Separation of Heavy-Flavour Production Mechanisms via Two-Particle Angular Correlations in pp Collisions at $\sqrt{s} = 2.76$ TeV


Erstgutachter: Priv.-Doz. Dr. C. Klein-Bösing
Zweitgutachter: Prof. Dr. A. Khoukaz
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Glossary of Abbreviations

<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ACD</td>
<td>Azimuthal-Correlation Distribution</td>
</tr>
<tr>
<td>ALICE</td>
<td>A Large Ion Collider Experiment</td>
</tr>
<tr>
<td>ATLAS</td>
<td>A Toroidal LHC Apparatus</td>
</tr>
<tr>
<td>CERN</td>
<td>European Organization for Nuclear Research</td>
</tr>
<tr>
<td>CMS</td>
<td>Compact Muon Solenoid</td>
</tr>
<tr>
<td>DCA</td>
<td>Distance of Closest Approach</td>
</tr>
<tr>
<td>EMCal</td>
<td>Electromagnetic Calorimeter</td>
</tr>
<tr>
<td>fe</td>
<td>flavour excitation</td>
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<tr>
<td>FSR</td>
<td>Final-State Radiation</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full Width Half Maximum</td>
</tr>
<tr>
<td>gs</td>
<td>gluon splitting</td>
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<tr>
<td>HFE</td>
<td>heavy-flavour hadron decay electrons</td>
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<tr>
<td>$HFE_{p-h}$ correlations</td>
<td>correlations of trigger HFEs from a certain heavy-flavour production process $p$ with hadrons</td>
</tr>
<tr>
<td>$HFE_{p-HFE}$ correlations</td>
<td>correlations of trigger HFEs from a certain heavy-flavour production process $p$ with other HFEs</td>
</tr>
<tr>
<td>ID</td>
<td>particle identity code</td>
</tr>
<tr>
<td>ISR</td>
<td>Initial-State Radiation</td>
</tr>
<tr>
<td>ITS</td>
<td>Inner Tracking System</td>
</tr>
<tr>
<td>LEP</td>
<td>Large Electron-Positron Collider</td>
</tr>
<tr>
<td>LHC</td>
<td>Large Hadron Collider</td>
</tr>
<tr>
<td>LHCb</td>
<td>Large Hadron Collider beauty</td>
</tr>
<tr>
<td>LO</td>
<td>Leading Order</td>
</tr>
<tr>
<td>MPI</td>
<td>Multi-Parton Interaction</td>
</tr>
<tr>
<td>NF technique</td>
<td>Normal Fit technique</td>
</tr>
<tr>
<td>NNPDF</td>
<td>Neural Network Parton Distribution Function</td>
</tr>
<tr>
<td>pc</td>
<td>pair creation</td>
</tr>
<tr>
<td>PDF</td>
<td>Parton Distribution Function</td>
</tr>
<tr>
<td>PHENIX</td>
<td>Pioneering High Energy Nuclear Interaction eXperiment</td>
</tr>
<tr>
<td>QCD</td>
<td>Quantum Chromodynamics</td>
</tr>
<tr>
<td>QED</td>
<td>Quantum Electrodynamics</td>
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<tr>
<td>QGP</td>
<td>Quark-Gluon Plasma</td>
</tr>
<tr>
<td>S technique</td>
<td>Slope technique</td>
</tr>
<tr>
<td>SM</td>
<td>Standard Model</td>
</tr>
<tr>
<td>TOF</td>
<td>Time Of Flight detector</td>
</tr>
<tr>
<td>TPC</td>
<td>Time Projection Chamber</td>
</tr>
<tr>
<td>TRD</td>
<td>Transition Radiation Detector</td>
</tr>
<tr>
<td>ZY technique</td>
<td>Zero Yield technique</td>
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<td>ZYAM</td>
<td>Zero Yield At Minimum</td>
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Introduction

Considering length scales of about $10^{-10}$ m, all matter surrounding us is formed by a complex composition of atoms. Atomic nuclei are built from nucleons ($\sim 10^{-15}$ m), i.e. protons and neutrons, and surrounded by a shell of electrons. Going to even smaller length scales, the constituents of the nucleons – the up and down quarks – can be resolved, which are bound by the strong interaction and belong to a group of six particles proclaimed by the so-called Standard Model of particle physics. Of these particles, the up, down and strange quarks exhibit a rather small mass ($\sim$ MeV/$c^2$) and are thus denoted as light flavours, whereas the charm, bottom and top quarks are comparably heavy ($\sim$ GeV/$c^2$) and therefore referred to as heavy flavours.

In high-energy particle collisions, heavy flavours are because of their large mass produced in interactions with large momentum transfer. Their production rates thus offer a possibility to test predictions from perturbative Quantum Chromodynamics, a branch of the Standard Model which describes the strong interaction at large momentum scales. As large momenta are only accessible at short time scales after the impact of the projectiles, heavy quarks are generated in the early stage of particle collisions and experience the evolution of the system. Therefore, they are expected to carry information on the entire propagation of the Quark-Gluon Plasma, the medium that is generated in collisions of heavy ions and in which quarks are no longer bound in nucleons.

For these reasons, it is of great interest how heavy flavours are produced in pp collisions, as the latter serve as a reference system for studies in heavy-ion physics. Results from previous analyses indicate that the production of charm and bottom quarks can be investigated via the angular correlations of their decay products\cite{7, 21, 40}. The individual production mechanisms are expected to produce a characteristic correlation pattern with respect to the particles’ azimuthal angle, by which the relative contributions of these production channels can be determined.

In this thesis, two-particle azimuthal correlations of heavy-flavour electrons (HFEs) from $B$ and $D$ meson decays with hadrons (HFE$_p$-h correlations) and with other HFEs (HFE$_p$-HFE correlations) are simulated using the Monte Carlo event generator PYTHIA 8.2\cite{42, 45} for pp collisions at a centre-of-mass energy of 2.76 TeV. The designation “electron”, in this context, refers to electrons as well as positrons and the term “hadron” shall denote all charged particles.\footnote{This definition has been chosen in accordance with corresponding analyses of the ALICE (A Large Ion Collider Experiment) collaboration\cite{1}.
}

One of the correlation partners is asked to have originated from a specific production mechanism $p$. The objective is to derive a method for the separation of the individual charm and bottom production mechanisms in data from high-energy particle collisions via the shape of Azimuthal-Correlation Distributions (ACDs) from PYTHIA 8.2 simulations.

This thesis is structured as follows: In Sec. 2, background information on the Standard Model of particle physics and on the strong interaction is provided, as well as an introduction to heavy-flavour physics at particle colliders. Recent analyses on heavy-flavour production are summarised. The framework of PYTHIA 8.2 and the concept of event generation are described in Sec. 3. An outline of the analysis strategy, the utilised PYTHIA set-up and a discussion of complications following from this set-up are given in Sec. 4. In Sec. 5, results from simulations of HFE$_{all}$-h correlations, for which HFEs irrespective of their production mechanisms have been
considered, are compared to data from the ALICE experiment and to simulation results from HFE_all-HFE correlations. The process separation in HFE_p-h and HFE_p-HFE correlations is examined in Sec. 6.
2. Theoretical and Experimental Background

2.1. The Standard-Model Particles and Their Interactions

The Standard Model (SM) of particle physics combines our present knowledge on how matter is built from its constituents and how these constituents interact via three of the four fundamental forces: the electromagnetic, the strong and the weak interaction. Only gravity, the fourth of these forces, cannot yet be described in the corresponding framework. It may, however, be neglected for most of the scenarios in particle physics, as it is by many orders of magnitudes weaker than the other three interactions.

Figure 2.1. – The elementary particles of the Standard Model of particle physics. The particle masses in GeV/$c^2$, which have been taken from [52], are given below the respective particles.

<table>
<thead>
<tr>
<th></th>
<th>Electric Charge</th>
<th>1st Gen.</th>
<th>2nd Gen.</th>
<th>3rd Gen.</th>
<th>Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>QUARKS</td>
<td>+2/3</td>
<td>u</td>
<td>c</td>
<td>t</td>
<td>γ, g, Z⁰, W⁺</td>
</tr>
<tr>
<td></td>
<td>-1/3</td>
<td>d</td>
<td>s</td>
<td>b</td>
<td></td>
</tr>
<tr>
<td>LEPTONS</td>
<td>-1</td>
<td>e</td>
<td>μ</td>
<td>τ</td>
<td>H, 125.7</td>
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<td></td>
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<td>νₑ, νₑ, νₑ</td>
<td></td>
<td>&lt;10⁻⁹ &lt;10⁻⁹ &lt;10⁻⁹</td>
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<td>&lt;10⁻⁹ &lt;10⁻⁹ &lt;10⁻⁹</td>
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</tbody>
</table>

Classification of Particles

A scheme of the particles described by the SM is provided in Fig. 2.1. They can be divided into twelve spin-$\frac{1}{2}$ fermions, four spin-1 bosons and the spin-0 Higgs boson. All (electrically) charged particles that are presented possess a corresponding antiparticle, which has the same mass as the original particle but inverted charge-like quantum numbers.¹

The fermions comprise of the six quarks (up u, down d, charm c, strange s, top t and bottom b) and the six leptons (electron $e^-$, muon $\mu^-$, tau $\tau^-$ and the corresponding neutrinos $\nu_e, \nu_\mu, \nu_\tau$) which are organised in three generations (see Fig. 2.1). Quarks are the constituents of the non-elementary hadrons. Hadrons built from three quarks are called baryons and those built from a quark and an antiquark are termed mesons. The most prominent examples of the baryons

¹The neutral photon and the $Z^0$ boson are their own antiparticles. Experimental findings are to this day consistent with antineutrinos and neutrinos being independent particles but the exceptional small mass of the neutrinos with respect to the other SM fermion masses could be explained if neutrinos are Majorana particles and thus their own antiparticles. The antiparticle of an individual gluon is one of the other gluons [38, 52].
are the proton \((uud)\) and the neutron \((udd)\), which form the atomic nuclei. For this thesis, the heavy-flavour mesons containing a charm or a bottom quark – the \(D\) mesons \((cx)\) and the \(B\) mesons \((bx)\) – will be of interest.

The spin-1 bosons provide the basis for the description of the mediation of the three interactions. They are assumed to couple to particles carrying the respective charge of the force and transfer momentum between the interacting particles. Their mass defines the reach of the corresponding force: according to Heisenberg’s uncertainty principle relating the uncertainty of the momentum with the uncertainty of the spatial position of a particle, the reach of the force decreases with increasing mass.

The Higgs boson, which completes the particle collection, occupies a special position as the masses of the other elementary particles come about by their interaction with the associated Higgs field [52].

**Introduction to the Three Interactions**

Each interaction is by quantum field theory defined as a quantum field, whose excitation is the corresponding SM spin-1 gauge boson by which the transfer of the interaction is described. The strength of an interaction is determined by its coupling constant and the mass of its mediator, whereby the coupling constant depends on the momentum exchange of the interacting particles. The electromagnetic interaction, inter alia, causes the attraction between shell electrons and atomic nuclei and is described in the framework of Quantum Electrodynamics (QED). It is transferred via the massless photons coupling to the electromagnetic charge. As its gauge boson is massless, the reach of the electromagnetic interaction is infinite.

The gauge bosons of the strong interaction are the gluons which couple to the colour charge. Even though gluons are massless, the reach of the strong interaction is short (about \(10^{-15}\) m [38]) as gluons themselves carry colour charge and can therefore interact with each other. The strong interaction manifests in the binding of quarks to hadrons and is mathematically formulated in the theory of Quantum Chromodynamics (QCD).

The weak interaction has the shortest reach with about \(10^{-18}\) m [38]. This is due to its force carriers, the neutral \(Z^0\) boson and the charged \(W^\pm\) bosons, being very massive. The interaction causes radioactive isotopes to decay via \(\beta\)-decays and acts on all fermions [23, 38, 52].

At typical energies of particle collisions, the coupling constant \(\alpha_s\) and likewise the strength of the strong interaction exceeds the electromagnetic and the weak coupling by some orders of magnitude. The coupling constants of the electromagnetic and the weak interaction are of comparable size, but due to the large mass of the weak gauge bosons, the strengths of both interactions only align at relatively large energies \(E \gtrsim m_{Z^0}, m_{W^\pm}\). Nevertheless, the weak and the electromagnetic interaction can be described by the unified electroweak theory of S. Glashow, A. Salam and S. Weinberg (GSW) in which both forces are interpreted as two manifestations of only one interaction [23, 38, 52].
Feynman Diagrams – Illustrating Particle Interactions

The interactions of particles can be illustrated by Feynman diagrams of which an example is presented in Fig. 2.2 (left panel): two light quarks annihilate into a gluon that splits into a heavy-flavour pair. A Feynman diagram is built from lines of different styles representing the interacting particles and from the interaction vertices (two in case of the diagram from Fig. 2.2), where the exchanged boson (a gluon in this case) couples to the particles’ charges. The SM vertices of the three interactions are displayed in Fig. 2.3: gluons are illustrated by curly, photons by curved and leptons by straight lines. To each component – the vertices, the initial and final state particles, and the gauge bosons – a mathematical term can be assigned according to the Feynman rules, from which it is possible to calculate the amplitude of the pictured process.

The exchanged gauge bosons can be thought of as carrying a four momentum corresponding to the momentum exchange $q$ of incoming and outgoing particles. In the case of the diagram in Fig. 2.2, the four momentum of the gluon is given by $q = p_1 + p_2 = p_3 + p_4$, whereby $p_1$ and $p_2$ are the four momenta of the incoming, and $p_3$ and $p_4$ the four momenta of the outgoing particles. Since the energy and momentum have, per definition to be conserved at each vertex, the exchanged gauge bosons themselves have to violate the energy-momentum relation

$$E = \sqrt{m^2 c^4 + p^2 c^2}. \quad (2.1.1)$$

They are therefore termed virtual particles and assigned a virtuality $Q^2$ which is defined by $Q^2 = -q^2$. The larger the virtuality, the larger is the probability for the generation of heavier particles within interaction processes [23, 52].

Feynman diagrams can be categorised in three groups according to the Lorentz-invariant Mandelstam variables $s, t$ and $u$ which give the energy of the exchanged gauge bosons and are defined by

$$s = (p_1 + p_2)^2, \quad (2.1.2)$$
$$t = (p_1 - p_3)^2, \quad (2.1.3)$$
$$u = (p_1 - p_4)^2. \quad (2.1.4)$$

The corresponding Feynman diagrams are denoted as $s$-, $t$- or $u$-channel and can be seen in Fig. 2.2 (right panel). While the $s$-channel diagram describes the annihilation of two particles, the $t$- and $u$-channels illustrate scattering processes [52].
2.2. Quantum Chromodynamics (QCD)

The theory of QCD predicts three colour charges – red, blue and green – together with the corresponding anticharges. While quarks (antiquarks) only carry one colour (anticolour), gluons can appear in eight pairings of a colour and an anticolour. Each charge state is mathematically represented by a so-called colour spinor\(^2\). The coupling of a gluon to a colour charge does not depend on the individual type of colour, i.e. the strong interaction is invariant under rotations of colour spinors in the colour space. These rotations are described by \(3 \times 3\) matrices from the SU(3) group\(^3\); one says, the theory of QCD contains an exact SU(3) colour symmetry\([52]\).

No freely moving particles with a net colour charge have been observed to this day. This suggests that coloured objects only exist in colour neutral combinations, i.e. quarks and gluons have to be bound in hadrons. Colour neutrality is either achieved by the pairing of a colour with its anticolour, as it is the case for mesons, or by the combination of three different colours or three different anticolours, as it is the case for baryons. Confinement is the name of this effect which can be ascribed to the running coupling constant of QCD. At leading order, the latter varies with the virtuality \(Q^2\) according to

\[
\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \ln \left(\frac{Q^2}{\Lambda^2}\right)} \quad (2.2.1)
\]

with \(\Lambda = 250\text{ MeV}/c\) being the scale parameter of QCD and \(n_f\) the number of quark flavours\([38]\).

At large \(Q^2\), corresponding to small spatial separations of the interacting particles, \(\alpha_s\) is low as it can be seen in Fig. 2.4. Consequently, quarks and gluons can move almost freely at high-energy scales which is called asymptotic freedom. Phenomenologically, this can be explained by the anti-screening of colour charges: the higher the momentum transfer and thus the resolution, the smaller is the charge of a colour-charged particle that is observed, as its charge can be separated more clearly from the surrounding colour charge of the gauge fields.

At small \(Q^2\), i.e. large separations, the coupling constant increases and becomes of \(\mathcal{O}(1)\). If, in a

\(^2\)Spinors are objects comparable to vectors and scalars that, however, transform differently under rotations and Lorentz transformations\([28]\).

\(^3\)The special unitary group SU(3) comprises all complex \(3 \times 3\) matrices \(M\) with \(\det M = 1\) and \(MM^\dagger = M^\dagger M = 1\)\([28]\).
Figure 2.4. – Summary of measurements of $\alpha_s(Q^2)$ in dependence of the momentum transfer, here denoted as $Q$. Also the world average value of $\alpha_s(M_Z^2)$ is provided, whereby $M_Z$ is the mass of the Z boson [37, modified].

Figure 2.5. – The phase diagram of QCD in dependence of the temperature $T$ and the baryochemical potential $\mu_B$. The different regions of the diagram that are investigated by the Large Hadron Collider (LHC), the Relativistic Heavy-Ion Collider (RHIC), the Super Proton Synchrotron (SPS), the Alternating Gradient Synchrotron (AGS) and the SIS18 at the Gesellschaft für Schwerionenphysik (GSI), are marked [36].
Gedanken experiment, the two quarks of a meson are pulled apart, a colour tube of gluons is stretched between them. Because of their colour charges, the gluons within this tube interact with each other, thereby minimising its transverse extent. Since the energy density stays constant, the potential between the two quarks grows linearly with the distance if the electromagnetic interaction is neglected. This can be seen as an illustrative explanation for the large coupling constant at small $Q^2$ and the confinement.

