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Stellar Structure and Evolution: Numerical Simulation and Its Potential for STEM Outreach with Teenagers

Master Thesis

by

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"Das Schönste, was wir erleben können, ist das Geheimnisvolle. Es ist das Grundgefühl, das an der Wiege von wahrer Kunst und Wissenschaft steht. Wer es nicht kennt und sich nicht mehr wundern, nicht mehr staunen kann, der ist sozusagen tot und sein Auge erloschen."

Albert Einstein, Wie ich die Welt sehe.

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Contents

List of Abbreviations

AGB	Asymptotic Giant Branch
CNO cycle	Carbon Nitrogen Oxygen cycle of nuclear fusion
HR diagram	Hertzsprung-Russell diagram:
	Double logarithmic plot of luminosity versus surface temperature
MLT	Mixing Length Theory
MS	Main Sequence
pp chain	Proton Proton chain for nuclear fusion
ρT **diagram**	Double logarithmic plot of central temperature versus central density
RGB	Red Giant Branch
RLOF	Roche Lobe Overflow
RLT diagram	Logarithmic plot of Radius, Luminosity, and Temperature versus time
SGB	Sub Giant Branch
WTTS	Window To The Stars
ZAMS	Zero Age Main Sequence
ZAHB	Zero Age Horizontal Branch

List of Figures

List of Tables

1 Introduction and Motivation

The fascination for stars is possibly as old as mankind itself. This fascination immediately becomes evident during a clear night far off artificial light sources, unveiling a sky filled with sparkling, blinking, colourful starlike phenomena, resembling all kinds of shapes and arrangements. Well-documented, detailed observations go back to at least the 14th century B.C.E. in ancient China (Shang dynasty), the 4th century B.C.E. in ancient Greece (famously Hipparchus and Ptolemy in the 2nd century B.C.E.) and in the 9th century in Arabia (e.g. al-Battani, Abd al-Rahman and al-Sufi). Other observations were conducted, partly even earlier, e.g. in Egypt, Mesopetamia, India and by the Maya. These, however, were primarily motivated by cultural, cultic, or religious purposes. ([Hos99], Ch. 1-3, [Bar94], Ch. 1 and [Key18], Ch. A and E)
The foundation of modern astronomy, characterized by the systematic observation of stars, attributing their properties to an evidence-based physical theory rather than a phenomenological, qualitative model, arose much later, starting in the 16th and 17th century with the work of Kepler, Galilei, Huygens and later Bessel. In the 19th and 20th century significant technological improvements regarding e.g. telescope technology and spectroscopy enabled much more precise observations (e.g. by Herschel, Leavitt and Hertzsprung), while theoretical work (e.g. Boltzmann, Planck, Einstein, Bethe and others) contributed greatly to the understanding of these phenomena. Finally, in the middle and late 20th century, the invention of electronic computers and their rapid increase in computational power revolutionized the field. Efficient simulation codes were developed, translating the understanding gained from astronomical observations, theoretical frameworks, and experimental data into quantitative models to predict the internal and evolutionary processes of stars.([Hos99], Ch. 5-8)

1.1 What is a Star?

Phenomenologically speaking, a star has two outstanding qualities: it can be observed as an isolated, spatially confined object, and radiates energy, predominantly in form of light from its surface. These qualities are not unique among astronomical objects, and thus clarification is necessary for what is understood as ‘star’. Stars are a product of the local collapse of a molecular cloud, driven by self-gravity, typically initiated by some form of external perturbation - e.g. a nearby supernova or gravitational interaction of some sort. Accordingly, a star is made of gas with a specific composition inherited from the molecular cloud. Furthermore, as the formation is driven by self-gravity and gravity can be described as a rotationally symmetric force field, the final product of this process will typically be approximately spheroidal. Even though gravitational collapse in general leads to an increase in temperature due to the conversion of gravitational (potential) energy to heat, just this energy reservoir proved to be insufficient to explain the lifetimes of stars. Stars thus require

some form of internal energy source. This turned out to be the conversion of matter into energy according to $E = mc^2$ via nuclear fusion reactions in stellar cores, m being the mass converted into energy E with the enormous conversion factor of speed of light in vacuum c squared. The exact distribution of matter is then determined by the balance of gravitational pull and sum of radiative and gas pressure. This balance is kept by an interplay of various heat transport mechanisms from the central region of the star, where it is generated, to the surface of the star, where it is radiated away into surrounding space.
The word ‘star’ thus refers to an amount of gas of a specific composition, that is confined by self-gravity and has collapsed to an object that generates heat from nuclear fusion of hydrogen or heavier constituent matter. Over major phases of a stars evolution, those processes are in precise equilibrium: any contraction leads to an increase of temperature resulting in increased nuclear fusion rates and a counteracting rise of radiative pressure. An expansion leads to cooling and again a counteracting effect: the reduction of nuclear fusion rates and thus radiative pressure. The details of the involved processes will be studied and further specified in sections 2 and 3.

1.2 Interest in Stars

Before discussing the details of stellar structure, it is worth elaborating on what drives the scientific interest in understanding their internal structure and processes. A key realization is that most observational evidence originates from the very surface of stars. It is impossible to reach the interior with probes and phenomena, allowing precise, direct measurements are typically rather limited (e.g. neutrinos, helioseismology). Furthermore stars and other astrophysical objects are in general by far out of reach of modern spacecrafts. Moreover they mostly evolve over timescales on the order of millions to billions of years, exceeding even very generous estimates of direct experimental accessibility of a few thousand years (from historic records mentioned above) by many orders of magnitude. Thus a considerable fraction of the evolution of a particular star can not possibly be observed and analyzed - instead, all observation is rather a snapshot of the population of stars overall or specific subsamples thereof. Therefore it is strictly speaking not possible to determine directly, whether any observed trend is caused by differences in parameters among subpopulations of stars or by the evolution of individual stars with time. As an analogy, an alien species observing only snapshots of all living humans at a single point in time is unable to determine whether humans undergo a period of growth followed by a longer phase of stable height around 1.75 ± 0.25 m or remain constant in size throughout their lives, but with a distinct smaller subpopulation. Any detailed understanding of the internal structure of stars and its evolution thus has to be obtained from theoretical studies of the underlying processes and numerical simulations of such, to relate the models to observables. Analysis of predicted properties has to predominantly happen on population level, not for individual stars. However, despite those restrictions, stars have gained immense scientific interest for a variety of reasons, e.g. ([Boe08], [Sal05]):

- determine properties of stars in clusters or groups to understand their formation and evolution;
- study such clusters and groups within galaxies, to gain insight into the formation and evolution of galaxies;
- use specific features of stars and their spectra such as redshift or the pulsation of Cepheid variables as so-called cosmic rulers to understand the structure and evolution of the universe;
- understand genesis of heavier elements through nuclear fusion, the so-called chemical evolution of the universe;
- understand the Sun and its evolution, as it is the basis for life as we know it, but may also pose a threat for technology such as satellites, electric grids, etc.

As will become evident in section 2 and 4, the processes driving stars are numerous, described by quite different theories and an attempt to 'solve' a star by combining fundamental theories or such on microscopic level fully algebraically is clearly not possible. Instead numerical simulation codes are available, that collect the driving influences, often in effective theories to deduce the resulting phenomena. Dedicated simulation codes exist for the formation, the atmosphere and its characteristic spectral details, the local dynamics on short timescales including magnetic effects, the final stages of stellar evolution including supernovae and formation of remnants and - which is the topic of this thesis - the structure and chemical evolution of stars driven by nuclear fusion. Moreover, the fascination with stars seen throughout human history across various cultures, religions and societies together with the absolute necessity for the emergence and persistence of life is supposed to be the driving motivation in a workshop for teenagers and young adults. This workshop aims to introduce participants to scientific methodologies, demonstrate the power of numerical simulations, and spark interest in physics. As space travel and specifically the search for extraterrestrial life is very present in modern media, such as news, movies, series, literature, games and more, this workshop will address the question for possibilities of intelligent life in distant star systems, to resonate with interests of teenagers and draw attention to the fascinating field of astrophysics.
The aim of this thesis is to evaluate the potential of using numerical simulations of stellar structure and evolution in outreach and teaching by introduction to the topic, providing a series of simulations and their analysis to establish a sophisticated understanding of the phenomenology of stars as well as an interactive and exciting workshop concept. First, stars will be described theoretically in section 2 and characteristic properties of stellar matter will

be introduced in section 3. Then numerical methods for the simulation of stellar structure and its evolution as used in Peter Eggleton's TWIN code as well as the software *Window to the Stars* (WTTS) are outlined and a number of simulations are presented and analysed in section 4. A concept for the workshop connecting the simulation of stellar evolution with the emergence of intelligent life as sketched above will be outlined in section 5. Finally, the feedback obtained from testing this workshop concept with different groups of highschool students will be evaluated and discussed in section 6.

2 Theory of Stellar Structure and Evolution

This chapter aims to provide a concise but comprehensive summary of the theoretical framework underlying the structure and evolution of stars, which will serve as a basis for the numerical simulations presented in subsequent sections. Specifically, theories describing the fundamental processes shaping stellar structure are introduced and combined to find the set of *basic equations of stellar structure*. These include: gravity, thermodynamics, heat transport via convection, radiation and diffusion and energy release due to nuclear fusion.
These influences dominate the evolution of general structure on timescales determined by composition changes due to nuclear fusion and thus the set of basic equations for stellar structure describes the star's behavior on in general long spatial and temporal scales. As was already mentioned in section 1.1, the formation of a star is caused by self-gravity, suggesting a high degree of spherical symmetry, which significantly simplifies the problem at hand. This symmetry is broken by angular momentum conservation, formally reducing the symmetry to a cylindrical one. The relevance of angular momentum in stars will turn out to be small enough to assume full spherical symmetry in the following and consider any phenomena breaking this symmetry in form of effective corrections only. This immediately has the consequence, that magnetic and electric fields can only be considered regarding their effective, radial influences, as spherically symmetric field configurations are associated with magnetic monopoles and a spherically symmetric net electric charge - which are not observed in stars.
Furthermore, it is assumed, that the star is isolated from external effects, so that its structure and evolution is determined solely by the star's internal properties. The observed luminosity, that is the rate of energy emission, is assumed to originate from a region called *photosphere*, that is located at a certain radius R from the center and contains the star's mass M.

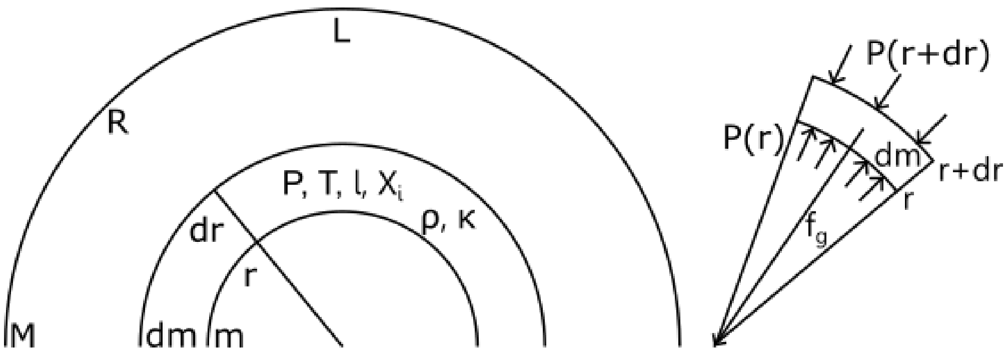

Figure 1: Sketch showing the coordinates r and m, describing mass shells in stars. Also shown are interior properties: temperature T, luminosity l, pressure P, composition X_i, the star's mass M, radius R, luminosity L, density ρ and opacity κ as well as the balance of gravitational force f_g and inwards and outwards pressure acting on a mass shell (right).

2.1 Coordinates and Equation of Motion

Full rotational symmetry essentially reduces the stellar structure to a one dimensional problem, that is, to its radial component and thereby significantly simplifies the theoretical framework required. This is a good approximation for stars that do not rotate too fast and are not deformed by external influences. In an *Eulerian* sense the star is then characterized by the radial distribution of the physical quantities describing it, first of its density profile of stellar matter. This can be expressed in form of the mass $m(r,t)$ contained within a sphere with radius r from the center of mass at time t. Therefore the variation of m with time and radius is considered:

$$dm = 4\pi r^2 \rho dr - 4\pi r^2 \rho v dt\,, \tag{2.1}$$

where ρ is the density of stellar matter and v is the velocity of outwards flow of material in that shell at that point in time. Globally it has to hold that $m(R,t) = M$. It can be derived directly, that:

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho\,, \qquad \frac{\partial m}{\partial t} = -4\pi r^2 \rho v\,, \tag{2.2}$$

are the partial derivatives of m with respect to radius r and time t. From this the continuity equation of hydrodynamics under spherical symmetry can be derived by differentiating the first equation with respect to t and the second with respect to r and equating them:

$$\begin{aligned} \frac{\partial}{\partial t}\frac{\partial m}{\partial r} &= 4\pi r^2 \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial r}\frac{\partial m}{\partial t} = -4\pi \frac{\partial\left(r^2 \rho v\right)}{\partial r} \\ \Leftrightarrow \quad \frac{\partial \rho}{\partial t} &= -r^{-2}\frac{\partial\left(r^2 \rho v\right)}{\partial r}\,. \end{aligned} \tag{2.3}$$

As stars can undergo considerable shrinking or expansion during their evolution it will turn out to be favorable to use a *Lagrangian* description with m and t as coordinates referring to a specific mass shell rather than a specific distance from the center (see fig. 1). The transformation law for this change of coordinates can be derived from the total differential with an arbitrary function f:

$$\left.\frac{\partial f}{\partial t}\right|_m dt + \left.\frac{\partial f}{\partial m}\right|_t dm = \left.\frac{\partial f}{\partial t}\right|_r dt + \left.\frac{\partial f}{\partial r}\right|_t dr = df\,, \tag{2.4}$$

For $dt = 0$:

$$\begin{aligned} \left.\frac{\partial}{\partial m}\right|_t dm &= \left.\frac{\partial}{\partial r}\right|_t dr \\ \Leftrightarrow \quad \frac{\partial}{\partial m} &= \left(\left.\frac{\partial m}{\partial r}\right|_t\right)^{-1}\frac{\partial}{\partial r} = \frac{1}{4\pi r^2 \rho}\frac{\partial}{\partial r}\,, \end{aligned} \tag{2.5}$$

For $dm = 0$:

$$\begin{aligned} \left.\frac{\partial}{\partial t}\right|_m dt &= \left.\frac{\partial}{\partial t}\right|_r dt + \left.\frac{\partial}{\partial r}\right|_t dr \\ \Leftrightarrow \quad \left.\frac{\partial}{\partial t}\right|_m &= \left.\frac{\partial}{\partial t}\right|_r + \left(\left.\frac{\partial r}{\partial t}\right|_m\right)\left.\frac{\partial}{\partial r}\right|_t\,, \end{aligned} \tag{2.6}$$

where the arbitrary function f is dropped. The benefit of a Lagrangian coordinate choice is that a partial derivative with respect to t implicitly follows a certain mass shell, that is specific stellar material in case of shrinking or expansion, without the need to consider the second, convection-like term on the right-hand side of (2.6). The derivative on the left-hand side is referred to as substantial time derivative of hydrodynamics. Equation (2.5) can be applied to r to derive the inverse of (2.2) explicitly, which is the first basic equation of stellar structure for the Lagrangian choice of coordinates:

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}\,. \tag{2.7}$$

The driving force of stellar formation and shrinking in general is self-gravity of the constituent matter, which can be described according to *Gauss' law* as

$$\nabla \vec{g} = -4\pi G \rho\,, \tag{2.8}$$

with $\vec{g}$ the gravitational field and G the gravitational constant. Then $\vec{g}$ can be expressed as the gradient of the scalar gravitational potential Φ: $\vec{g} = -\nabla\Phi$, so that

$$\nabla^2 \Phi = 4\pi G \rho\,. \tag{2.9}$$

For spherical symmetry the Laplace operator simplifies, giving

$$\nabla^2 \Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) = 4\pi G \rho\,. \tag{2.10}$$

This results in a radial acceleration g towards the coordinate origin, that is in $-\vec{e_r}$-direction, $\vec{e_r}$ being the unit vector along the radial direction, with

$$g = |-\nabla\Phi| = \frac{\partial \Phi}{\partial r} = \frac{Gm}{r^2}\,. \tag{2.11}$$

In abscence of magnetic fields and net charges, the motion of stellar matter is driven by gravity and pressure differences. Considering a mass shell dm at distance r (see fig. 1, right), the force f_P exhibited by the difference of outwards and inwards pressure per unit area can be expressed in differential form as

$$f_P = -\frac{\partial P}{\partial m} dm\,. \tag{2.12}$$

The force f_g exhibited by the gravitational pull per surface area $4\pi r^2$ on the other hand is:

$$f_g = -g \cdot \frac{dm}{4\pi r^2} = -\frac{Gm}{r^2}\frac{dm}{4\pi r^2}\,. \tag{2.13}$$

The equation of motion for the mass element per surface area thus is:

$$\frac{dm}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} = f_P + f_g = \left(-\frac{\partial P}{\partial m} - \frac{Gm}{4\pi r^4}\right) dm$$

$$\Leftrightarrow \qquad \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} = -\frac{\partial P}{\partial m} - \frac{Gm}{4\pi r^4}\,, \tag{2.14}$$

where both sides were divided by dm. During significant stages of evolution the mass shells are efficiently in *hydrostatic equilibrium*, meaning that the forces due to pressure gradient and gravitational pull cancel out almost exactly and the acceleration on the mass shell is negligible:

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \,. \tag{2.15}$$

Equation (2.14) and (2.15) are the general form and the hydrostatic form of the second basic equation of stellar structure. It is worth pointing out, that the differential equation for the pressure P in the Eulerian description in the hydrostatic case is underdetermined, until a specific equation of state $\rho(P)$ is provided:

$$\frac{\partial P}{\partial r} = \frac{\partial P}{\partial m}\frac{\partial m}{\partial r} = -\frac{Gm}{4\pi r^4} \cdot 4\pi r^2 \rho = -\frac{Gm\rho}{r^2} \,. \tag{2.16}$$

To provide an accurate equation of state for all regimes of stars in full generality careful considerations are required, which are carried out in section 3.
Associated with the forces described above are time scales for the gravitational collapse and explosion, that is the systems reaction to the absense of one or the other force in (2.14). Of more general interest is the time scale τ_{hydr} for the systems reaction to changes in pressure or distribution of stellar material in the hydrostatic case. An estimate can be given by the ratio of the absolute value of the force terms relative to the size of the total system. Therefore the equation of motion is solved for constant gravitational acceleration equal to its surface value GM/R^2 for the complete collapse from $r(0) = R$ to $r(\tau_{\text{hydr}}) = 0$:

$$\begin{aligned} \frac{\partial^2 r}{\partial t^2} &= -\frac{GM}{R^2} \\ \Rightarrow \qquad R &= \frac{1}{2}\frac{Gm}{R^2}\tau_{\text{hydr}}^2 \\ \Leftrightarrow \qquad \tau_{\text{hydr}} &\approx \left(\frac{R^3}{GM}\right)^{1/2} \,. \end{aligned} \tag{2.17}$$

This can only be an estimate, as the actual characteristic time scale for changes of hydrostatic equilibrium are given by the ratio of the systems size to the propagation speed for perturbations, which is the speed of sound and requires specification and evaluation of the equation of state.[Hen72]

2.2 Virial Theorem and Energy Balance

Associated with the balance of gravitational pull and inner pressure in hydrostatic equilibrium is the *virial theorem*. It puts the internal, thermal energy into relation to the gravitational energy and thereby is a useful tool to describe conversion of these energy forms into each other during contraction or expansion of stars. It can be derived from (2.15) by multiplying both sides with $4\pi r^3$ to obtain the energy required to expand an entire mass shell to its actual radius in equilibrium. Then integration over the entire star, evaluating the integral on the left-hand side using integration by parts for the functions $P(m)$, $4\pi r^3$ and their derivatives $\partial P/\partial m$ and $12\pi r^2 \partial r/\partial m$, gives:

$$\int_0^M 4\pi r^3 \frac{\partial P}{\partial m} dm = \left[4\pi r^3 P\right]_0^M - \int_0^M 12\pi r^2 \frac{\partial r}{\partial m} P dm = 4\pi R^3 P_s - 3\int_0^M \frac{P}{\rho} dm \tag{2.18}$$

$$= -\int_0^M \frac{Gm}{r^2} dm =: E_g < 0\,, \tag{2.19}$$

where (2.7) was used, P_s is the surface pressure $P(m = M)$ and the gravitational energy E_g was introduced. The remaining integral can be evaluated when assuming a rather general equation of state of the form

$$\zeta \cdot u := 3\frac{P}{\rho}\,, \tag{2.20}$$

with ζ a constant and u the internal energy per unit mass of the stellar matter. Both, the ideal gas and a photon gas are special cases of this form, with $\zeta = 2$ for a monoatomic ideal gas and $\zeta = 1$ for the photon gas. A mixture of monoatomic ideal gases is in many cases already a reasonably good approximation for stellar material to derive an understanding of its internal processes, especially for regions sufficiently hot for full dissoziation of all molecules and a constant degree of ionization.
The total internal energy of the star is given by

$$E_\mathrm{i} := \int_0^M u\,dm\,, \tag{2.21}$$

so that the integral above can be rewritten as ζE_i, if ζ is constant for the entire star. The virial theorem then can be stated as

$$\zeta E_\mathrm{i} + E_\mathrm{g} = 4\pi R^3 P_s\,. \tag{2.22}$$

One can then define the total energy content of the star as:

$$E_\mathrm{tot} = E_\mathrm{i} + E_\mathrm{g} = \left(1 - \frac{1}{\zeta}\right) E_\mathrm{g} < 0, \tag{2.23}$$

where the contribution from surface pressure was neglected. The last relation holds for gravitationally bound systems (such as stars). The change of the total energy by emission of radiation is called the luminosity L of the star:

$$L := \left.\frac{dE_\mathrm{tot}}{dt}\right|_\mathrm{rad}\,. \tag{2.24}$$

With the gravitational energy E_g a second typical time scale of general interest can be defined, the *Kelvin-Helmholtz* time scale τ_KH. This is the time, a star could sustain its luminosity for, if the only energy reservoir is its gravitational energy via contraction. This was a mechanism for explaining the luminosity observed in stars historically proposed by

