2.1.5 Massive gauge bosons (8.2, 8.4)

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Free Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 A_\mu A^\mu$$

Euler-Lagrange equations: Proca equations

$$\boxed{\partial_\mu F^{\mu\nu} + M^2 A^\nu = 0}$$

Taking the divergence ∂_ν leads to $\partial_\nu A^\nu = 0$ $\Rightarrow$ for $M \neq 0$ a necessary condition!

Then the Proca equations are

$$\boxed{\Box A^\mu + M^2 A^\mu = 0}$$

Polarization vectors of massive spin-1 particles automatically satisfy $p \cdot \varepsilon = 0$,

but since there is no gauge invariance, there is an additional d.o.f. (longitudinal)

$$\varepsilon^\mu_{(L)} = \frac{1}{M} (|\vec{p}|, 0, 0, p_0)$$

Polarization vectors satisfy completeness relations

$$\sum_\lambda \varepsilon^{*\mu}(\lambda) \varepsilon^\nu(\lambda) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2} =: \Pi^{\mu\nu}_1$$

Feynman rules for massive spin-1 gauge boson: External same as massless.

Propagator in unitary gauge:

$$\boxed{\sim\!\sim\!\sim \qquad \frac{i}{p^2 - M^2 + i\varepsilon} \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2} \right]}$$

All SM gauge interactions mediated by spin-1 vector bosons (QED, QCD massless; GSW massive).

2.1.6 Feynman rules for scalar QED (8.3, 9.2)

Derivation of Feynman for $i\mathcal{M}$:

- Vertices come from interaction terms in $i\mathcal{L}$ [expansion of $\exp(i\mathcal{L})$]
- Replace derivatives by $(-i)$ × incoming momenta of the field [Fourier transform]
- Sum over indices and momenta of equal external fields [symmetrization]
- Remove external fields [functional derivative]
- Impose four-momentum conservation at each vertex [remainders of Fourier transform]

Lagrangian:

Complex scalar field ϕ of mass m and charge e (e.g. $\pi^\pm$), coupling to photon A_μ:

$$\boxed{\mathcal{L} = |D_\mu \phi|^2 - m^2 |\phi|^2 - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 - \frac{1}{4} F^2_{\mu\nu}}$$

Field strength tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Invariant by itself under $A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$ $\longrightarrow$ $\varepsilon_\mu \to \varepsilon_\mu + p_\mu$

leads to Ward identity: $p_\mu \mathcal{M}^\mu = 0$, also imposed by Lorentz invariance (see below).

Local $U(1)$ gauge transformation: $\phi \to e^{i\alpha(x) Q}\phi$. Generator Q: $Q\phi = e_\phi \phi$.

Mass term is invariant

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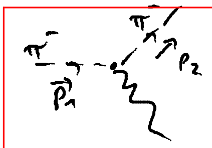
Gauge covariant derivative: $D_\mu\phi := (\partial_\mu - ieA_\mu Q)\phi$.

Transforms like ϕ: $D_\mu\phi \to e^{i\alpha(x)Q} D_\mu\phi$.

Expanded Lagrangian:

$$\boxed{\mathcal{L} = \phi^*(\Box - m^2)\phi - ieA_\mu[\phi^*(\partial_\mu\phi) - (\partial_\mu\phi^*)\phi] + e^2 A_\mu^2 |\phi|^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \frac{1}{4}F_{\mu\nu}^2}$$

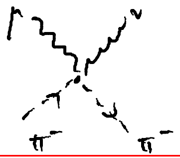
Three-point vertex:



$= ie(-p_\mu^1 - p_\mu^2)$

Derivatives in interaction vertices act on quantized scalar fields → four-momenta.
Therefore $(ie)\times$[sum of incoming and outgoing scalar particle momenta].