As another consequence of the running coupling, only hard QCD processes (large $Q^2$) can be described perturbatively, i.e. by employing series expansions in $\alpha_s$. For soft processes (small $Q^2$), phenomenological models have to be applied as $\alpha_s$ gets too large and higher-order terms can no longer be neglected [17, 23, 38, 52].

The Quark-Gluon Plasma

When the effect of asymptotic freedom was established in particle physics, this lead to the prediction of a new phase of matter at high temperatures and densities: the Quark-Gluon Plasma (QGP). Quarks and gluons are deconfined in this thermalised state and are expected to show collective behaviour. A QGP can either be produced by heating up hadronic matter to temperatures larger than the critical temperature $T_c \approx 160$ MeV [24] or by compressing it to generate large baryon densities. In both cases, the hadrons start to overlap and quarks and gluons are dissolved at energy densities larger than 1-10 GeV/fm$^3$ [24]. A transition from the nucleon gas to a QGP can therefore proceed at low baryochemical potentials $\mu_B$ and large temperatures or lower temperatures and large $\mu_B$ and is indicated by a rapid rise of the energy density in a narrow temperature interval. The schematic phase diagram of QCD is presented in Fig. 2.5. The black line indicates the transition from the hadronic phase to the QGP, which is expected to be a first order transition at high $\mu_B$ and a crossover with a rapid change of thermodynamical properties at low $\mu_B$. The critical point of the change from a smooth cross over to a first order transition is indicated as well as the ground state of nuclear matter [17, 24, 36, 53].

2.3. Heavy-Flavour Physics

Since the mass of the top quark is larger than the $W$ boson, it can decay into a $W$ boson and a bottom quark, which is why it has a short lifetime of $\tau \approx 0.5 \cdot 10^{-24}$ s and decays before hadronisation [37]. The analyses presented in the following are therefore restricted to the investigation of charm and bottom quark observables and the term “heavy flavours” shall hereafter only refer to these two particles.

2.3.1. Relevance and Detection in High-Energy Collisions

The predictions of the SM can be tested via high-energy particle collisions, whereby particles of different species are accelerated to energies up to some GeV or TeV and brought to collision. At these large energies, a variety of new particles is formed whose identity, momentum and energy can be reconstructed by the characteristics of their interaction with the detector material. A short discussion of relevant observables for the analysis of particle collisions can be found in Sec. A.1.

Heavy Flavours and the QGP

In high-energy collisions of heavy ions like lead (\(^{208}\text{Pb}^{82+}\)) or gold nuclei (\(^{197}\text{Au}^{79+}\)), such high densities and temperatures are achieved that a QGP is expected to form. The first evidence for the emergence of such a medium was proclaimed in 2000 after the analysis of data from

\^4The baryochemical potential is a measure for the net baryon density of the system.
Pb-Pb collisions with beam energies of 160 GeV per nucleon at the European Organization for Nuclear Research (CERN) [25, 33]. Nowadays, the different sectors of the QCD phase diagrams indicated in Fig. 2.5 are studied by a variety of experiments. The largest center of mass energies can currently be achieved at the Large Hadron Collider (LHC), where ALICE (A Large Ion Collider Experiment) is especially involved in the investigations of Pb-Pb collisions. As heavy flavours are generated in the early stage of the collision ($\tau \approx 1/2(m_Q) \lesssim 0.1 \text{fm}/c$ [35]), they experience the evolution of matter and in heavy-ion collisions they interact with the emerging QGP. Consequently, one can obtain information about the properties of the medium from heavy-flavour observables. Information on the QGP’s energy density can, for example, be deduced from the energy loss of heavy flavours in the medium. According to predictions from QCD calculations, heavy flavours lose less energy via gluon radiation than light quarks which is referred to as the dead cone effect. This suppression of low-angle gluon emissions is expected to translate into a larger ratio of D mesons and pions in heavy-ion collisions, which can be measured [11, 20].

Collisions of protons provide reference measurements for heavy-ion collisions, since no QGP is expected to emerge for this collision system. In the field of heavy-flavour physics, a comparison of data from pp and heavy-ion collisions can provide information on the production of bottom and charm quarks [5] and on modified angular correlations [7] in heavy-ion collisions as well as on the interaction of heavy flavours with the QGP [40].

ALICE at the Large Hadron Collider

Two beams of protons or heavy ions circulate in opposite directions in the collider ring of the LHC at the CERN. The four main experiments – ATLAS$^5$, CMS$^6$, ALICE and LHCb – are located at the four interaction points where the beams are brought to collision. Particles are accelerated by radiofrequency cavities and kept on their circular paths by superconducting magnets. The whole acceleration proceeds stepwise via an accelerator complex of one linear and three circular accelerators until the particles are injected in the LHC and brought to their final energies. In 2009, the LHC started its operation with pp collisions at $\sqrt{s} = 0.9$ TeV; in the course of LHC Run 1 (2009-2013), pp collisions at $\sqrt{s} = 2.76, 7$ and 8 TeV, Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV were achieved [29]. For Run II, data from pp and Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV is taken; thus it will be possible to investigate all three collision systems at the same centre-of-mass energies. Besides, pp collisions at 13 TeV and p-Pb collisions at 8.16 TeV are explored.

ALICE is the LHC experiment designed to investigate the properties of the QGP generated in Pb-Pb collisions. Also, reference data from pp and p-Pb collisions is investigated. The amount of particles generated in Pb-Pb collisions by far exceeds the particle yield in pp collisions. Therefore, ALICE is dealing with comparably large particle multiplicities and requires a high granularity. Particle identification is possible over a large momentum range from $p_T^{\text{min}} \approx 0.15$ GeV/c up to $20$ GeV/c [6]. This is of great advantage for the study of heavy flavours, as the charm and bottom cross section are largest$^7$ at low $p_T$ [6, 30].

The ALICE detector system displayed in Fig. 2.6 has dimensions of $16 \times 16 \times 26$ m$^3$ and consists of a variety of sub-detectors which embrace the beam pipe. For electrons from semi-leptonic heavy-flavour decays, the particle identification is mainly performed by the Time Projection Chamber (TPC) measuring the specific energy loss $dE/dx$ of the particles. It is at low particle momenta supported by the Inner Tracking System (ITS) and the Time Of Flight detector (TOF) and at high momenta by the Transition Radiation Detector (TRD) and the ElectroMagnetic

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$^5$ATLAS – A Toroidal LHC ApparatuS.
$^6$CMS – Compact Muon Solenoid.
$^7$Measurements with $p_T^{\text{min}} = 0.5$ GeV/c cover $\sim 50\%$ of the total charm and $\sim 90\%$ of the total bottom cross section [30].
Calorimeter (EMCal). The detectors are embedded in a solenoid magnet; its magnetic field of 0.5 TeV deflects charged particles on circular orbits from which the particle momentum can be reconstructed.

All mentioned detectors cover the full azimuth – thereby facilitating measurements on angular correlations – and a rapidity range of $|\eta| < 0.9$ apart from the EMCal covering $|\eta| < 0.7$ and $107^\circ$ in azimuth. According to the pseudorapidity coverage of the detectors, the study of semi-electronic decays is at ALICE restricted to midrapidity [5, 6, 30].

Reconstruction of HFEs with ALICE

The main background sources of HFEs in pp collisions are electrons from photon conversions in the detector material and Dalitz decays of $\pi^0$ and $\eta$ mesons (e.g. $\pi^0 \rightarrow e^+e^-\gamma$) as it can be seen in Fig. 2.7. These decays each produce two electrons with unlike charge signs and a small invariant mass ($\equiv$ centre-of-mass energy/$c^2$), whereas the invariant mass of HFE pairs is comparably large. A pairing of unlike-sign electrons and a selection on the corresponding invariant mass thereby allows for the separation of non-HFEs. The contribution from HFEs is then determined from subtracting the yield of non-HFEs from the inclusive electron yield. A correction for the amount of HFEs is performed, which have accidentally been paired with, and therefore been misidentified as electrons from the background source.

For the correlation of HFEs and hadrons, an adequate signal-to-noise ratio can be achieved with this reconstruction technique. In the case of HFE-HFE correlations, however, only events with two HFEs are selected for the analysis and thus the signal quality decreases. Investigations are ongoing whether the resulting signal-to-noise ratio allows for sound analysis. Correlations of the leading track particle\(^8\) with HFEs as well as HFE-h correlations with a more rigorous selection on the transverse momentum of the hadron might offer an alternative field for the investigation of heavy-flavour production mechanisms. With ALICE, it would also be possible to examine correlations of HFEs

\(^8\)The leading particle is the particle with the largest transverse momentum in the event, which is most likely emitted in a jet.
Figure 2.7. – Differential cross section as a function of the invariant mass $m_{ee}$ for the production of $e^+e^-$ pairs in pp collisions at 7 TeV. The cocktail contributions from different HFE background sources are presented as well as the cross section for the production of $e^+e^-$ pairs from $c\bar{c}$ pairs [39].

emitted at midrapidity with muons emitted at forward rapidity to gain further information on the individual process contributions. Investigations on a suitable $p_T$ range of the muons would, however, be necessary to keep background contributions low [1, 2, 5].

For the separation of HFEs from bottom and charm decays, the transverse impact parameter $d_0$ can be employed. The latter is defined via the Distance of Closest Approach (DCA), which is the shortest distance of the particle trajectory to the primary vertex, i.e. the interaction point of the particle collision. If the DCA is projected onto the plane perpendicular to the beam pipe, $d_0$ is obtained (see Fig. 2.8). The lifetime of $B$ mesons ($c\tau \sim 500 \mu m$ [5]) is larger than for $D$ mesons ($c\tau \sim 150$-$300 \mu m$ derived from [37]), which is why in particle collisions, the secondary vertex of heavy-flavour decays is for $B$ mesons further separated from the primary vertex of the collision. This provides with the possibility to discriminate between $B$ and $D$ mesons and HFEs from $B$- and $D$-meson decays on the basis of $d_0$. Also, a separation of bottom decay electrons from background electrons is possible by this approach. The secondary vertex of heavy-flavour meson decays is for ALICE reconstructed by the ITS and allows for the determination of $d_0$ with a high resolution ($> 85 \mu m$ for $p_T > 1 \text{ GeV/c}$ [30]) [5].

Figure 2.8. – Definition of the transverse impact parameter $d_0$ [4].
2.3.2. Production Mechanisms

The production cross section of charm and bottom quarks in pp collisions at 2.76 TeV has been simulated with the Monte Carlo event generator PYTHIA 8.2 [42, 45]. As one would expect from the smaller mass of the charm quark, its cross section is about one order of magnitude larger than the bottom quark production cross section:

\[
\sigma_{\text{proc}}^{c} = (163.956 \pm 0.142) \text{mb} \quad \text{and} \quad \sigma_{\text{proc}}^{b} = (14.954 \pm 0.038) \text{mb}. \tag{2.3.1}
\]

In Fig. 2.9, the \(p_{T}\)-differential cross section for the production of charm and bottom quarks derived from PYTHIA 8.2 simulations is presented. At low \(p_{T}\) the charm cross section exceeds the one for bottom production by far; the curves approach each other for higher \(p_{T}\). A significantly harder \(p_{T}\)-spectrum for \(b\) quarks with the maximum production taking place at about 3 GeV/c in contrast to 1 GeV/c for charm quarks can be observed.

**Figure 2.9.** – The \(p_{T}\)-differential cross sections for the production of charm and bottom quarks simulated for pp collisions at 2.76 TeV with PYTHIA 8.2 (Monash tune).
Figure 2.10. – Exemplary Feynman diagrams corresponding to the three heavy-flavour production mechanisms as they are defined in [34]. Gluon fusion and quark annihilation (upper row left and right) are subsumed under the term pair creation.

Figure 2.11. – The different production cross sections of charm and bottom quarks for pp collisions in dependence of the centre-of-mass energy $\sqrt{s}$. The contributions from pair creation, flavour excitation and gluon splitting are shown [34].
Figure 2.12. – PYTHIA 6 simulations for the rapidity distribution of a bottom quark with $p_{T,1} > 5 \text{ GeV}/c$ for which the corresponding anti-bottom quark, that has been generated in the same process, has a rapidity of $|y_2| < 0.5$ and a transverse momentum of $p_{T,2} > 5 \text{ GeV}/c$ for $pp$-collisions at $\sqrt{s} = 1.8 \text{ TeV}$. In this figure, flavour creation represents pair-creation processes and contributions from shower/fragmentation can be interpreted as those from gluon-splitting processes [22].

Heavy-flavour production mechanisms can be classified by how many heavy-flavour particles take part in the hardest sub-process which is the one with the largest virtuality $Q^2$. Three categories can be distinguished [3, 34]:

1. **Pair creation**: The fusion of two gluons $g$ or the annihilation of two light quarks $q$ and the subsequent splitting into a heavy-flavour quark-antiquark pair $Q\bar{Q}$, i.e.

$$gg \rightarrow Q\bar{Q}; \quad qq \rightarrow Q\bar{Q}.$$  \hspace{1cm} (2.3.2)

This is a leading order (LO) process with two heavy flavours taking part in the hardest sub-process.

2. **Flavour excitation**: An initial-state gluon splits into a quark and an antiquark ($g \rightarrow Q\bar{Q}$) of which one then takes part in the hardest process. The latter embodies the scattering of a gluon or a light quark with the heavy flavour, i.e.

$$Qg \rightarrow Qg; \quad Qq \rightarrow Qq.$$  \hspace{1cm} (2.3.3)

Flavour excitation is a higher order process.

3. **Gluon splitting**: No heavy flavour takes part in the hardest sub-process. A gluon originating from initial- or final-state radiation splits into a heavy-flavour quark-antiquark pair, i.e.

$$g \rightarrow Q\bar{Q}.$$  \hspace{1cm} Gluon splitting is also a higher order process.

Examples for the Feynman diagrams corresponding to these three mechanisms are shown in Fig. 2.10. Heavy flavours can also be produced by the decay of heavy resonances, e.g. of the Higgs ($H^0 \rightarrow b\bar{b}$) or the $Z^0$ boson ($Z^0 \rightarrow b\bar{b}$). These production mechanisms are, however, not considered for this thesis.

Perturbative calculations predict that the three production mechanisms contribute to the overall heavy-flavour yield in different proportions depending on the centre-of-mass energy of the
collision according to Fig. 2.11. At low energies, pair-creation processes dominate the heavy-flavour production followed by contributions from flavour excitation and a small fraction from gluon-splitting processes. This ordering is reversed for high energies $\sqrt{s} \sim 10$ TeV with gluon splitting becoming the prominent source. In the region from 2 to 3 TeV, which is of interest for this analysis, pair creation is dominating the charm production whereas for the bottom production pair creation and flavour excitation are comparable [34].

A softer $p_T$ spectrum is predicted for bottom quarks from pair-creation processes with respect to quarks originating from the higher order processes, i.e. the proportion of the contribution from pair-creation processes on the overall heavy-flavour production is expected to decrease with increasing $p_T$ of the emerging quarks (See Fig. A.2 and Fig. A.3) [22, 34].

In Fig. 2.12, PYTHIA 6 simulations for the rapidity distribution of bottom quarks originating from the different heavy-flavour production mechanisms in p\overline{p}-collisions at $\sqrt{s} = 1.8$ TeV are shown. Only bottom quarks with a momentum $p_{T,1} > 5$ GeV/c are considered of which the corresponding anti-bottom quark with $p_{T,2} > 5$ GeV/c has been generated in the same production process and emitted at midrapidity ($|y_2 < 0.5|$). The distributions for pair creation and gluon splitting roughly obey the form of a Gaussian function, with the one for pair creation being much wider than the one for gluon splitting. This follows from the fact, that pair creation is dominated by $t$-channel quark-antiquark production while gluon splitting can be interpreted as the $s$-channel exchange of a gluon. The cross section for $s$-channel processes is proportional to $s^{-2}$ whereas the cross section for $t$-channel processes goes with $t^{-2}$. Consequently, gluon splitting is suppressed at high partonic centre-of-mass energies and quark-antiquark pairs which are generated in gluon-splitting process exhibit a smaller separation in rapidity [34]. The distribution for flavour excitation exhibits a large dip at $y_1 = 0$, i.e. the majority of bottom quarks generated in these processes with an anti-bottom partner emitted at midrapidity is itself emitted at rapidities $|y_1| > 0.9$ [22].

In this thesis, the selections on the $p_T$ of the trigger and the associated particles differ from the cuts of the analysis that has just been described. Nevertheless, a similar behaviour with respect to the rapidity of the correlation partners is expected. As the simulations from this thesis are performed with pseudorapidity cuts of either $|\eta| < 0.7$ or $|\eta| < 0.9$ for the correlated particles, the relative contributions from flavour excitation and pair creation are likely to be reduced with respect to the production cross sections from Fig. 2.11.

2.3.3. Heavy-Flavour Meson Decays

One distinguishes between open and hidden heavy-flavour mesons, whereby the former contain one charm and/or one bottom quark ($D$ and $B$ mesons) and the latter two charm or two bottom quarks (quarkonia). In this thesis, only the decays of open heavy-flavours mesons are considered.