Kelvin and Helmholtz, which however turned out to be in conflict with the lifetimes of stars as indicated by other means. With (2.19) and the mean values $m \approx \bar{m} = M/2$ and $r \approx \bar{r} = R/2$ one finds:

$$\tau_{\text{KH}} := \frac{|E_{\text{g}}|}{L} \approx \frac{GM\bar{m}}{\bar{r}L} = \frac{GM^2}{RL} \,. \tag{2.25}$$

The luminosity L as introduced above on long time scales is the consequence of the energy released due to nuclear fusion. To describe energy transport in the interior a local description of this energy balance is required. Let $l(r)$ be the energy passing through the mass shell at radius r, then clearly $l(R) = L$ and $l(0) = 0$, where the energy carried by neutrinos is explicitly excluded, as stellar material in general is close to transparent for neutrinos (this does not hold during supernova stages). The local energy balance has to consider energy release rate per unit mass ϵ, energy flow l, neutrino losses ϵ_ν and heat dq taken up. Using the first law of thermodynamics and general relations given in the appendix this gives:

$$dq = \left(\epsilon - \epsilon_\nu - \frac{\partial l}{\partial m}\right) dt \,, \tag{2.26}$$

$$\Leftrightarrow \quad \frac{\partial l}{\partial m} = \epsilon - \epsilon_\nu - \frac{\partial u}{\partial t} - P\frac{\partial v}{\partial t} \tag{2.27}$$

$$= \epsilon - \epsilon_\nu - c_P\frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \,, \tag{2.28}$$

$$= \epsilon - \epsilon_\nu + \epsilon_{\text{g}} \,, \qquad \text{with } \epsilon_{\text{g}} := -T\frac{\partial s}{\partial t} \,, \tag{2.29}$$

which is the third basic equation of stellar structure. Often the neutrino losses are considered to be a correction to the energy release rate and included in ϵ. The total energy is locally conserved:

$$0 = \frac{d}{dt}\left(E_{\text{kin}} + E_{\text{g}} + E_{\text{i}} + E_{\text{n}}\right) + L + L_\nu \,, \tag{2.30}$$

where $L_\nu = \int \epsilon_\nu \, dm$ is the neutrino luminosity. Integration of $\partial l/\partial m$ over the entire star on the other hand gives:

$$L = \int_0^M \frac{\partial l}{\partial m} dm = \int_0^M \epsilon + \epsilon_{\text{g}} \, dm - L_\nu \,, \tag{2.31}$$

$$= -\frac{dE_{\text{n}}}{dt} + \int_0^M \epsilon_{\text{g}} \, dm - L_\nu \,, \tag{2.32}$$

where the definition of L_ν and ϵ as rate of energy release were used. This can be inserted into (2.30) to find:

$$\frac{d}{dt}\left(E_{\text{kin}} + E_{\text{i}} + E_{\text{g}}\right) = -\int_0^M \epsilon_{\text{g}} \, dm = \int_0^M \frac{\partial u}{\partial t} + P\frac{\partial v}{\partial t} dm \,, \tag{2.33}$$

$$\Leftrightarrow \quad \frac{d}{dt}\left(E_{\text{kin}} + E_{\text{g}}\right) = -\int_0^M \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} dm \,, \tag{2.34}$$

where (2.21) and $v = 1/\rho$ were used. This is both, the integrated form of (2.14) and the virial theorem for temporal changes of the gravitational and internal energy.
Again, there is a characteristic time scale associated with the process of nuclear fusion, τ_n:

$$\tau_\mathrm{n} := \frac{E_\mathrm{n}}{L} \tag{2.35}$$

For all stars burning hydrogen it will hold that $\tau_\mathrm{n} \gg \tau_\mathrm{KH} \gg \tau_\mathrm{hydr}$. For changes on a scale $\tau \gg \tau_\mathrm{KH}$ the time derivatives in (2.28) are small and the equation simplifies significantly as $\epsilon \gg |\epsilon_\mathrm{g}|$, so that $\partial l/\partial m \approx \epsilon$. This is the case of *complete equilibrium* and especially holds for stars during hydrogen and helium burning. For changes on a scale $\tau \ll \tau_\mathrm{KH}$ the time derivatives are no longer negligible, but change much faster than the structure of the star can adopt. Both terms have to approximately cancel each other out: $c_P \partial P/\partial t \approx \delta/\rho \cdot \partial P/\partial t \gg \epsilon$, so that still $\partial q/\partial t \approx 0$ on this time scale. This is the case of adiabatic change on short time scales, such as pulsation phenomena.

2.3 Radiative Energy Transport and Conduction

To couple energy production, thermodynamic quantities and structure it is necessary to describe mechanisms of heat transport. The mechanisms to be considered are radiative transport, conduction and convection. Due to the phenomenologically quite different character of convection, it will be handled seperately in section 2.4. The transport of energy by radiation is essentially linked to the concept of opacity. Opacity is the ability of a medium to hold back radiation and can be expressed in terms of the mean free path $\ell_\mathrm{Ph} = 1/(\kappa\rho)$. It is effectively an absorbtion coefficient, that is a cross section for absorbtion of radiation averaged over the frequency of radiation. The phenomenology will be discussed in section 3.2. As in stars $\ell_\mathrm{Ph} \ll R$ the stochastic nature of absorbtion processes can be neglected and heat transport by radiation can instead be treated as a diffusive process as outlined in the following. The flux $\vec{j}$ of a diffusing particle can be written as

$$\vec{j} = -D\nabla n\,, \tag{2.36}$$

with the diffusion coefficient $D = v_\mathrm{part}\ell_\mathrm{part}/3$, where v_part is the velocity, ℓ_part the mean free path of the particle and n the particle density. The factor of 1/3 arises due to averaging over the three spatial dimensions. To find the flux of radiative energy $\vec{F}_\mathrm{rad} = F_\mathrm{rad} \cdot \vec{e}_r$ the particle density n needs to be replaced by the radiative energy density $U = a \cdot T^4$ with a the radiation density constant, and the velocity v_part by the speed of light c, as well as ℓ_part by ℓ_Ph. Using $\nabla U = 4aT^3 \frac{\partial T}{\partial r}\vec{e}_r$ one obtains:

$$F_\mathrm{rad} = -D\,|\nabla U| = -D \cdot 4aT^3 \frac{\partial T}{\partial r} = -\frac{1}{3}c\ell_\mathrm{Ph} 4aT^3 \frac{\partial T}{\partial r} \tag{2.37}$$

$$= -\frac{4ac}{3}\frac{T^3}{\kappa\rho}\frac{\partial T}{\partial r} \tag{2.38}$$

$$= -k_\mathrm{rad}\frac{\partial T}{\partial r}\,, \qquad \text{with: } k_\mathrm{rad} := \frac{4ac}{3}\frac{T^3}{\kappa\rho}\,. \tag{2.39}$$

In the last line, the relation was written in a form typical for conduction of heat, with $k_{\rm rad}$ the conduction coeffitient for radiative heat transport. This gives a relation for the temperature gradient due to this type of heat transport:

$$\frac{\partial T}{\partial m} = \frac{\partial T}{\partial r}\frac{\partial r}{\partial m} = -\frac{F_{\rm rad}}{k_{\rm rad}} \cdot \frac{1}{4\pi r^2 \rho} = -\frac{3F_{\rm rad}\kappa\rho}{4acT^3} \cdot \frac{1}{4\pi r^2\rho} \tag{2.40}$$

$$= -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}\,, \tag{2.41}$$

where $l = 4\pi r^2 F$ was used. This is the fourth basic equation of stellar structure, assuming fully radiative energy transport. It only holds, where the free mean path of photons is much smaller than the distance from the surface, $\ell_{\rm ph} \ll (R - r)$, which is certainly fulfilled in the deep interior of stars.

So far, the material was treated as fully homogenous, having one specific, frequency averaged opacity. As the stellar material has variable composition and the frequency dependence for the opacity of different components is not uniform, the frequency dependent opacities $\kappa_{\nu,i}$ have to be combined before performing the averaging. These frequency dependent opacities combine according to

$$\kappa_\nu = \sum_i \kappa_{\nu,i} \cdot X_i\,, \tag{2.42}$$

with X_i is the mass fraction for the i-th component. The spectrum of the energy density follows the Boltzmann equation with Boltzmann function $B(\nu, T)$ according to

$$U_\nu = \frac{4\pi}{c}B(\nu,T) = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT} - 1}\,, \tag{2.43}$$

$$|\nabla U_\nu| = \frac{4\pi}{c}\frac{\partial B}{\partial r} = \frac{4\pi}{c}\frac{\partial B}{\partial T}\frac{\partial T}{\partial r}\,, \tag{2.44}$$

where h is Planck's constant. Consider then the spectrum of the energy flux by radiation in radial direction $F_\nu = F_{[\nu,\nu+d\nu]}$ for ν being in the inveral $[\nu, \nu + d\nu]$. Integration over F_ν has to recover the total energy flux F giving the correct averaging procedure for the opacities by comparing to (2.39):

$$F_{\rm rad} = \int_0^\infty F_\nu d\nu = -\frac{4\pi}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B}{\partial \nu}d\nu\frac{\partial T}{\partial r}\,, \tag{2.45}$$

$$\Rightarrow k_{\rm rad} = \frac{4\pi}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B}{\partial T}d\nu\,, \tag{2.46}$$

$$\frac{1}{\kappa} = \frac{\pi}{acT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B}{\partial T}d\nu\,. \tag{2.47}$$

This *Rosseland Mean* is an harmonic mean with a weighting proportional to $\partial B/\partial T$, meaning that frequencies are weighted according to their spectral energy flux. This concept is an essential part of the opacity tables, that are used in numerical simulations.

As indicated by the subscript, $F_{\rm rad}$ describes only the radiative part of F. The conductive contribution $F_{\rm cd}$ is associated with an even shorter mean free path for the movement of particles and the stochastic nature of the process can thus again be neglected. This leads to an analogous, effective description of the same form as for the radiative process with a heat conduction coefficient k_{cd}:

$$F_{\mathrm{cd}} = -k_{\mathrm{cd}} \frac{\partial T}{\partial r} \, . \tag{2.48}$$

In this form, the two contributions can be combined directly to give the total energy flux due to conduction and radiative heat transport:

$$F = F_{\mathrm{rad}} + F_{\mathrm{cd}} = -\frac{4ac}{3} \frac{T^3}{\rho} \left(\frac{1}{\kappa_{\mathrm{rad}}} + \frac{1}{\kappa_{\mathrm{cd}}} \right) \frac{\partial T}{\partial r} \, , \quad \text{with: } \kappa_{\mathrm{cd}} := \frac{4ac}{3} \frac{T^3}{\rho} \, . \tag{2.49}$$

The form of κ_{cd} was chosen such, that the phenomenologically similar contributions are given in mathematically analogous form.
For convenience it is useful to rewrite (2.41) by dividing it by (2.15), which only holds assuming hydrostatic equilibrium, giving a relation for the variation of temperature with depth, expressed in terms of the local pressure:

$$\left(\frac{dT}{dP} \right)_{\mathrm{rad}} = \frac{\partial T / \partial m}{\partial P / \partial m} = \frac{3}{16 \pi a c G} \frac{\kappa l}{m T^3} \, . \tag{2.50}$$

This quantity is typically given in a modified way, using the abbreviation

$$\nabla_{\mathrm{rad}} := \left(\frac{\partial \ln T}{\partial \ln P} \right)_{\mathrm{rad}} = \frac{3}{16 \pi a c G} \frac{\kappa l P}{m T^4} \, . \tag{2.51}$$

The maximum temperature gradient, that can be maintained by complete heat transport via radiation is quantified by ∇_{rad}, which will be used to deduce stability criteria for a stellar region with radiative energy transport.

2.4 Stability Criteria and Convection

Another significant mechanism of heat transfer is convection via local convective currents. Local convective currents are inconsistent with the full spherical symmetry assumed and convective elements in general also are dynamic on temporal scales, that are significantly shorter than the scales considered so far. Thus an effective theory for the contribution of convection, averaged over the full sphere and long time scales has to be found, to complete the description of heat transfer. In this sense convection phenomenologically is a local perturbation of the structure described so far. To prepare the discussion of convection considerations regarding stability against local, non-spherical perturbations are presented.
First, so-called dynamic instability is described - that is dynamics on time scales short enough, not to allow relevant exchange of heat with surroundings - the adiabatic case. The previously derived structure functions are still assumed to hold, but in a sense of proper averages over the entire sphere over sufficiently long time scales. In this sense one can consider specific, spatially small mass elements on the sphere in a noticably different state

from the surrounding on that sphere. Quantities refering to the mass element will be labeled with subscript e and refering to the surroundings with subscript s. The difference in a quantity A is then noted as

$$DA := A_\mathrm{e} - A_\mathrm{s} \,. \tag{2.52}$$

As the driving factors for the structure in a star are in the first place gravity and pressure, especially deviations in density and pressure will be relevant, leading to a net force and thus acceleration of a mass element. As the element is assumed to be small, any excess in pressure $DP > 0$ will immediately lead to an expansion of besaid element, restoring $DP = 0$, while undergoing a change in temperature due to the work done during expansion. Therefore differences in density are of special interest and cause a so-called buoyancy force $\vec{K} = K_r \cdot \vec{e}_r = -gD\rho\vec{e}_r$. The reaction of a mass element, if it gets displaced from the position at which it is in equilibrium with the surrounding mass shell determines, whether the state is stable or not. In a stable scenario this displacements results in a returning force, restoring the equilibrium configuration, while for instability, a net effect occurs, that increases the displacement. The discrepancy in density $D\rho$ for the displaced element relative to its new, non-equilibrium surroundings can be written as:

$$D\rho = \left[\left(\frac{d\rho}{dr} \right)_\mathrm{e} - \left(\frac{d\rho}{dr} \right)_\mathrm{s} \right] \Delta r \,. \tag{2.53}$$

Here Δr is the spatial displacement, $(d\rho/dr)_\mathrm{e}$ is the change of density for the mass element while undergoing the spatial displacement and $(d\rho/dr)_\mathrm{s}$ is the change in density of the surroundings along this path. Stability then corresponds to $K_r < 0$ and thus

$$\left(\frac{d\rho}{dr} \right)_\mathrm{e} - \left(\frac{d\rho}{dr} \right)_\mathrm{s} > 0 \,. \tag{2.54}$$

The density gradients appearing in this condition are not directly described in the basic equations of stellar structure as derived so far, so that it is practical to transform those into temperature gradients. Therefore a general equation of state is assumed:

$$\frac{d\rho}{\rho} = \alpha \frac{dP}{P} - \delta \frac{dT}{T} + \varphi \frac{d\mu}{\mu} \,. \tag{2.55}$$

Here α and δ are analoge to those defined in the appendix and additionally a change of composition is considered, charaterized by the change of mean molecular weight μ. With this, one finds:

$$\alpha := \left(\frac{\partial \ln\rho}{\partial \ln P} \right)\bigg|_{T,\mu} ,\ \delta := \left(\frac{\partial \ln\rho}{\partial \ln T} \right)\bigg|_{P,\mu} ,\ \varphi := \left(\frac{\partial \ln\rho}{\partial \ln\mu} \right)\bigg|_{T,P} . \tag{2.56}$$

Here μ is supposed to represent actual changes in composition, rather than degree of ionization. The latter are assumed to be expressed in terms of P and T and included in changes of

α and δ respectively. With these notations and (2.54) the stability criterion for a displaced element with conserved composition can be expressed as

$$\left(\frac{\alpha}{P}\frac{dP}{dr}\right)\bigg|_{\mathrm{e}} - \left(\frac{\delta}{T}\frac{dT}{dr}\right)\bigg|_{\mathrm{e}} - \left(\frac{\alpha}{P}\frac{dP}{dr}\right)\bigg|_{\mathrm{s}} + \left(\frac{\delta}{T}\frac{dT}{dr}\right)\bigg|_{\mathrm{s}} - \left(\frac{\varphi}{\mu}\frac{d\mu}{dr}\right)\bigg|_{\mathrm{s}} > 0\,. \tag{2.57}$$

With $DP = 0$ as explained above and after multiplication with

$$H_P := -\frac{dr}{d\mathrm{ln}P} = -P\frac{dr}{dP}\,, \tag{2.58}$$

the *scale height of pressure*, one finds in the case of hydrostatic equilibrium:

$$H_P = \frac{P}{\rho g}\,, \quad \left(\frac{d\ln T}{d\ln P}\right)\bigg|_{\mathrm{s}} < \left(\frac{d\ln T}{d\ln P}\right)\bigg|_{\mathrm{e}} + \frac{\varphi}{\delta}\left(\frac{d\ln \mu}{d\ln P}\right)\bigg|_{\mathrm{s}}\,. \tag{2.59}$$

This is typically expressed, using shorthands as follows:

$$\nabla := \left(\frac{d\ln T}{d\ln P}\right)\bigg|_{\mathrm{s}}\,, \quad \nabla_{\mathrm{e}} := \left(\frac{d\ln T}{d\ln P}\right)\bigg|_{\mathrm{e}}\,, \quad \nabla_{\mu} := \left(\frac{d\ln \mu}{d\ln P}\right)\bigg|_{\mathrm{s}}\,, \tag{2.60}$$

$$\nabla < \nabla_{\mathrm{e}} + \frac{\varphi}{\delta}\nabla_{\mu}\,. \tag{2.61}$$

If this relation holds, then the layer is stable against convection and the full energy transport is realized by conduction and radiation: $\nabla = \nabla_{\mathrm{rad}}$. As a convective element in general is hotter than its surroundings, it also radiates stronger than its surroundings and thus undergoes radiative cooling to some degree. Therefore $\nabla_{\mathrm{e}} > \nabla_{\mathrm{ad}}$ and a practical estimate for the criterian above can be given by assuming $\nabla_{\mathrm{e}} = \nabla_{\mathrm{ad}}$ and then testing for radiative energy transport:

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_{\mu}\,. \tag{2.62}$$

This relation is known as *Ledoux criterion.* If the relation holds, the assumption of radiative energy transport is self-consistent meaning the layer is stable against convection. Otherwise, convection will occur and contribute to the overall heat transport. Note, that $\nabla_{\mu} > 0$ is the usual case for production of heavier elements in the core by nuclear fusion, which tends to stabilize against convection. A typical sOverflowituation for instability would be large fluxes F in an opaque region. As all terms in (2.62) are defined by local quantities only, it is easy to calculate numerically. However this is also reflecting the fact, that this condition was derived considering local effects only, while the actual system would adjust non-locally to any change in heat transport, which might make convection not self-consistent. Thus this relation only holds approximately and a more sophisticated calculation would have to consider non-local effects also, making it computationally more taxing.
In the absence of chemical gradients the relation further simplifies to the so-called *Schwarzschild criterion*:

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}}\,. \tag{2.63}$$

If convection occurs, then one finds for the convective region:

$$\nabla_{\mathrm{rad}} > \nabla > \nabla_{\mathrm{e}} > \nabla_{\mathrm{ad}} \,. \tag{2.64}$$

Dynamical stability can lead to oscillation of a convective element around its equilibrium position and in a regime with $\Delta\mu > 0$ also oscillation with increasing amplitude can occur, which is referred to as *vibrational instability*. Lastly in a situation with $D_\mu \neq 0, \nabla_\mu = 0$ a convective element ($DP = D\rho = 0$) would result in $\delta DT/T = \phi D\mu/\mu$ and for $D_\mu > 0$ a situation is found, in which a convective element with heavier composition and higher temperature first levitates, while cooling due to radiation, until it sinks. This situation can be found when e.g. a helium core ignites off-center, so that a hotter layer with heavier composition forms above the helium core. Then blobs of carbon can enter the core region and only if they cool radiatively before the buoyance force lets them rise again will they sink to the very center. This situation is referred to as *secular instability*.

