3.1.4 Gluon propagator (cont.) (25.4)

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Becchi-Rouet-Stora-Tyutin (BRST) transformations:

$$\psi_i \to \psi_i + i(\theta c^a) T^a_{ij} \psi_j$$

$$A^a_\mu \to A^a_\mu + \frac{1}{g_s}\theta D_\mu c^a$$

$$\bar{c}^a \to \bar{c}^a - \frac{1}{g_s}\theta \frac{1}{\xi}\partial_\mu A^{a\mu}$$

$$c^a \to c^a - \frac{1}{2}\theta f^{abc} c^b c^c$$

Since we just redefined the gauge transformation, $\mathcal{L}_0$ is invariant.

Transformation of $\partial_\mu \bar{c}^a$ cancels transformation of $\frac{1}{2\xi}(\partial_\mu A^{a\mu})^2$.

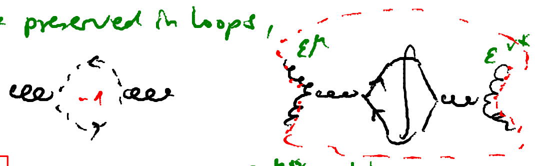
Transformation of A^a_μ induces $D_\mu c^a \to D_\mu c^a + \theta f^{abc}(D_\mu c^b) c^c$,

canceled by the additional transformation of c^a.

Note that we did not require c and $\bar{c}$ to be related!

Since BRST is an exact symmetry, it will be preserved in loops,

constraining the appearance of ghosts.



$\sum \varepsilon\varepsilon^{\mu\nu} \to g^{\mu\nu}$

Gluon propagator

$$\boxed{\nu,b \;\text{eee}\; \mu,a \;(p) = i\,\frac{-g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2}}{p^2 + i\varepsilon}\,\delta_{ab}}$$

"Covariant gauge"

Gauge-fixing and ghost Lagrangian in axial gauge:

$$\mathcal{L}_{gf.} + \mathcal{L}_{gh} = -\frac{1}{2\xi}(n^\mu A_\mu)^2 + \bar{c}\, n^\mu (\delta^{ac}\partial_\mu + g_s f^{abc} A^b_\mu) c^c$$

Choosing $n^\mu := (1,0,0,0)$ leads to $-\frac{1}{2\xi}(A^a_0)^2$ and thus $A_0{}^\mu \to 0$ ("axial gauge") for $\xi \to 0$.

Gluon propagator in axial gauge:

$$\boxed{i\Pi^{\mu\nu}_{ab} = \frac{i}{p^2 + i\varepsilon}\left[-g^{\mu\nu} + \frac{n^\mu p^\nu + n^\nu p^\mu}{n\cdot p} - \frac{(n^2 + \xi p^2)p^\mu p^\nu}{(n\cdot p)^2}\right]\delta_{ab}}$$

"axial gauge"

Satisfies $n_\mu \Pi^{\mu\nu}_{ab} = 0$, when $\xi = 0$, i.e. ghost-antighost-gluon vertex contraction (see above) vanishes.

Also satisfies $p_\mu \Pi^{\mu\nu}_{ab} = 0$, when $p^2 = 0$, i.e. when p_μ is on-shell, so the ghosts decouple, when $\xi = 0$.

Gluon propagator in lightcone gauge: $n^2 = 0$, i.e. n^μ is light-like

$$\boxed{i\Pi^{\mu\nu}_{ab} = \frac{i}{p^2 + i\varepsilon}\left[-g^{\mu\nu} + \frac{n^\mu p^\nu + n^\nu p^\mu}{n\cdot p}\right]\delta_{ab}}$$

"light-cone gauge"

Numerator is polarization sum of transverse modes in a particular basis.

Only two polarizations propagate, so there is no need for ghosts.

3.1.5 Feynman rules (26.1)

Expanded Lagrangian with gauge-fixing terms:

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{2\xi}(\partial_\mu A^{a\mu})^2 + (\partial_\mu \bar{c}^a)(\delta^{ac}\partial^\mu + g_s f^{abc} A^{b\mu})c^c$$

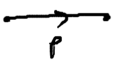
$$+ \bar{\psi}_i(\delta_{ij}\, i\partial\!\!\!/ + g_s A\!\!\!/^a T^a_{ij} - m\delta_{ij})\psi_j$$

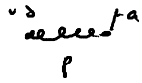
where c^a and $\bar{c}^a$ are Faddeev-Popov ghosts and antighosts.