A large fraction of charm-anticharm pairs directly fragments into $D^0$ mesons ($c \rightarrow D^0 + X$, $\mathcal{BR} = 56.5 \pm 3.2\%$ [32]) while bottom quarks mainly hadronise into $B$ mesons which then decay into $D^0$ mesons ($b \rightarrow B^\pm / B^{\mp} / \bar{B}^0 \rightarrow D^0 + X$, $\mathcal{BR} = 59.6 \pm 2.9\%$ [32]). Bottom quarks can, however, likewise produce $B$ mesons that directly decay into electrons with the dominant decay channels being $B^0 / B^{\pm} \rightarrow \nu_e e^+ D^*$. The latter decays, that shall be denoted as prompt $B$-meson decays, can thereby produce up to two HFEs in one decay chain as the $D$ mesons might again decay into HFEs. The overall branching ratios of charm and bottom quarks into electrons are 9.6\% and 10.86\%, respectively [32]. Schematics of a $c\bar{c}$ and a $b\bar{b}$ pair fragmenting into $D^0$ mesons are shown in Fig. 2.13.

The production rates $\sigma_{\text{prod}}$ of the $D$ and $B$ mesons in 2.76 TeV pp collisions have been simulated with the Monte Carlo event generator PYTHIA 8.2 and the results, together with information on the meson properties utilised by PYTHIA and the cross section $\sigma_{\text{HFE}}$ of prompt HFE production
Figure 2.13. – Schematics of a $c\bar{c}$ pair (a) and a $b\bar{b}$ pair (b) fragmenting into decaying mesons [32].

from the individual mesons, are listed in Tab. 2.1. The cross section $\sigma_{\text{prod}}^{\text{HFE}}$ has thereby been determined from the product of the cross section for the meson production and the branching ratio $BR$ of the corresponding meson into HFEs, i.e.

$$\sigma_{\text{prod}}^{\text{HFE}} = \sigma_{\text{prod}} \cdot BR.$$  \hspace{1cm} (2.3.4)

For all mesons a distinct mass ordering of the production rates can be observed. The $D$ and $B$ mesons with the largest branching ratios for prompt decays into electrons are the $D^\pm$ and the $B_c$ mesons with $BR = 17\%$ and $BR = 13\%$. Since, however, the production cross section of $B_c$ mesons is by two to four orders of magnitude smaller than the other cross sections, its contribution to the overall amount of HFEs is negligible and will not be considered in this thesis. Altogether, the largest contribution to HFEs are expected to originate from $D^\pm$ and $D^0$ decays.

Table 2.1. – $D$ and $B$ meson properties as they are implemented in PYTHIA 8.2. The total production cross section $\sigma_{\text{prod}}$ of the mesons, their branching ratios for prompt decays into electrons and the total production cross section $\sigma_{\text{prod}}^{\text{HFE}}$ of HFEs from the individual prompt $D$- and $B$-meson decays are given. $\sigma_{\text{prod}}$ has been derived from PYTHIA 8.2 (Monash 2013 tune) simulations.

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>$D^0$</th>
<th>$D^+$</th>
<th>$D_s^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>quark content</td>
<td>$c\bar{c}$</td>
<td>$c\bar{u}$</td>
<td>$c\bar{s}$</td>
<td></td>
</tr>
<tr>
<td>mass (in MeV)</td>
<td>1500</td>
<td>1864.86</td>
<td>1869.62</td>
<td>1968.49</td>
</tr>
<tr>
<td>$BR$ in $e^-$</td>
<td>$0.0677$</td>
<td>$0.1699$</td>
<td>$0.0692$</td>
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</tr>
<tr>
<td>$\sigma_{\text{prod}}$ (in mb)</td>
<td>$7.156 \pm 0.029$</td>
<td>$4.189 \pm 0.022$</td>
<td>$2.173 \pm 0.016$</td>
<td>$0.752 \pm 0.009$</td>
</tr>
<tr>
<td>$\sigma_{\text{prod}}^{\text{HFE}}$ (in $\mu$b)</td>
<td>$283.6 \pm 1.5$</td>
<td>$369.2 \pm 2.7$</td>
<td>$52.1 \pm 0.7$</td>
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</tbody>
</table>

<table>
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<tr>
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<th>b</th>
<th>$B^0$</th>
<th>$B^+$</th>
<th>$B_s^0$</th>
<th>$B_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>quark content</td>
<td>$d\bar{b}$</td>
<td>$u\bar{b}$</td>
<td>$s\bar{b}$</td>
<td>$c\bar{b}$</td>
<td></td>
</tr>
<tr>
<td>mass (in MeV)</td>
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<td>5279.58</td>
<td>5279.25</td>
<td>5366.77</td>
<td>6277.00</td>
</tr>
<tr>
<td>$BR$ in $e^-$</td>
<td>$0.1148$</td>
<td>$0.1062$</td>
<td>$0.0930$</td>
<td>$0.1280$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{prod}}$ (in mb)</td>
<td>$116.9 \pm 3.5$</td>
<td>$118.5 \pm 3.5$</td>
<td>$26.5 \pm 1.7$</td>
<td>$0.3 \pm 0.2$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{prod}}^{\text{HFE}}$ (in $\mu$b)</td>
<td>$13.6 \pm 0.4$</td>
<td>$12.4 \pm 0.4$</td>
<td>$2.5 \pm 0.2$</td>
<td>$0.04 \pm 0.02$</td>
<td></td>
</tr>
</tbody>
</table>
2.3.4. Analyses Considering Azimuthal-Correlation Distributions

One possibility to study the angular correlation of heavy flavours is via two-particle Azimuthal-Correlation Distributions (ACDs), whereby the number of correlation pairs is examined in dependence on their difference in azimuthal angle $\Delta \varphi$. Of each correlation pair, one particle is denoted as the trigger particle and the other as the associated particle, such that $\Delta \varphi$ is given by $\Delta \varphi = \varphi_{\text{trig}} - \varphi_{\text{assoc}}$.

From the Feynman diagrams in Fig. 2.10, one can formulate expectations about how the ACDs of quarks and antiquarks should appear for the different heavy-flavour production processes. Pair creation induces a two-jet event and the correlation partners have to be emitted back-to-back because of momentum conservation. In Fig. 2.14, this is reflected by the peak at $\Delta \varphi \approx \pi$ for the ACD of bottom and antibottom quarks from pair-creation processes in 2 TeV pp-collisions. For flavour excitation, two processes contribute to the angular separation of the generated quark-antiquark pair, thus the distribution in $\Delta \varphi$ is rather flat, i.e. there is only little correlation. The difference in azimuthal angle in the case of gluon-splitting processes largely depends on the initial momentum of the gluon: the larger the momentum, the smaller is $\Delta \varphi$ and vice versa. Also the ACD for gluon-splitting processes therefore does not show a distinct correlation pattern as long as no selection on the transverse momentum is applied to the quarks. If, however, only high-$p_T$ quarks are considered for the analysis, the emergence of a peak centred around $\Delta \varphi = 0$ is expected for gluon splitting and the peak for pair creation should be even more distinct than in Fig. 2.14. The hard process will more strongly dominate the angular separation of quarks from flavour-excitation processes leading to an increased number of correlation partners with a large $\Delta \varphi$ for this production mechanism [34].

The objective of this thesis is to investigate to what extend the shape of the quark-antiquark ACDs is reflected in the ACDs of HFE-h and HFE-HFE correlations and how the selection on the $p_T$ of the trigger particle affects the overall shapes.

**HFE-h Correlations**

In Fig. 2.15, the ACDs of HFEs and charged hadrons are shown as they have been measured in $\sqrt{s} = 2.76$ TeV pp collisions with ALICE for two $p_T$-intervals: $1.5 < p_T < 2.5$ GeV/c and $4.5 < p_T < 6$ GeV/c. The near-side peak centred around $\Delta \varphi = 0$ results from correlation partners which have been emitted in the same jet and whose difference in azimuthal angle has therefore been small. If two particles have however emerged from back-to-back jets, they appear in the ACD as part of the away-side peak centred around $\Delta \varphi = \pi$. The more particles are emitted in jets with respect to the rest of the event, the more distinct are the peaks of the ACD and the smaller is the baseline of uncorrelated particle pairs.

For both $p_T$-intervals the contribution of HFEs from charm decays exhibit a considerably higher near-side and a slightly higher away-side peak than those from bottom decays. This allowed for the determination of the relative bottom contribution to all HFEs from fits of PYTHIA 6 simulations for both heavy-quark contributions to the data. The results which are displayed in

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9 For a definition of the azimuthal angle see Sec. A.1.
Figure 2.15. – Azimuthal-Correlation Distributions for correlations of heavy-flavour electrons and charged hadrons in pp collisions at $\sqrt{s}=2.76$ TeV together with Pythia 6 simulations for the contributions of electrons from charm meson ($c \rightarrow e$) and bottom meson decays ($b \rightarrow (c \rightarrow e)$). The results are presented for two $p_T$-intervals of the HFEs. Also shown is the fit of the different Monte Carlo contributions to the data from which the relative bottom contributions to all HFEs has been derived [5].

Figure 2.16. – The relative bottom contribution to the HFE production in 2.76 TeV pp collisions. Data from two ALICE analyses is compared to model calculations [5, modified] (For original figure see Fig. A.4).
Correlation of Two Particles Originating from Heavy-Flavour Hadron Decays

Analyses considering the correlation of two particles that originate from heavy-flavour hadron decays tend to point more clearly to a possible separation of heavy-flavour production mechanisms via ACDs.

In [21], azimuthal correlations of HFEs and neutral $D^0$ mesons in 7 TeV pp collisions are discussed. The analysis differs between correlation pairs of which both particles originate from charm decays ($cD^0$-$cEl$) or from bottom decays ($bD^0$-$bEl$). The results of the corresponding PYTHIA 6 simulations are given in Fig. 2.18. The $bD^0$-$bEl$ correlation distribution is about one order of magnitude larger than the $cD^0$-$cEl$ distribution. The near-side peak is dominant in $bD^0$-$bEl$ correlations while the away-side peak dominates for $cD^0$-$cEl$ correlations [21]. A distinct separation of contributions from bottom and charm meson decays should be possible for these correlations.

$DD$-correlation distributions in pp collisions at $\sqrt{s} = 200$ GeV show, if only $D$ mesons originating from charm decays are considered, a similar qualitative shape as the $cD^0$-$cEl$ correlations from the previous analysis. The analysis in [40] utilises PYTHIA 8 simulations to examine the ACDs for three different momentum cuts, which are displayed in Fig. 2.18. Apart from an increasing away-side peak with increasing transverse momentum of the correlation partners, the authors observed the emergence of a small near-side peak. This evolution is associated with the growing contribution of gluon splittings to the charm production cross section as the particle-$p_T$ rises [40].

10The origin of these characteristics is in more detail described in Sec. 6.2.1, where the distributions from Fig. 2.18 are compared to simulation results from this thesis.
**Figure 2.18.** – **Left:** PYTHIA 6 simulations of $D^0$-HFE correlations in 7 TeV pp collisions for all $D^0$ mesons and electrons ($D^0$-AllEl), $D^0$ mesons and electrons originating from charm quarks (c$D^0$-cEl), $D^0$ mesons and electrons originating from bottom quarks (b$D^0$-bEl) and all $D^0$ mesons with non-heavy-flavour electrons ($D^0$-nonHFEl) [21]. **Right:** $D\bar{D}$-correlation distributions from PYTHIA 8 simulations for pp collisions at $\sqrt{s} = 200$ GeV for three different momentum cuts. Only $D$ mesons from events containing a charm quark are considered [40].

The ACD for like-sign heavy-flavour electron-muon correlations with $p_T^e > 0.5$ GeV/c and $p_T^\mu > 1$ GeV/c measured by PHENIX\(^{11}\) in $\sqrt{s} = 200$ GeV pp collisions exhibits a similar shape as the $D\bar{D}$-correlations in Fig. 2.18 for $p_T,\text{trig} > 2$ GeV/c. Corresponding simulations with PYTHIA 6 suggest that the away-side peak is dominated by LO heavy-flavour production processes, especially from gluon fusion [7].

Heavy-flavour production mechanisms can also be studied via other observables than those from particle correlations. Their contributions to the amount of di-jets with a certain flavour composition, for example, differs for each process. The ATLAS collaboration therefore initiated investigations on the abundance of flavour-symmetric and antisymmetric di-jets to separate the fractions of events that originate from pair creation, flavour excitation and gluon splitting [3]. Furthermore, distributions of transverse-momentum asymmetries ($A = (p_T^1 - p_T^2)/(p_T^1 + p_T^2)$) derived from PYTHIA 6 simulations differ for the three production processes suggesting that they can be utilised to select individual mechanisms [22].

In summary, simulations show azimuthal-correlation distributions for contributions from charm and bottom meson decays to HFE-h and HFE-$D^0$ correlations that are different in shape. The relative bottom contribution to all HFEs has been determined, which exceeds the charm contribution for $p_T^B > 3$ GeV/c. The separation of the different heavy-flavour production processes is challenging in HFE-h correlations at $2.5 < p_T^B < 3.5$ GeV/c. In $D\bar{D}$-correlations with $D$ mesons from charm decays a rising near-side peak is observed with growing $p_T$ of the correlation partners which is associated with an increasing fraction of gluon splitting heavy-flavour production processes. Correlations of HFEs and muons suggest on the other hand that the away-side is dominated by contributions from LO processes.

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\(^{11}\)PHENIX – Pioneering High Energy Nuclear Interaction eXperiment. PHENIX is one of the experiments at the Relativistic Heavy Ion Collider (RHIC) of the Brookhaven National Laboratory (BNL).
3. PYTHIA 8.2

PYTHIA [42, 45] is a tool to simulate high-energy lepton-lepton and hadron-hadron collisions. The findings from these simulations contribute to the understanding of experimental data or can help to put constraints on observables that are to be measured, thereby simplifying the search for new physics.

The event generation is based on the Monte Carlo approach, which, by utilising random number generation and the probability distributions of the individual processes, allows for the investigation of properties of a collision that cannot be calculated analytically. In PYTHIA, the generation of the hardest process determining the nature of the collision, is accurate to leading order.\(^1\) The subsequent parton-shower evolution, which covers the radiation and scattering of particles before and after the hardest process, has leading logarithmic accuracy.\(^2\) Interactions with detector material are not considered, thus particles are assumed to evolve in vacuum\([14, 45]\).

3.1. Three Steps of Event Generation in PYTHIA

A scheme of how individual sub-processes contribute to the matter evolution in high-energy collisions is shown in Fig. 3.1. The initial-state partons radiate and scatter inside the incoming projectiles which is denoted as Initial-State Radiation (ISR). Partons from one projectile will then interact with partons from the other projectile according to the partonic interaction cross section. The process with the largest momentum transfer is referred to as the hardest process, whereas the interactions at less hard momentum scales are subsumed under the term Multi-Parton Interactions (MPIs). Particles leaving these hard processes will again radiate and scatter – Final-State Radiation (FSR) – until the quarks fragment into hadrons and secondary hadron decays conclude the particle production of the event.

In PYTHIA, the event generation can be subdivided into three major steps: the generation of the hardest process, the propagation of initial-state and final-state radiation, i.e. the parton-shower evolution, and the fragmentation into hadrons.

3.1.1. The Hardest Process

The first step of the event generation is the generation of the hardest sub-process, which is the \(2\rightarrow 1, 2\rightarrow 2\) or \(2\rightarrow 3\) partonic process with the largest virtuality \(Q^2\) in the event. It determines the overall nature of the collision and its cross section can be calculated from

\[
\sigma = \sum_{a,b} \int_{0}^{1} dx_a dx_b \int d\Phi_n f_{i}^{b} (x_a, \mu_F) f_{h}^{b} (x_b, \mu_F) \frac{1}{2s} |M_{ab \rightarrow n}|^2 (\Phi_n; \mu_F, \mu_R).
\] (3.1.1)

The different factors represent:

- the parton distribution functions (PDFs) \(f_{i}^{b} (x, \mu_F)\) describing the probability to find a parton \(i\) with momentum fraction \(x_i\) of the corresponding hadron momentum bound in

\(^1\)A leading-order approximation only considers the first term in a perturbative series expansion.

\(^2\)In the leading logarithmic approximation (LLA), the perturbative series is not terminated at some order of the coupling constant but the leading logarithms are considered for each order\([14]\).
Figure 3.1. – The different sub-processes of a collision event shown schematically. The dark red blob represents the hard parton interaction, i.e. the process determining the nature of the event, from which initial-state showers (blue) and final-state showers (red) unfurl. The light green blobs correspond to the fragmentation processes of partons into hadrons and the dark green blobs to hadron decays or final-state hadrons. A second hard interaction is shown as a violet oval [27].
hadron $h_j$ with $j = a, b$. The factorisation scale $\mu_F$ is necessary to regularise collinear divergences;

- the parton flux $1/2 \hat{s}$ with the partonic centre-of-mass energy $\hat{s}$;

- the matrix element squared $|M_{ab \rightarrow n}^2(\Phi_n; \mu_F, \mu_R)|$ of the parton-level process, which can be derived perturbatively as the momentum transfer in the hard sub-process is large. The renormalisation scale $\mu_R$ on which it depends is introduced to cancel ultraviolet divergences and $\Phi_n$ expresses the phase space of the $n$ final-state particles; and

- the phase space element $d\Phi_n$ for the $n$ final-state particles.

The contributions from the soft interactions between partons in the initial-state and the hard scattering factorise in Eqn. 3.1.1, as the former are happening at larger length scales than the latter [14, 17, 34, 46].

3.1.2. Parton-Shower Evolution and MPI Generation

In PYTHIA, parton showers, i.e. ISR and FSR, are described as successive splittings of colour dipoles. Each quark entering a hard process can be assigned a colour which is propagated towards one of the outgoing quarks, the respective colour partner. Assuming an infinite number of different colour charges, i.e. utilising the large-$N_c$ limit, each process can be decomposed in individual colour lines that span between a quark and its colour partner. The ensemble of colour lines is called colour flow and each of these lines is interpreted as a colour dipole, that itself can branch into further dipoles thereby generating new particles. As gluons carry two colour charges, a quark emitting a gluon can be described as a colour dipole splitting into two colour dipoles (see Fig. 3.2).