Convection and Mixing Length Theory

If convection occurs, it contributes to heat transport, as hotter, displaced convective elements at some point dissolve into the surrounding and thereby dissipate the energy density excess into the surrounding material. As the time scales involved are much shorter than those considered in the stellar structure basic formulas, the interest for stellar evolution is not an exact description of the convective flows, but an effective treatment of the contribution to heat transport on average, which then influences the structure. For the flux F one finds with ∇_{rad} the gradient required for transport by radiation only:

$$F = F_{\mathrm{rad}} + F_{\mathrm{conv}} = \frac{4acG}{3}\frac{T^4 m}{\kappa P r^2}\nabla_{\mathrm{rad}} \,, \tag{2.65}$$

$$F_{\mathrm{rad}} = \frac{4acG}{3}\frac{T^4 m}{\kappa P r^2}\nabla \,, \tag{2.66}$$

$$F_{\mathrm{conv}} = \rho v c_p DT \,, \tag{2.67}$$

for a single convective element. The global contribution of convection is then to be obtained from this contribution of a single convective element using an efficient theory. In particular this means that a mean for vDT needs to be calculated. Therefore a theory called *Mixing Length Theory* (MLT) is used, which assumes that all convection occurs due to convective elements that start as infinitesimal elements with $DT_0 = 0$, $v_0 = 0$ and move for a total distance l_{m} called *mixing length* before dissolving in the surrounding. Thus, when considering all convective elements passing through the full sphere at a certain mass coordinate m, those on average have moved $l_{\mathrm{m}}/2$. When passing through the mass shell, they have a temperature difference DT and velocity v. Using the definition of the pressure height (2.58) and (2.60) the temperature difference and corresponding buoyancy force can be determined:

$$\frac{DT}{T} = \frac{1}{T}\frac{\partial(DT)}{\partial r}\frac{l_{\rm m}}{2} = (\nabla - \nabla_{\rm e})\,\frac{l_m}{2H_P}\,, \tag{2.68}$$

$$k_r = -g\frac{D\rho}{\rho} = g\frac{\delta DT}{T}\,, \tag{2.69}$$

as $DP = 0$, $D\mu = 0$. Assuming that half of the buoyancy force per unit mass acting over the distance of $l_{\rm m}/2$ is converted into work on the convective element, while the other half is transfered to the surroundings and dillutes, one finds:

$$\frac{1}{2}k_r\frac{l_m}{2} = g\delta\,(\nabla - \nabla_e)\,\frac{l_{\rm m}^2}{8H_P} \tag{2.70}$$

$$\Rightarrow \qquad v^2 = g\delta\,(\nabla - \nabla_e)\,\frac{l_{\rm m}^2}{8H_P}\,. \tag{2.71}$$

Inserting (2.68) and (2.71) into (2.67) gives:

$$F_{\rm con} = \rho c_P T\sqrt{g\delta}\frac{l_{\rm m}^2}{4\sqrt{2}}H_P^{-3/2}\,(\nabla - \nabla_e)^{3/2}\,. \tag{2.72}$$

The convective elements experience radiative cooling in the colder surrounding material while rising. The radiative energy loss per unit time $\lambda_{\rm rad}$ can be calculated as product of the elements surface area S and the radiative flux perpendicular to this surface f as obtained from (2.38):

$$\lambda_{\rm rad} = S\cdot f = S\frac{4acT^3}{3\kappa\rho}\left|\frac{\partial T}{\partial n}\right|\,. \tag{2.73}$$

Here $|\partial T/\partial n|$ is the temperature gradient in perpendicular direction, which for a spherical convective element with diameter $r_{\rm e}$ can be approximated as

$$\left|\frac{\partial T}{\partial n}\right| = \frac{DT}{r_{\rm e}}\,. \tag{2.74}$$

The temperature change due to radiative losses per unit length travelled by the rising convective element is then (with $m_{\rm e}$ its mass and $V_{\rm e}$ its volume):

$$\frac{\lambda_{\rm rad}}{m_{\rm e}c_P}\frac{1}{v} = \frac{\lambda_{\rm rad}}{\rho V_{\rm e}c_P v} \tag{2.75}$$

and the temperature change of the element per unit length due to both, radiative losses and adiabative expansion:

$$\left(\frac{dT}{dr}\right)_{\rm e} = \left(\frac{dT}{dr}\right)_{\rm ad} - \frac{\lambda}{\rho V_{\rm e}c_P v}\,. \tag{2.76}$$

As pressure was assumed to be in equilibrium with the surrounding in all cases, one finds:

$$\left(\frac{d\ln T}{d\ln P}\right)_{\rm e/ad} = \frac{P}{T}\left(\frac{dT}{dP}\right)_{\rm e/ad} = \frac{P}{T}\left(\frac{dT}{dr}\right)_{\rm e/ad}\frac{dr}{dP} = -\frac{H_P}{T}\left(\frac{dT}{dr}\right)_{\rm e/ad} \tag{2.77}$$

Thus (2.76) can be written as

$$\begin{aligned}\nabla_\mathrm{e} - \nabla_\mathrm{ad} &= \frac{\lambda H_P}{\rho V_\mathrm{e} c_P v T} = \frac{SDT}{V r_\mathrm{e} v}\frac{4acH_P T^2}{3\kappa\rho^2 c_P}\\ &= \frac{S}{V_\mathrm{e} r_\mathrm{e}}\frac{2acl_m T^2}{3\kappa\rho^2 c_P}\left(\nabla - \nabla_\mathrm{e}\right)\\ &= \frac{9}{l_m^2}\frac{2acl_m T^2}{3\kappa\rho^2 c_P}\left(\nabla - \nabla_\mathrm{e}\right)\,,\\ \Leftrightarrow \qquad \nabla_\mathrm{e} - \nabla_\mathrm{ad} &= \frac{6acT^3}{\kappa\rho^2 c_P l_\mathrm{m} v}\left(\nabla - \nabla_\mathrm{e}\right)\,. \end{aligned} \tag{2.78}$$

Here $S/(V_\mathrm{e} r_\mathrm{e}) \approx 9/l_m$ was used, which is a common approximation in literature that determines r_e.[Kip12] The set of equations (2.65), (2.66), (2.71), (2.72), (2.78) can be solved for the quantities F_rad, F_conv, v, ∇_e, ∇ given the local quantities P, T, ρ, l, m, c_P, ∇_ad, ∇_rad and g. These equations can be rewritten using dimensionless variables:

$$U := \frac{3acT^3}{c_P\rho^2\kappa l_\mathrm{m}^2}\sqrt{\frac{8H_P}{g\delta}}\,, \tag{2.79}$$

$$W := \nabla_\mathrm{rad} - \nabla_\mathrm{ad}\,. \tag{2.80}$$

Then (2.78) with (2.71) and (2.79) can be written as

$$\nabla_\mathrm{e} - \nabla_\mathrm{ad} = 2U\sqrt{\nabla - \nabla_\mathrm{e}}\,. \tag{2.81}$$

On the other hand (2.65), (2.66) with (2.72) becomes

$$\begin{aligned}\rho c_P T\sqrt{g\delta}\frac{l_\mathrm{m}^2}{4\sqrt{2}}H_P^{-3/2}\left(\nabla - \nabla_e\right)^{3/2} &= \frac{4acG}{3}\frac{T^4 m}{\kappa P r^2}\left(\nabla - \nabla_\mathrm{rad}\right)\\ \Leftrightarrow \quad \left(\nabla - \nabla_e\right)^{3/2} &= \frac{acT^3}{c_p\rho\kappa l_\mathrm{m}^2\sqrt{g\delta}}H_P^{3/2}\frac{16\sqrt{2}Gm}{3Pr^2}\left(\nabla - \nabla_\mathrm{rad}\right)\\ \Leftrightarrow \quad \left(\nabla - \nabla_e\right)^{3/2} &= U\rho\frac{8GmH_P}{9Pr^2}\left(\nabla - \nabla_\mathrm{rad}\right)\,, \end{aligned} \tag{2.82}$$

and then with (2.16) and (2.58)

$$\left(\nabla - \nabla_\mathrm{e}\right)^{3/2} = \frac{8}{9}U\left(\nabla_\mathrm{rad} - \nabla\right)\,. \tag{2.83}$$

Expending (2.81) on the left hand side to $(\nabla - \nabla_\mathrm{ad}) - (\nabla - \nabla_\mathrm{e})$ it can be read as a quadratic equation that has the solution

$$\sqrt{\nabla - \nabla_\mathrm{e}} = -U + \zeta\,, \tag{2.84}$$

$$\zeta := +\sqrt{\nabla - \nabla_\mathrm{ad} + U^2}\,. \tag{2.85}$$

Inserting this into (2.83) with the definition for ζ gives

$$\left(\zeta - U\right)^3 + \frac{8U}{9}\left(\zeta^2 - U^2 - W\right) = 0\,, \tag{2.86}$$

which can be solved for ζ for given combinations of U and W. ζ will have (without proof) only one real solution, from which the gradient ∇ is found using the definition of ζ. The only quantity in these equations that is undetermined is the mixing length l_m, which is typically expressed as

$$l_\mathrm{m} = \alpha_\mathrm{MLT} \cdot H_P \,, \tag{2.87}$$

with α_MLT being a parameter. It is important to point out, that the MLT as introduced here makes a variety of rather arbitrary assumptions regarding shape, size, internal temperature gradient, and behavior of the convective element. Different versions of MLT exist, that differ in the details of these assumption, but lead to qualitatively similar results. This arbitrariness demands careful comparison between the effective energy transport by convection and experimental data and results from more sophisticated, microscopic simulations to ensure that the real phenomenology is described well. This arbitrariness however also allows to introduce modification factors to reproduce the phenomenology found in this comparison and/or numerical convenience. As explained in section 4 this can be done in form of a *overshooting parameter*. There is also more theoretically sophisticated versions of the mixing length theory with less strict assumptions, which however do not reach a fundamental description yet. The consequence is, that they involve a list of parameters, which can not be determined by measurement, so that the introduced uncertainty by fitting with the limited set of observables is comparable to this very simple version of MLT. Therefore a common choice is to use simple versions of MLT which incorporate all uncertainties in one parameter α_MLT as demonstrated above and use expressions motivated by more sophisticated theory for the overshooting parameter only.[Egg72][Egg83]

2.5 Chemical Composition

It is intuitively clear, that convection leads to a quick mixing of stellar material, which needs to be treated alongside with diffusion and the change of composition by nuclear fusion. The stellar material will be described by a set of n_sp functions $X_i(m,t)$, $i = 1, \ldots, n_\mathrm{sp}$, each giving the mass fraction of a specific nuclear species. With m_i the mass of a particle of species i and n_i its number density:

$$X_i = \frac{m_i n_i}{\rho} \,, \tag{2.88}$$

$$\sum_{i=1}^{n_\mathrm{sp}} X_i = 1 \,. \tag{2.89}$$

The composition is sometimes described by just a set of three numbers $X = X_\mathrm{H} \approx X_{^1\mathrm{H}}$, $Y = X_\mathrm{He} \approx X_{^4\mathrm{He}}$, $Z = 1 - X - Y$, where Z is called *metallicity* and often used to characterize the initial composition of a star. Empirical values for the ratios of all species heavier than helium are then used to gain the mass fractions of individual heavy species. Typical values are $X \approx 0.7$, $Y \approx 0.25$ and $Z \in [0.0001, 0.05]$.

Radiative Region

Neglecting effects of diffusion for now, composition changes are only due to conversion by fusion and described with the nuclear reaction rates r_{ij} for generation of a particle of species i from a particle of species j. Then the change of composition with time is:

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho} \left(\sum_j r_{ji} - \sum_k r_{ik} \right) , \tag{2.90}$$

where the sums go over all species that are involved in creating or can be created from species i. The nuclear fusion processes occuring in stars are discussed in section 3.1. In such reaction a certain energy e_{ij} is released per particle i, contributing to the overall released energy rate per unit mass according to

$$\epsilon = \frac{1}{\rho} \sum_{i,j} r_{ij} e_{ij} = \sum_{i,j} \epsilon_{ij} , \tag{2.91}$$

with ϵ_{ij} the contribution from reaction $i \to j$. Let $q_{ij} = e_{ij}/m_i$ be the energy released per unit mass of particle i due to conversion to particle j, so that

$$\frac{\partial X_i}{\partial t} = \sum_j \frac{\epsilon_{ji}}{q_{ji}} - \sum_k \frac{\epsilon_{ik}}{q_{ik}} . \tag{2.92}$$

The collection of associated species together with their corresponding reactions is called a *nuclear (reaction) network*. Typically those will be described explicitly while the normalisation condition is used for verification.

Diffusion

To include diffusion into the composition change rates, three types of diffusion can be distinguished: *concentration diffusion*, *temperature diffusion* and *pressure diffusion*. Concentration diffusion can be described by the flux of concentration

$$\vec{j}_{\mathrm{D},i} = -D \nabla c_i , \tag{2.93}$$

where $c_i = n_i / \sum_j n_j$ is the concentration of species i and D the diffusion coefficient. Note that this can not be treated as a one dimensional problem in the same way as before, where the total flux through a mass shell was considered. With $\vec{v}_\mathrm{D}$ the diffusion velocity and $\vec{j}_{\mathrm{D,i}} = c_i \vec{v}_\mathrm{D}$ one finds

$$\vec{v}_\mathrm{D} = -\frac{D}{c_i} \nabla c_i \tag{2.94}$$

and then with the continuity equation

$$\frac{\partial c_i}{\partial t} = -\nabla \cdot \vec{j}_\mathrm{D} = \nabla \cdot (D \nabla c_i) = D \nabla^2 c , \tag{2.95}$$

$$= D \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial c}{\partial r} \right) . \tag{2.96}$$

Concentration and temperature diffusion are caused by the random motion of particles at finite temperature. To illustrate how D can be derived from a microscopic description, consider the one dimensional case and fluxes $j^{\pm}$ in $\pm z$-direction due to a concentration and temperature gradient in this direction. The flux j^{+} can be found when assuming, that the particles on average had their last collision one mean free path length l_{part} away from $z = 0$, with $\bar{v}_z$ the mean velocity in $+z$-direction:

$$j^{\pm} = c(\mp l_{\text{part}})\bar{v}_z(\mp l_{\text{part}}) = \frac{1}{6}c(\mp l_{\text{part}})\bar{v}(\mp l_{\text{part}})\,. \tag{2.97}$$

Expanding $c(\Delta z) = c(0) + \partial c/\partial z \cdot \Delta z$ and analogously for the mean velocity gives:

$$j^{\pm} = \frac{1}{6}\left(c(0) \mp \frac{\partial c}{\partial z}l_{\text{part}}\right)\left(\bar{v}(0) \mp \frac{\partial \bar{v}}{\partial z}l_{\text{part}}\right)\,, \tag{2.98}$$

$$\Rightarrow j := j^{+} - j^{-} = \frac{1}{3}\left(\frac{\partial c}{\partial z}l_{\text{part}}\bar{v} + \frac{\partial \bar{v}}{\partial z}l_{\text{part}}c +\right)\,. \tag{2.99}$$

For two different elements, the difference of their diffusion velocities $v_{\text{D}_1} - v_{\text{D}_2}$ will be non-zero, if their molecular masses μ_i differ (assuming an ideal gas):

$$\bar{v}_i = \left(\frac{3\Re T}{\mu_i}\right)^{1/2}\,, \tag{2.100}$$

$$\Rightarrow v_{\text{D}_1} - v_{\text{D}_2} = \frac{j_1}{c_1} - \frac{j_2}{c_2} = -\frac{D}{c_1 c_2}\left(\frac{\partial c_1}{\partial z} + k_T\frac{\partial \ln T}{\partial z}\right)\,, \tag{2.101}$$

where D and k_T are functions of T, c_i, $l_{\text{part},i}$, v_i and μ_i for $i = 1,\,2$, which can be found in [Kip12]. In a similar way also pressure diffusion arises from the fact, that elements with different molecular weights μ_i in the case of ideal gas have specific pressures $P_i = \Re\rho_i T/\mu_i$ and pressure height scales $H_{P_i} = \Re T/(g\mu_i)$ resulting in smaller pressure scales for heavier species. A rigorous quantitativ treatment as found in [Tho94] gives a relation for the one dimensional case of the form:

$$(v_{\text{D,tot}})_i = -\frac{1}{c_i}D\left(\frac{\partial c_i}{\partial z} + k_T\frac{\partial \ln T}{\partial z} + k_P\frac{\partial \ln P}{\partial z}\right)\,, \tag{2.102}$$

with coefficients D, k_T and k_P found as illustrated above in the case of conzentration and temperature diffusion. The exact effect of temperature and pressure diffusion can vary with conditions, however the concentration diffusion always counteracts any imbalance building up due to pressure and temperature diffusion. Typically the time scales for diffusion are long, but the effects are not always negligible. Diffusion processes for the spherically symmetric three dimensional case can qualitatively be described as:

$$\left(\frac{\partial X_i}{\partial t}\right)_{\text{dif}} = -\frac{\partial}{\partial m}\left(r^4\rho\sigma_{\text{diff}}X_i\left(A_{P,i}\frac{\partial P}{\partial m} + A_{T,i}\frac{\partial T}{\partial m} + \sum_k A_{k,i}\frac{\partial c_i}{\partial m}\right)\right)\,, \tag{2.103}$$

with some factors $\sigma_{\text{diff}}(T)$, $A_{P,i}$, $A_{T,i}$ and $A_{k,i}$ that can be found by rigorous theoretical treatment or comparison to literature (see e.g. [Tho94]). The factor $r^4\rho$ originates from the laplace operator in spherical coordinates and the transformation to derivatives with respect to m.

Composition and Convection

Convective currents in general transport stellar material on short time scales compared to both, diffusion and nuclear composition changes. In a first approximation, the difference of time scales, if convection occurs, is significant enough to treat convective mixing as quasi instantaneous, causing a homogenous composition in the entire convective zone, if convection occurs. Thus composition changes are described by the composition change due to nuclear fusion within the convective zone and the change of its borders in this simplified picture. The change of mean mass fraction $\bar{X}_i$ is then described by

$$\bar{X}_i = \frac{1}{m_2 - m_1} \cdot \int_{m_1}^{m_2} X_i dm \,, \tag{2.104}$$

$$\left(\frac{\partial \bar{X}_i}{\partial t}\right)_{\text{conv}} = \frac{1}{m_2 - m_1} \cdot \left(\left[\frac{\partial m_2}{\partial t}\right] \left(X_{i,m_2} - \bar{X}_i\right) + \left[\frac{-\partial m_1}{\partial t}\right] \left(X_{i,m_1} - \bar{X}_i\right) + \int_{m_1}^{m_2} \frac{\partial X_i}{\partial t} dm \right) , \tag{2.105}$$

$$[x] := \max\left(x, 0\right) . \tag{2.106}$$

Here composition change at the borders of the convective zone only occur, if those expand into a previously non-convective region, as expressed in terms of square brackets. The rate $\partial X_i / \partial t$ has to be expressed in terms of the corresponding nuclear rates of the entire nuclear network contributing. If the convective region shrinks, composition change in the previously convective region is described by diffusive mixing and composition change due to nuclear fusion instead, beginning with an initially discontinuous composition profile. In practise this simplified picture is however often insufficient to describe the dynamics in stars during phases of rapid changes and convection has to be effectively treated as another diffusive process. The overall change of composition due to diffusion and convection is then described by a combined coefficient σ, which typically is dominated by convective contribution:

$$\left(\frac{\partial X_i}{\partial t}\right)_{\text{conv, dif}} = \frac{\partial}{\partial m}\left(\sigma \frac{\partial X_i}{\partial m}\right) , \tag{2.107}$$

where all factors depending on the state of matter as well as r are defined to be included in σ (see [Sch99]). Typically this situation occurs due to the time scale for composition change by nuclear reaction becoming small, rather than the scale for convective composition change becoming large, so that in this case the contribution of the diffusion processes are small. Outside convective regions σ is determined by diffusive processes only.

2.6 Mass Loss and Binaries

Mass loss in stars happens due to two reasons: i) conversion of mass to energy that is radiated away, and ii) material lost at the surface due to so-called *wind*. The first effect gathers neutrino losses as well as nuclear fusion converting mass to energy according to $E = mc^2$. The second effect is especially relevant for stars with loosely bound outermost layers, which is the case for very luminous and heavy stars with big radii and for lower mass stars during late stages of their evolution. Thus significant winds coincide with scenarios for relatively high mass loss by the first effect. In all these cases loss due to wind dominates. Stellar wind is a complex surface effect and involves local processes on short time scales and therefore typically effective, empirical models are used. These have substantial uncertainties that overshadow the overall effect for mass loss due to nuclear fusion and neutrinos. In the following the mostly used empirical formulas for mass loss by stellar wind are given:

$$\text{Reimers:} \quad \frac{\dot{M}_{\mathrm{R}}}{M_\odot} = -4 \cdot 10^{-13} \left(\frac{L}{L_\odot}\right)\left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right) \mathrm{y}\,, \tag{2.108}$$

$$\text{Schröder\&Cuntz:} \quad \frac{\dot{M}_{\mathrm{SC}}}{M_\odot} = 0.2 \cdot \frac{\dot{M}_{\mathrm{R}}}{M_\odot}\left(\frac{T_{\mathrm{eff}}}{4000\,K}\right)\left(1 + \frac{g}{4300 g_\odot}\right)\,, \tag{2.109}$$

$$\text{Blöcker:} \quad \frac{\dot{M}_{\mathrm{B}}}{M_\odot} = 4.83 \cdot 10^{-9} \frac{\dot{M}_{\mathrm{R}}}{M_\odot}\left(\frac{M}{M_\odot}\right)^{-2.1}\left(\frac{L}{L_\odot}\right)^{2.7}\,, \tag{2.110}$$

$$\text{Lamers:} \quad \frac{\dot{M}_{\mathrm{L}}}{M_\odot} = -1.48 \cdot 10^{-5} \left(\frac{L}{1000 L_\odot}\right)^{1.42}\left(\frac{R}{30 R_\odot}\right)^{0.61}\left(\frac{30 M_\odot}{M}\right)^{0.99} \mathrm{y}\,. \tag{2.111}$$

Reimers' formula applies to stars with sun-like composition in their red giant phase and was improved to higher accuracy by Schöder and Cuntz. Blöckers formula is based on theories of dust driven winds and Lamers formula fits stars of spectral classes O and B.

Though so far only isolated, single stars were considered, binaries of stars are of significant relevance for astrophysics, as on the one hand a significant fraction of stars is part of multi star systems and on the other hand orbital parameters can give additional information regarding the constituents radii and masses, that are otherwise challenging to obtain. A particular interest is in close binaries in which a considerable mass transfer occurs and tidal interactions influence both, the rotation and the mass transfer. A considerable mass transfer perturbs the equilibrium state stars are in for substantial parts of their existance, adding another way to check the understanding of dynamic processes. Also certain products of such final stages like white dwarfes, neutron stars and black holes are challenging to study unless through their interaction with their surrounding. Lastly certain phenomenologically relevant events such as type 1a supernovae or emission of gravitational waves are directly coupled to interactions in binaries.[Egg07]

In order to simulate stars in interacting binaries, mass transfer due to winds and so-called *Roche Lobe Overflow* (RLOF) must be included. RLOF is the situation, that the outer layers of a star exceed the region in which they are firmly bound to that star gravitationally, so that the material can be accreted by the companion due to radiative pressure or dynamic processes. More over the transfer of material by a stream of gas and its impact or accretion from a disk, the material mixing on the accreting stars surface, transfer of angular momentum, change of orbital parameters, etc. need to be considered. Due to these complications the very interesting aspect of stellar binaries and their simulation is omitted in this thesis.

