-free theory.

$\mathbb{Z}_2$ still there, but hidden, since $\tilde{\phi} \to -\tilde{\phi} - 2v$

Classical expectation: $\phi = v$ everywhere (satisfies equations of motion and dominates path integral)

4.2.2 Global symmetries

Global symmetry $\phi \to e^{-i\alpha}\phi$

$$0 = \frac{\delta\mathcal{L}}{\delta\alpha} = \sum_m \left\{ \left[\frac{\partial\mathcal{L}}{\partial\phi_m} - \partial^\mu \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_m)} \right] \frac{\delta\phi_m}{\delta\alpha} + \partial^\mu \left[\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_m)} \frac{\delta\phi_m}{\delta\alpha} \right] \right\}$$

Noether Current:

∂S $\quad$ $\delta\Phi$

$$J_\mu = \sum_m \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_m)} \frac{\delta \phi_m}{\delta \alpha}$$

is conserved on-shell, when equations of motion are satisfied, i.e. $\partial^\mu J_\mu(x) = 0$.

Conserved charge:

$$Q = \int d^3x \, J_0(x) = \int d^3x \sum_m \frac{\partial \mathcal{L}}{\partial \dot\phi_m} \frac{\delta \phi_m}{\delta \alpha}$$

with $\partial_t Q = \frac{1}{i}[H, Q] = 0$ is an operator in QFT, since fields are operators.

Conjugate field $\pi_m = \frac{\partial \mathcal{L}}{\partial \dot\phi_m}$ and its commutation relation

$$[\phi_n(\vec{x}), \pi_m(\vec{y})] = i\, \delta^{(3)}(\vec{x}-\vec{y})\, \delta_{nm}$$

lead to

$$[Q, \phi_n(\vec{y})] = \sum_m \int d^3x \, [\pi_m(\vec{x}), \phi_n(\vec{y})] \frac{\delta \phi_m(\vec{x})}{\delta \alpha} = -i \frac{\delta \phi_n(\vec{y})}{\delta \alpha}$$

i.e. the charge Q generates the symmetry transformation!

Spontaneous symmetry breaking

The symmetric vacuum $Q|0\rangle = 0$ is unstable.

Only the charged vacuum $Q|\Omega\rangle \neq 0$ is stable.

If the physical vacuum $|\Omega\rangle$ has energy E_0, i.e. $H|\Omega\rangle = E_0|\Omega\rangle$,

$$H\, Q|\Omega\rangle = [H, Q]|\Omega\rangle + QH|\Omega\rangle = E_0 Q|\Omega\rangle$$

shows that $Q|\Omega\rangle$ is degenerate with this vacuum.

Goldstone theorem

Spontaneous breaking of continuous global symmetries implies existence of massless particles.

Goldstone bosons with energy $E = E(\vec{p}) + E_0$ and momentum $\vec{p}$ are generated by

$$|\pi(\vec{p})\rangle = \frac{-2i}{F} \int d^3x \, J_0(x)|\Omega\rangle$$

(prefactor for convenience, $[F] = m$).

Since their energy $E(\vec{p}) \to 0$ for $\vec{p} \to 0$, they must be massless.

So, when $\langle \pi(\vec{q}) | J_\mu(y) | \Omega\rangle = i q_\mu F e^{i\vec{q}\vec{y}}$, then $\pi(\vec{q})$ is a Goldstone boson.

4.2.3 Linear sigma model

Complex scalar field ϕ with Lagrangian:

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) + m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4$$

Symmetric under global $U(1)$: $\phi(x) \to e^{i\alpha}\phi(x)$.

Unstable for $m^2 > 0$ around $\phi = 0$.

Potential: $V(\phi) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$

Particle masses: $m_{ij}^2 = \left.\frac{\partial^2 V}{\partial\phi_i \partial\phi_j}\right|_{\langle\Omega_\theta|\vec{\phi}|\Omega_\theta\rangle}$.

Minima at $|\phi|^2 = \frac{2m^2}{\lambda}$ have massive (radial direction) and massless (flat direction) excitations

Vacua: Infinitely many with $\langle\Omega_\theta|\phi|\Omega_\theta\rangle = \sqrt{\frac{2m^2}{\lambda}}\, e^{i\theta}$

Choose $|\Omega\rangle$ with $\langle\Omega|\phi|\Omega\rangle = v = \sqrt{\frac{2m^2}{\lambda}} \in \mathbb{R}$.

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Field expansion: $\phi(x) = v + \tilde{\phi}(x)$ with $\tilde{\phi}(x) \in \mathbb{C}$ or (more convenient)

$$\boxed{\phi(x) = \left(\sqrt{\frac{2m^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma(x)\right) e^{i\frac{\pi(x)}{F_\pi}}}$$

with $F_\pi \in \mathbb{R}$

Expanded Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 + \left(\sqrt{\frac{2m^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma(x)\right)^2 \frac{1}{F_\pi^2}(\partial_\mu\pi)^2 - \left(-\frac{m^4}{\lambda} + m^2\sigma^2 + \frac{1}{2}\sqrt{\lambda}\, m\sigma^3 + \frac{1}{16}\lambda\sigma^4\right)$$

Canonic normalization: $F_\pi = \sqrt{2}\, v$

Field $\pi(x)$ is massless → Goldstone boson, closely related to physical pion.

$\boxed{4.2.4 \quad SU(2)_L \times SU(2)_R}$

Application of these ideas to chiral symmetry in QCD:

$$\begin{pmatrix} u^L \\ d^L \end{pmatrix} \to g_L \begin{pmatrix} u^L \\ d^L \end{pmatrix} \qquad \begin{pmatrix} u^R \\ d^R \end{pmatrix} \to g_R \begin{pmatrix} u^R \\ d^R \end{pmatrix}$$

where

$$\mathcal{L}_{QCD} = i\bar{u}\not{D}u + i\bar{d}\not{D}d - m_u\bar{u}u - m_d\bar{d}d - \frac{1}{4}(F_{\mu\nu})^2 .$$

is symmetric (apart from mass terms).