For the simulation of a complete FSR parton shower, one possible colour flow scenario is selected for the hardest process providing the initial condition for the shower. The parton showers are then ordered in some variable $p_\perp^{\text{evol}}$ in PYTHIA meaning that the individual dipole branchings are evolved starting from a maximum momentum scale $p_{\perp \text{max}}$ down towards lower momenta until hadronisation completes the generation of the final-state. The $p_{\perp \text{max}}$ of one branching is either given by the momentum scale of the hard process or by a previous particle branching. In this way, energy and momentum are conserved in the shower as each branching is constrained by an upper limit $p_{\perp \text{max}}$. An infra-red cut-off scale $p_{\perp \text{min}}$ treats low-energy dipole branchings as unresolvable such that divergences arising from the emission of soft or collinear gluons are avoided.

The description of ISR proceeds in a similar way as for FSR but whereas for FSR the shower is evolved forwards in time, it expands backwards in time for ISR. Starting from the partons which are entering the hardest process the probability whether a parton emerges in form of

---

3This approximation is legitimate as corrections to it are suppressed by $1/N_c \approx 0.10$ [14].
Figure 3.3. – Left: In the Lund String Model, string breaks produce new quark-antiquark pairs that fragment into hadrons [52]. Right: Scheme of temporal and spatial evolution of successive string breaks. Quarks from adjacent breakup vertices merge to form hadrons [9].

a mother\textsuperscript{4} particle branching is calculated. The simulation stops when no further resolvable branching can be found and the resulting particle is identified with a parton confined in the incoming projectiles [14, 18, 44].

MPIs are usually relatively soft and therefore rarely producing particle jets. They still contribute to the overall energy scattering and production of particles. Thereby, events with larger multiplicities and larger amounts of transverse energy are produced. In PYTHIA, also MPIs are ordered in $p_T$ with the largest-$p_T$ interactions being generated before the lower-$p_T$ ones. Their simulation is interleaved with the ISR and FSR generation according to a common evolution equation for all three effects [14, 44].

3.1.3. Hadronisation

If no further resolvable branching can be found the perturbative showering is terminated and the non-perturbative fragmentation process initiated. Hadronisation in PYTHIA is implemented according to the Lund String Model [10], for which the picture of colour dipoles is replaced by one-dimensional strings that span between a quark and an anti-quark. These strings represent the tube-like colour field between quarks and anti-quarks that forms because of the gluon self-interaction. The energy per unit length that is stored in these strings is given by the string constant $\kappa \approx 1 \text{ GeV/fm}$, that also serves as the proportionality constant in the approximate description of the potential between quarks and antiquarks, i.e. $V(r) \approx \kappa r$ [14].

String breaks can produce further particle-antiparticle pairs (see Fig. 3.3, left panel). The production of heavy quarks, because of their large mass, is however largely suppressed and can be neglected.\textsuperscript{5} The probability for the production of a hadron from a quark and an antiquark of two adjacent string breaks (see Fig. 3.3, right panel) is given by the Lund Symmetric Fragmentation Function [9], which depends on the mass of the produced hadron and the momentum fraction $z$ it carries with respect to the whole system of produced particles [9, 14, 47].

\textsuperscript{4}If a particle decays into further particles, the original particle is the mother particle of the decay products.

\textsuperscript{5}Neglecting the quark masses, the quark and the anti-quark emerging from string breaks are produced in one point of space-time. For the production of heavy quarks, however, energy has to be taken from the colour field and the quark and the anti-quark have to be produced with a certain distance from one another. This distance is overcome by the quarks tunnelling towards their allowed separation according to the rules of quantum mechanics, which is why heavy-quark production is strongly suppressed in soft fragmentation processes according to $u : d : s : c \approx 1 : 1 : 0.3 : 10^{-11}$ [14].
3.2. Computational Framework

Overview on Important Classes

The basic framework of PYTHIA is shown in Fig. 3.4. The administrative Pythia class, inter alia, initialises the variables for the collision system according to user commands and settings read from the Settings and the ParticleData databases. It also calls the three main classes, ProcessLevel, PartonLevel and HadronLevel, which can be identified with the three major steps of the event generation introduced in the previous chapter: the ProcessLevel class being responsible for the generation of the hardest process, the PartonLevel class for the parton-shower evolution and, finally, the HadronLevel class for the hadronisation process and the secondary hadron decays. All three classes interact with the Event class that has two instances: process and event. The process type objects are vectors of Particle objects of which the corresponding particles have participated in the hardest process. The event type objects list the whole yield of particles in the event.

The Event Listing

An extract of the particle listing of an event type object is displayed in Fig. 3.5. Each particle produced in the event appears with its particle index \(i\), which is in the event listing denoted as “no”. In the first and second line, general information about the event and the incoming projectiles are given. The particles entering and leaving the hardest process, in this case a charm pair-creation process, are listed in lines three to six. They can, in each event listing, be referred to by their status codes. In PYTHIA, each particle is assigned a status code which provides information on the particles’ production and function in the event, e.g. whether it is an initial-state or final-state particle or whether it takes part in a hard interaction or MPI. Non-final-state particles have negative status codes, whereas to final-state particles a positive status is assigned. This can in Fig. 3.5 exemplarily be seen for the final-state electron in line 290 and its mother particles. Particles entering and leaving the hardest process (an MPI) can be found via their status codes -21 (-31) and -23 (-33).

Each particle species is assigned a particle identity code (ID) that corresponds to the codes of the Particle Data Group (PDG) for Monte Carlo simulations [37] and is positive for a particle and negative for an antiparticle. This particle ID is given in the second column of the event listing in Fig. 3.5. The third column presents the translation of the particle IDs to the particle names, i.e. the PDG code “-421” refers to a \(\bar{D}^{0}\), the PDG code “4”, refers to a charm quark etc.

In columns five to eight of the event listing, the indices of the first and the second mother and daughter\(^6\) particles are provided. As it was mentioned in Sec. 3.1.2, ISR is evolved backwards in time whereas FSR is evolved forwards in time. This leads to initial-state daughter particles having smaller indices than their mother particles, while the indices of final-state daughters are larger than their mothers indices.

The subsequent columns of the event listing in Fig. 3.5 would give the particles colour, its x-, y- and z-momentum, its energy \(e\) and mass \(m\) but are not shown in this reduced event listing [45, 46].

The same particle can appear several times in the event listing only with its momentum changed. These so-called carbon copies are indications of PYTHIA shuffling around momentum between all particles generated in the event to contain the overall momentum and energy [46].

Knowing all this, one can track one of the charm quarks generated in the pair-creation process of lines three to six through the final-state showers until it fragments into a \(\bar{D}^{0}\) meson in line 124 only by a loop over the particle indices from the beginning to the end of the event listing.

\(^6\)If a particle decays into further particles, the decay products are the daughter particles of the original particle.
Figure 3.4. – Scheme of the information flow between the major PYTHIA classes [43, modified].

<table>
<thead>
<tr>
<th>no</th>
<th>id</th>
<th>name</th>
<th>status</th>
<th>mothers</th>
<th>daughters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90</td>
<td>(system)</td>
<td>-11</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2212</td>
<td>(p+)</td>
<td>-12</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>2212</td>
<td>(p+)</td>
<td>-12</td>
<td>0</td>
<td>91</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>(g)</td>
<td>-21</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
<td>(g)</td>
<td>-21</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>(c)</td>
<td>-23</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>-4</td>
<td>(cbar)</td>
<td>-23</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
<td>(g)</td>
<td>-42</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
<td>(g)</td>
<td>-41</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>(c)</td>
<td>-44</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>-4</td>
<td>(cbar)</td>
<td>-44</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>21</td>
<td>(g)</td>
<td>-43</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

...  

122 1 (d) -71 118 118 125 132
123 21 (g) -71 110 110 125 132
124 -4 (cbar) -71 93 93 125 132

...  

132 -423 (D^0) -84 122 124 239 240

...  

239 -421 (D^0) -91 132 0 290 292

...  

290 11 e^- 91 239 0 0 0
291 -12 nu_ebar 91 239 0 0 0
292 321 K+ 91 239 0 0 0

Figure 3.5. – Example for an event listing in PYTHIA. Each particle that is generated in the simulation of a collision event appears in this event record with its properties: ‘no’ represents the particle index, ‘id’ the particle ID defined in the PDG Monte Carlo numbering scheme, ‘name’ the name of the particle and the columns ‘mothers’ and ‘daughters’ provide the particle indices of the first and last mother and daughter particles [46].
The decay of the $D^{0\ast}$ into a $\bar{D}^0$ meson followed by a semi-electronic $\bar{D}^0 \rightarrow e^- \nu_e K^+$ decay can thus be observed.

**Tunes – The Monash 2013 Tune**

A variety of parameters is needed to describe the evolution of matter after a collision. These parameters are often correlated or anti-correlated with one another and if one is modified this demands for careful adjustment oft the remaining. Because of this, parameter sets are combined to *tunes* in *PYTHIA* so that they can be changed as a whole. The individual tunes have been assembled and optimised until they adequately described the data sets from various collider experiments. The first tunes have been adjusted according to measurements from the Large Electron-Positron (LEP) collider\(^7\), whereas for more recent tunes like the tune 4C (standard tune from version 8.150 ongoing) also LHC Run 1 data has been involved. From *PYTHIA* version 8.2 ongoing, the default tune is the Monash 2013 tune which provides a more adequate description of hadronisation, ISR and MPI than the former ones. A new leading order PDF\(^8\) – NNPDF2.3 – is employed to describe the parton densities for the generation of hard processes, parton showers and MPIs\([46, 47]\).

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\(^7\)The LEP at CERN was the predecessor of the LHC and operated from 1989 to 2000 at energies from 91 GeV up to 209 GeV\([49]\).

\(^8\)NNPDF – Neural Network Parton Distribution Function. NNPDFs are constructed with the help of Monte Carlo methods and neural network training\([19, 46]\).
4. Strategy of Analysis

Two types of correlations will be studied via PYTHIA 8.219 (Monash 2013 tune) simulations: the azimuthal correlation of HFEs and hadrons (HFE$_p$-h correlations) and the correlation of HFEs with other HFEs (HFE$_p$-HFE correlations). Only the trigger electron is asked to originate from an individual heavy-flavour production mechanism, which can either be flavour excitation ($p = fe$), pair creation ($p = pc$) or gluon splitting ($p = gs$). Also, HFE$_{all}$-h and HFE$_{all}$-HFE correlations are simulated, for which all HFEs are considered as trigger particles irrespective of their production process.

HFE$_{all}$-h ACDs for five $p_T$ bins$^1$ of the HFEs are simulated and the relative bottom contribution to all HFEs is derived from fits to the contributions of bottom and charm decays. The results are compared to data from ALICE$^5$, whereby the correlation procedure and the normalisation of the utilised code can be checked, and to HFE$_{all}$-HFE correlations.

The focus is set on the analysis of HFE$_p$-HFE correlations, since previous analyses on correlations between two particles originating from heavy-flavour hadron decays show characteristic properties which can be attributed to certain HFE production mechanisms (see Sec.2.3.4). Four $p_T^{\text{trig}}$ intervals of the trigger HFEs are investigated for each process to find the regime in which the shapes of the individual ACDs differ the most. The yields of the near- and away-side peaks are determined for each ACD and the results are compared. Two $p_T^{\text{trig}}$ intervals are selected for the separation of the heavy-flavour production mechanisms.

4.1. PYTHIA Settings

4.1.1. Process Selection

The cross section for high-$p_T$ HFEs is small (see Sec.2.3.2), which is why it is beneficial to manipulate the generation of the hardest process. In PYTHIA, individual groups of processes can be turned on via process flags. For this analysis, the group SoftQCD:inelastic is utilised to simulate events in the low-$\hat{p}_T$ region, whereby $\hat{p}_T$ denotes the maximum $p_T$ that particles outgoing from the hardest process can carry. This process group appropriately represents the minimum-bias$^2$ production cross section in data combining all inelastic processes, i.e. diffractive$^3$ as well as non-diffractive events$^4$.

Using only the soft-process flags leads to low statistics for the generation of HFEs with higher $p_T$. Therefore, the hard QCD group HardQCD:all is utilised in the high-$\hat{p}_T$ regime.$^4$ As this flag

$^1$On the notation: For HFE$_p$-h correlations, the $p_T$ of the HFE is referred to by $p_T^p$ and the $p_T$ of the hadron by $p_T^h$ following the notation in [5]. For HFE$_p$-HFE correlations, the $p_T$ of the trigger HFE is referred to by $p_T^{\text{trig}}$ and the $p_T$ of the associated HFE by $p_T^{\text{assoc}}$.

$^2$The term minimum bias denotes an event selection which is most inclusive with only a few essential cuts.

$^3$In particle collisions with a relatively soft hardest process the original projectiles might survive the collision event or only get excited and dissociate. The particle production of these diffractive events exhibits large rapidity gaps in the final state[41].

$^4$Also process flags for gluon fusion ($gg \rightarrow Q\overline{Q}$), quark annihilation ($qq \rightarrow Q\overline{Q}$) and quark scattering ($qQ \rightarrow qQ$) are available in PYTHIA. Using these flags would decrease the computation time drastically. For HFE$_p$-HFE correlations, the trigger electrons from a certain heavy-flavour production mechanism are, on the other hand, to be correlated with HFEs from all production processes. The proportion of associated HFEs from the different production mechanisms will, however, always be biased if events are generated with one of the above mentioned
Figure 4.1. – Generation of the hardest process in dependence on its $\hat{p}_T$ in PYTHIA 8.2 simulations for pp collisions at 2.76 TeV. The results for the two process flags SoftQCD:inelastic and HardQCD:all are compared and the ratio of them is displayed.

employs the perturbative QCD cross section, that diverges for low $p_T$, it is necessary to carefully define a value $p_{sw}^T$ at which on switches from using the SoftQCD:inelastic group to using the HardQCD:all group.

Fig. 4.1 shows the predicted cross section\cite{46} for generating a hard process with a certain $\hat{p}_T$ when turning on the flags SoftQCD:inelastic and HardQCD:all. Up to a $\hat{p}_T$ of 11 GeV/c, the cross section using the HardQCD:all group by far exceeds the one from the SoftQCD:inelastic group, whereas for $\hat{p}_T = 11$ GeV/c the curves become compatible. Therefore, the value $\hat{p}_{sw}^T = 11$ GeV/c is chosen for the analysis.

Since the particle production is manipulated by this PYTHIA set-up, the contributions from the individual $\hat{p}_T$ bins have to be scaled according to the generation cross section $\sigma_{Gen}$ before they are summed up. Only in this way, simulation results satisfying the correct production cross section are obtained\cite{46}.

4.1.2. Forcing Semi-Leptonic Decays of Heavy-Flavour Mesons

To further increase the statistics of HFEs, the decay channels of the heavy-flavour mesons can be manipulated in the decay tables of the ParticleData database such that only semi-electronic decay flags as HFEs from the employed process group are generated more frequently.

\footnote{Also for the multiplicity distribution generated with the HardQCD:all flag, limited statistics is observed for $\hat{p}_T > 15$ GeV/c which does not meet the initial expectations. This can, however, be ascribed to the unphysical large cross section at low $p_T$ and is considerably improved by simulations using two $\hat{p}_T$ bins.}
decays are simulated. The branching ratios of the selected decay channels in PYTHIA are rescaled to unity before the event generation. To preserve the correct relation of HFEs from the individual decay channels, each contribution of a correlation pair to the ACD has to be weighted according to the probability $P$ that the two meson mothers decay into HFEs in the same event. This probability is given by the product of the branching ratios $BR_i$ into electrons of the HFE mother particles:

$$ P = BR_1 \cdot BR_2. $$

(4.1.1)

The branching ratios are for each meson provided in Tab. 2.1. Prompt $B$-meson decays are simulated separately from secondary $B$-meson decays. For the former only the $B$-meson decay channels are manipulated, while for the latter, as well as for charm decays, only the $D$-mesons decay channels are manipulated. It is by this avoided that two mesons in one decay chain have modified decay channels, which would require a more complex amendment of the probability $P$.

In the course of the analysis, it appeared that contributions of correlation pairs with particles from the same type of meson mother have not been weighted correctly. A detailed discussion of how this incorrect weighting affects the results can be found in Sec. 4.5.

4.2. The Different Steps of the Simulation

For this analysis, heavy-flavour production via the hardest sub-process and via MPIs shall be considered. To simplify the following discussion the hardest sub-process and the MPIs will be subsumed under the term hard process. The simulation of the correlation is done in four major steps:

1. observation of a heavy-flavour production
2. tracking of the HFE throughout the event
3. correlation of the selected trigger HFEs with charged hadrons/all HFEs
4. merging of the results from individual contributions/normalisation.

The specific procedures differ for the first step while the others are similar for each production process. A scheme of the underlying decision tree for the HFE selection is shown in Fig. A.5.

Observation of a Heavy-Flavour Production

For pair creation, the heavy-flavour quark-antiquark pair is generated in the hard process. To find a pair-creation process in PYTHIA, one therefore has to look for particles with a status code appropriate to particles participating in a hard process (see Sec. 3.2). One also has to check whether the particle ID of the particles entering and leaving the hardest process is suitable (i.e. two incoming gluons/light quarks, two outgoing heavy flavours).

A flavour-excitation process can be found by tracking a heavy flavour that has been generated by a gluon splitting through the ISR until it participates in a hard process. The latter can again be identified via the status codes and the IDs of the particles. Scatterings of two heavy flavours that have been generated by gluon splittings, i.e. double flavour excitation is not taken into account for this analysis.