3 Stellar Material

Within the numerical simulation additionally to the dependent and independent variables several quantities are required, that are essentially material functions such as κ and ϵ. Though theoretical models based on first principles partly exist, these will typically be taken as external input and not calculated or simulated directly. The same holds for certain numerical values used in the equation of state. This avoids complex computation and provides interchangeability for different methods to obtain these quantities by combinations of experimental data and theory. In practise the material functions are provided in the form of tabulated data for a range of values of P, T and X_i and then interpolated within the stellar simulation code. Regarding the equation of state $\rho(P, T, X_i)$ the theoretical framework to include the effect of ionization and degeneracy will be derived in this section.

3.1 Nuclear Fusion

The basic equations of stellar structure describe local and total energy conservation, including the effect of nuclear fusion without specifying the process other than by the rate of energy release and the composition change induced. The released energy originates from the *mass defect*, the total binding energy of a number of nucleons in different configurations. For the fusion process of a number of nuclei of mass m_i forming a nucleus of mass m_j this is

$$E = \Delta mc^2 = \left(\sum_i m_i - m_j \right) c^2 \,, \tag{3.1}$$

with c the speed of light in vacuum. Here the mass of a nucleus ${}^A_Z X$ with A nucleons, Z of them protons, is given as the sum of the masses of its constituents reduced by the binding energy E_B:

$$m_j = (A - Z)\, m_\mathrm{n} + Z m_\mathrm{p} - E_\mathrm{B} c^{-2} \,, \tag{3.2}$$

where E_B depends on the configuration of neutrons and protons and interactions with electrons are excluded. The binding energy per nucleon E_B/A for a variety of nuclei is shown in fig. 2. It can be seen, that there is a maximum of binding energy per nucleon around ^{56}Fe, so that heavier atoms tend to reach energetically favorable configurations by undergoing nuclear fission, while lighter nuclei might reach such configuration by undergoing nuclear fusion. To undergo nuclear fusion however the nuclei have to overcome the repulsive coulomb force between the positively charged nuclei, until the short ranged nuclear forces dominate and can mediate the fusion process ultimatively leading to an energy release. In fact the nuclei overcome the coulomb barrier by a combination of their kinetic energy and the quantum mechanic tunnel effect. The tunnel effect originates from the *energy-time-uncertainty* essentially stating, that particles can overcome a potential barrier that exceeds their energy, with a probability that is exponentially damped with the so-called *Gamow factor*. This

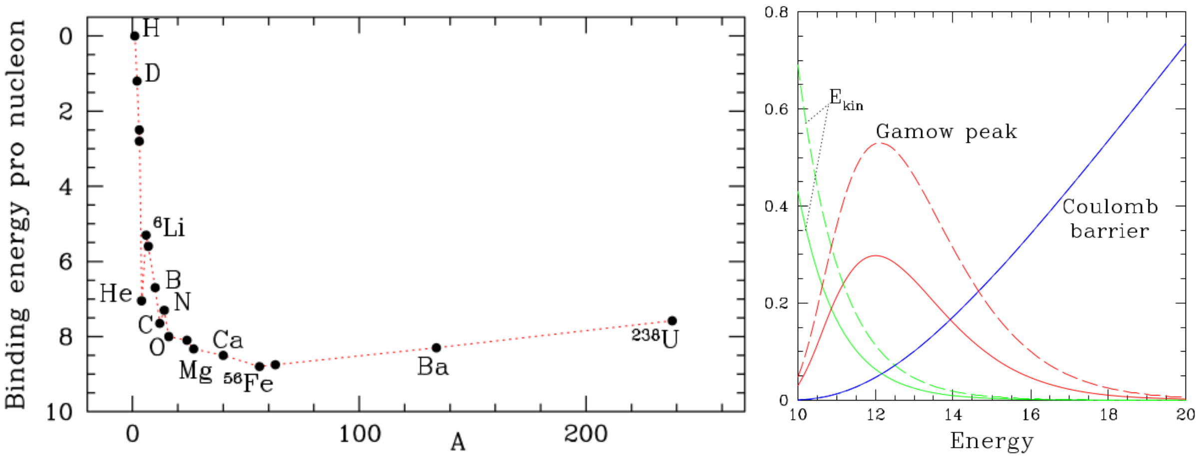

Figure 2: Binding energy per nucleon for increasing number of nucleons (left) and sketch of the Gamow peak (red, scaled with a factor 100) showing probabilities to overcome a coulomb barrier (blue) due to maxwell-distributed kinetic energy (green) for two different temperatures (dashed and solid curves) shown on the right (taken from [Boe08]).

factor increases with the difference of the particle energy and the height of the potential barrier, the barriers thickness and the particles mass. The convolution of this factor with the maxwellian energy distribution is shown in fig. 2 (right). The probability to undergo fusion is further complicated by the occurance of resonances due to the existance of excited nuclear states. For the evaluation of reaction rates as used in stellar simulations thus a combination of experimental data and theoretical calculations is used to also include regions of low energies, that are less accessible in experiment (see [Kip12], ch. 18). The reaction rate for a nucleus of species j moving through nuclei of species k, which have an energy dependent individual cross section $\sigma_{\text{nuc}}(E)$, with a relative velocity v, the collision rate per volume is given by:

$$r_{jk} = \frac{1}{1+\delta_{jk}} n_j n_k \langle \sigma_{\text{nuc}} v \rangle \,. \tag{3.3}$$

Here n denotes the nuclei density for the two species and $\langle \sigma_{\text{nuc}} v \rangle$ is the averaged product of cross section and relative velocity. Assuming that each reaction of these species releases an energy Q_{jk} and using $\rho X_i = n_i m_i$, with X_i the mass fraction and m_i the mass per nucleus, the energy generation rate per unit mass is

$$\epsilon_{jk} = \frac{r_{jk}}{\rho} = \frac{1}{1+\delta_{jk}} \frac{Q}{m_j m_k} \rho X_j X_k \langle \sigma v \rangle \,. \tag{3.4}$$

Here Q and $\langle \sigma v \rangle$ have to be obtained from a combination of experimental data and theory and are taken as external input depending on P, T and X_i. In the following the most important fusion reactions are outlined.

Hydrogen Burning

The biggest increase in binding energy per nucleon is observed in the steep region for small A in fig. 2 (left), that is by fusion of hydrogen to helium. As stars in general initially predominantly consist of hydrogen ${}^1_1\mathrm{H}$ and the coulomb barrier for hydrogen nuclei is also minimal, the fusion of hydrogen is dominating most stars during major stages of their existence.

$$ {}^1_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^2_1\mathrm{H} + e^+ + \nu_\mathrm{e} \tag{3.5} $$

$$ {}^2_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + \gamma \tag{3.6} $$

It is important to notice, that in the first reaction, the conversion of a proton into a neutron by the weak interaction occurs, which happens in general on much longer time scales, than processes mediated by the strong interaction. Thus the time scale for this reaction is typically $\sim 10^{10}$ y compared to $\sim$ s.[Gui19] As can be seen in fig. 3, also the temperature dependence for the second reaction is less steep and as a result its rate dominates over the first reaction by many orders of magnitude, especially for less hot stars. As a consequence, even less massive objects that do not reach temperatures sufficient to undergo the first reaction ($\sim 3{\cdot}10^6$ K) can still generate energy by fusion from the fusion of hydrogen and deuterium already existing during their formation. There is three reaction chains, called *pp chains* leading from ${}^3_2\mathrm{He}$ to

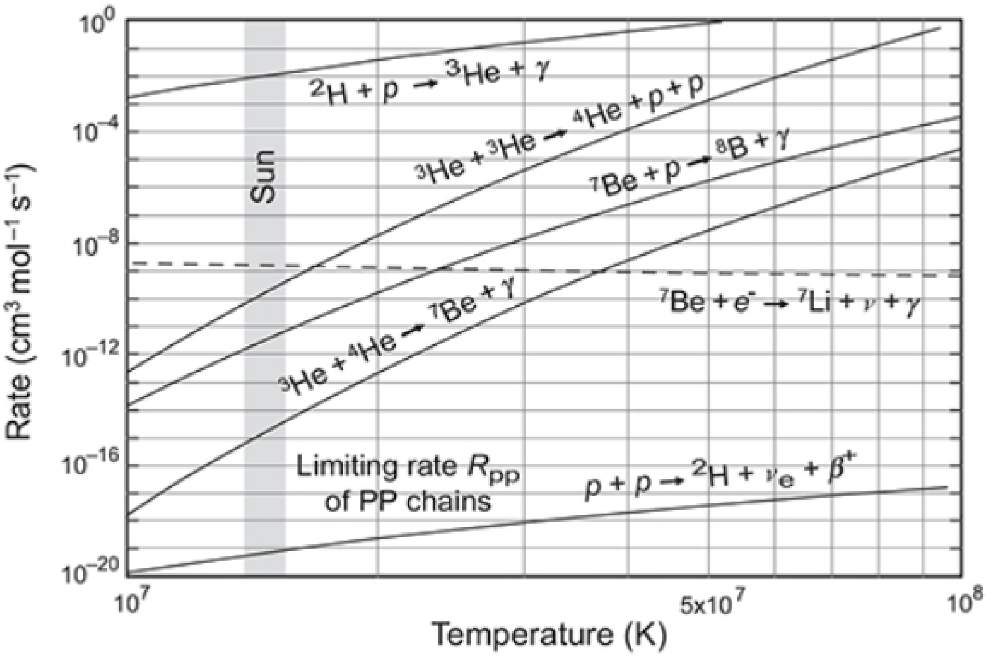

Figure 3: Dependence of reaction rates relevant for the pp chains on temperature, taken from [Gui19].

${}^4_2\mathrm{He}$, which are shown in fig. 4 and all have slightly different overall mean energy release Q per produced ${}^4_2\mathrm{He}$ due to different particles generated and fractions of energy being carried away by neutrinos and not included in the energy release: $Q_\mathrm{pp1} = 26.50\,\mathrm{MeV}$, $Q_\mathrm{pp2} = 25.97\,\mathrm{MeV}$ and $Q_\mathrm{pp3} = 19.59\,\mathrm{MeV}$.[Kip12] The reactions required for these chains also have different rates and i.e. temperature dependence of rates as seen in fig. 3, so that their branching depends on the exact conditions (composition, density, temperature) and the exact energy release due to hydrogen fusion via the pp chains is given as an weighted average for a specific situation. For the sun at its current stage, pp1 dominates ($\sim$ 69%), while pp3 only contributes with $\sim$ 0.1%. The importance of pp2 and pp3 increases with increasing temperature.[Boe08] An interesting finding is, that the neutrino generated in the pp2 chain has a sharply defined energy, while the one produced in the pp3 chain has an energy spectrum, due to the nature of two and three body decays. A second mechanism for the formation of

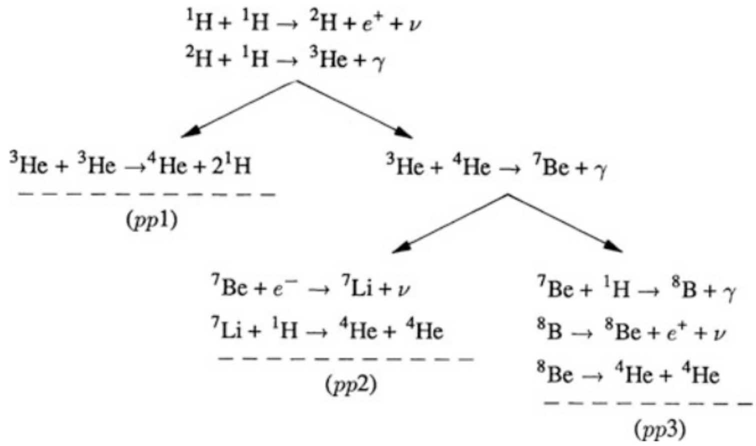

Figure 4: Reactions occuring in the three different pp chains, taken from [Kip12].

${}^4_2$He is the *CNO cycle*, which dominates at above $\sim 2 \cdot 10^7\,$K:

$$\begin{aligned}
{}^{12}_{6}C + {}^{1}_{1}H &\rightarrow {}^{13}_{7}N + \gamma\,, \\
{}^{13}_{7}N &\rightarrow {}^{13}_{6}C + e^+ + \nu_e\,, \qquad T_{1/2} \approx 10\,\text{min} \\
{}^{13}_{6}C + {}^{1}_{1}H &\rightarrow {}^{14}_{7}N + \gamma\,, \\
{}^{14}_{7}N + {}^{1}_{1}H &\rightarrow {}^{15}_{8}O + \gamma\,, \\
{}^{15}_{8}O &\rightarrow {}^{15}_{7}N + e^+ + \nu_e\,, \qquad T_{1/2} \approx 2\,\text{min} \\
{}^{15}_{7}N + {}^{1}_{1}H &\rightarrow {}^{12}_{6}C + {}^{4}_{2}He\,.
\end{aligned} \tag{3.7}$$

For the last two steps in (3.7) a second branch exists:

$$\begin{aligned}
{}^{15}_{7}N + {}^{1}_{1}H &\rightarrow {}^{16}_{8}O + \gamma\,, \\
{}^{16}_{8}O + {}^{1}_{1}H &\rightarrow {}^{17}_{9}F + \gamma\,, \\
{}^{17}_{9}F &\rightarrow {}^{17}_{8}O + e^+ + \nu_e\,, \qquad T_{1/2} \approx 1\,\text{min} \\
{}^{17}_{8}O + {}^{1}_{1}H &\rightarrow {}^{14}_{7}N + {}^{4}_{2}He\,.
\end{aligned} \tag{3.8}$$

Both are sketched in fig. 5 (right) and the temperature dependence of partial reactions are shown in fig. 5 (left). The second branch only contributes with $\approx$ 0.1% and the energy released per helium nucleus can be averaged to $Q_{\text{CNO}} = 24.97\,$MeV, which again is not precise due to the three body decays involving neutrinos. It is important to notice, that all involved reaction rates, except the decay scale with high powers of the temperature, while all pp chains involve at least one reaction, that scales with a significantly smaller power of the temperature. This has the consequence, that the CNO cycles as shown above overall scale with a significantly higher power of the temperature ($\sim T^{13}$ to T^{23}) than the pp chains ($\sim T^{6}$) and dominate in stars, that are somewhat more massive than the sun ($\approx 1.2\,M_{\odot}$), while the pp chains dominate in lower mass stars.[Kip12] The bottle neck in the CNO cycle is typically the reaction of ${}^{14}_{7}\mathrm{N}$, so that an equilibrium is reached and the overall reaction rate is driven by this process. At very high temperatures and densities even the β^{+}-decay halftimes can become the bottleneck. However, the involved nuclear network is in general complicated and thus exact temperature dependencies can not be derived without precisely specifying the conditions. There also exist two additional CNO cycles, that are relevant only in very massive stars and another three so-called hot CNO cycles, which are only relevant in novae (when the β^{+}-decay half time dominates the reaction rate) and thus not explicitly given here.[Boe08][Gui19]

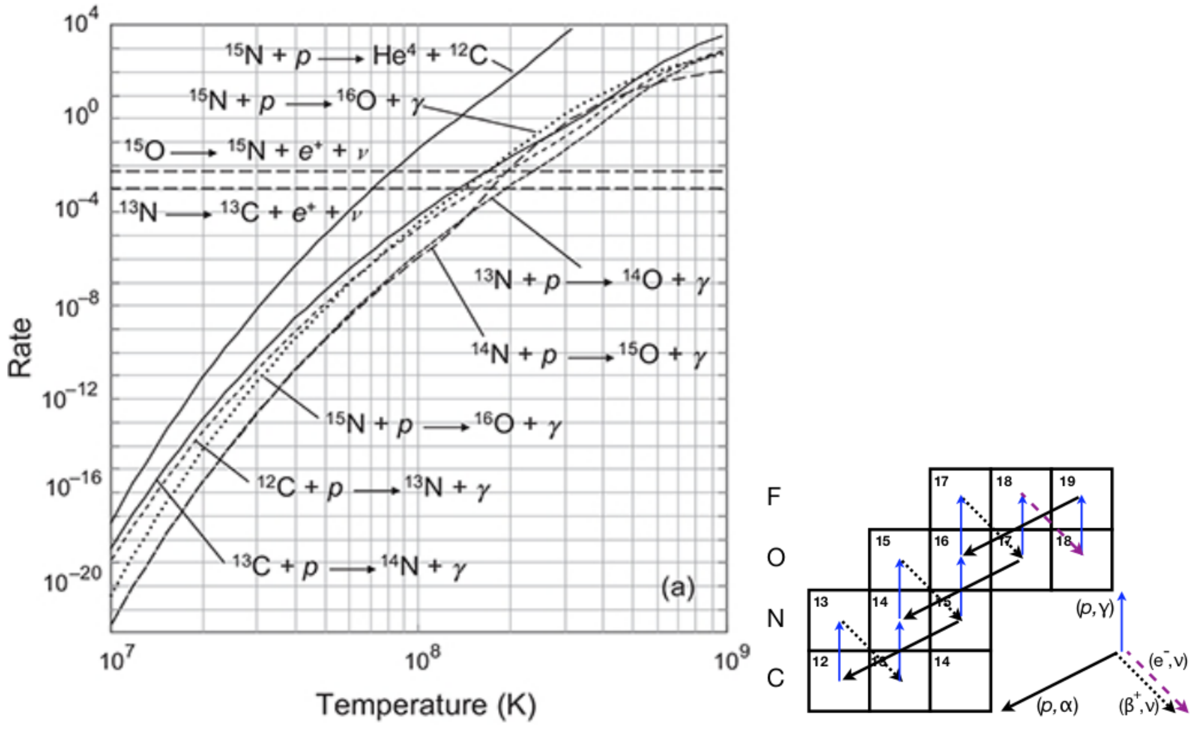

Figure 5: Temperature dependence of a variety of reactions involved in the CNO cycle (left), taken from [Gui19] and a sketch of the involved reactions for the two involved branches (right), taken from [Boe08].

Helium Burning

At even higher temperatures of above $\sim 10^{8}\,\mathrm{K}$ the *triple alpha process* occurs:

$$\begin{aligned} {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} &\rightleftharpoons {}^{8}_{4}\mathrm{Be}\,, \\ {}^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} &\to {}^{12}_{8}\mathrm{C} + \gamma\,. \end{aligned} \tag{3.9}$$

As can be seen in fig. 2 (left) $^{8}_{4}$Be is higher in energy, than two seperate $^{4}_{2}$He nuclei so that Be nuclei decay on a time scale of 10^{-16} s, as is indicated by the reaction arrow going in both directions. The energy release per carbon atom is $Q_{3\alpha} = 7.274\,\text{MeV}$ and thus the energy release per mass is only about 10% of the value for hydrogen burning. Due to the requirement of this short lived transient state to undergo one additional reaction before decay, the triple alpha process is only relevant at very high temperatures and densities, so that the second reaction in (3.9) is sufficiently likely to occur. Additionally the Gamow peak mentioned above overlaps with an excited state of the carbon atom, which drastically increases the reaction probability and thereby reduces the required temperatures significantly. The combination of increased rate of beryllium nuclei produced, the increasing collision rate and the increased overlap of the Gamow peak with the excited state in carbon leads to an extreme dependence on temperature of up to $\sim T^{40}$.[Gui19] As soon as $^{12}_{6}$C is generated in macroscopic amounts, further α captures can occur in parallel to the triple alpha process, mainly:

$$\begin{aligned} {}^{12}_{6}\text{C} + {}^{4}_{2}\text{He} &\rightarrow {}^{16}_{8}\text{O} + \gamma\,, \\ {}^{16}_{8}\text{O} + {}^{4}_{2}\text{He} &\rightarrow {}^{20}_{10}\text{Ne} + \gamma\,. \end{aligned} \tag{3.10}$$

The energy release per produced nucleus is $Q_{C\alpha} = 7.162\,\text{MeV}$ and $Q_{O\alpha} = 4.73\,\text{MeV}$. Further reactions are possible, require a consecutive mixing though, providing a supply of $^{4}_{2}$He when sufficient levels of heavier nuclei such as $^{20}_{10}$Ne already build up, as otherwise the concentration of additional helium nuclei at that point is too low. This situation of consecutive mixing can be found in AGB stars (Asymptotic Giant Branch).[Boe08]

Carbon and Heavier Nuclei

For even higher temperatures of $\sim 5 \cdot 10^{8}\,\text{K}$ to $10^{9}\,\text{K}$ carbon burning and above $10^{9}\,\text{K}$ even oxygen burning can occur. For both reactions, several channels are availble, some of which release α-particles, neutrons or protons, which due to the extreme temperatures immediately fuse with the carbon, oxygen or other species present. Also neutron capture is available then, which circumvents overcoming the coulomb barrier for production of heavier species. Complex nuclear networks are the consequence, with a broad variety of species and isotopes being populated at the same time, unlike in the case of hydrogen and helium burning, where one dominating reaction at a time takes place, clearly seperated by different required temperatures. As a consequence the amount of reaction channels as a whole increases drastically. Additionally *photodisintegration* becomes relevant, that is the absorbtion of high energetic γ quanta, inducing decays in some constituent species. All in all the description of a nuclear network at these temperatures becomes increasingly challenging due to the multitude of involved species, processes and reaction channels available. Therefore a discussion of specific reactions is omitted here.

3.2 Opacity

Another important aspect of the stellar matter, that was already seen in the equations for heat transport by radiation is its opacity $\kappa(\rho, T)$. Though there is basic theoretical framework for explaining individual effects observed, there is no approximation that can describe the arising properties of κ analytically with reasonable accuracy over extended ranges of ρ and T. Therefore *opacity tables* are used, that is tabulated values for a wide range of densities and temperatures, that are themselves found by numerical simulation of all involved processes. Such tables are also used in the numerical simulation of stellar structure and evolution, so that here only the essential processes leading to interaction of radiation and stellar matter will be briefly sketched, rather than theoretically discussed. Figure 6 shows an example for a spectral opacity (left) and a mean opacity (right). The mean opacity is calculated using the *OPAL* opacity table, which is also implemented in the simulation code used in this thesis.[Igl96]

In the presence of free, charged particles incident radiation constantly undergoes scattering by *Thompson scattering* in the non-relativistic case and *Compton scattering* in case of relativistic momentum transfer. Both give a constant background, that only depends on temperature due to the relativistic correction included in Compton scattering. These corrections are negligible for moderate temperatures, but become important ($> 20\%$) above 10^8 K.[Kip12] Other than that, scattering on free electrons essentially only depends on the density of free electrons, which inturn depends on the state of ionization.