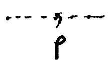
QCD field strength:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

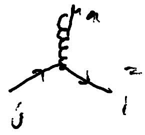
Propagators:

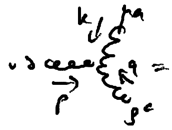
 $= \dfrac{i\,\delta^{ij}}{\not p - m + i\varepsilon}$

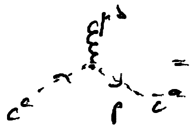
 $= i\,\dfrac{-g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2}}{p^2 + i\varepsilon}\,\delta_{ab}$

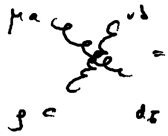
 $= \dfrac{i\,\delta^{ab}}{p^2 + i\varepsilon}$

Vertices:

 $= i g_s \gamma^\mu T^a_{ij}$

 $= g_s f^{abc}\left[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\right]$

 $= -g_s f^{abc} p^\mu$

 $= -i g_s^2 \left[f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \right]$

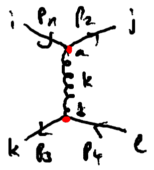
3.1.6 Attractive and repulsive potentials (26.2)

First identity:

$$\sum_a T^a_{ij} T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}\right)$$

Check by multiplying with δ_{ij} or δ_{kl}, which give zero, since the Gell-Mann matrices are traceless.

Quark-antiquark scattering at tree-level:



$$= T^a_{ji} T^a_{kl} (i g_s)^2 \left[\bar u_j(p_2)\gamma^\mu u_i(p_1)\right] \frac{-i\left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]}{k^2} \left[\bar v_k(p_3)\gamma^\nu v_l(p_4)\right]$$

Identical to QED except for strong(er) coupling constant ($e \to g_s$) and color factor.

a) For red up-quark ($i=1$) and anti-green anti-down ($k=2$):

$$T^a_{j1} T^a_{2l} = \begin{pmatrix} 0 & -1/6 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{jl} = -\frac{1}{6}\delta_{j1}\delta_{2l}$$

So $j = i = 1$ and $k = l = 2$, i.e. color is conserved.

Similarly for all other combinations of color octet in $3 \otimes \bar 3 = 8 \oplus 1$.

Negative sign → repulsive Coulomb potential:

$V(r) = +\frac{1}{6}\frac{g_s^2}{4\pi r}$ for color octet states

b) For red up-quark ($i=1$) and anti-red anti-down ($k=1$):

$$T^a_{j1} T^a_{1l} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}_{jl}$$

So 3 possible final states: $(R\bar R, G\bar G, B\bar B)$.

So 3 possible final states: $(R\bar{R}, G\bar{G}, B\bar{B})$.
Positive signs → attractive potentials.
For a color-singlet state $|u\rangle = \frac{1}{\sqrt{3}}|R\bar{R}, G\bar{G}, B\bar{B}\rangle$, we must take the trace:

$$V(r) = -\frac{4}{3}\frac{g_s^2}{4\pi r} \quad \text{for color-singlet states}$$

3.1.7 Accidental symmetries (28.2)

QCD Lagrangian possesses also a number of global symmetries (parity P, charge C, time reversal T)
Also baryon number, quark number for each individual flavor, strong isospin $SU(2)$, if $m_u = m_d$, etc.

They are called accidental, since adding terms, that violate them, requires introduction of new fields.

Chiral symmetry

Can write Lagrangian in terms of chiral fields $q_{L,R} = \frac{1 \mp \gamma_5}{2} q =: P_{L,R}\, q$

$$\mathcal{L}_{QCD} = \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R - \bar{q}_L M q_R - \bar{q}_R M q_L - \frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})$$

When $M \to 0$, this Lagrangian is invariant under separate groups of unitary transformations

$$q_L \to U_L q_L \quad , \quad q_R \to U_R q_R$$

Broken explicitly by quark mass terms, spontaneously by quark condensates (see below).

Example: Flavor $U(3)$ (include s quarks)
9 conserved Noether currents for L and R chiral quarks:
Change basis from $L, R \to V(=R+L)$ and $A(=R-L)$ currents:

$$SU(3)_V \times U(1)_V : \quad J_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \qquad J_B^\mu = \bar{q}\gamma^\mu q$$

$$SU(3)_A \times U(1)_A : \quad J_{5,a}^\mu = \bar{q}\gamma^\mu\gamma^5 \frac{\lambda_a}{2} q \qquad J_5^\mu = \bar{q}\gamma^\mu\gamma^5 q$$

Extra $U(1)_V$ corresponds to global baryon number conservation.
When $m_q \to 0$, we expect all currents to be conserved, i.e. $\partial_\mu J^\mu_{\{a, B, 5, a5\}}$
Both flavor and axial symmetries broken by $m_q \neq 0$.