---

6Representatively for the manipulation of the $D^0$-meson decays:

- $411:\text{onMode}=\text{off}$ \ turns off all decays of the $D^0$ meson (PDG code: 411)
- $411:\text{onIfAny}=\text{11}$ \ turns on only those leading to an electron/positron (PDG code: 11/-11).
To avoid an overlap with flavour-excitation processes, the selection for gluon-splitting processes\(^7\) only considers heavy flavours generated by a gluon splitting that do not participate in a hard process. This can easily be done by only allowing \(1 \rightarrow 2\) processes for these quarks until they fragment into mesons.

**Tracking of Heavy Flavours Throughout Event**

The heavy flavours are tracked through the event listing until they hadronise. Since – following the evolution of the system in time – for ISR the particle indices are decreasing towards the hard process whereas for FSR the particle indices are increasing from the hard process ongoing, two loops are required: one loop from the end of the event listing to the beginning covering ISR and one loop from the beginning of the event listing to the end covering FSR (Fig. A.5).

The heavy flavours are allowed to emit or absorb gluons and photons in ISR and FSR. They are, however, not allowed to take part in another hard process as then heavy flavours from gluon-splitting processes would have to be handled separately to avoid overlap with flavour excitation.

When the last entry of a quark before the hadronisation is located in the event listing, it is searched for \(D\) and \(B\) mesons among its daughter particles. Depending on which decay channel is simulated, some particles have to be excluded from the analysis:

- for charm decays, \(D\) mesons from \(B\) meson decays are not considered;
- for prompt \(B\)-meson decays, no HFEs from \(D\)-meson decays are considered;
- for secondary \(B\)-meson decays, no HFEs from prompt \(B\)-meson decays are considered as well as no HFEs from \(D\)-meson decays, of which the \(D\) meson has no \(B\)-meson mother;
- \(B\) mesons with two possible bottom-quark mothers or \(D\) mesons with two possible charm-quark mothers\(^8\) are not considered, as for these particles it cannot be reconstructed in which process the particles bottom or charm constituents have been generated.

**Correlation**

Having selected suitable trigger electrons, first basic cuts on the transverse momentum \(1.5 < p_T^{\text{trig}} < 6\ \text{GeV/c}\) and the pseudorapidity \(|\eta| < 0.9\) are applied for the HFEs and they are paired with charged hadrons with \(p_T^h > 0.3\ \text{GeV/c}\) and \(|\eta| < 0.9\) from the same event. By these cuts, the subsequent run time of evaluation programs is reduced. The important information about the correlation pairs is stored for each event and written to file when all events have been generated. This data file is then read by a program that performs the final \(p_T\) and \(\eta\) cuts and decides on the correlation type, e.g. selects only correlation pairs of which both correlation partners are HFEs if \(\text{HFE}_{p\rightarrow \text{HFE}}\) are to be simulated. For each correlation pair that passes these selection criteria, the difference in azimuthal angle is calculated according to \(\Delta \varphi = \varphi_{\text{HFE}_p} - \varphi_{\text{assoc}}\).

If necessary, the contributions of the correlation pairs are scaled with respect to the branching ratio of the mother particles of the two partners (see Sec. 4.1.2).

\(^7\)The heavy-flavour production process “gluon splitting” as it is defined in [34] shall here be referred to as “gluon-splitting process”. The general splitting of a gluon, e.g. as part of a flavour-excitation process, shall be denoted as “gluon splitting”.

\(^8\)A scenario like this can occur during the production of primary hadrons: PYTHIA does not provide separate mother-particle indices for each produced hadron but lists several possible mother particles (among which there are two charm or bottom quarks) for a whole group of hadrons (among which there are two \(B\) or \(D\) mesons).

For gluon-splitting processes simulated with the `HardQCD:all` flag in the high-\(p_T\) bin, e.g. roughly about 8\% of the HFEs would originating from respective decays if they were not excluded from the analysis.
Merging of Individual Contributions and Normalisation

As it was mentioned previously, different process flags are used to simulate the hard- and the soft-$p_T$ bins. Since also contributions from bottom and charm hadron decays are simulated separately and it is distinguished between prompt and secondary $B$-meson decays, this leads to six contributions $(dN/d\Delta\phi)_i$ to the HFE$^{all-h}$ ACDs:

<table>
<thead>
<tr>
<th>$0 &lt; \hat{p}_T &lt; 11$ GeV/$c$</th>
<th>$11 &lt; \hat{p}_T &lt; 1000$ GeV/$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>soft, $c \rightarrow e$</td>
<td>hard, $c \rightarrow e$</td>
</tr>
<tr>
<td>soft, $b \rightarrow c \rightarrow e$</td>
<td>hard, $b \rightarrow c \rightarrow e$</td>
</tr>
<tr>
<td>soft, $b \rightarrow e$</td>
<td>hard, $b \rightarrow e$</td>
</tr>
</tbody>
</table>

Each of these contributions needs to be scaled according to the generation cross section $\sigma_{\text{Gen},i}$ as stated in Sec. 4.1.1. To get the same statistics in the soft- and hard-$p_T$ bins, more events have to be generated in the soft bins with respect to the hard bins, which is why a scaling according to the number of events $N_{\text{ev},i}$ is necessary. The six contributions $i$ are then merged and normalised by the overall number $N_{\text{trig}}^p$ of trigger electrons originating from the considered heavy-flavour production process $p$ according to

$$\frac{1}{N_{\text{trig}}^p} \left( \frac{dN}{d\Delta\phi} \right) = \frac{1}{N_{\text{trig}}^p} \left( \sum_{i=1}^{6} \sigma_{\text{Gen},i} \frac{dN}{d\Delta\phi}_i \right), \quad (4.2.1)$$

whereby $N_{\text{trig}}^p$ is given by

$$N_{\text{trig}}^p = \sum_{i=1}^{6} \sigma_{\text{Gen},i} N_{\text{ev},i}^p. \quad (4.2.2)$$

The quantity $N_{\text{trig}}^i$ represents the total amount of suitable trigger electrons that appear for each of the six contributions, irrespective of whether a correlation partner can be found. ACDs of individual heavy-flavour production processes which are normalised according to Eqn. 4.2.1 are referred to as average contributions to HFE$^{all-h}$ or HFE$^{all-HFE}$ correlations. For some applications, it has been beneficial to normalise by the total yield $N_{\text{sum}}^{\text{trig}} = N_{\text{trig}}^{pc} + N_{\text{trig}}^{fe} + N_{\text{trig}}^{gs}$ of trigger HFEs from all production processes thereby presenting the absolute contributions. Together with the overall ACDs for charm and bottom decays, also the contributions from only bottom or only charm decays are provided. These can accordingly be normalised by $N_{\text{trig}}^{p=\text{b}}$ or $N_{\text{trig}}^{p=\text{c}}$ representing the average contributions or by $N_{\text{trig}}^{b+c}$ representing the absolute contributions.

4.3. On the Uncertainties Presented for this Analysis

All uncertainties provided for the simulated ACDs are statistical uncertainties adopted from ROOT 5.34/30[13], that serves as a tool for the evaluation of the simulation results. In ROOT, the statistical uncertainties $\Delta_{\text{stat}}$ are for each bin in a histogram calculated from the square root of the sum of weights in this very bin, whereby the latter is given by the quadrature sum of the weights $w_i$ with which individual entries $i$ are filled into histograms, i.e.

$$\Delta_{\text{stat}} = \sqrt{\sum_i^N w_i^2}. \quad (4.3.1)$$

Consequently, the bin errors correspond to the square root $\Delta_{\text{stat}} = \sqrt{N}$ of respective bin entries $N$ as long as the histograms are filled without any weighting (i.e. $w_i = 1$) [15]. If histograms are rebinned in ROOT, the uncertainties on the new bins are calculated from the
square root of the quadrature sum of the errors from the original bins [15]. The uncertainties on observables like the peak yields determined in Sec. 6.2.2 which are calculated from summing up bins contents are in the following analyses defined according to this concept.

Whereas the uncertainties on the ACDs presented for HFE$_p$-h correlations in Sec. 6.2.1 and Sec. 5.1 appear to be of reasonable size, the uncertainties of the ACDs for HFE$_p$-HFE correlations are large with respect to the overall fluctuation of individual points (e.g. see Fig. 6.6). Unfortunately, no adequate explanation could be found for this effect; mistakes in the actual simulations and in the normalisation procedure could be excluded. Possible errors most probably occur during the selection process of suitable correlation pairs from those pairs which passed the selection criteria illustrated by Fig. A.5. The misestimation of uncertainties does not influence the actual arrangement of simulation points but leads to small $\chi^2$/NDF values of the fit functions utilised for the separation of heavy-flavour production processes in Sec. 6.2.3.

4.4. On the Fits to the Simulations

Several steps in the analysis, e.g. the determination of the peak yields in Sec. 6.2.2 or the process separation in Sec. 6.2.3, require that fits are applied to the simulation results. Since ACDs usually exhibit a near-side (NS) and an away-side (AS) peak, the predominant fit functions that are utilised are a combination of two Gaussian distributions, i.e.

$$f_{2\text{gaus}}(\Delta \varphi) = \frac{\alpha_{\text{NS}}}{\sqrt{2\pi} \sigma_{\text{NS}}} \exp\left(\frac{(\Delta \varphi - \varphi_{\text{NS}})^2}{2\sigma_{\text{NS}}^2}\right) + \frac{\alpha_{\text{AS}}}{\sqrt{2\pi} \sigma_{\text{AS}}} \exp\left(\frac{(\Delta \varphi - \varphi_{\text{AS}})^2}{2\sigma_{\text{AS}}^2}\right) + C.$$  \hspace{1cm} (4.4.1)

and a combination of a Lorentzian function for the near-side and a Gaussian distribution for the away-side peak, i.e.

$$f_{\text{gaus+lor}}(\Delta \varphi) = \frac{1}{2\pi} \frac{\alpha_{\text{NS}} \Gamma}{(\Delta \varphi - \varphi_{\text{NS}})^2 + \frac{1}{4} \Gamma^2} + \frac{\alpha_{\text{AS}}}{\sqrt{2\pi} \sigma_{\text{AS}}} \exp\left(\frac{(\Delta \varphi - \varphi_{\text{AS}})^2}{2\sigma_{\text{AS}}^2}\right) + C.$$ \hspace{1cm} (4.4.2)

The latter is used for distributions with sharper near-side peaks. In Eqn. (4.4.1) and Eqn. (4.4.2), $\alpha_i$ gives the peak yield on the near- or away-side and $\varphi_i$ the position of the peaks. The parameter $\Gamma$ in Eqn. (4.4.2) represents the full width at half maximum (FWHM) of the Lorentzian peak and from $\sigma_i$ the FWHM of Gaussian peaks can be derived. The constant $C$ describes the baseline of the ACDs.

For some distributions, no near- or no away-side peak can be recognised and the fits assuming

\footnote{The $\chi^2$/NDF test provides the possibility to validate the goodness of fits. In ROOT, the $\chi^2$ for a histogram with $i$ bins is defined by

$$\chi^2 = \sum_i \left( \frac{y(i) - f(x(i))}{e(i)} \right)^2,$$ \hspace{1cm} (4.3.2)

whereby $y(i)$ is the bin contents, $x(i)$ the bin centre and $e(i)$ the bin error of the $i$th bin. NDF represents the number of degrees of freedom of the fits [15]. For data with normally distributed uncertainties, an adequate fit result yields a $\chi^2$/NDF that approaches one. However, large error bars with respect to the overall fluctuation of points of the fitted histograms can make the $\chi^2$ decrease even though the histogram is well described by the corresponding fit.}
two peaks fail. In this case, fits with a single Gaussian or Lorentzian function are used, i.e.

\[ f^{\text{gaus}}(\Delta \varphi) = \frac{\alpha}{\sqrt{2\pi} \sigma} \exp \left( \frac{(\Delta \varphi - \varphi)^2}{2\sigma^2} \right) + C, \]  
\[ f^{\text{lor}}(\Delta \varphi) = \frac{1}{2\pi} \frac{\alpha \Gamma}{(\Delta \varphi - \varphi_0)^2 + \frac{1}{4}\Gamma^2} + C. \]  

All fits are performed with the TF1::Fit Method of ROOT 5.34/30 [13].

4.5. Effects of Wrong Weighting on HFE-HFE\(_p\) Correlations

As it was mentioned previously, errors occurred with respect to the weighting of contributions from individual correlation pairs: pairs of which both correlation partners have meson mothers of the same type\(^{10}\) are mistakenly scaled with an additional factor 0.5 with respect to pairs with partners from different types of meson mothers. For HFE-h correlations a negligible effect is expected as the contributions from correlation pairs of which both partners are HFEs should be low. HFE-HFE\(_p\) correlations from Sec. 5.2, Sec. 6.2.1 and Sec. 6.2.2 are more strongly affected.

The impact of this incorrect weighting on the ACDs has been investigated in HFE\(_p\)-HFE correlations for all heavy-flavour production processes with \(1.5 < p_T^{\text{trigger}} < 2.5\) GeV/c. In Fig. 4.2, the correctly weighted ACDs for trigger HFEs from flavour excitation, gluon splitting, pair creation and all processes are compared to the incorrectly weighted ones. The left panel of Fig 4.3 shows the ratios of the correctly and incorrectly weighted ACDs and the right panel the ratios of ACDs from different production processes for the correct weighting and the incorrect weighting. It appears that for all correctly weighted curves in Fig. 4.2 a larger baseline can be observed. While the shape of the distribution remains unchanged for the gluon splitting ACD, for all other processes a larger away-side peak is apparent. This indicates that the probability to find two mesons of the same type within one jet is smaller than to find them in back-to-back jets. Apart from some small deviations on the away-side for the ratio of flavour excitation and pair creation and on the near-side for the ratio of flavour excitation and gluon splitting, the relations of the ACDs from the different production mechanisms are preserved in the right panel of Fig. 4.3. Investigations of the ACDs from charm decays for \(3 < p_T^{\text{trigger}} < 6\) GeV/c show similar behaviour with even less alteration following from the incorrect weighting (see Figs. A.6 and A.7). It is therefore assumed that also investigations of the ACDs with the incorrect weighting can provide reliable predictions of whether ACDs can be utilised to separate individual processes. Consequently, only the ACDs that are used for the final process separation have been corrected. Plots that display results with incorrect weighting are nevertheless marked with a (*) for clarity.

\(^{10}\)The term type shall in this case for example distinguish between \(D^\pm\) and \(D^0\) mesons. Particles and Antiparticles belong to the same type. Consequently, two \(D^0\) mesons or a \(D^0\) and a \(D^0\) meson are of the same type whereas a \(D^\pm\) and a \(D^0\) meson are of different types.
Figure 4.2. – Comparison of the ACDs with correctly weighted correlation pairs with those for which correlation partners with the same type of meson mother are mistakenly weighted with an additional factor 0.5 for HFE$_p$-HFE correlations with $1.5 < p_T^{\text{assoc}} < 2.5$ GeV/$c$.

Figure 4.3. – Left: The ratio of the correctly and incorrectly weighted curves from Fig. 4.2 for each production process. Right: The ratios of the ACDs with trigger HFEs from different production mechanisms. Compared are the ratios for the corrected and the uncorrected ACDs.
5. Results on HFE_{all}-h and HFE_{all}-HFE Correlations

5.1. HFE_{all}-h Correlations

HFE_{all}-h correlations have been simulated for HFEs with 1.5 < p_T < 2.5 GeV/c and 4.5 < p_T < 6 GeV/c and hadrons with p_T > 0.3 GeV/c. Fig. 5.1 compares the results to data from ALICE [5]. The average correlation distributions for HFEs from bottom and charm decays are also presented. It appears that the simulated distributions for the low-p_T (high-p_T) interval are shifted by a constant k_{low} (k_{high}) towards higher (lower) multiplicities with respect to the data. These deviations imply that the particle multiplicity of the underlying event is not well described by PYTHIA. As the simulated curves resemble the data in form, this misestimation only leads to a larger number of uncorrelated electron-hadron pairs while the character of the correlated particles is described properly.

To determine the constants k_{low} and k_{high}, the simulated HFE_{all}-h distributions for both p_T intervals have been fitted according to the fit functions described in Sec. 4.4. The resulting functions with an additional term +k_i have then been fitted to the ALICE data and k_{low} as well as k_{high} have been deduced from these fits

\[ k_{low} = -0.36 \pm 0.02 \text{ and } k_{high} = 0.55 \pm 0.03. \]  

The HFE_{all}-h distribution for 1.5 < p_T < 2.5 GeV/c in Fig. 5.1 resembles the distribution considering only charm decays in form, suggesting that contributions from charm decays dominate the HFE production in this p_T interval. To verify this statement, the ACDs considering HFEs from bottom and charm decays have each been fitted and a combined fit of the resulting fit functions \( f(\Delta \phi)_b \) and \( f(\Delta \phi)_c \) according to

\[ \frac{1}{N_{trig}^{all}} \frac{dN}{d\Delta \phi}_{all} = r_b \cdot \frac{1}{N_{trig}^b} \frac{dN}{d\Delta \phi}_b + (1 - r_b) \cdot \frac{1}{N_{trig}^c} \frac{dN}{d\Delta \phi}_c \]  

has been performed to the simulated HFE_{all}-h distributions [cf. 5]. The same procedure is repeated for the other p_T interval and three further ones. The relative bottom contributions \( r_b \) can be derived from the corresponding fits and the results are given in Tab. 5.1. A monotonous increase of \( r_b \) with increasing p_T of the trigger HFE can be observed.