Unbound electrons in the electric field of an nucleus can absorb photons and thereby contribute to the opacity by *free-free transitions*. This effect was studied by Kramers and found to result in a spectral opacity

$$\kappa_\nu \sim Z^2 \rho T^{1/2} \nu^{-3} \,. \tag{3.11}$$

Inserting this dependence into (2.47) and performing the integration leads to a mean opacity

$$\kappa_{\mathrm{ff}} \sim \rho T^{-7/2} Z^2 \,, \tag{3.12}$$

which is often called *Kramers opacity* and only a classical approximation, often multiplied with a correction factor to account for quantum-mechanical corrections. The contributions of different species with different Z still need to be added up according to the composition of the stellar medium. This effect dominates in dense, hot stellar medium, such as in the stellar interior, near the core.

Another contribution comes from *bound-free transitions*, that is the absorbtion of photons by bound electrons leading to ionization. The probability depends on the energy state the electron is in and drops off with ν^{-3} for photon frequencies $\nu \geq \chi_i/h$ where χ_i is the ionization energy of the state i the electron is in. For a mixture of partially ionized stellar matter, all possible electron states must be considered and summed up. This effect is important in less hot regions, where the matter is not mostly ionized, such as outer envelops and stellar

atmospheres.
Additionally *bound-bound transitions* can occur, in which a photon is absorbed and excites a bound electron to a higher bound state. As the electron will reemit the absorbed energy into an arbitrary direction, this effect also contributes to opacity for discrete photon energies matching such transitions. Calculations of these contributions require detailed knowledge of the stellar materials composition and all excitation and ionization levels and becomes very important in cooler stellar atmospheres, leading to the immense complications of simulation of (cool) stellar atmospheres due to the coupling to the equation of state by ionization and excitation states. In this scenario, also molecules may be present, further adding to the multitude of available excitation states and complexity of such simulations.
Additional contributions are transitions involving *plasmons*, that is collective excitations of electrons in plasma, which can be relevant in dense, hot stellar cores. Yet another factor is the so-called *negative hydrogen atom*, that is loosely bounded H^- systems, which are present in rather cool regions of stars, where heavier nuclei are already considerably partially ionized. The neutral hydrogen density can still be high though, due to its relatively large ionization energy and bound-free transitions involving H^- increase the opacity. This effect yet again is relevant in the cool surface region of stars, especially such that have high metallicity.

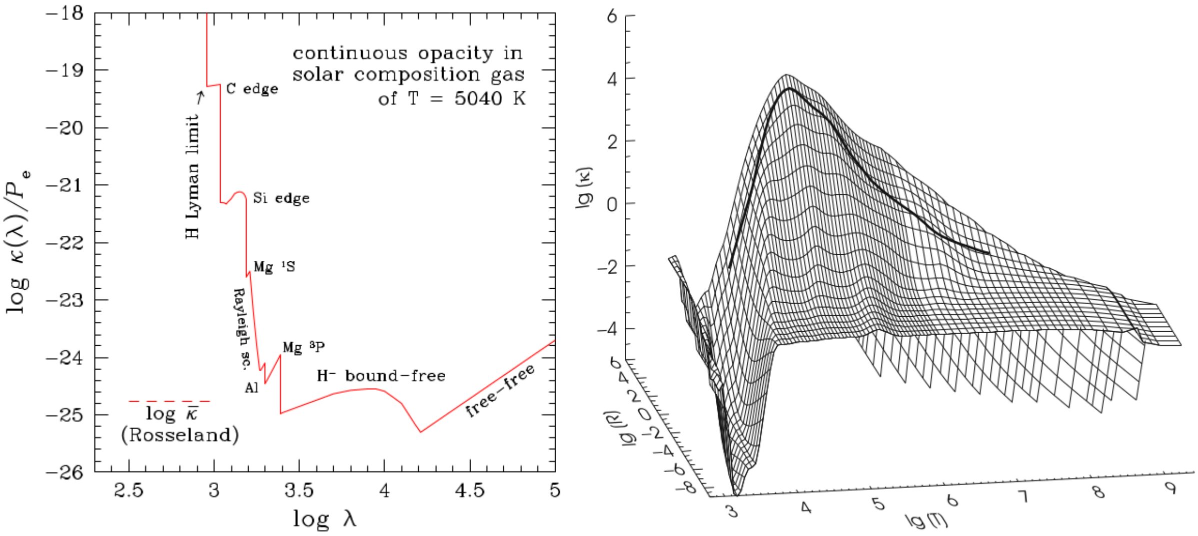

Figure 6: Shown is the spectral opacity $\kappa(\lambda)$ over electron gas pressure P_e against the photon wavelength λ for a star with solar composition at $T = 5040\,\mathrm{K}$ in a double logarithmic plot with labels indicating specific contributions (taken from [Boe08]) and $\lg \kappa/[\mathrm{cm}^2\,\mathrm{g}^{-1}]$ for the Rosseland mean of opacity κ as function of $\lg T/[\mathrm{K}]$ and $\lg R$ where $R = \rho{\cdot}10^6/T$ in $\mathrm{K\,g/cm^{-3}}$, taken from [Kip12], [Igl96].

3.3 Equation of State

Ionization

Due to the immense changes of pressure P and temperature T from the central region to its outer layers the constituent gas is not necessarily fully ionized. Towards the surface partial ionization will in general occur and in cooler stars even molecule formation is possible. Here the influence of partial ionization will be derived, by analogy to excitation. Consider a single species of constituent particles, having a number s of excited states which may or may not be degenerate, such that the *statistical weight* of the state g_s gives the number of actual states (so that $g_s = 1$ for a not degenerate state). If the transition energy from the ground state to state s is $\Psi_s > 0$, then in equilibrium the ratio of number of particles in those states is

$$\frac{n_s}{n_0} = \frac{g_s}{g_0} \exp\left(-\frac{\Psi_s}{kT}\right), \tag{3.13}$$

which is the *Boltzmann formula*, with n_0, n_s the number of particles in the ground state and excited state s respectively. With $n := \sum_s n_s$ the number of particles in all states and

$$u_\mathrm{P} := g_0 \frac{n}{n_0} = \frac{g_0}{n_0} \sum_{s=0}^{\infty} \frac{n_s}{n_0} = g_0 + g_1 \exp\left(-\Psi_1/kT\right) + g_2 \exp\left(-\Psi_2/kT\right) + \ldots\,, \tag{3.14}$$

the partition function, it follows

$$\frac{n_s}{n} = \frac{g_s}{u_\mathrm{p}} \exp\left(-\frac{\Psi_s}{kT}\right). \tag{3.15}$$

Unlike in the case of excitation, the transitions in case of ionization are not to another bound state, but to an unbound state of the electron, thus having a continuous energy spectrum, rather than discrete excitation energies. The same argument holds for the inverse process of recombination. Let (r) be the r-th ionization state, that is the state after losing r electrons due to ionization with all remaining electrons being in their ground states and χ_r the energy to remove the $r+1$-th electron from the ion. After the $r+1$-th ionization the newly freed electron is in an unbound state with momentum in the interval $[p_\mathrm{e},\ p_\mathrm{e} + \delta p_\mathrm{e}]$ and $n_{(r),0}$, $\delta n_{(r+1),0}$ the number densities of the ion after this process in again the ground state for the remaining electrons. For the statistical weight of this specific, $r+1$ times ionized state the weight $g_{(r+1),0}$ has to be multiplied with the phase space volume of the electron $dg(p_\mathrm{e})$. Taking infinitesimal intervals, that is $\delta \to d$ the Boltzmann formula in equilibrium then reads

$$\frac{dn_{(r+1),0}}{n_{(r),0}} = \frac{g_{(r+1),0}dg(p_\mathrm{e})}{g_{(r)},0} \exp\left(-\frac{\chi_r + p_\mathrm{e}^2/2m_\mathrm{e}}{kT}\right). \tag{3.16}$$

The phase space volume can be expressed as $dg(p_\mathrm{e}) = 2dV\, d^3p_\mathrm{e} h^{-3}$ due to the Pauli exclusion principle, with h the Planck constant. Introducing n_e, the density of electrons, and the volume of a sphere of radius p_e as $4\pi p_\mathrm{e} dp_\mathrm{e}$, the overall equation reads:

$$\frac{dn_{(r+1),0}}{n_{(r),0}} = \frac{g_{(r+1),0}}{g_{(r),0}} \frac{8\pi p_\mathrm{e}^2 dp_\mathrm{e}}{n_\mathrm{e} h^3} \exp\left(-\frac{\chi_r + p_\mathrm{e}^2/2m_\mathrm{e}}{kT}\right) . \tag{3.17}$$

Integration over dp_e then gives

$$\begin{aligned}\frac{n_{(r+1),0}}{n_{(r),0}} &= \int \frac{dn_{(r+1),0}}{n_{(r),0}} = \frac{g_{(r+1),0}}{g_{(r),0}} \frac{8\pi}{n_\mathrm{e} h^3} \exp\left(\frac{-\chi_r}{kT}\right) \int_0^\infty dp_\mathrm{e} p_\mathrm{e}^2 \exp\left(\frac{-p_\mathrm{e}^2}{2m_\mathrm{e} kT}\right) \\ &= \frac{g_{(r+1),0}}{g_{(r),0}} \frac{8\pi}{n_\mathrm{e} h^3} \exp\left(\frac{-\chi_r}{kT}\right) \frac{\sqrt{\pi}}{4} (2m_\mathrm{e} kT)^{3/2}\end{aligned}$$

where the integral was evaluated using the known integral $\int_0^\infty dx x^2 \exp -x^2/a = \sqrt{\pi a^3}/4$ for $a > 0$. This then is the so-called *Saha equation*

$$\frac{n_{(r+1),0}}{n_{(r),0}} n_\mathrm{e} = \frac{g_{(r+1),0}}{g_{(r),0}} f_r(T) , \qquad f_r(T) := \frac{2}{n_\mathrm{e}} \exp\left(-\frac{\chi_r}{kT}\right) \left(\frac{2\pi m_\mathrm{e} kT}{h^2}\right)^{3/2} . \tag{3.18}$$

Equation (3.13), considering also the ionization state, reads

$$\frac{n_{(r),s}}{n_{(r),0}} = \frac{g_{(r),s}}{g_{(r),0}} \exp\left(-\frac{\Psi_{(r),s}}{kT}\right) , \tag{3.19}$$

and with $u_{\mathrm{p},(r)} := g_{(r),0} \sum_s n_{(r),s}/n_{(r),0}$ and the number densities for all excitation states $n_{(r)} = \sum_s n_{(r),s}$, equation (3.18) can be written as

$$\frac{n_{(r+1)}}{n_{(0)}} n_\mathrm{e} = \frac{u_{\mathrm{p},(r+1)}}{u_{\mathrm{p},(r)}} f_r(T) . \tag{3.20}$$

So far only one species of constituents was considered. For X_i the weight fraction of the i-th species, Z_i its charge number, μ_i its molecular weight, $x_{i,(r)}$ the fraction of particles of species i, that is in the r-th ionization state, and $X_i \mu_0/\mu_i$ the fraction of particles of species i of all particles, the total number of electrons is

$$N_\mathrm{e} = \sum_i \frac{\mu_0}{\mu_i} X_i \sum_{r=0}^{Z_i} x_{i,(r)} r . \tag{3.21}$$

The Saha equation then becomes

$$\frac{x_{i,(r+1)}}{x_{i,(r)}} \frac{N_\mathrm{e}}{N_\mathrm{e}+1} = K^i_{(r)} , \quad i = 1, \ldots, l , \quad r = 0, \ldots, Z_i - 1 \qquad \text{with} \tag{3.22}$$

$$K^i_{(r)} = \frac{u_{\mathrm{p},(r+1)}}{u_{\mathrm{p},(r)}} \frac{f^i_{(r)}}{P_\mathrm{gas}} . \tag{3.23}$$

Here $f^i_{(r)}$ is given as in (3.18) with the r-th ionisation energy $\chi^i_{(r)}$ for the i-th species. This is a set of Z_i equations and one additional equation, as the sum of all $x_{i,(r)}$ for any species has to give 1. There also is $Z_i + 1$ quantities $x_{i,(r)}$ that need to be determined, so that the equations found determine them uniquely. Expressions for the internal energy and related quantities can be found in [Kip12]. For the calculation of thermodynamic quantities, derivatives of the ionization functions x need to be determined numerically. Thereby, the overall pressure has to be considered constant, so that changes in P_{gas} need to be compensated by P_{rad}, further complicating the calculations. Also the Saha equation is only an approximation, which assumed at least local thermodynamic equilibrium, that is particle collisons dominating over absorbtion/emission of photons. This may not be fulfilled in the entire stellar atmosphere. Additionally implicitly the particles were considered to have a mean seperation at which the coulomb potentials of the ions do not overlap, as the ionization energies will be reduced otherwise. This effect is called pressure ionization and is in fact significant in the very dense, central region of stars.

Degenerate Electron Gas

Yet another influence to consider arises from the Pauli exclusion principle for the electron gas, when the constituent matter is ionized: degeneracy. For n_{e} the number density of free electrons assuming their velocities are following the Boltzmann statistic, then the number of electrons with momentum p in the interval $[p, p + dp]$ is

$$f(p)dp\,dV = n_{\mathrm{e}} \frac{4\pi p^2}{(2\pi m_{\mathrm{e}} kT)^{3/2}} \exp\left(-\frac{p^2}{2m_{\mathrm{e}}kT}\right) dp\,dV \tag{3.24}$$

For low temperatures, this distribution becomes increasingly narrow and centered at low momenta, but according to Heisenbergs uncertainty relation and the Pauli exclusion principle, there can be a maximum of two electrons per phase space cell of volume $dp^3\,dV = h^3$. The shell in momentum space for $[p, p + dp]$ can thus contain a maximum number of electrons given by

$$f(p)dp\,dV \leq 4\pi p^2 dp\,dV \cdot \left(\frac{2}{h^3}\right) = \frac{8\pi}{h^3} p^2 dp\,dV\,, \tag{3.25}$$

which is simply the phase space volume for p in the interval given above divided by the phase space cell volume times the number of electrons per cell. For sufficiently low temperatures, the distribution given by the Boltzmann statistic is thus quantum mechanically forbidden and the electrons must populate states with higher momenta. This is called *degeneracy*. If the occupied momentum states have a sharp edge at the Fermi momentum p_{F}, the electron gas is called completely degenerate. At finite temperature this however will not be the case and the electron gas is partially degenerate.

$$f(p)dp\,dV = \frac{8\pi}{h^3} p^2 dp\,dV \frac{1}{1 + \exp\left(E_{\mathrm{kin}}/kt - \Psi\right)} \qquad \text{with} \tag{3.26}$$

$$E_{\mathrm{kin}} = E_{\mathrm{tot}} - m_{\mathrm{e}}c^2 = m_{\mathrm{e}}c^2 \left(\sqrt{1 + \frac{p^2}{m_{\mathrm{e}}^2 c^2}} - 1\right) \tag{3.27}$$

The electron density is then obtained by dividing by dV and integration over dp:

$$n_{\mathrm{e}} = \int f(p)dp = \int_0^\infty dp \frac{8\pi}{h^3} p^2 \frac{1}{1 + \exp\left(E_{\mathrm{kin}}/kT - \Psi\right)}\,. \tag{3.28}$$

Chandrasekhar gave expansions for the electron density for several regions of Ψ and T, which can be used to construct an approximation for Ψ as explained in [Egg73a]. There, two functions f, g are introduced:

$$\Psi = \ln \frac{\sqrt{1+f}-1}{\sqrt{1+f}+1} + 2\sqrt{1+f}\,, \tag{3.29}$$

$$g = \left(\frac{kT}{mc^2}\right)\sqrt{1+f}\,, \tag{3.30}$$

$$n_\mathrm{e} = \frac{f}{1+f}(g(g+1))^{3/2}\frac{\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} f^i g^j}{(1+f)^3(1+g)^3}\,. \tag{3.31}$$

The coefficients a_{ij} are determined by fitting and are given in the paper cited above.

Polytropic Gaseous Spheres

The first two structure equations (4.1) and (4.2) do not explicitly depend on the temperature T, only indirectly through $\rho(P,T)$ and with that the mass distribution $m(r)$. In a simple equation of state $\rho(P,T) = \rho(P)$ thus these two structure equations become independent of T. A specific form of such equation of state are the so called *polytropic relations*:

$$P = K\rho^\gamma = K\rho^{1+1/n}\,, \tag{3.32}$$

with γ the *polytropic exponent*, n the *polytropic index* and K a constant. Polytropes can serve as a significantly more simple approximation or test model. In the early days of simulation of stellar evolution, multiple polytropes with adjusted γ and K sometimes were used to describe different regions in a star and to generate stellar models by fitting those partial models at their intersections. Currently they are only used as simplified models for benchmarks, debugging or as initial approximations to generate starting models from. The stellar structure function (4.1) in Eulerian description reads

$$\begin{aligned} \frac{dP}{dr} &= -\rho\frac{d\Phi}{dr} \\ \Leftrightarrow \frac{d\Phi}{dr} &= -\gamma K\rho^{\gamma-2}\frac{d\rho}{dr} \\ \Leftrightarrow \rho &= \left(\frac{-\Phi}{(n+1)K}\right)^n\,, \end{aligned} \tag{3.33}$$

which with (2.9) gives a second order differential equation, that can be transformed into the *Lane-Emden-equation* using dimensionless variables. This equation can be solved numerically with physical boundary conditions for $\rho(r=0) = \rho_\mathrm{c}$ and $\Phi(r=0) = \Phi_\mathrm{c}$. In the context of simulation of stellar evolution, tabularized values for the dimeonsionless variables and their derivatives are used.

4 Simulation of Stellar Structure and Evolution

After these considerations regarding the physical processes and quantities that appear in the description of the structure and evolution of stars the numerical simulation itself is outlined in the following. The set of basic equations of stellar structure as derived in section 2 is:

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{4.1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{\partial^2 r}{\partial t}\frac{1}{4\pi r^2} \tag{4.2}$$

$$\frac{\partial l}{\partial m} = \epsilon_n - \epsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t}\,, \qquad \text{with } c_P = \left.\frac{dq}{dT}\right|_P,\ \delta = -\left.\frac{\partial \ln \rho}{\partial \ln T}\right|_P \tag{4.3}$$

$$= \epsilon_n - \epsilon_\nu - T\frac{\partial s}{\partial t}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla\,, \qquad \text{with } \nabla = \frac{d\ln T}{d\ln P} \tag{4.4}$$

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho}\left(\sum_j r_{ji} - \sum_k r_{ik}\right) + \left(\frac{\partial X_i}{\partial t}\right)_{\text{conv,diff}}\,. \tag{4.5}$$

Here ∇ has to be replaced with a suited expression based on the mode of energy transport, which is determined by the stability criteria of Schwarzschild (2.63) or Ledoux (2.62):

$$\text{Radiative region:} \qquad \nabla = \nabla_{\text{rad}} = \frac{3}{16\pi acG}\frac{\kappa l}{m}\frac{P}{T^4}\,, \tag{4.6}$$

$$\text{Convective region:} \qquad \nabla = \zeta^2 - \nabla_{\text{ad}} + U^2,\ (\zeta - U)^3 + \frac{8U}{9}\left(\zeta - U^2 - W\right) = 0\,, \tag{4.7}$$

where U and W are given by (2.79) and (2.80) and the second equation is solved for ζ. Additionally there is one equation describing temporal changes of composition due to convection and diffusion in the form of a diffusion equation:

$$\left(\frac{\partial X_i}{\partial t}\right)_{\text{conv, dif}} = \frac{\partial}{\partial m}\left(\sigma\frac{\partial X_i}{\partial m}\right)\,. \tag{4.8}$$

4.1 Numerical Procedure in the TWIN Code

In these equation a number of functions (ϵ, $\epsilon_n u$, κ, c_P, r_{ij}, ∇_{ad}, δ) appear, which together with the equation of state for $\rho(P, T, X_i)$ all depend on the pressure P, temperature T and the set of mass fraction functions $X_i(m, t)$. Furthermore there is $4 + n$ variables (l, r, P, T and X_i for $i = 1, \ldots n$) which depend on the coordinates m and t and need to be solved for in the regions $0 \leq m \leq M$ and $t > 0$. This is possible, if suited boundary conditions and initial values are provided, as the above set of equations gives also $4 + n$ equations.