Four-point ("seagull") vertex:



$= 2ie^2 g_{\mu\nu}$

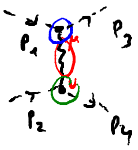
Comes from $|D_\mu\phi|^2$ ("gauge kinetic term"), i.e. forced by gauge invariance.
Factor 2 from symmetrization.

2.1.7 Scattering in scalar QED (9.3)

Example: Scalar Møller scattering ($e^-e^- \to e^-e^-$)

First diagram:

$i\mathcal{M}_t =$



$$(-ie)(p_1^\mu + p_3^\mu)\,\frac{-i[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}]}{k^2}\,(-ie)(p_2^\nu + p_4^\nu)$$

where $k = p_3 - p_1$

$$\mathcal{M}_t = e^2\,\frac{(p_1+p_3)\cdot(p_2+p_4)}{t}$$

Second diagram:

$i\mathcal{M}_u =$

$= \ldots$

$$\mathcal{M}_u = e^2\,\frac{(p_1+p_4)(p_2+p_3)}{u}$$

Differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 E_{cm}^2}\left[\frac{(p_1+p_3)\cdot(p_2+p_4)}{t} + (p_3 \leftrightarrow p_4)\right]^2 = \frac{\alpha^2}{4s}\left[\frac{s-u}{t} + \frac{s-t}{u}\right]^2$$

where $\alpha = \frac{e^2}{4\pi}$ is the fine structure constant and $s = (p_1+p_2)^2 = (p_3+p_4)^2 = E_{cm}^2$.

2.1.8 Ward identity and gauge invariance (8.4, 9.4)

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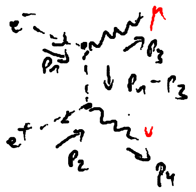
Matrix elements with external photons:

Can be written as $\mathcal{M} = \mathcal{M}^\mu \epsilon_\mu$. Gauge invariance (Ward identity) requires: $\mathcal{M}^\mu p_\mu = 0$.

Example: $e^+ e^- \to \gamma\gamma$

First diagram:

$i\mathcal{M}_t =$ 

$$= (-ie)^2 \frac{i(2p_1^\mu - p_3^\mu)(p_4^\nu - 2p_2^\nu)}{(p_1 - p_3)^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}$$

Using $p_1^2 = 0$, but $p_3^2 = p_4^2 = p_3 \cdot \epsilon_3 = p_4 \cdot \epsilon_4 = 0$, this simplifies to

$$\mathcal{M}_t = e^2 \frac{(p_3 \cdot \epsilon_3^* - 2p_1 \cdot \epsilon_3^*)(p_4 \cdot \epsilon_4^* - 2p_2 \cdot \epsilon_4^*)}{p_3^2 - 2p_3 \cdot p_1}$$

Second diagram: (cross $1 \leftrightarrow 2$ or $3 \leftrightarrow 4$)

$i\mathcal{M}_u =$ 

$$= ie^2 \frac{(p_3 \cdot \epsilon_3^* - 2p_2 \cdot \epsilon_3^*)(p_4 \cdot \epsilon_4^* - 2p_1 \cdot \epsilon_4^*)}{p_3^2 - 2p_3 \cdot p_2}$$

Sum: Check Ward identity with $\epsilon_3 \to p_3$

$$\mathcal{M}_t + \mathcal{M}_u = 2e^2 \epsilon_4^* (p_4 - p_2 - p_1) \neq 0$$

Missing diagram:

$i\mathcal{M}_4 =$ 

$$= 2ie^2 g_{\mu\nu} \epsilon_3^{*\mu} \epsilon_4^{*\nu}$$

Sum: Check Ward identity again with $\epsilon_3 \to p_3$

$$\mathcal{M}_t + \mathcal{M}_u + \mathcal{M}_4 = 2e^2 \epsilon_4^* (p_4 - p_2 - p_1 + p_3) = 0 \;!$$

Did not use conditions on real/transverse photons !

Applies also to unphysical photons (e.g. photons).