In Fig. 5.2, the fits and the fitted histograms associated with the determination of \( r_b \) are shown exemplary for HFE_{all}-h correlations with 1.5 < p_T < 2.5 GeV/c and p_T > 0.3 GeV/c. The distributions for HFEs from charm and bottom hadron decays have, in this case, each been fitted with a combination of two Gaussian distributions. The absolute contributions of charm and bottom decays to HFE_{all}-h correlations, i.e. the corresponding distributions from Fig. 5.1 multiplied with \( r_c \) and \( r_b \), are also presented.

The relative bottom contribution \( r_b \) can also directly be calculated from the ratio of the number of trigger HFEs from bottom decays and from bottom and charm decays, i.e. from \( N_{trig}^b/N_{trig}^{all} \), whereby the corresponding values of trigger HFEs are provided by the PYTHIA simulations. The results of this method and the results on \( N_{trig}^b \) and \( N_{trig}^{all} \) for 712 million simulated PYTHIA
Figure 5.1. – Correlation distributions for HFEs with $1.5 < p_T^e < 2.5 \text{ GeV}/c$ and $4.5 < p_T^e < 6 \text{ GeV}/c$ and hadrons with $p_T^h > 0.3 \text{ GeV}/c$ compared to ALICE data from [5]. The distributions are weighted by $N_{\text{trig}}^p$, i.e. the average contributions of bottom and charm decays are displayed.
Table 5.1. – The results for the relative bottom contributions \( r_b \) to ACDs of HFEs from different \( p_T^b \) intervals and hadrons with \( p_T^h > 0.3 \text{ GeV}/c \). These results have been determined from fits to the ACDs and via the ratio \( N_{\text{trig}}^b/N_{\text{all}}^b \). The number of trigger HFEs from bottom decays \( N_{\text{trig}}^b \) and the number of trigger HFEs from bottom and charm decays \( N_{\text{trig}}^{b\text{all}} \) taken from 712 million simulated PYTHIA events are also provided.

<table>
<thead>
<tr>
<th>( p_T^b ) interval (( \text{GeV}/c ))</th>
<th>( r_b ) (Fits)</th>
<th>( r_b ) (( N_{\text{trig}}^b ))</th>
<th>( N_{\text{trig}}^b )</th>
<th>( N_{\text{trig}}^{b\text{all}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1.5, 2.5]</td>
<td>0.244 ± 0.011</td>
<td>0.244 ± 0.002</td>
<td>30798 ± 175</td>
<td>126417 ± 356</td>
</tr>
<tr>
<td>[3.0, 4.0]</td>
<td>0.472 ± 0.013</td>
<td>0.473 ± 0.006</td>
<td>3744 ± 61</td>
<td>21798 ± 148</td>
</tr>
<tr>
<td>[3.5, 4.5]</td>
<td>0.531 ± 0.014</td>
<td>0.535 ± 0.008</td>
<td>10321 ± 102</td>
<td>13379 ± 116</td>
</tr>
<tr>
<td>[4.0, 5.0]</td>
<td>0.583 ± 0.016</td>
<td>0.587 ± 0.010</td>
<td>7164 ± 85</td>
<td>8628 ± 93</td>
</tr>
<tr>
<td>[4.5, 6.0]</td>
<td>0.640 ± 0.017</td>
<td>0.642 ± 0.013</td>
<td>5065 ± 71</td>
<td>5831 ± 76</td>
</tr>
</tbody>
</table>

events are also presented in Tab.5.1. As they are in good agreement with the findings from the method utilising fits to the ACDs, the latter technique is confirmed to give accurate results. This is why a similar approach will in Sec.6.2.3 be used to disentangle the contributions of the individual heavy-flavour production processes to azimuthal correlations of HFEs and HFEs.

Fig.5.3 compares the results for \( r_b \) determined via the fitting technique to findings of two separate ALICE analyses\[5, 26\]. Whereas the first analysis, similarly to the procedure described in this paragraph, fits ACDs for charm and bottom decays derived from PYTHIA 6 simulations to ALICE data, the second method employs a selection on the transverse impact parameter \( d_0 \) of the HFEs to determine the bottom proportion.\(^1\) At low \( p_T^b \), the results presented for this thesis predict a larger fraction of HFEs from charm decays than the ALICE results using HFE\(_{\text{all}}\)-h correlation distributions for the analysis. They are, however, within the uncertainties compatible with the results of this very analysis for \( p_T^b > 0.3 \text{ GeV}/c \) as well as with the findings from the ALICE impact parameter analysis for the whole \( p_T^b \) range.

The uncertainties that are provided for the two ALICE analyses correspond to the square root of the quadrature sum of statistical and systematic uncertainties. The latter are dominated by uncertainties with respect to the HFE and background identification\[5\]. In contrast, the uncertainties presented for the results of this thesis correspond to the uncertainties of the combined fit with Eqn.5.1.2. As no systematic uncertainties are considered, they only provide a lower limit on the absolute uncertainties and are therefore much smaller than for the ALICE analyses.

The ACDs from Fig.5.1, which have been shifted about the constants \( k_i \), can be seen in Fig.5.4, where they show a good agreement with the ALICE data. The reliability of the PYTHIA 8.2 simulations and the utilised correlation procedure is thereby confirmed. Also the absolute contributions of bottom and charm decays to HFE\(_{\text{all}}\)-h correlations are displayed.

The individual contributions to HFE\(_{\text{all}}\)-h correlations can be further split up into those from soft- and hard-\( p_T \) bins. Also contributions from prompt and secondary \( B \)-meson decays can be considered separately. Fig.5.5 provides the unshifted results for these individual contributions to correlations with \( 1.5 < p_T^b < 2.5 \text{ GeV}/c \) and \( p_T^b > 0.3 \text{ GeV}/c \). All distributions are normalised by the total yield of trigger HFEs of the overall ACD, thus they represent the absolute contributions to HFE\(_{\text{all}}\)-h correlations.

From Fig.5.5, it can be concluded that in this \( p_T^b \) range HFE-h correlations are dominated by trigger HFEs originating from charm decays, of which the corresponding quark has been generated in a rather soft process. This result is in accordance with findings of the analysis in\[5\]

\(^1\)For the original plot displaying the ALICE results see Fig.2.16.
Figure 5.2. – Fits and fitted histograms associated with the determination of the relative bottom contribution \( r_b \) to the HFE production for HFE_{all-h} correlations with \( 1.5 < p_T^e < 2.5 \text{ GeV}/c \) and \( p_T^h > 0.3 \text{ GeV}/c \). The average distributions for charm and bottom meson decays are presented with open markers whereas the absolute contributions to HFE_{all-h} correlations are shown with closed markers.

Figure 5.3. – The results for the relative bottom contribution \( r_b \) to the HFE production compared to findings from two ALICE analyses [5].
Figure 5.4. – Simulations of azimuthal-correlation distributions for HFEs with $1.5 < p_T^e < 2.5$ GeV/$c$ and $4.5 < p_T^e < 6$ GeV/$c$ and hadrons with $p_T^h > 0.3$ GeV/$c$ compared to ALICE data from [5]. The PYTHIA simulations have each been shifted by suitable constants \( k \) to match the data. The 95% confidence intervals of the fits to determine the \( k \) are displayed. All distributions are weighted by \( N_{b+c}^{\text{trig}} \), i.e. the absolute contributions of bottom and charm decays are displayed.
stating that up to a $p_T$ of 3 GeV/c, the HFE production is dominated by charm decay processes (see Sec. 2.3.2). HFEs from soft, prompt $B$-meson decays provide a notable contribution together with HFEs from charm decays originating from a harder process. All distributions exhibit a larger away-side and a smaller near-side peak apart from the contribution from soft, prompt $B$-meson decays. The latter is rather flat for this $p_T$ interval but shows growing near-side and away-side peaks with selecting higher-$p_T$ trigger electrons. It is thus expected that electrons from prompt $B$-meson decays in general exhibit a large $p_T$ and by the applied $p_T$ cut, only particles are selected which have not been emitted in jets and are thus uncorrelated.

Remarkable is the double-peak structure for the near-side peak that can be observed for hard, prompt $B$-meson decays. A similar pattern has been perceived for HFE-$D^0$ correlations when the HFEs on the one hand and the kaons from the respective $D^0$-meson decays on the other hand were asked to carry a like-sign charge [31]. The effect can most probably be ascribed to the large branching fraction of the decay channels $B^0/B^\pm \to \nu_\ell e^+D^*$ (see Sec. 2.3.3). As the $B$ meson has about three times the mass of the $D$ meson, the angular separation of the $B$-meson decay products will be larger than for the decay products of the $D$ meson, which are expected to be emitted almost collinearly. If the $e^+$ and the $D^*$ meson should exhibit a typical, non-zero angular separation, this separation would also be apparent for this very $e^+$ and a possibly generated HFE from the following $D$-meson decay. To confirm this assumption, further investigations with PYTHIA would be necessary.

5.2. HFE$_{all}$-HFE Correlations

HFE$_{all}$-HFE correlations have been simulated for trigger HFEs with $1.5 < p_T^{\text{trig}} < 2.5$ GeV/c and $p_T^{\text{trig}} > 0.3$ GeV/c. The results are presented in Fig. 5.6 together with the average correlations for charm and bottom decays in these $p_T$ intervals.
Since the cross sections for the generation of HFEs are small, the average number of correlation partners per trigger electron is much smaller for HFE\textsubscript{all}-HFE correlations with respect to HFE\textsubscript{all}-h correlations, which is why the ACDs for HFE\textsubscript{all}-HFE correlations are shifted towards lower multiplicities. The decay chain of a $B$ meson on average produces more HFEs than the decay chain of a $D$ meson. Therefore, the correlation distribution for charm decays is in the low-$p_T$ interval about four times smaller than the one for bottom decays and the near-side peak of the latter is more pronounced. For the high-$p_T$ interval the baselines of both ACDs approach and especially the near-side peaks get more distinct. Presumably, high-$p_T$ particles are more likely to have been emitted in jets and therefore exhibit a more distinct correlation leading to more narrow peaks.

The simulation results for ACDs of charm and bottom decays can be compared to PYTHIA 6 simulations on azimuthal correlations of HFEs and neutral $D^0$ mesons\cite{21} whereby either both particles have been originating from events containing a charm ($cD^0$-cEl) or a bottom quark ($bD^0$-bEl). The latter simulations are shown for $2 < p_T^{D^0} < 16\text{ GeV}/c$ and $1 < p_T^e < 4\text{ GeV}/c$ in Fig. 2.18. It is remarkable that even though these simulations ask both correlated particles to emerge from charm and bottom quark events, the distributions resemble the ACDs for charm and bottom decays for the low-$p_T$ interval which are presented in Fig. 5.6. This suggests on the one hand, that the latter distributions are dominated either by correlation pairs with both partners originating from a charm event, or with both partners originating from a bottom quark event. On the other hand, the correlation structure seems to be propagated from the $D^0$ mesons to their HFE daughters.

As for HFE\textsubscript{all}-h correlations, the different contributions from soft- and hard-$\hat{p}_T$ bins can be disentangled and the results for trigger HFEs with $1.5 < p_T^{\text{trig}} < 2.5\text{ GeV}/c$ and associated HFEs with $p_T^{\text{assoc}} > 0.3\text{ GeV}/c$ are shown in Fig. 5.7. In contrast to the HFE\textsubscript{all}-h correlations, HFEs from prompt $B$-meson decays and from $D$-meson decays with quarks from soft processes contribute in an equal manner. The overall shapes of the distributions are similar to HFE\textsubscript{all}-h correlations. Noteworthy is however the different appearance of the contributions for charm-decay processes in the soft- and hard-$\hat{p}_T$ bin: The latter shows a distinct near-side and a small away-side peak while for the former almost no near-side structure can be observed. Potentially, this can be ascribed to the larger percentage of gluon-splitting processes on production processes with high $\hat{p}_T$ in comparison to production processes with low $\hat{p}_T$. Gluon-splitting processes produce particles with small angular separations thus leading to narrow near-side peaks (see Sec. 6.2.1).
Figure 5.6. – Simulations of azimuthal-correlation distributions for HFEs with $1.5 < p_T^{\text{trig}} < 2.5$ GeV/$c$ and $4.5 < p_T^{\text{trig}} < 6$ GeV/$c$ and associated HFEs with $p_T^{\text{assoc}} > 0.3$ GeV/$c$. The distributions are weighted by $N_{\text{trig}}$, i.e., the average contributions of bottom and charm decays are displayed.
Figure 5.7. – The absolute contributions to HFE_{all}-HFE azimuthal correlations for $1.5 < p_T^{\text{trig}} < 2.5$ GeV/c and $p_T^{\text{assoc}} > 0.3$ GeV/c. For clarity, the error bars are shown only for the overall ACD.
6. Results on HFE$\!\!_{p}$-h and HFE$\!\!_{p}$-HFE Correlations

6.1. HFE$\!\!_{p}$-h Correlations

HFE$\!\!_{p}$-h correlations, of which the trigger HFE has been originating from either flavour excitation, gluon splitting or pair creation, have been simulated for $1.5 < p_{T} < 2.5 \text{ GeV/c}$ and $p_{T}^{h} > 0.3 \text{ GeV/c}$. The results are displayed in Fig. 6.1. Comparisons of the ACDs for the different heavy-flavour production processes for bottom decays, charm decays and the overall distributions are provided in Figs. A.8, A.9 and A.10.

All ACDs in this figure exhibit a higher near-side and a smaller away-side peak. For gluon-splitting processes a larger baseline than for the other two mechanisms can be observed. This is because higher centre-of-mass energies $\sqrt{s}$ of the hardest process are necessary in order for a gluon-splitting process to come about. Since the particle multiplicity scales with $\sqrt{s}$, events containing a gluon-splitting process also exhibit larger multiplicities, i.e. more possible correlation partners.

The shape of the peaks is – in case of the overall ACDs and the ACDs for charm decays – similar for all production processes: the process separation based on these distributions seems to be challenging for the considered $p_{T}$ interval. Potentially, a separation of bottom-quark production processes can be achieved on the basis of the ACDs for bottom decays as in this case the distributions for the higher order mechanisms are flatter than for the LO mechanisms. The latter assumption would need to be confirmed by further investigations.

As already stated in Sec.2.3.4, the production mechanisms can, in all probability, also not conclusively be distinguished in HFE$\!\!_{p}$-h correlations for $2.5 < p_{T} < 3.5 \text{ GeV/c}$, $|\eta_{e}| < 0.7$ and $|\eta_{h}| < 0.9$ [50]. Together with the findings from the present thesis, this indicates that, in general, HFE$\!\!_{p}$-h azimuthal correlations are not suited for this kind of investigations. This is most likely because there are for each HFE too many possible correlation partners per event. No characteristic structure can develop for the individual processes, as the correlation pairs which have been generated in the same production process and which exhibit an angular separation typical for this kind of process are screened by the amount of arbitrary correlation pairs. Therefore, the focus will hereafter be set on the analysis of ACDs for HFE$\!\!_{p}$-HFE correlations.
Figure 6.1. — (*) Simulations of HFE\(p\)-h azimuthal correlations of which the trigger HFEs with \(1.5 < p_T^c < 2.5 \text{ GeV}/c\) are generated either via flavour excitation, gluon splitting or pair creation. Also, the results for HFE\(all\)-h correlations are displayed. All ACDs for the individual processes have been simulated with the correct weighting, apart from the ACD for HFE\(all\)-h correlations which is displayed for incorrect weighting (see Sec. 4.5). The distributions are weighted by \(N_{\text{trig}}^p\), i.e. the average contributions of charm and bottom decays are displayed.
6.2. HFE<sub>p</sub>-HFE Correlations

6.2.1. Comparison of ACDs from Different Production Processes

The HFE<sub>p</sub>-HFE correlations for different production mechanisms of the trigger HFE have been simulated in four \( p_{\text{trig}}^{\text{assoc}} > 0.3 \text{ GeV/c} \):

- \( 1.5 < p_{\text{trig}}^{\text{trig}} < 2.5 \text{ GeV/c} \),
- \( 2.0 < p_{\text{trig}}^{\text{trig}} < 4.0 \text{ GeV/c} \),
- \( 3.0 < p_{\text{trig}}^{\text{trig}} < 6.0 \text{ GeV/c} \) and
- \( 1.5 < p_{\text{trig}}^{\text{trig}} < 6.0 \text{ GeV/c} \).

The simulation results for pair creation, gluon splitting and flavour excitation are presented in Fig.6.2, Fig.6.3 and Fig.6.4, respectively. The deviations caused by the wrong weighting of correlation pairs (see Sec.4.5) are exemplarily shown as error rectangles for the overall curves with \( 1.5 < p_{\text{trig}}^{\text{trig}} < 2.5 \text{ GeV/c} \).

For pair creation, the generated HFEs are expected to be emitted predominately with a difference in azimuthal angle of about \( \pi \) due to momentum conservation. In accordance with this expectation, the corresponding ACDs for the charm-decay contributions in Fig.6.2 exhibit only an away-side peak. Since, in contrast to a fragmenting charm quark, a fragmenting bottom quark can produce up to two HFEs, the bottom contribution shows an additional near-side peak, evoked by correlated HFEs originating from the same \( B \) meson. The overall ACD is dominated by the charm contribution at low \( p_{\text{trig}}^{\text{trig}} \) and is approaching the bottom contribution at higher \( p_{\text{trig}}^{\text{trig}} \). For the lowest \( p_{\text{trig}}^{\text{trig}} \) interval, the near-side peak for the overall ACD is compatible with a constant.

A boosted gluon splitting into a heavy-flavour pair evokes a small angular separation of the emerging quarks, which is manifested in the near-side peaks of the ACDs from bottom and charm contributions and the overall ACD in Fig.6.3. Correlation pairs with trigger HFEs from flavour-excitation processes are expected to show a rather large separation in azimuthal angle because of the hard-process scattering. This is reflected by the away-side peaks of the respective ACDs for charm contributions in Fig.6.4. The contributions of bottom decays exhibit large near-side peaks that conceal contributions on the away-side. This is due to the decay characteristics which have already been discussed for pair-creation processes.