Time Scales and Boundary Conditions

Specifically for functions of which time derivatives explicitly appear in the set of equations, initial values need to be provided ($s(m, 0)$, $X_i(m, 0)$, $r(m, o)$ and $\dot{r}(m, 0)$). Each term including a time derivative is associated with a time scale as was discussed before. Those are τ_{hyd}

for (4.2) for the adjustment of hydrostatic equilibrium, τ_{KH} for (4.3) for thermal adjustmend and τ_{nuc} for (4.5) driven by composition changes due to nuclear fusion. If the evolution of the star is slow compared to τ_{hyd}, then the inertia term in (4.2) can be neglected, which holds, if the evolution is driven by either nuclear reaction or thermal adjustment: $\tau_{\text{KH}} \gg \tau_{\text{hyd}}$ or $\tau_{\text{nuc}} \gg \tau_{\text{hyd}}$. Then the star is in hydrostatic equilibrium, which typically is a good approximation for stellar evolution and thus initial values for r and $\dot{r}$ are not required. Hydrostatic equilibrium is also a condition for (4.4) to hold. If further $\tau_{\text{nuc}} \gg \tau_{\text{hyd}}$ also (4.3) simplifies, as the first order time derivative can be neglected. This situation is called *complete equilibrium* and initial values for the composition only are sufficient. For the quantities involving spatial derivatives, boundary conditions are required for $m = 0$ and $m = M$. As $l(0) \neq 0$ is unphysical, boundary conditions at the center are found as

$$r(m=0) = 0\,, \quad l(m=0) = 0\,, \quad P(m=0) = P_{\text{c}}\,, T(m=0) = T_{\text{c}}\,. \tag{4.9}$$

To find the boundary conditions at the surface ($m = M$) for the pressure P and temperature T it needs to be clarified, what is meant by the term surface:

$$r(m=M) = R\,, \quad l(m=M) = L\,, \quad P(m=M) = P_{\text{s}}\,, \quad T(m=M) = T_{\text{s}} \tag{4.10}$$

Phenomenologically the surface is the region from which the light as observed from outside originates. This region is called *photosphere* and can be defined reasonably using the optical depth τ

$$\tau(r) := \int_r^\infty \kappa\rho d\tilde{r} \tag{4.11}$$

by requiring, that $\tau(R) = 2/3$. Spectral features of the stellar regions above the photosphere (*stellar atmosphere*) are neglected in the simulation of stellar evolution and the resulting spectrum is instead treated as black body radiation and linked to the luminosity by the Boltzmann law for a sphere with radius R as:

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4\,, \tag{4.12}$$

$$T_{\text{eff}}(\tau = 2/3) = T(\tau = 2/3)\,. \tag{4.13}$$

For the stellar atmosphere the *grey atmosphere* approximation is used. That is the opacity is considered constant and frequency independent above $r = R$: $\kappa = \langle\kappa\rangle$. Neglecting also the change in gravitational acceleration in this region, $g = g(R)$, one finds:

$$\tau(R) = \int_R^\infty \kappa\rho d\tilde{r} \approx \langle\kappa\rangle \int_R^\infty \rho d\tilde{r} = \frac{2}{3}\,, \tag{4.14}$$

$$P_{\text{s}} = \int_R^\infty g\rho d\tilde{r} \approx \frac{GM}{R^2} \int_R^\infty \rho d\tilde{r} \tag{4.15}$$

$$\Leftrightarrow \quad P_{\text{s}} = \frac{GM}{R^2} \frac{2}{3\langle\kappa\rangle}\,. \tag{4.16}$$

Henyey Method

To solve the differential equations with boundary conditions on both sides, the so-called *Henyey method* is used, which is described in the following (see [Hen64] and [Sch99]). The problem is treated using a so-called *mesh*, dividing the region $m \in [0, M]$ into a fixed number of mass shells with interfaces at $m = m_j$ for $j = 1, \ldots, n_{\text{mesh}}$. Therefore the differential equations given above must be converted into *difference equations*, that is equations for the change in the physical quantities between neighboring mesh points depending on physical quantities at neighboring mesh points or averages of these. The mesh points are not fixed, but found using a mesh point function to spread the overall change of variables as equally as possible among the mesh points, to improve accuracy and numerical stability of the procedure. This approach is qualitatively described in [Egg71]. Therefore an additional variable q and one additional differential equation is introduced to the set of equations, effectively rendering the values of m_j variables. The problem is also treated in a time discretized way with a dynamic time step size Δt. The difference equations describing structure and composition change are then solved for iteratively. Furthermore, the mass loss is assumed to be small enough, to be neglected within a time step, so that M is a constant for each time step. The set of values for the variables at all mesh points is called a *stellar model* and the evolution, that is the series of sets for consecutive time points is called a *stellar model sequence*. The Henyey method is then a procedure to improve an initial guess for the model at time $t + \Delta t$ until some accuracy criteria for the overall solution of all variables are met. The initial guess is typically found by extrapolating the rate of change of variables from the previous time step. In most cases a sequence is started from the output of a previous simulation, such as the so-called *Zero Age Main Sequence* (ZAMS) models - that is the states of a star of specific mass and composition, when stable fusion in its core is established and it reached thermal equilibrium. To use such a procedure that is discretized in time steps and mesh points, all derivatives appearing in the basic equations of stellar structure must be converted into discrete forms. First this is presented in a general form with only one variable describing the composition, to introduce the principle of the Henyey method. Time derivatives in this discretized form are evaluated as:

$$\left(\frac{\partial P}{\partial t}\right)_j = \dot{P}_j = \frac{P_j - P_j^*}{\Delta t} , \tag{4.17}$$

and analogously for T, where P_j^* denotes the value for P at mesh point j from the previous time step. The differential equations (4.1) - (4.4) are rewritten as difference equation:

$$\frac{dy_i}{dm} = f_i(y_1, \ldots, y_4) , \qquad i = 1, \ldots, 4 , \tag{4.18}$$

$$\Rightarrow \frac{y_i^{j+1} - y_i^j}{m^{j+1} - m^j} = f_i^{j+1/2}(y_1^{j+1/2}, \ldots, y_4^{j+1/2}) , \qquad j = 1, \ldots, n_{\text{mesh}} - 1 , \tag{4.19}$$

$$= \frac{1}{2}\left(f_i^j(y_1^j, \ldots, y_4^j) + f_i^{j+1}(y_1^{j+1}, \ldots, y_4^{j+1})\right) , \tag{4.20}$$

where $y_1, \ldots, y_4$ are r, P, T, l and the arithmetic mean was used to evaluate $f_i^{j+1/2}$. These equations are then written in the form required for the *Newton-Raphson method*:

$$\frac{y_i^{j+1} - y_i^j}{m^{j+1} - m^j} - \frac{1}{2}\left(f_i^j(y_1^j, \ldots, y_4^j) + f_i^{j+1}(y_1^{j+1}, \ldots, y_4^{j+1})\right) = 0\,. \tag{4.21}$$

In the TWIN code an additional equation describing the position of the mesh points m_j is used. Therein, the process of convection couples the difference equations not just to the next, but also the previous mesh point, involving f_i^{j-1}, f_i^j, and f_i^{j+1} to describe composition changes. Functions h_i^j are defined to find approximate solutions to these equation based on an initial guess $y_i^{j,(0)}$ by the Newton-Raphson method. The equations to be solved are

$$h_i^j(y_k^{j-1}, y_k^j, y_k^{j-1}) = 0\,, \qquad i,\, k = 1, \ldots, n_{\text{var}},\ j = 2, \ldots, n_{\text{mesh}} - 1\,, \tag{4.22}$$

$$y_k^j = y_k^{j,(0)} + \Delta y_k^j\,, \tag{4.23}$$

where y_k^j refers to the set of dependent variables on the respective mesh point. In this example that is six variables: r^j, P^j, T^j, l^j, m^j, X^j, where X would represent the mass fraction of one species for simplicity. With $h_i^{j,(0)}$ the value of h_i^j for the initial guesses $y_k^{j,(0)}$, linearization leads to:

$$h_i^j(y_k^{j-1}, y_k^j, y_k^{j-1}) \approx h_i^{j,(0)} + \frac{\partial h_i^j}{\partial y_k^{j-1}}\Delta y_k^{j-1} + \frac{\partial h_i^j}{\partial y_k^j}\Delta y_k^j + \frac{\partial h_i^j}{\partial y_k^{j+1}}\Delta y_k^{j+1}\,, \tag{4.24}$$

$$=: h_i^{j,(0)} + A_{ik}^j \Delta y_k^{j-1} + B_{ik}^j \Delta y_k^j + C_{ik}^j \Delta y_k^{j+1}\,, \quad j = 2, \ldots, n_{\text{mesh}} - 1\,. \tag{4.25}$$

This defines a set of three $n_{\text{var}} \times n_{\text{var}}$ matrices A, B, C for each meshpoint $j = 2, \ldots, n_{\text{mesh}} - 1$. For the two remaining mesh points B^{c}, C^{c} for $j = 1$ and A^{s}, B^{s} are defined analogously to (4.25) by leaving out the there undefined expression for A^1 and $C^{n_{\text{mesh}}}$. B^{c}, C^{c} and A^{s}, B^{s} represent the boundary conditions (4.9) and (4.10) at the center and the surface for $j = 1$ and $j = n_{\text{mesh}}$ respectively. The entire set of coupled, discretized, linearized equations can then be expressed as

$$\vec{h}^{(1)} = \vec{h}^{(0)} + H \cdot \Delta\vec{y}\,, \tag{4.26}$$

where $\vec{h}^{(1)}$ is the vector of $(h_1^1, h_2^1, \ldots, h_{n_{\text{var}}}^1, h_1^2, \ldots h_{n_{\text{var}}}^{n_{\text{mesh}}})$ and analogously $\Delta\vec{y}$ is the vector of the corrections to the values of all dependent variables of all meshpoints. H is the so-called *Henyey matrix.* The Henyey matrix is a $n_{\text{var}} \cdot n_{\text{mesh}}$ dimensional square matrix, that has a block-wise diagonal form and can be solved by gaussian elimination or numerically:

$$H = \begin{bmatrix} B^c & C^c & 0 & \cdots & 0 \\ A^2 & B^2 & C^2 & \cdots & 0 \\ 0 & A^3 & B^3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & C^{n_{\text{mesh}}-1} \\ 0 & 0 & 0 & A^s & B^s \end{bmatrix}\,. \tag{4.27}$$

The corrections $\Delta\vec{y}$ found thereby are then applied to all dependent variables (including all m_j) and the procedure is repeated with the newly founed y_k^j as inital guesses. The method is applied iteratively, until a quality criterion is fullfilled, that typically is all $h_i^j < \epsilon_{\mathrm{a}}$ individually as well as $\sum h_i^j < \epsilon_{\mathrm{b}}$ overall, for some chosen values ϵ_{a}, ϵ_{b}.
In the version of the TWIN code used here, the equations are solved for the mass fractions of five species: X_{H}, X_{He}, X_{C}, X_{O}, X_{Ne} and a sixth, X_{N}, can optionally be added. To increase numerical stability, that is avoid an overestimation of the required change in the dependent variables, a parameter $\lambda < 1$ can be set, with which the corrections are multiplied, before they are applied. This favors a steady approximation to an accurate solution.

Mesh Point and Time Step Control

The adaptive choice of mesh points is an essential step to provide good accuracy with few mesh points, avoiding unnecessary computational effort and thereby reducing computational time for a simulation. Therefore the spacing of mesh points must follow the dynamics of effects driving the evolution of stellar structure. The approach is to identify the quantities that indicate regions, which drive the evolution and to then choose the mesh point spacing such, that changes in these quantities are equally distributed over the mesh. For a set of such quantities at the j-th mesh point Q_i^j the sum over the quadratic differences over all functions Q_i and the entire mesh,

$$\sum_{j=1}^{n_{\mathrm{mesh}}} \sum_i \left(Q_i^{j+1} - Q_i^j\right)^2 , \tag{4.28}$$

is thus to be minimized in order to find a optimal mesh. In order to minimize this double sum, a quantity $q_k = (k-1)/(n_{\mathrm{mesh}} - 1)$ is introduced, which splits the mass range into equidistant intervals. The sum is then approximated by an integral

$$\sum_{j=1}^{n_{\mathrm{mesh}}-1} \sum_i \left(\frac{\Delta Q_i^j}{\Delta m_j}\Delta m_j\right)^2 \approx \int_0^M \sum_i \left(\frac{dQ_i}{dm}\frac{dm}{dq}\right)^2 \left(\frac{dm}{dq}\right)^{-1} dm = \int_0^M \sum_i \left(\frac{dQ_i}{dm}\right)^2 \frac{dm}{dq} dm \tag{4.29}$$

and minimized using the lagrange formalism leading to

$$\frac{dq}{dm} = \Phi \cdot \left[\sum_i \left(\frac{dQ_i}{dm}\right)^2\right]^{1/2} , \quad \frac{d\Phi}{dm} = 0\,. \tag{4.30}$$

Reasonable functions Q_i are $\lg P$ to account for changes of structure throughout the outer regions of a star, l for changes in released energy in central regions and m in an inert core, where other physical quantities are approximately constant.[Egg71] The function $q(m)$ serves to rescale the mass shells according to their importance for the evolution of the star as specified by the coeffiecients. As described in [Egg71], this approach is computationally inefficient and instead all important influences for the evolution of the star can be gathered in one function. The mesh points are then evenly distributed according to this function. This is then equivalent to solving one additional differential equation along with the stellar structure functions, which describes the change in mass coordinate for the different mesh points:

$$\begin{aligned} Q := Q^1 =& c_4 \lg P + c_5 \lg(P + c_9) + c_7 \lg T - c_7 \lg(T + c_{10}) - c_3 \lg(1 + r^2/c_8) \\ & - \lg(1 + (c_6 m^{1/3})^{-1}) \, , && (4.31) \\ \frac{dq}{dm} =& C \left(\frac{dQ}{dm} + c_2 \frac{3l}{2} \right) \, , && (4.32) \end{aligned}$$

where all variables are considered to be given in dimensionless code units. C is a scaling constant to ensure, that $q \in [0, 1]$ with $q_k = (k-1)/(n_{\mathrm{mesh}} - 1)$ describes the mass range $m \in [0, M]$.

The time step control is implemented by summing the absolute values of the changes of a number of variables over the full mesh and comparing the value to a user-specified target value. The time step size is scaled with the ratio of this sum to the user-specified value, while also a maximum and a minimum value for the relative change is provided. Typical variables to be included in this sum are $\lg T$, $\lg r$, X_{H}, X_{O}. Additionally there is user-specified factors with which the target value is multiplied during specific stages of stellar evolution.

Difference Equations

As explained above linearization is used in the Henyey method. Therefore a choice of dependent variables is favorable, in which quantities change approximately linearly in critical regions. Due to the radius r appearing in the denominator in multiple expressions, the central region is of special interest regarding numerical stability. A reasonable choice is $m^{2/3}$ and r^2 instead of m and r as is explained in [Egg03]. Temperature T and pressure P are expressed as $\lg T$ and $\lg P$. With these choices the difference equations as required for the Henyey method can be described in the following. Three shorthands

$$\begin{aligned} m' &= \frac{dm}{dk} = C \left(\frac{dQ}{dm} \right)^{-1} \, , && (4.33) \\ \sigma &= \frac{(4\pi r^2 \rho)^2}{m'} (vl)_{\mathrm{MLT}} k_{\mathrm{OS}} \, , && (4.34) \\ \dot{f} &= \frac{df}{dt} && \text{non lagrangian, const. mesh point} \, , \quad (4.35) \\ k_{\mathrm{OS}} &\sim (\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}} + \nabla_{\mathrm{OS}})^2 && \text{if } (\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}} + \nabla_{\mathrm{OS}}) > 0 \, , \quad (4.36) \\ &= 0 && \text{otherwise} \, , \quad (4.37) \end{aligned}$$

are introduced, where ∇_{OS} is a so-called *overshooting parameter* to resolve inconsistencies if a convection zone forms outside of a radiative core region as explained and specified in [Egg72] and [Egg03]. For the change of composition the following equation is used, which is derived in [Egg72]:

$$\sigma_{k+1/2}\left(X_{i,k+1}-X_{i,k}\right)-\sigma_{k-1/2}\left(X_{i,k}-X_{i,k-1}\right)=\left(\dot{X}_{i,k}+R_{\mathrm{nuc},k}\right)m'_k-$$
$$\left(X_{i,k+1}-X_{i,k}\right)[\dot{m}_k]-\left(X_{i,k}-X_{i,k+1}\right)[\dot{m}_{k+1}]\,,\quad i=1,\ldots,5\,. \tag{4.38}$$

Here $R_{\mathrm{nuc},k}$ is the absolute rate of nuclear reactions changing the number of particles of species i in the mesh interval. The term on the left-hand side is the contribution of diffusion and convection as in (4.8) and the right-hand side describes composition change due to nuclear fusion and a shift in mesh point interval as in (2.105) using $[f]$ as defined in (2.106). The remaining difference equations can be derived directly and are given in the appendix. In total there thus is a set of $10\cdot n_{\mathrm{mesh}}$ variables that need to be determined X_i, r^2, $m^{2/3}$, T, P, l for $i=1,\ldots 5$ and a set of $5(n_{\mathrm{mesh}}-2)$ composition change equations (4.38), $5(n_{\mathrm{mesh}}-1)$ difference equations (A.22) and (A.19)-(A.24). Additionally there are 10 equations for the next-to center and next-to surface composition change[Egg03]:

$$\sigma_{k\pm1/2}\left(X_{i,k}-X_{i,k\pm1}\right)=\left(\dot{X}_{i,k}+R_{\mathrm{nuc},k}\right)\left(m_k-m_{k\pm1}\right), \tag{4.39}$$

as well as five boundary conditions:

$$l_{\mathrm{c}}=0,\quad m_{\mathrm{c}}=0,\quad r_{\mathrm{c}}=r_{\mathrm{min}},\quad l_{\mathrm{s}}=L=4\pi r_{\mathrm{s}}^2T_{\mathrm{s}}^4,\quad P_{\mathrm{s}}=\frac{GM}{R^2}\frac{2}{3\langle\kappa\rangle}\,. \tag{4.40}$$

Here subscript c refers to the innermost and s to outermost mesh point, while r_{min} is a small parameter to avoid divergence at the center and (4.13) was used for the surface boundary condition. The set of difference equations and boundary conditions fits the number of overall variables to be determined.

The overall simulation then follows this pattern:

- Either an input model is read or an initial model is generated using a polytrope.
- C, M and Δt are calculated for the next time step.
- Given the input provided for κ, ϵ, ϵ_ν, etc. the set of difference equations is iteratively solved by the Henyey method.
- Stop conditions are evaluated and if not met, the previous two steps are repeated.

4.2 Window to the Stars (WTTS)

The TWIN code is operated using command line input, configuration files and the different inputs described in the section above, usually including a full stellar model to start from, to generate a final stellar model and optionally intermediate models or a full model sequence as output. This is convenient for scientists, as it gives full control and access to both input and output, especially allowing for manual manipulation of the stellar model. It is unfortunately not a convenient solution for outreach though, as command line inputs are often perceived as deterrent and the use of external software for visualization purposes adds further complication. Thus the lack of direct access to the output to gain insights into stellar structure and evolution may be considered very hinderous. This problem is addressed by the WTTS software, which combines a graphic user interface to initialize simulations using the TWIN code and brief descriptions of all options with a variety of tools to directly visualize the output.[Izz06a][Izz06b]

Layout

WTTS consists of a series of tabs, which are typically used in order from left to right to conduct and analyse a simulation. The status of the current simulation is always shown at the bottom. The *Options* tab (see fig. 11) allows to adjust the parameters given to the simulation code. That is the stellar masses for one or two stars, their shared metallicity, basic input regarding the mode of operation (single or binary star, length of simulation, etc.) and a list of the various options and parameters grouped into categories. The arrangement allows direct access to key parameters such as the masses without the need to explicitly go through the more advanced options, which are in general less relevant for application in outreach. In the *Evolve* tab the simulation can be started or terminated and a reduced output is shown during the simulation process (see fig. 12). Then a tab follows, that shows the *Hertzsprung-Russell diagram* (HR diagram), which is a double logarithmic plot of luminosity relative to the solar luminosity $\lg L/L_\odot$ against surface temperature $\lg T_\mathrm{eff}/[\mathrm{K}]$ with inverted x-axis, see fig. 13). Next, a double logarithmic plot of the central temperature $\lg T_\mathrm{c}/[\mathrm{K}]$ against the central density $\lg \rho/[\mathrm{g\ cm}^3]$ (ρT diagram) is given ($\rho - T$ tab, see fig. 14). Both are used to visualise basic features of the stars, regarding external observables (luminosity and temperature, HR diagram) and internal quantities which determine the available fusion reaction in the center (central temperature and density, ρT diagram). Both tabs offer a variety of options to illustrate the changes in these quantities, like labeling with model number, age etc. Furthermore the software has a *Structure* tab (see fig. 15) to plot a variety of internal and external quantities against each other and an *Internals* tab to generate plots of internal quantities for a number of models (e.g. temperature and density against mass coordinate for every 500th model, see fig. 16), either as an animation or in one combined plot. An outstanding feature is the generalized Kippenhahn plot, which can be generated in the *Kippenhahn* tab. This feature allows to show any internal quantity as a false colour plot for two quantities, such as the helium mass fraction as false colour plot for the radial coordinate against the stellar age. Thereby it is possible to visualize the temporal and spatial evolution of an internal quantity simultaneously for the entire star over the entire model sequence (see fig. 17). The last three tabs are of no importance for the use of WTTS here. Using the *Layers* option, multiple simulations can be run from one instance of WTTS, combining their output in the HR and $\rho - T$ tabs and giving access to individual output in the other tabs.

4.3 Insights into Stellar Structure and Evolutions using WTTS

To demonstrate the use and potential of the TWIN code and WTTS various simulations are conducted. This in particular will be a series of shorter simulations partly reproducing features of the HR diagram and demonstrating the flexibility of the TWIN code. So-called *ZAMS series* are generated to discuss the influence of metallicity and convection. Furthermore, a long model sequence of a sun-like star is presented, discussed and compared to the model sequence of a heavier star with $M = 8\,M_\odot$ and solar composition.

Series of Simulations

Stellar model sequences for a series of short simulations with solar metallicity $Z = 0.02$ and various stellar masses in the range 0.1 to 18 $M_\odot$ are shown in fig. 18 and 19. The sequences have 400 to 800 time steps, except for $M = 0.1\,M_\odot$ (the simulation stops at $t = 1.2 \times 10^{12}$ y) and were conducted with slightly altered constants for the mesh weighting function Q. Figure 18 resambles a HR diagram, with the difference, that evolution tracks for individual stars are shown instead of the momentary states for a population of stars. For reference the stellar ages are shown along those tracks. Several features can be identified:

- The position of the starting points follow a predominantly linear relation, with T_{eff} and L increasing with increasing stellar mass.
- Very low mass stars follow this trend during the entire sequence, reaching immense stellar ages.
- Above $0.5\,M_\odot$ a deviation from this trend to lower temperatures at later stages is seen.
- Above $1.7\,M_\odot$ the shape of this deviation changes, showing a significantly bigger change in T_{eff} over very short time spans followed by an increase in luminosity.