Only minor deviations are expected to follow from the wrong weighting of correlation pairs as it was discussed in Sec.4.5: the baseline of incorrectly weighted distributions is most probably smaller with respect to correctly weighted ones and a decrease in size of the away-side peaks might be apparent as it was observed for HFE<sub>fe</sub>-HFE and HFE<sub>pc</sub>HFE correlations with \( 1.5 < p_{\text{trig}}^{\text{trig}} < 2.5 \text{ GeV/c} \). The general behaviour of the individual ACDs with \( p_{\text{trig}}^{\text{trig}} \) is, however, assumed to be correctly represented by the distributions shown in Figs.6.2 to 6.4.

6.2.2. Determination of Peak Yields

The yields of the near-side and the away-side peaks of all distributions presented in Figs.6.2 to 6.4 have been determined via the zero yield at minimum (ZYAM) method, assuming that at the minimum \( \Delta \varphi_{\text{min}} \) of each ACD particles are not correlated [8]. The widths \( \sigma \) of the peaks are then defined by the interval between the respective minima, whereby the latter have been determined by fitting the ACDs with the fit functions described in Sec.4.4. The contents of the bins within the respective intervals are summed up and the contribution from the baseline \( b \)

\[ b = \sigma \cdot C(\Delta \varphi_{\text{min}}), \]  

(6.2.1)
Figure 6.2. – (*) Simulations of HFE_{pc}-HFE correlations of which the trigger HFEs are generated by pair-creation processes for four $p_T^{\text{trig}}$ intervals. All distributions are weighted by $N_p^{\text{trig}}$, i.e. the average contributions of charm and bottom decays are displayed. For the overall distributions with $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV}/c$, the deviations caused by the wrong weighting of correlation pairs are shown as boxes.
Figure 6.3. (*) Simulations of HFE_{g\rightarrow HFE} correlations of which the trigger HFEs are generated by gluon-splitting processes for four $p_T^{\text{trig}}$ intervals. All distributions are weighted by $N^p_{\text{trig}}$, i.e. the average contributions of charm and bottom decays are displayed. For the overall distributions with $1.5 < p_T^{\text{trig}} < 2.5$ GeV/c, the deviations caused by the wrong weighting of correlation pairs are shown as boxes.
Figure 6.4. – (*) Simulations of HFE_{fe}-HFE correlations of which the trigger HFEs are generated by flavour-excitation processes for four $p_T^{\text{trig}}$ intervals. All distributions are weighted by $N^{p\text{trig}}_{\text{trig}}$, i.e. the average contributions of charm and bottom decays are displayed. For the overall distributions with $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV}/c$, the deviations caused by the wrong weighting of correlation pairs are shown as boxes.
which is calculated from the yield at the minimum $C(\Delta \varphi_{\text{min}})$, is subtracted. This method for the peak-yield determination shall be denoted as Normal Fit (NF) technique.

Having a closer look at the ACDs for contributions from gluon-splitting processes in Fig. 6.3, these curves exhibit only a near-side peak and one minimum, of which the latter is clearly defined. In this case, the peak has been fitted with a sole Gaussian or Lorentzian function to find the minimum and the yield has been derived by adding up the yields of all bins and subtracting the baseline (Zero Yield (ZY) technique). On the other hand, for the overall ACD for pair creation in the interval $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV}/c$ no distinct minimum could be determined from the fits. Therefore, the reach of the corresponding away-side peak has been defined by the difference in azimuthal angle where the first derivative of the fit function is smaller than 0.0001 (Slope (S) technique).

The statistical uncertainties on the peak yields are calculated as the square root of the quadrature sum of the uncertainties of the individual bins (see Sec. 4.3). In addition, systematic uncertainties can be specified for this approach. It has been investigated for how many bins adjacent to $\Delta \varphi_{\text{min}}$ the corresponding values of the fit function are compatible with the yield at $\Delta \varphi_{\text{min}}$ within the uncertainties. From this an uncertainty for $\Delta \varphi_{\text{min}}$ could be constructed. The peak yields have then been calculated for the central values of the minima as well as for the upper and the lower edges. The larger value of the deviations of the peak yields calculated for the upper and for the lower edges with respect to those derived from the central value at $\Delta \varphi_{\text{min}}$ are given as systematic uncertainty for each yield.

In cases where either no near-side or no away-side peak has been recognised from the fits, i.e. if either the Zero Yield or the Slope technique are applied, the yield of the corresponding peak is taken to be zero. An estimate for the upper limit on the uncertainty of this yield is derived from the overall yield on the near-side ($-\pi/2 < \Delta \varphi < \pi/2$) or away-side ($\pi/2 < \Delta \varphi < 3\pi/2$) with the baseline subtracted. The latter is in this case defined by the product of the minimal bin content and the number of bins within the corresponding interval, similar to Eqn. 6.2.1. Systematic uncertainties are not estimated for respective yields as they are expected to be small with respect to the uncertainties derived by the abovementioned technique.

In Tab. A.1, it is listed for all individual distributions which methods have been used to determine the peak yields and uncertainties.

The results of the peak-yield determination are shown in Fig. 6.5. For the near- and away-side peaks of all processes, a trend towards rising yields with selecting higher-$p_T$ trigger particles can be observed. For the interval $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV}/c$, the major contribution to the away-side peak of the overall ACDs is provided by pair-creation processes. This suggests that LO and higher order processes can be separated in this $p_T^{\text{trig}}$-interval. The contributions from flavour excitation and gluon splitting on the near-side are, however, to closely resembling each other such that a further separation of the higher order processes seems challenging.

The contributions from charm decays with $3 < p_T^{\text{trig}} < 6 \text{ GeV}/c$ appear more promising for a complete separation of all three heavy-flavour production mechanisms, since mainly gluon splitting is contributing to the near-side peak and pair-creation contributions are more peaked on the away-side than those from flavour excitation.

Concerning the ACDs of bottom decays, a process separation should best be possible for $2 < p_T^{\text{trig}} < 4 \text{ GeV}/c$ and $1.5 < p_T^{\text{trig}} < 6 \text{ GeV}/c$. For the ACDs with $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV}/c$, the indefinite shape of the near-side peak for pair creation and for the ACDs with $3 < p_T^{\text{trig}} < 6 \text{ GeV}/c$, the small contribution of flavour excitation to the away-side peak would complicate the analysis.

The incorrect weighting of correlation pairs (see Sec 4.5) most likely produces smaller peak yields especially for the away-side peaks of the ACDs. This should not affect the selection of suitable $p_T^{\text{trig}}$ intervals for the process separation since the overall behaviour of the ACDs with $p_T^{\text{trig}}$ is expected to be only slightly influenced by the wrong weighting. If, however, the separation of production
processes is based e.g. on the distinct shape of the away-side peak of HFE\textsubscript{pc}-HFE correlations – as it was proposed for the separation of LO and higher order processes in correlations with $1.5 < p_T^{trig} < 2.5$ GeV/c – growing away-side peaks for HFE\textsubscript{fc}-HFE or HFE\textsubscript{gs}-HFE correlations could make the determination of relative contributions more difficult or even impossible. To not distort the final results of this thesis, the ACDs for which the individual process contributions are to be determined are thus re-evaluated with the correct weighting.

6.2.3. Determination of Process Contributions

From the PYTHIA simulations, the number $N_{trig}^p$ of trigger HFEs originating from a specific heavy-flavour production process can be gathered. The relative contribution $r_p$ of a process $p$ to the overall heavy-flavour production can be calculated from the proportion of $N_{trig}^p$ on the sum $N_{trig}^{sum}$ of trigger HFEs from all processes, i.e.

$$r_p = \frac{N_{trig}^p}{N_{trig}^{sum}} = \frac{N_{trig}^p}{N_{trig}^{pc} + N_{trig}^{gs} + N_{trig}^{fe}}. \quad (6.2.2)$$

In the following, a possible method for the determination of the relative process contributions from azimuthal-correlation distributions is introduced, which can likewise be applied to data. It is based on a combination of fits to the data, whereby the PYTHIA simulations for the average ACDs of the individual processes serve as templates for the process separation. Simulation results with the correct weighting of correlation pairs (see Sec. 4.5) are utilised for this part of the analysis and simulations for HFE\textsubscript{all}-HFE correlations are employed substitutionally for real data. The reliability of the obtained findings is confirmed by a comparison to the results which are provided by Eqn. 6.2.2. All production processes are separated for HFE\textsubscript{pc}-HFE correlations from charm decays with $3 < p_T^{trig} < 6$ GeV/c. For HFE\textsubscript{pc}-HFE correlations from charm and bottom decays with $1.5 < p_T^{trig} < 2.5$ GeV/c, higher order processes have been distinguished from LO processes. The process separation with respect to contributions from bottom decays is left for further analysis due to the limited statistics of the corresponding ACDs.

Separation of Charm-Production Mechanisms for $3 < p_T^{trig} < 6$ GeV/c

In Fig. 6.6, the sum of the contributions from flavour excitation, gluon splitting and pair creation is compared to the ACD for HFE\textsubscript{all}-HFE correlations with trigger HFEs from charm decays and $3 < p_T^{trig} < 6$ GeV/c. Also, the absolute contributions of the individual processes are presented. The ACD for the process sum is in good agreement with the ACD for all processes. Thus the contribution to the azimuthal correlation is negligible for charm-production processes which are not considered in this thesis. Fig. 6.6 indicates that the charm production is dominated by gluon-splitting processes with smaller contributions from flavour excitation and pair creation.

To obtain a quantitative statement on the relative process contributions, the simulation results for all production processes have been fitted and the resulting functions shall be denoted by $f(\Delta \phi)_gs$ for gluon splitting, $f(\Delta \phi)_fe$ for flavour excitation and $f(\Delta \phi)_pc$ for pair creation. The simulation results for HFE\textsubscript{pc}-HFE and HFE\textsubscript{fc}-HFE correlations have been added according to

$$\frac{1}{N_{trig}^{pc+fe}} \left( \frac{dN}{d\Delta \phi} \right)_{pc+fe} = \frac{(dN/d\Delta \phi)_{pc}}{N_{trig}^{pc} + N_{trig}^{fe}} + \frac{(dN/d\Delta \phi)_{fe}}{N_{trig}^{pc} + N_{trig}^{fe}}. \quad (6.2.3)$$
Figure 6.5. – (*) Peak yields of the ACDs from HFE_{p-HFE} correlations presented in Fig. 6.2, Fig. 6.3 and Fig. 6.4 for contributions from pair creation, gluon splitting and flavour excitation processes. The ACDs for bottom ("B") and charm ("C") decays and the overall ACDs ("C+B") are shown. Statistical uncertainties are given as error bars, systematic uncertainties as hooks.
**Figure 6.6.** – Comparison of the ACD for all processes to the sum of contributions from flavour excitation, pair creation and gluon splitting for HFE$_p$-HFE correlations. Also, the contributions from the individual processes normalised by the total yield $N_{\text{sum}}$ of trigger HFEs are presented. **Upper Panel:** HFE$_p$-HFE correlations with trigger HFEs from charm decays and $3 < p_T^{\text{trig}} < 6 \text{ GeV/c}$ and associated HFEs with $p_T^{\text{assoc}} > 0.3 \text{ GeV/c}$. **Lower Panel:** HFE$_p$-HFE correlations with trigger HFEs from charm and bottom decays with $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV/c}$ and associated HFEs with $p_T^{\text{assoc}} > 0.3 \text{ GeV/c}$. 
Table 6.1. – Number $N^p_{\text{trig}}$ of trigger HFEs originating from pair creation (pc), flavour excitation (fe) and gluon splitting (gs) processes in PYTHIA 8.2 (Monash tune) simulations with 600 million events.

<table>
<thead>
<tr>
<th>$p_T^{\text{trig}}$ (GeV/c)</th>
<th>pc</th>
<th>fe</th>
<th>gs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.0 &lt; p_T^{\text{trig}} &lt; 6.0$</td>
<td>$542 \pm 23$</td>
<td>$1455 \pm 38$</td>
<td>$2117 \pm 46$</td>
</tr>
<tr>
<td>only charm decays</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1.5 &lt; p_T^{\text{trig}} &lt; 2.5$</td>
<td>$4269 \pm 65$</td>
<td>$10884 \pm 104$</td>
<td>$24149 \pm 155$</td>
</tr>
<tr>
<td>charm &amp; bottom decays</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and a further fit has been performed to this combined distribution. The fit function $f(\Delta \varphi)_{pc+fe}$ derived from this latter fit and $f(\Delta \varphi)_{gs}$ are then utilised for a combined fit with

$$\frac{1}{N_{\text{all}}^{\text{trig}}} \left( \frac{dN}{d\Delta \varphi} \right)_{\text{all}} = r_{gs} \cdot \frac{1}{N_{\text{gs}}^{\text{trig}}} \left( \frac{dN}{d\Delta \varphi} \right)_{gs} + (1 - r_{gs}) \cdot \frac{1}{N_{\text{pc+fe}}^{\text{trig}}} \left( \frac{dN}{d\Delta \varphi} \right)_{pc+fe}$$

(6.2.4)

to the ACD for HFE_{all}-HFE correlations. The relative contribution $r_{gs}$ of gluon-splitting processes has been determined from this fit. The absolute gluon-splitting contribution could then be subtracted from the HFE_{all}-HFE correlations by the bin-wise subtraction of $f(\Delta \varphi)_{gs} \cdot r_{gs}$ from the corresponding histogram. This leaves the combined contributions from flavour excitation and pair creation. A fit with

$$\frac{1}{N_{\text{pc+fe}}^{\text{trig}}} \left( \frac{dN}{d\Delta \varphi} \right)_{pc+fe} = r_{pc} \cdot \frac{1}{N_{\text{pc}}^{\text{trig}}} \left( \frac{dN}{d\Delta \varphi} \right)_{pc} + (r_{gs} - r_{pc}) \cdot \frac{1}{N_{\text{fe}}^{\text{trig}}} \left( \frac{dN}{d\Delta \varphi} \right)_{fe}$$

(6.2.5)

served to separate the relative contributions from these two processes.

In Fig.6.7 the individual fit functions and the fitted histograms are shown. The $\chi^2$/NDF of the fits is low at $O(10^{-3})$, which can be ascribed to the large error bars of the fitted histograms (see Sec.4.3). The results for the contributions from the different heavy-flavour production processes are

$$r_{pc} = 0.12 \pm 0.09, \quad r_{gs} = 0.53 \pm 0.05 \quad \text{and} \quad r_{fe} = 0.35 \pm 0.07.$$  

(6.2.6)

The uncertainties are taken from the errors on the fit results provided by the TF1::Fit Method of ROOT.

The process contributions $r_i$ can also be deduced from the yields of trigger HFEs as described by Eqn.6.2.2, whereby the results on $N^p_{\text{trig}}$ for 600 million generated PYTHIA events are provided in Tab.6.1. The values, that can be derived in this way are, within the uncertainties, in agreement with the results from Eqn.6.2.6:

$$r_{pc} = 0.13 \pm 0.01, \quad r_{gs} = 0.51 \pm 0.01 \quad \text{and} \quad r_{fe} = 0.35 \pm 0.01.$$  

(6.2.7)

The uncertainties in Eqn.6.2.7 are derived via Gaussian error propagation from the statistical uncertainties of the $N^p_{\text{trig}}$. 

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Figure 6.7.  – **Upper Panel:** Combined fit with the fit functions for flavour excitation and pair creation $f(\Delta \varphi)_{pc+fe}$ on the one hand and gluon splitting $f(\Delta \varphi)_{gs}$ on the other hand to the simulation results for the charm contribution to HFE\textsubscript{all}-HFE correlations with $3 < p_{\text{T}}^\text{trig} < 6 \text{ GeV}/c$. From this fit the relative contribution of gluon-splitting processes $r_{gs}$ has been derived. **Lower Panel:** Combined fit of the fit functions for flavour excitation $f(\Delta \varphi)_{fe}$ and pair creation $f(\Delta \varphi)_{pc}$ to the HFE\textsubscript{all}-HFE ACD of which the fit function for gluon splitting weighted with $r_{gs}$ has been subtracted. From this fit the relative contributions of flavour excitation $r_{fe}$ and pair-creation processes $r_{pc}$ have been derived.
Separation of Higher Order and LO Production Mechanisms for $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV}/c$

The sum of the contributions from pair creation, gluon splitting and flavour excitation for HFE_{pT}-HFE correlations with $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV}/c$ are in Fig. 6.6 compared to the ACD for all processes. As for the other $p_T^{\text{trig}}$ interval, the sum of contributions agrees with the ACD for all processes and the heavy-flavour production is dominated by contributions from gluon-splitting processes.

To separate the higher order processes flavour excitation and gluon splitting from the LO contribution to the heavy-flavour production, the simulation results for HFE_{gs}-HFE and HFE_{fe}-HFE correlations have been added equivalently to Eqn. 6.2.3. The contributions from the higher order processes and pair creation have been fitted and a combination of the resulting fit functions has been used to fit the results for HFE_{all}-HFE correlations and determine the relative contribution $r_{pc}$ from LO processes. The corresponding fits and fitted histograms can be found in Fig. A.11 and the fit results are

$$r_{pc} = 0.15 \pm 0.02 \quad \text{and} \quad r_{fe+gs} = 0.85 \pm 0.02. \quad (6.2.9)$$

As for the fits in the other $p_T^{\text{trig}}$ interval, the $\chi^2/NDF$ of the fits is low at $O(10^{-2})$.