Above $10\,M_\odot$ the simulation does no longer maintain sufficiently long time steps to reach this phase of the evolution. To investigate the origin of the transitions between $0.5\,M_\odot$ and $1.7\,M_\odot$ the evolution of the central density and temperature is shown in fig. 19. As nuclear fusion first occurs in the hot, dense center of stars, these central quantities allow insight into their energy production. Zones for conditions compatible with hydrogen and helium fusion are shown. Additional findings can be made:

- Very low mass stars start with only deuterium fusion available.
- Heavier stars in general have a higher core temperature but lower core density.
- While the starting points for low mass and high mass stars appear to lie on a linear line, there again is a clear transition between $0.5\,M_\odot$ and $1.8\,M_\odot$.
- Sequences for stars with $0.5\,M_\odot < M \leq 1\,M_\odot$ show two almost linear phases, both with constantly increasing core temperature and density, the first phase being significantly longer in time.
- Sequences for stars with $1\,M_\odot < M \leq 10\,M_\odot$ also show a constant increase in core density with two similar, almost linear phases. The linear phases are seperated by a characteristic non linear phase with a decrease in core temperature.

With these findings, the general feature of HR diagrams being densely populated around the *main sequence* (MS) and sparsely populated at later stages can be explained, as the time span spent on the MS dominates the evolution of all stars shown. The general trend for decreasing core density with increasing mass can be explained with the high temperature dependence of nuclear fusion, described in section 3, leading to a strong increase of radiative pressure and an expansion of the core region with increasing mass. While luminosity and radius of a star are expected to increase with increasing mass, from the linear increase in the HR diagram with a slope of approximately 7 it can be estimated that $RT_{\text{eff}}^2 \sim T_{\text{eff}}^{3.5}$ as $L \sim (RT_{\text{eff}}^2)^2$, so that $R \sim T_{\text{eff}}^{3/2}$. The clear divergence of starting points in the ρT diagram from a linear relation in the transition zone mentioned above indicates, that the qualitative change in evolutionary tracks is driven by properties of the central region. The zone for nuclear hydrogen burning shown coincides with the onset of the transition zone. These considerations indicate a qualitative difference for the mode of nuclear fusion and the evolution of central conditions of low mass stars compared to higher mass stars. As such changes are not only associated with nuclear fusion directly, but also with possible mixing of stellar material in the presence of convection, the mode of nuclear fusion and heat transport will be investigated further. Therefore ZAMS sequences will be simulated in the following.

Change of Convective Regions

The TWIN code allows to artificially add mass to the surface of a star with a specified rate while bringing the models into hydrostatic equilibrium. It additionally allows to prohibit changes of time step size and of composition due to nuclear reactions, while still simulating the energy release by fusion. Therefore it is possible to effectively generate ZAMS runs, in which the stellar age corresponds to a change in mass rather than of evolutionary stage,

while the star is kept in hydrostatic equilibrium and at constant composition, that is on the main sequence. Three such stellar model sequences were simulated for masses increasing exponentially from 0.1 to 100 $M_\odot$ (that is constant increase of $\lg M$ with model number) for the metallicities $Z = 10^{-2}$, 10^{-3}, 10^{-5}. The evolutionary tracks are shown in fig. 20 and 21 and confirm the trends observed in the series of model sequences described above. The approximately linear increase of luminosity in the HR diagram with deviations especially for lower masses and around the transition zone is observed again. The transition mass range in the latter diagram is clearly seen for all three metallicities. For an increase by 9 over a span of 1.4 on the x-axis in the HR diagram, $R \sim T_{\text{eff}}^{1.2}$ can be estimated. More precise estimates for the dependencies of the observable quantities R, T_{eff} and L on each other and on M can be obtained from such ZAMS runs and are studied in detail in literature. These results can be compared to theoretical considerations and also measurements, to verify the precision of numerical simulation codes.[Kip12][Sal05]
The following observations can be made from the ZAMS runs presented here:

- Stars with lower metallicity have a higher core density.
- For high masses they additionally show a significantly higher core temperature.
- The hydrogen nuclear burning zone only coincides with the onset of the transition zone for $Z = 10^{-2}$.
- The onset of the transition zone occurs at very similar masses for $Z = 10^{-2}$ and $Z = 10^{-3}$.
- The onset of the second transition starts at very similar masses for $Z = 10^{-3}$ and $Z = 10^{-5}$.

The significantly lower core temperature required to balance high mass stars with higher metallicity hints, that the origin of this finding is connected with hydrogen burning via the CNO cycle, which is more efficient in presence of higher mass fractions for the heavy, catalytic elements involved. Therefore, the fraction $\epsilon_{\text{CNO}}/\epsilon_{\text{ges}}$ is shown in fig. 25 for $Z = 10^{-3}$ and additionally the Schwarzschild stability criterion is evaluated and shown in fig. 22 ($Z = 10^{-2}$), 23 ($Z = 10^{-3}$) and 24 ($Z = 10^{-5}$). One finds:

- A clear change from fully convective to mostly radiative stars around $M = 10^{-0.5}\, M_\odot = 0.3\, M_\odot$.
- This change occurs at almost identical mass for $Z = 10^{-2}$, $Z = 10^{-3}$ and a noticeably higher mass for $Z = 10^{-5}$.
- The formation of a small convective core around 1 $M_\odot$ for $Z = 10^{-2}$, while for the other metallicities a small convective region persists.

- The CNO cycle contributes noticeably above $M = 10^{0.2}\,M_\odot \approx 1.6\,M_\odot$ and dominates above $M = 10^{0.3}\,M_\odot \approx 2\,M_\odot$.

Combining these findings with the observations from the ρT diagram the simulations suggest the following interpretation:

- The first, steep transition coincides with a change from fully convective to radiation dominated energy transport.
- The onset of the second transition seems to be associated with a persistence/emergence of a central convective core.
- The significant change occuring during the second transition is due to a change in mode of hydrogen burning from pp chain to the CNO cycle.

So far only the relations and properties for the ZAMS, that is on the MS were investigated. To gain insights into the qualitative changes in evolutionary tracks in the HR and ρT diagram, stellar model sequences for two individual stars are analysed in the following.

Stellar Model of the Sun

First, a model sequence for a star with $1\,M_\odot$ and solar metallicity is simulated. The external observables are shown in a HR diagram in fig. 26 and a logarithmic plot of $\lg R/R_\odot$, $\lg L/L_\odot$, $\lg T_{\text{eff}}/[\text{K}]$ and $\lg T_{\text{c}}/[\text{K}]$ against t/y (RLT diagram) in fig. 28. Additionally a ρT diagram is shown in fig. 27. During the evolution a number of distinctly different phases can be identified, their transition points being indicated with labels A to E:

A ($\approx 7 \times 10^9$ y): Increase of T_{eff} stops (HR diagram). Distinct change in slope (ρT diagram);

B ($\approx 11 \times 10^9$ y): L begins to increase (HR diagram). L begins to increase, increase of T_{c} and decrease of T_{eff} slow down (RLT diagram).

C ($\approx 11.75 \times 10^9$ y): The steep increase of R and L slows down significantly, similarly the increase of T_{c} and decrease of T_{eff} slow down noticeably (RLT diagram).

D ($\approx 11.85 \times 10^9$ y): Increase of L and decrease of T_{eff} stop (HR diagram). L and R have a sharp maximum, T_{eff} a sharp minimum and the steep increase of T_{c} pauses (RLT diagram).

E ($\approx 11.95 \times 10^9$ y): Increase of T_{eff} stops (HR diagram).

To understand the origin of these features certain interior quantities are analysed. The change of helium mass fraction and energy release rate with radius and time is shown in fig. 29 and 30. With this additional information the evolutionary stages listed above can be explained as follows:
Initially the sun-like star fuses hydrogen to helium in its core. The composition change

leads to an increasing core temperature, core density and luminosity. At point A, a small almost pure helium core forms and hydrogen fusion preceeds in a shell around this core. The shell produces additional helium and the helium core mass increases, while the hydrogen burning shell gradually moves towards increasing radii and closer to the surface. This causes the surface to expand and thereby cool. At point B the helium core begins to contract significantly, releasing gravitational energy providing additional heat to the hydrogen shell, which due to its increasing surface area emitts an increasing luminosity and can not maintain sufficient temperature otherwise. This further expands and thereby cools the stellar envelope - the star is on the *Red Giant Branch* (RGB). In fig. 30 between points B and C above $r = 10^{-0.5}\,R_\odot$ a discontinuity of helium mass fraction is seen, which is the signiture of a convective region extending deeper into the star. This region extends almost down to the hydrogen burning shell before point C, which is the *first dredge-up*. When this discontinuity reaches the hydrogen burning shell, it reduces the nuclear fusion efficiency, which can be seen in fig. 28. Furthermore material from the central region of the star is transported to the surface, where it is potentially detectable in the spectrum features of the stars surface. Additionally the star undergoes a substantial mass loss by stellar wind due to the immense radius and immense luminosity, which in the model sequence reaches a total of almost $0.25\,M_\odot$. The core reaches densities at which it becomes dominated by degeneracy, leading to the so-called helium flash at point D. The degenerate core initially has a temperature inversion due to neutrino losses cooling its center, which can form as the pressure decoupled from the temperature in cores dominated by degeneracy. When the hydrogen shell burning can no longer support the pressure from the outer layers, this leads to a further compression of the helium core which finally reaches sufficient conditions for the triple alpha process. These are first reached in the outer region of the helium core due to the temperature inversion and the release of heat due to the helium fusion causes a thermonuclear runaway. Significant mass fractions of the helium core undergo nuclear fusion on a time scale of seconds. This process can not be modelled by the TWIN code directly, but its occurance is detected and the simulation is then continued with a scaled, preconstructed so-called *Zero Age Horizontal Branch* (ZAHB) model. In this phase the star undergoes helium fusion in its core and hydrogen fusion in a thin shell as can be seen in fig. 30. The star develops on the ZAHB with a rapidly increasing surface temperature and reaches point E, where a loop in the HR diagram starts to form. This loop indicates a rather unstable evolutionary phase, which gives rise to pulsation effects and is associated with the phenomen of Cepheid variables (see [Vas93]). Here the time step size reaches such small values that the evolution terminates.

Stellar Model $M = 8\,M\odot$

To understand the characteristically different evolution tracks of higher mass stars observed in the HR diagrams (fig. 18), a stellar model sequence for a $M = 8\,M_\odot$ star with stellar metallicity is simulated. The corresponding HR, ρT and RLT diagrams are shown in fig. 31, 32 and 33. The characteristic changes can be described as:

A ($\approx 3.05 \times 10^7$ y): T_{eff} begins to increase (HR diagram).

B ($\approx 3.13 \times 10^7$ y): T_{eff} begins to decrease again, after a loop is seen (HR diagram).

C ($\approx 3.18 \times 10^7$ y): L has a narrow minimum (HR and RLT diagram) and R increases rapidly (RLT diagram).

To investigate these observations, the change of helium mass fraction, energy release rate ϵ and convective regions with time and radius are shown in fig. 34, 35 and 36. As seen in the ZAMS run already stars of this mass do have an extended convective core, which mixes the stellar material in the central region and thus the formation of a small, almost pure helium core is prohibitted. This leads to a hydrogen depleted, extended region forming as a consequence of the very efficient hydrogen fusion by the CNO cycle. Unlike for the sun-like star, there is no gradual transition to hydrogen shell burning, but the entire central region becomes so hydrogen depleted, that it finally contracts significantly at point A, while a thin hydrogen burning shell forms (see fig. 35 between point A and B). The convection in the core stops as the hydrogen fusion stops and the almost pure helium core contracts further, while the stellar envelops expands and cools. Finally at point C the core reaches sufficient temperatures for helium fusion to ignite without reaching densities sufficient for a dominant degeneracy. Thus no temperature inversion occurs and no helium flash is observed. The core region becomes convective again and stable helium core fusion and hydrogen shell burning establishes before the evolution terminates due to a decrease in time step size.
The characterstic difference in the evolutionary tracks described above thus is the continuous transition from core to shell burning in stars around $1\,M_\odot$, which can not occur in significantly heavier stars due to their convective core region. In the latter, hydrogen shell burning is established in a discontinuous way at a distance from the center when a significant, almost pure helium core already formed. This discontinuity gives rise to the characteristic hook-like feature in the HR diagram.

Dating of Stellar Clusters

As outlined in the very beginning, one possible application of stellar evolution simulations is the dating of stellar clusters. Therefore the so-called *turn-off* point is determined. That is the point, at which the stars of the population are located in the HR diagram, which just left the MS. As the position on the MS is linked to the mass and thereby to the time span before leaving the MS, this turn-off point is associated with the age of those stars - and thereby the stellar cluster. To illustrate this process, the HR diagrams for two stellar clusters are shown in fig. 37 in which the turn-off point is indicated by lines. As the evolution tracks when leaving the MS are clearly non-linear, this method can only give first estimates. To find more accurate estimates, so-called *isochrones* are determined by simulation. That is the

track in the HR diagram for stars at a specific age with varying mass. Isochrones depend on stellar metallicity, so that a stellar populations HR diagram can be compared to these isochrones, if its metallicity is known.[Sal05]
To outline the procedure, the turn-off points indicated in fig. 37 are compared to fig. 18:

- M3: $T_{\text{off}} \approx 10^{3.8}\,\text{K}$, $L_{\text{off}} \approx 10^{0.4}\,L_{\odot}$
- Hyades: $T_{\text{off}} \approx 10^{4}\,\text{K}$, $L_{\text{off}} \approx 10^{1.2}\,L_{\odot}$

By comparison to the tracks shown in fig. 18, the time span for the stellar model sequence closest in the HR diagram can be given as estimated age:

- M3: $M_{\text{off}} \approx 1\,M_{\odot} \rightarrow t_{\text{M3}} \approx 8 \times 10^{9}\,\text{y}$
- Hyades: $M_{\text{off,low}} \approx 1.77\,M_{\odot} \rightarrow t_{\text{M3,max}} \approx 1.2 \times 10^{9}\,\text{y}$
- $M_{\text{off,high}} \approx 2.37\,M_{\odot} \rightarrow t_{\text{M3,min}} \approx 0.59 \times 10^{9}\,\text{y}$

As neither isochrones, nor stellar model sequences with reasonable mass resolution were determined in this thesis, these values are only given to illustrate the method and no uncertainties for the turn-off point, corresponding mass or age are given. In the literature estimates can be found as $t_{\text{M3}} = (11.63 \pm 0.07) \times 10^{9}\,\text{y}$[Gon24] and $t_{\text{Hyades}} = (6.25 \pm 0.50) \times 10^{8}\,\text{y}$[Per98], showing an agreement in the order of magnitude with the estimates provided above.

5 Workshop: Can Intelligent Life emerge in Distant Solar Systems?

As outlined in the introduction the goal of this thesis is to make modern scientific methods accessible for outreach and science communication in a way that is suited to amaze teenagers and draw their interest to the fascination physics, and i.e. astrophysics offers. This is demonstrated for the numerical simulation of stellar structure and its evolution using the graphic user interface and analyzation software WTTS to access the TWIN stellar evolution code as part of a workshop for teenagers. To enhance interest and motivation for this topic I chose to place it in the context of emergence of extraterrestrial life, which is currently very prominent in popular media (*'Interstellar'*, *'Three body problem'*, *'Infiltration'*, . . .) and news and gaining increasing attention. This makes the workshop on one hand interdisciplinary, broadening the directly addressed audience by relating to physics, biology, as well as mass media and public interest, but on the other hand also adds to the challenges by increasing complexity and amount of topics to be covered. Therefore I designed a workshop concept to present stellar structure and evolution in the context of the specific question 'Which stars are potentially relevant for the emergence of intelligent life?'. Thereby the fascination for astrophysics and stars in particular as described in section 1 is connected with the open question of emergence of life, current developments in space travel technology (as i.e. driven by *SpaceX*) and also the search for habitable planets recently driven by the *James Webb Space Telescope* (JWST). The workshop is designed to be interactive and to showcase the scientific method of numerical simulation as is omnipresent in modern science and especially physics. The concept was prototyped with a group of 8 highschool students (Q2, last year of high school) from a local Gymnasium in central Münster, Germany, on the 29th of August 2024. The time schedule of the workshop is outlined in tab. 1. A reduced version of the workshop was tested at besaid school with two 9th grade classes on 9th of October 2024. The different parts of the workshop will be outlined in the following. The impressions and feedback are discussed at the end of this section. To cover and present the concepts in physics and biology needed to address the underlying question stated above, a collection of material was developed, which is included in the appendix. As the workshop was given in german language, so is this material and the feedback received by the participants.

5.1 Introduction Talks

The basics of the topics 'Emergence of Life' and 'Stars' are given in form of two input sessions supported by slides. These sessions are kept as interactive and as phenomenologically oriented as possible, reducing theoretical and technical details to a minimum. The two topics are seperated by a group work phase, giving handouts to the participants, in which examples for life forms of different evolutionary stages are briefly presented. This change in method

Part	Duration [min]	Content
Introduction	5	-
Phenomen Life	10	Presentation regarding life and its emergence
Group work	10	Different life forms and evolutionary stages
Discussion	10	Possibility of life on Mars, Titan, Venus
Stars	10	Presentation formation of and processes in stars
Break	10	-
Experiments	25	Two experiments: convection and convective currents
WTTS	20	Explain WTTS, workflow and set up the software
Simulation	20	Conducting simulations in groups of two
Lifetime	10	Combine simulation results to find masses of stars compatible with the emergence of ‘intelligent’ life
Conclusion	10	Sum up the workshop, questions
Total	140	

Table 1: Time schedule for the different phases of the workshop, giving their duration and a brief description of the content.

is chosen to keep the attention of the participants and the engagement high. The handouts prepare the participants for a short, open discussion with the entire group regarding the possibility for emergence of extraterrestrial, in particular intelligent life, which follows before the second presentation regarding stars is given.

Emergence and Persistence of Life as we know it

The difficulty to properly define the term life/living is outlined. It is motivated to work with a minimum set of physical requirements for emergence and persistence of life, which is found to be:

- energy uptake,
- (conditions compatible with) liquid water,
- material to generate/grow cells.

A schematic representation of characteristic processes and interactions with an environment is presented as shown in fig. 38 to give a qualitative understanding of the term life. Additionally the course of evolution on Earth is sketched as shown in fig. 39 including the time scale for the formation of the solar system and Earth to illustrate both, the course of evolution, but especially the time scales needed for the emergence of the different stages of complexity in life. A qualitative definition of the term *intelligent life* is given, as species that

- do not act predominantly based on instincts,

- show some level of problem solving abilities: 'try and error' or appearingly planned.

It is important to pointed out, that a unique definition of the term 'intelligence' is again far from trivial. In terms of evolutionary time scales the vague definition given here is sufficient though, to clearly distinguish emergence of intelligent from simple life. The participants are then divided into groups of two and given handouts, that present three life forms. The specific life forms differ between the groups, but contain one extremophile, one multicellular, primitive organism and a complex, higher form of life each (see appendix). Required conditions for the survival of the organisms and typical behaviour is sketched, as well as an estimate for when they emerged, if known. This serves the purpose of putting the before shown time scales into a more specific, relateable context to enable the participants to apply this rather complex, abstract information to open questions in the next part. The examples are additionally chosen in such a way, that at least one group gains information regarding an extremophile, that is able to survive certain challenging conditions, which will be found in the extraterrestrial worlds presented to them next (extreme cold, heat, acidity and radiation). The groups are asked to briefly discuss the examples in the context of the upcoming considerations regarding extraterrestrial life.
Next, three examples for such worlds are presented, providing relevant information regarding conditions, mostly, but not exclusively referring to criteria introduced before, such as temperatures, chemical composition and water content in atmosphere, surface and under ground (see fig. 40, 41 and 42). If relevant, changes regarding those conditions as indicated by climate models are mentioned. The information presented is sketched in the appendix. After each of those examples is presented, the participants are asked to openly give their ideas whether, and if where and what kind of life forms they think could have emerged and persisted. The worlds chosen are Mars, Titan, Venus in this order, ranging from rather mild conditions on Mars to very hostile conditions on Venus. The discussions are to be held interactive among the particitants and open regarding speculative ideas, especially as these questions are still debated in science, too. This part serves the purpose to summarize the new information regarding emergence and persistence of life and to apply it to a specific scientific question in a creative, interactive way.

Stars

Next the basics of what a star is and what kind of processes are relevant for its formation and properties are introduced in a second input session. Again a formal, theoretical definition of the phenomen is avoided and instead a simulation of the collapse of a molecular cloud to protostars is shown. Hereby the process of self-gravity as driving force of star formation is presented and basic characteristics are outlined. The heat generated by gravitational collapse is qualitatively explained and a simplified representation of nuclear fusion in hot, dense hydrogen is given. Therefore high temperatures are associated with rapid movement of nuclei, to motivate the possibility of overcoming repulsive interaction until nuclear, very short range forces dominate and result in fusion and energy release (see fig. 43). For this explanation the fusion of deuterium to alpha particles is presented, to avoid the complications of weak interaction in the pp chain and complex nuclear network in case of the CNO cycle. An extension to include these processes is of course possible and should be considered for groups of participants with a strong background and/or interest in physics. Different modes of heat transfer by conduction, convection and radiation are introduced qualitatively with examples from everyday life and a star is characterized as a self-regulating system in which the gas and radiative pressure counteracts the inwards gravitational pull. The sensitivity of nuclear fusion for temperature and density is identified to balance inwards and outwards forces. The emission of radiation in the photosphere giving the luminosity as rate of total energy release and the concept of habitable zones are explained (see fig. 44). The relevance of atmospheres, as was obvious in the conditions of the extraterrestrial worlds is pointed out to connect the presented concepts and information to the earlier parts of the workshop.