A comparison to the values derived by Eqn. 6.2.2, shows compatibility of the results from both analyses techniques

$$r_{pc} = 0.171 \pm 0.002, \quad r_{fe} = 0.290 \pm 0.002 \quad \text{and} \quad r_{gs} = 0.539 \pm 0.002. \quad (6.2.10)$$

Discussion of the Results

The results on the relative process contributions derived via the fits to the ACDs are in good agreement with the findings from the trigger HFE yields. The presented analysis technique is therefore confirmed to provide reliable results for the considered $p_T^{\text{trig}}$ intervals. In general, good statistics of the ACDs from data will, however, be necessary for adequate results as well as Monte Carlo templates for the individual heavy-flavour production processes, that show characteristic features for each mechanism.

Since the bottom contribution to all HFEs is low for $p_T^{\text{trig}} < 2.5 \text{ GeV}/c$ (about 20-30% according to Fig. 2.16), the analyses performed in this sub-chapter mainly investigate the production of charm quarks. The results are compatible with a decreasing contribution of pair-creation processes with increasing $p_T^{\text{trig}}$, which would be in accordance with findings concerning the dependence of process contributions on the $p_T$ of generated bottom quarks (See Fig. A.2 and Fig. A.3) [22, 34]. Fits in further $p_T^{\text{trig}}$ intervals are necessary to confirm this behaviour.

Perturbative calculations predict comparable contributions from pair creation and gluon splitting to the charm production at the given centre-of-mass energy together with a dominant proportion of flavour excitation (see Fig. 2.11). The process contributions to the total yield of HFEs from charm decays are expected to behave similarly as the probability for a charm quark to decay into a HFE is the same for all production mechanisms. In the analyses presented here, however, a selection on the $p_T$ of the trigger electron and a pseudorapidity cut $|\eta| < 0.9$ have been performed. According to Fig. 2.12, the rapidity cut presumably eliminates a considerable fraction of pair
creation and flavour-excitation processes with respect to gluon splitting. This effect is expected to lead to the substantial contribution from gluon-splitting processes to the HFE production in the considered $p_T^{\text{trig}}$ intervals.
Summary and Outlook

In high-energy particle collisions, the production of the heavy flavours charm and bottom by the strong interaction can be classified in three major mechanisms: pair creation, gluon splitting and flavour excitation. In this thesis, it has been investigated whether the contributions of these processes to azimuthal correlations of heavy-flavour electrons and hadrons (HFE-h correlations) as well as HFEs and other HFEs (HFE-HFE correlations) can be separated in pp collisions at 2.76 TeV via the shape of their azimuthal-correlation distributions (ACDs). HFE$_p$-h and HFE$_p$-HFE correlations with trigger HFEs originating from individual production processes $p$ have been simulated with the Monte Carlo event generator PYTHIA 8.2 for different $p_T$ intervals of the trigger HFE.

The contributions from $b \rightarrow c \rightarrow e$, $b \rightarrow e$ and $c \rightarrow e$ decays have been simulated separately to disentangle the contributions of the individual channels to the overall ACD. The detection conditions in terms of the acceptance in pseudorapidity have been included and $p_T^{\text{trig}}$ cuts for the trigger HFEs of $1.5 < p_T^{\text{trig}} < 6$ GeV/c and for the associated charged particles of $p_T^{\text{assoc}} > 0.3$ GeV/c have been applied; the choice of the correlation type and the final selection of suitable particles were performed as part of the evaluation of the simulation results. This is why the obtained results can also be employed for analyses on related correlation systems, e.g. correlations of HFEs with the leading particle of the event.

In the course of the analysis, some correlation pairs have mistakenly been weighted with an additional factor of 0.5 with respect to other pairs. Investigations on the impact of this incorrect weighting indicate only negligible effects on the qualitative findings of this thesis.

The simulated ACDs for HFE$_{\text{all}}$-h correlations are compatible with data from ALICE for trigger HFEs with $1.5 < p_T^{\text{trig}} < 2.5$ GeV/c and $4.5 < p_T^{\text{trig}} < 6$ GeV/c and hadrons with $p_T^{\text{h}} > 0.3$ GeV/c. Results on the relative bottom contribution $r_b$ to the total yield of HFEs are within the uncertainties in accordance with findings of two ALICE analyses. The latter analyses predict a rise of $r_b$ with increasing $p_T$ of the HFEs until for $p_T^{\text{e}} > 3$ GeV/c, HFEs are primarily produced by bottom decays.

Only for bottom decays it might be possible to separate contributions from higher order and leading order processes in HFE$_p$-h correlations for HFEs with $1.5 < p_T^{\text{trig}} < 2.5$ GeV/c and hadrons with $p_T^{\text{h}} > 0.3$ GeV/c. The overall ACDs and the ACDs for charm decays are similar in shape for all heavy-flavour production processes. The same behaviour can be observed regarding PYTHIA 6 simulations from [51] for HFE$_p$-h correlations with slightly different selection criteria for the transverse momentum and the pseudorapidity of the correlation partners. Therefore, it is concluded that HFE$_p$-h correlations with correlation partners from the considered kinematic regime are not well suited for the study of heavy-flavour production mechanisms.

HFE$_p$-HFE correlations have been simulated for four $p_T$ intervals of the trigger electron. Based on the shapes and yields of near-side and away-side peaks, the following two $p_T^{\text{trig}}$ regimes have been chosen for the separation of the individual heavy-flavour production processes: $3 < p_T^{\text{trig}} < 6$ GeV/c and $1.5 < p_T^{\text{trig}} < 2.5$ GeV/c with $p_T^{\text{assoc}} > 0.3$ GeV/c. The process selection proceeded via a combination of fits to simulation results for HFE$_{\text{all}}$-HFE correlations, whereby the basic analysis technique could likewise be applied to respective data from ALICE.
HFE$_{p}$-HFE correlations for trigger HFEs from charm decays with $3 < p_{T}^{\text{trig}} < 6\text{ GeV}/c$ are predicted to exhibit the following contributions from pair creation, gluon splitting and flavour excitation to the overall HFE production

$$r_{pc} = 0.13 \pm 0.07, \quad r_{gs} = 0.53 \pm 0.05 \quad \text{and} \quad r_{fe} = 0.34 \pm 0.07.$$  

A contribution from higher order processes of $r_{fe+gs} = 0.79 \pm 0.01$ to HFE$_{p}$-HFE correlations for trigger HFEs from charm and bottom decays with $1.5 < p_{T}^{\text{trig}} < 2.5\text{ GeV}/c$ has been found. In this $p_{T}^{\text{trig}}$ interval, the yield of HFEs is dominated by HFEs originating from charm decays and the obtained proportions can be compared to the results from the other $p_{T}^{\text{trig}}$ interval. The results might indicate a decreasing fraction of contributions from pair-creation processes to the charm production with increasing trigger $p_{T}$. Investigations in other $p_{T}^{\text{trig}}$ intervals would be required to confirm this behaviour.

The investigation of bottom-quark production processes is, at this point, left for further analysis. From the simulation results, it is expected that a separation is possible for associated HFEs with $p_{T}^{\text{assoc}} > 0.3\text{ GeV}/c$ and trigger HFEs with $2 < p_{T}^{\text{trig}} < 4\text{ GeV}/c$ as well as $1.5 < p_{T}^{\text{trig}} < 6\text{ GeV}/c$.

At the time of writing, investigations are ongoing whether azimuthal correlations of two heavy-flavour electrons can be measured by ALICE with an adequate signal-to-noise ratio [1, 2]. If this should be possible, the results presented in this thesis can indicate which kinematic properties of the correlation partners are suitable for the analysis of heavy-flavour production via ACDs. For reliable statements on all $p_{T}^{\text{trig}}$ intervals, a renewed evaluation with a correct weighting of all correlation pairs would however be necessary to confirm the simulation results.

Correlations of HFEs with the leading particle of the event as well as with hadrons from a stricter $p_{h}^{\text{T}}$ cut might offer an alternative basis for the study of heavy-flavour production mechanisms. Respective examinations are currently ongoing whereby, inter alia, the simulation results obtained in this thesis provide a first testing ground. Correlations of HFEs and muons could also provide further insight into the production of heavy flavours [1, 2].
A. Appendix

A.1. Particle Observables for High-Energy Collisions

Several observables can be studied to investigate the characteristics of heavy-flavour production. Among the central quantities is the centre-of-mass energy $\sqrt{s}$ of the collision system, which is defined as the square root of the Mandelstam variable $s$. It determines the multiplicity of the emerging particles and the proportion of how the different heavy-flavour production mechanisms contribute (see Sec. 2.3.2). For heavy-ion collisions, usually the centre-of-mass energy $\sqrt{s_{\text{NN}}}$ per nucleon pair is given.

A selection on the transverse momentum $p_T$, which is given as the particle momentum perpendicular to the beam line, i.e.

$$p_T = p \cdot \cos \theta$$  \hspace{1cm} (A.1.1)

allows to study whether heavy-flavour production mechanisms can better be separated in a group of particles with certain kinematic properties. In Eqn. A.1.1, $p$ is the absolute momentum of the particle and $\theta$ the angle of its track with respect to the beam line.

Quantities like the rapidity $y$, the pseudorapidity $\eta$ or the azimuthal angle $\varphi$ are considered to examine the angular correlation of particles. The rapidity is additive under Lorentz transformations and a measure of the longitudinal momentum $p_L = p \cdot \sin \theta$ of the particles

$$y = \frac{1}{2} \ln \frac{E + p_L}{E - p_L}.$$  \hspace{1cm} (A.1.2)

The pseudorapidity

$$\eta = \frac{1}{2} \ln \frac{p + p_L}{p - p_L} = - \ln \left( \tan \left( \frac{\theta}{2} \right) \right)$$  \hspace{1cm} (A.1.3)

is easier to measure than the rapidity, since it only depends on the scattering angle $\theta$. For high energies $E \ll m$, at which the mass $m$ of the particle can be neglected, the pseudorapidity approximates the rapidity. One has $\eta = +\infty$ or $\eta = -\infty$ if the particle is emitted in $+z$ or $-z$ direction parallel to the beam line or $\eta = 0$ (midrapidity) if it is emitted perpendicular. For this thesis, the azimuthal angle $\varphi$ is of great interest. Looking at the plane of the particle collision which is perpendicular to the beam line, a two-dimensional coordinate system can be defined according to Fig. A.1. The azimuthal angle is given by the angle of the particle trajectory with respect to the $x$-axis of this coordinate system.

Figure A.1. – Definition of the azimuthal angle here denoted as $\phi$. The impact parameter $b$ of the collision defines the event plane $\Psi_R$ [16].

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A.2. Additional Plots

A.2.1. Dependence of Process Contributions on $p_T$ of Produced Bottom Quarks

As it was mentioned in Sec. 2.3.4, bottom quarks produced by pair-creation processes are expected to be softer than those originating from higher order mechanisms. Accordingly, the results for the relative process contributions to the charm production, which have been determined in Sec. 6.2.3, might indicate a decrease of the relative contribution from pair creation processes with increasing trigger $p_T$. Fig. A.2 shows the results of perturbative calculations for the $p_T$ distributions of bottom quarks from individual production channels in $pp$ collisions at 2 TeV. PYTHIA 6 simulations on the same issue but for $p\bar{p}$ collisions at 1.8 TeV are presented in Fig. A.3.

![Figure A.2](image1.png)

**Figure A.2.** – Perturbative calculations for the $p_T$ distributions of bottom quarks from different production processes in $pp$ collisions at 2 TeV. The distributions are normalised to unit area [34].

![Figure A.3](image2.png)

**Figure A.3.** – PYTHIA 6 simulations for the $p_T$-differential cross sections of bottom-quark production processes in $p\bar{p}$ collisions at 1.8 TeV and a rapidity $|y| < 1$ of the emerging bottom quarks [22].
A.2.2. Relative Bottom Contribution to Heavy-Flavour Production

In Sec. 2.3.4, an ALICE analysis [5] is addressed that determines the relative bottom contribution to all HFEs in pp collisions at 2.76 TeV via fits to ACDs from bottom and charm contributions. In this context, the modified Fig. 2.16 is provided of which the original plot can be seen in Fig. A.4.

**Figure A.4.** – **Upper Panel:** The relative bottom contribution to the HFE production in 2.76 TeV pp collisions. Data from two ALICE analyses is compared to model calculations. **Lower Panel:** The $p_T$-differential production cross section of HFEs from bottom decays [5].
A.2.3. Decision Tree of HFE Selection

In Fig. A.5, a scheme for the selection of the trigger HFEs for \( \text{HFE}_{p}\text{-h} \) and \( \text{HFE}_{p}\text{-HFE} \) correlations, as it is described in Sec. 4.2, is provided.

**Figure A.5.** – Decision tree of HFE selection. The selection process can be divided in the observation (orange) of the heavy-flavour generation and the tracking (green) of the heavy-flavour (HF) through the event until a HFE is produced. A basic cut on the \( p_T \) of the HFE is performed and it is paired with all hadrons of the event with \( p_T > 0 \text{ GeV}/c \) (violet). Double counting of HFEs is suppressed by asking whether the particle index of the respective HFEs is already saved each time a suitable HFE is found.
A.2.4. Effects of Wrong Weighting on HFE$_T$-HFE ACDs

In Sec. 4.5, the effects of the incorrect weighting of correlation pairs with correlation partners from the same kind of mother particles is discussed for ACDs with $1.5 < p_T^{\text{trig}} < 2.5 \text{ GeV/c}$ and $p_T^{\text{assoc}} < 0.3 \text{ GeV/c}$. For charm decays and trigger HFEs with $3 < p_T^{\text{trig}} < 6 \text{ GeV/c}$ and hadrons with $p_T^{\text{assoc}} < 0.3 \text{ GeV/c}$, the impact of the wrong weighting shows the same qualitative behaviour but to a smaller extend. This can derived from Fig. A.6 and Fig. A.7.

**Figure A.6.** Comparison of the ACDs for which the correlation pairs are correctly weighted according to the branching ratio of the HFEs mothers with ACDs for which correlation partners with the same type of meson mother are mistakenly weighted with an additional factor of 0.5. Contributions from charm decays with $3 < p_T^{\text{trig}} < 6 \text{ GeV/c}$ and $p_T^{\text{assoc}} > 0.3 \text{ GeV/c}$ are considered.
A.2.5. Further Plots on HFE\_p-h Correlations

In Sec 6.1, it is claimed that for HFE\_p-h correlations with 1.5 < p\_T^f < 2.5 GeV/c a separation of heavy-flavour production processes on the basis of ACDs appears to be difficult. This impression is supported by Figs. A.8, A.9 and A.10 in which the ACDs for all processes, flavour excitation, gluon splitting and pair creation are compared for bottom decays, charm decays and the overall ACDs.

**Figure A.7.** – Left: The ratio of the correctly and incorrectly weighted curves from Fig. A.6 for each production process. Right: The ratios of the ACDs with trigger HFEs from different production mechanisms. Compared are the ratios for the corrected and the uncorrected ACDs from charm decays with 3 < p\_T^{\text{trig}} < 6 GeV/c and p\_T^{\text{assoc}} > 0.3 GeV/c.

**Figure A.8.** – Comparison of PYTHIA 8.2 simulations for HFE\_p-h correlations with HFEs from all production processes, flavour excitation, gluon splitting and pair creation and 1.5 < p\_T^f < 2.5 GeV/c and hadrons with p\_T^h > 0.3 GeV/c. Only the ACDs for bottom decays are shown.
Figure A.9. – Comparison of PYTHIA 8.2 simulations for HFE$_p$-h correlations with HFEs from all production processes, flavour excitation, gluon splitting and pair creation and $1.5 < p_T^h < 2.5$ GeV/$c$ and hadrons with $p_T^h > 0.3$ GeV/$c$. Only the ACDs for charm decays are shown.

Figure A.10. – Comparison of PYTHIA 8.2 simulations for HFE$_p$-h correlations with HFEs from all production processes, flavour excitation, gluon splitting and pair creation and $1.5 < p_T^h < 2.5$ GeV/$c$ and hadrons with $p_T^h > 0.3$ GeV/$c$. The ACDs show the combined contributions from charm and bottom decays.
A.2.6. On the Determination of the Peak Yields

In Sec.6.2.2, the peak yields of the ACDs for correlations of HFEs from individual heavy-flavour production processes and other HFEs are determined using the three different techniques Normal Fit (NF), Zero Yield (ZY) and Slope (S) depending on the shape of the respective distributions. For further information on these techniques see Sec.6.2.2. The methods via which the corresponding peaks yields have been determined are for each ACD listed in Tab. A.1.

### Table A.1.

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A.2.7. Fit Functions for the Separation of LO and Higher Order Processes

In Sec. 6.2.3, the separation of LO and higher order processes in HFE$_{trig}$-HFE correlations with $1.5 < p_{T}^{\text{trig}} < 2.5$ GeV/$c$ and $p_{T}^{\text{assoc}} < 0.3$ GeV/$c$ is discussed. The fit functions and fitted histograms via which the relative contribution of LO processes has been determined, are presented in Fig. A.11.

Figure A.11. – Combined fit of the fit functions for flavour excitation and gluon splitting $f(\Delta \phi)_{fe+gs}$ on the one hand and pair creation $f(\Delta \phi)_{pc}$ on the other hand to the simulation results for HFE$_{all}$-HFE correlations with $1.5 < p_{T}^{\text{trig}} < 2.5$ GeV/$c$ and $p_{T}^{\text{assoc}} > 0.3$ GeV/$c$. From this fit the relative contribution of pair creation processes $r_{pc}$ has been derived.
Bibliography


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Ich erkläre, die vorliegende Arbeit selbstständig verfasst, keine anderen als die angegebenen Quellen und Hilfsmittel verwendet, sowie alle Zitate als solche kenntlich gemacht zu haben.

Katharina Garner
Münster, 25.07.17