5.2 Experiments

The influences leading to structure formation inside stars as well as heat transport are then presented in an interactive and direct way. Therefore one experiment will be shown to the group and a second experiment will be conducted by the participants themselves. The break indicated in tab. 1 is used to prepare the materials used in the experiments.

Structure Formation due to Convection

One of the challenges for introducing the participants to the phenomenology of stars is, that thermodynamics often is not covered by physics courses in school. So a qualitative understanding of the coupling between density, pressure and temperature should be motivated. As derived in section 2 stars are driven by the interplay of gravity and pressure, the latter being determined by radiation and thermodynamic pressure of the gas. The coupling between gravitational and thermodynamical influences is essentially realized through the physical quantity of density. So for an understanding of the formation of a collapsed gaseous sphere with a certain density and temperature profile in the sense of a protostar, before onset of nuclear fusion this coupling needs to be shown at least qualitatively. Furthermore also on smaller scales the very same interplay of gravity and thermodynamics leads to the heat transport mechanism of convection through the dependence of density on temperature in a gaseous system. So first an experiment is presented, which puts the importance of density differences for structure formation by convection into contrast with diffusion driving temperature and material exchange. Therefore a thin container of transparent acrylic glas with a removable separation in the middle is filled with dyed hot tap water (about 50 °C, red) on one side and dyed room temperatured tap water (blue) on the other, as shown in fig. 47 (top).

After removing the separation, both begin to rearrange by convection, showing turbulence at the interface as seen in fig. 47 (middle), but little actual mixing. Both phases remain clearly seperated, with the hotter water rising to the top and a dampened, oscillation-like movement of the interface is seen for two to three minutes (compare 47, bottom). The participants are asked to describe their observations first and then guess which liquids were used. After gathering some suggestions a poll is conducted, before it is revealed that both liquids are water, but with different temperatures. The participants are then asked wheter the observation matches their expectation given this information and to interpret deviations. It is important to point out the clear contrast of (presumably) expected mixing of liquids/colours and the clear seperation with minor mixing found in experiment. The driving factors for this effect are identified to be a density difference induced by a temperature difference and the sorting characteristic of gravity in presence of such density differences. This is used to qualitatively motivate the clear separation of phenomena in stars according to their thermodynamical properties temperature, pressure and their density. The dynamic component, being the movement induced by local differences in density, observed in the rearrangement is introduced as the phenomenon of convection. Unlike in stars, the sorting in this simplified model is not due to self-gravity and a liquid is used, not a compressed gas. The induced density difference in a star arises from the pressure gradient overcompensating the difference induced by temperature gradient, which is not reproduced here. This phenomenological difference is pointed out to result in a reversed ordering with the hot phase being the less dense one, rising to the top in the experiment. The container can be put aside and presented again at a later stage, as the clear separation will persist for half an hour or more, showcasing the relevance of density difference compared to diffusion.

Convective Currents

The just observed phenomen of convection is then to be seen in the context of local currents transporting heat by generating convective currents in a beaver.

(i) The participants are split into groups of two and handed one heating plate, a beaver filled with room temperatured tap water and a pipette per group.

(ii) The beaver is put partly on the heating plate, using a metal block as shown in fig. 46.

(iii) A drop of dye is placed at the bottom of the beaver over the heating plate using the pipette and the heat plate is turned on.

(iv) Thin convective currents of dyed water soon begin to rise, showing complex small scale features (compare fig. 46).

(v) It can be seen that the dyed water is dropping on the opposing side of the beaver and overall a clear circular flow forms (see fig. 46, right). If initially induced turbulences make this difficult to see, a second drop of dye can be added to the water on the opposing side.

The experiment is rather sensitive for careful placement of the dye drop in the beaver, but all participants reported some level of experience in working with pipettes from chemistry or biology classes and thus were able to conduct the experiment by themselves without relevant complications. Once the effects were seen by all groups, the experiment is ended and the participants are asked to describe their observations and explain them in context of the before found effect of convection. It is important to clarify the connection to again a density difference due to temperature with a phenomenological contrast between the static structure seen before and the dynamic movement of material here. Thus the relevance of convective flows as a way to transport heat and the challenging nature of a description of the small (spatial and temporal) features is pointed out.

5.3 Window To The Stars

With these qualitative observations regarding structure and heat transport by convective currents at hands, the general situation is concluded to involve a number of phenomenologically quite different aspects and processes. These are challenging to tackle in generality in mathematical, analytical form, but can be simulated, as the individual aspects can be described analytically. The participants are reminded of the overall goal, to determine the life time of stars to provide long term, stable conditions over billions of years for the evolution of intelligent life. A star consuming hydrogen by nuclear fusion to generate luminosity is then compared to a motor consuming fuel from a limited size tank at a certain rate. This analogy is characterized as a simplified model with limitations and a simple calculation of stellar life time based on the rule of three is motivated. The required quantities for this calculation are pointed out as the stars mass (representing fuel), its luminosity (representing the motors output power) and a reference value (to determine the conversion factor). An estimate of the lifetime of the Sun $\tau_{\text{sun}} = 1.2 \times 10^{10}$ y as result of a long runtime simulation is presented. Furthermore it is pointed out, that this would be a possible way to determine life times in general, however numerical stability and runtimes do not allow this method to be applied for a series of simulations in the context of a workshop.

The participants are once more divided into groups of two, handed one laptop per group with the preinstalled software and lead through the process of how to boot up the required simulation software (WTTS), which requires the use of a virtual machine on *Windows* systems. This process may take some time, during which the use of the software and the workflow are explained: The steps to check the correct settings, set a specific stellar mass, start the simulation and determine the time averaged luminosity from the simulation output are shown and explained for the case of a star with one solar mass. Before having the participants start their own simulations, the understanding of the meaning of the luminosity is emphasized.

Therefore several questions are raised and discussed in plenum:

(i) Why is the averaged luminosity for the sun not one? What does *solar luminosity* mean?

(ii) Find the time, when the sun has one solar luminosity in the $L/L_\odot$ vs. t diagram (see fig. 45, top right).

(iii) Compare this value with the age of the solar system as shown in fig. 39.

(iv) How does the luminosity change over time? What is the implication for the *habitable zone* in the early solar system (compare fig. 44)?

(v) Did we find a deviation between distance from the Sun and the observed temperature on a planet earlier (compare fig. 42)?

(vi) Is there a process indicated in fig. 39, that could explain this finding?

It is discussed, that the age of the solar system claimed earlier agrees with the simulation and that the atmospheric effects of CO_2 as observed on Venus, together with the emergence of *photo synthesis* removing it from Earths atmosphere, can qualitatively explain stable temperatures on Earth. It is important to point out these connections of the different topics presented so far and their function as cross checks for the understanding of the evolution of the solar system and emergence of life on Earth. Then the participants are asked to do simulations themselves with about four to six different masses of their choice in the range of 0.7 to 20 solar masses. This mass range provides very high numerical stability, short runtimes and avoids the region of low mass with extremly long life time estimates, which would hinder the graphical representation and interpretation of the results.
Finally, the pairs of $M/M_\odot$ and time averaged luminosities $\bar{L}/L_\odot$ found by all groups are combined in one table, the estimated life times are calculated using the rule of three:

$$\tau_M = \tau_{\text{sun}} \cdot \frac{M}{\bar{L}}\,, \tag{5.1}$$

with $\tau_{\text{sun}} = 1.2 \times 10^{10}$ y as explained above. The obtained data points are plotted as is shown in fig. 7. The participants are asked to describe the behaviour of stellar life times with the stars mass and to interpret this result. The implication of the steep drop with increasing mass for emergence of life is discussed and concluded to be, that stars significantly heavier than the Sun (roughly $1.3\,M_\odot$) are not expected to be relevant for emergence of intelligent life, as their life time is too short. At this point the initial question guiding the work shop is answered very conclusively based on simulation results generated during the workshop by the participants themselves. This is the intended result and end point for the workshop.
However, the very logical question, whether stars with significantly lower masses are thus expected to be especially suited for emergence of intelligent life is brought up by the participants, showcasing their high level of engagement and understanding gained during the

$M/M_\odot$	$\bar{L}/L_\odot$	$\tau[10^9\,\mathrm{y}]$
1.00	1.1	12
0.7	0.3	28
3.5	155	0.271
10	8500	0.014
11.89	14000	0.01
19.95	50000	0.005
0.71	0.275	30.982
5.62	1200	0.056
8.4	450	0.022
16.7	3200	0.006
1.3	3.2	4.875

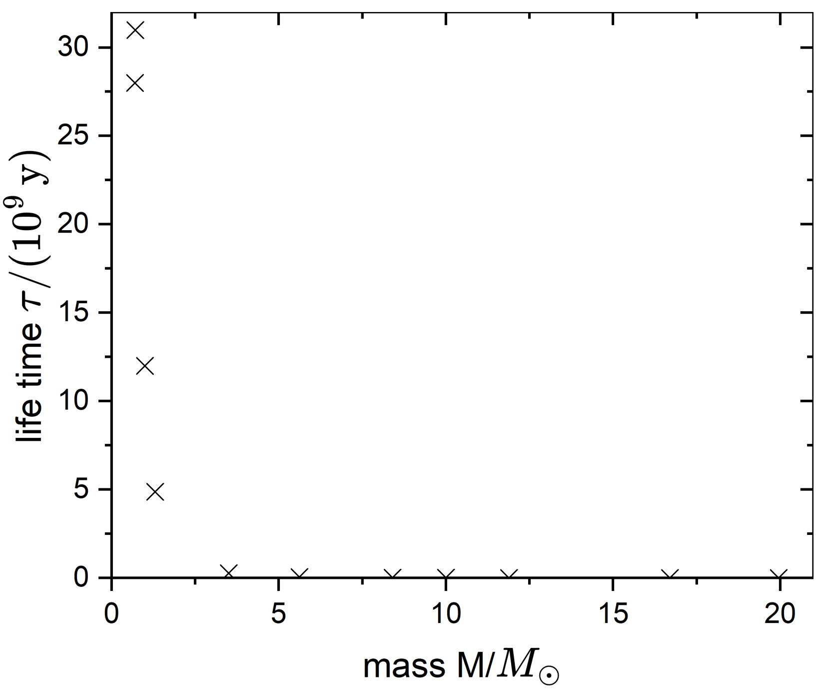

Figure 7: Averaged simulated luminosities $\bar{L}/L_\odot$ for stars of specific masses $M/M_\odot$ and corresponding calculated life times (left); calculated life times plotted against stars mass (right, same units as in the table).

workshop. A brief, qualitative explanation of convection leading to violent solar storms and coronal mass ejections in low mass stars, combined with the very proximate habitable zone and tidal locking counteracting the effect of long life time is given. Thus also significantly less massive stars than the Sun are expected to have little relevance for emergence of intelligent life, ending the workshop with an even more complete conclusion than intended.

5.4 Reduced Version for 9th Graders

As mentioned above, a reduced version of the workshop was seperately given to two 9th grades with about 25 pupils each at the same local high school in Münster, Germany, in October 2024 as a 90 minute intervention. The contents of the workshop were adjusted to fit the younger audience and shorter available time due to the stiff schedule at schools, being organized in lessons of 45 minutes. The adjusted time schedule for the workshop is shown in tab. 2. Compared to the workshop as described above, the handouts presenting examples for the different life forms were replaced by a presentation and interactive discussion of four environments for extremophiles (arctic, deep sea - black smoker, salt lake and Chernobyl) as well as three examples for life forms of increasing complexity (slime mold, tardigrade and craws). The discussion of possibilities for extraterrestrial life was reduced to one example: Mars. Thereby the complexities of splitting the significantly bigger group were avoided. The priority in this setting is to showcase an interdisciplinary insight into science, offering hands on experiments rather than a deeper insight into the method of numerical simulation.

Therefore no simulations were conducted during this workshop. Instead the results obtained in the previous, full version of the workshop on 29th of August were presented. This also avoids the challenge to introduce the younger audience to the simulation software itself, provide the hardware and assist the entire group, given the significantly bigger group size and strict time schedule at school.

Part	Duration [min]	Content
Introduction	5	-
Phenomen Life	10	Presentation regarding life and its emergence
Different life forms	10	Different life forms and evolutionary stages
Discussion	5	Possibility of life on Mars
Stars	15	Presentation formation of and processes in stars
Break	5	-
Experiments	25	Two experiments: convection and convective currents
WTTS	5	Explain the need for simulation
Lifetime	5	Explain the calculation of stellar life times
Conclusion	10	Interpret life times, questions, feedback
Total	95	

Table 2: Time schedule for the different parts of the workshop for 9th grade giving their duration and a brief description of the content.

6 Evaluation and Outlook

For evaluation purposes, feedback in form of a questionnaire and an interview was gathered. To put the workshop experience and feedback obtained into context, a brief profile of the school is provided. The local Gymnasium the participants attend, is a well established school, that strongly engages in national scientific competitions and educational programs for their students, as well as international cooperations and educational programmes, such as the *international baccalaureate* (IB).
At the end of the workshop, the participants were asked to fill in a feedback form on fully voluntary, anonym basis, which serves soley the purpose of evaluation. Therefore a *Google* feedback form was used, which is shown in the appendix in its full version. For evaluation of the reduced version of the workshop also a shortened version of this feedback form was used, reducing the statements to a minimum set of the most relevant ones. In both cases the aim of the questionnaire is to evaluate the general suitability of the workshop and to identify possibly problematic parts therein. Questions regarding the participants knowledge or interest are only relevant to put the obtained feedback into context, not to gain a detailed profile of the participants abilities or to compare subgroups of these.
The participants were asked to state their level of agreement to a series of statements. The level of agreement was expressed using a scale with four levels, which for graphical representation and quantitative analysis was translated into the numerical values given in parenthesis (*Likert scale*): *'I disagree.'* (0), *'I rather disagree.'* (1), *'I rather agree.'* (2), *'I agree.'* (3). In the feeback form for the full version free text comments could be given to each part of the workshop, which was omitted for the shortened form.

6.1 The Workshop in Its Full Form

The participants are in their final year of high school and working towards the supplemental science qualification (IB) alongside their Abitur (German secondary school diploma). Among these, three had enrolled in a basic physics course, seven in either a basic or advanced biology course. Specifically, five students attended only a biology course, two were enrolled in both, a biology course and the basic physics course, and one student attended only the physics course. Out of eight participants six were female.
The feedback form was filled in by seven out of eight participants, the results of which are analyzed in the following. Additionally two volunteers gave a detailed exploratory interview, a transcription of which is included in the appendix and a summary of which is discussed here. During their stay at the university after the workshop the participants were also briefly asked about their prior knowledge regarding stars and emergence of life. The response was rather heterogenic, with mostly only some terminology and basics being known due to basic biology classes (e.g. structure of a cell, importance of energy and water) and some fragmentary knowledge from documentaries or topics briefly touched in popular media (e.g.

'Netflix-series'). Two participants, currently having advanced biology courses, mentioned that some of the information regarding life and its emergence was familiar, but in a quite different context. Other prior knowledge mentioned included early geography classes (9th grade, structure of the solar system), an integrated science class (very basics of stars) and a year abroad in England (astrophysics basics). The participants stated that simulations in especial were not used or introduced and if at all, only briefly mentioned in school prior to this workshop.

Feedback Form

Following items that were developed and evaluated in [Liu23] to access the scientific knowledge and interest of teeenagers, the participants' self-perceived abilities in physics and interest in sciences were tested asking for their level of agreement to the following three statements:

- 'I think, that i am good in physics.'
- 'Sciences amaze me.'
- 'I am completely unsuited for physics.'

The results are shown in fig. 8. These items alone do not allow a detailed or nuanced estimate of the actual interest and abilities in physics and sciences, but only give a general tendency. As mentioned above this is only used for contextualization of the obtained feedback and impression given prior knowledge or specific interest. This is used for the evaluation of the workshop presented here, regarding applicability to a general audience. A detailed assessment of the participants abilities and interests in science is neither possible nor attended. The questionnaire then tested on one hand whether the different parts of the workshop were informative/comprehensive and on the other hand interesting/fun, for each topic and each method (presentation, discussion, experiment, etc.) seperately. A short, paraphrased english version of each statement is shown in tab. 6.

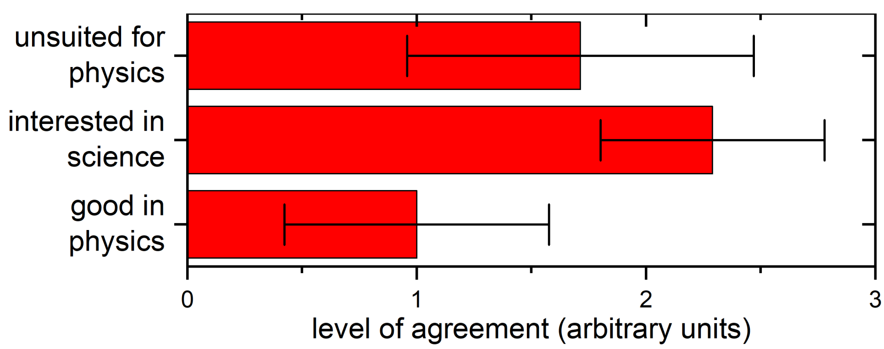

Figure 8: Participants self-perception regarding their abilities in physics and interest in science as stated in an anonymous form at the end of the workshop ($n = 7$).

The participants tend to consider themselves to be not good in physics (1.00 ± 0.58) and averagely suited for physics (1.71 ± 0.76), both with big variance, overlapping with a neutral mean response. They clearly state having interest/amazement in/for science (2.29 ± 0.49). This finding is in agreement with them having chosen the IB for their Abitur (a scientific supplement), but tending to choose biology (7 out of 8) rather than physics (three out of eight, no advanced course) as a course. The overall feedback is positive throughout all items

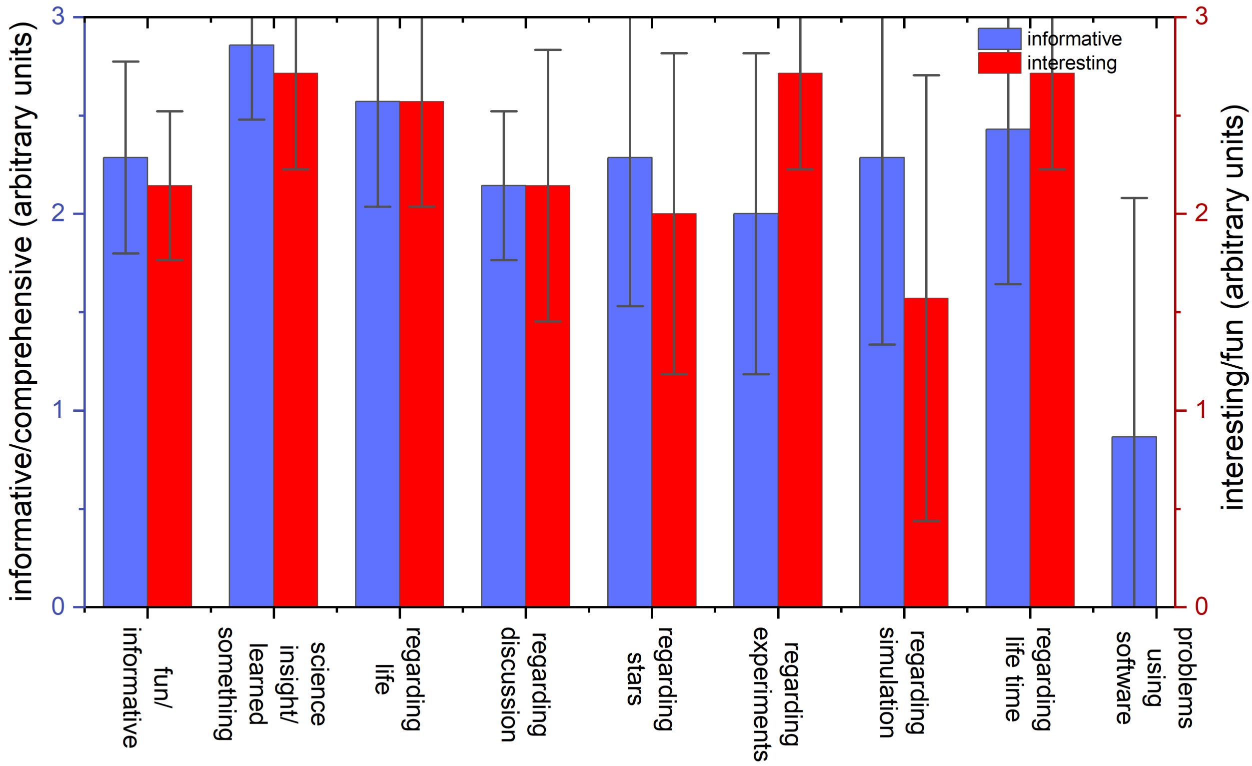

Figure 9: Results of the questionnaire as shown in tab. 8 regarding the various items explained in tab. 6 and dimensions informative/comprehensive (blue) and interesting/fun (red) with $n = 7$.

in both dimensions, informative and interesting, the only item averaging below neutral being *'problems using the simulation software'* (0.86 ± 1.21). The overall perception of the workshop was clearly positive, with the strongest agreement overall with the statements *'I learned something new'/'I gained new insights into science'* ($2.86 \pm 0.38/2.71 \pm 0.49$). This highlights the workshop as very successful overall in communicating science in an informative, comprehensive and interesting way for teenagers with varying prior knowledge and no specific affinity with physics. All particular stages of the workshop were perceived as both, comprehensive and interesting, with a trend to favor the biology related topics (emergence of life and possibilities of extraterrestrial life) over the physics related ones (stars and their simulation). The only clear deviation between the dimensions informative/comprehensive and interesting/fun are seen for the experiments, favoring the latter (2.00 ± 0.82 compared to 2.71 ± 0.49) and the simulation favoring the former (2.29 ± 0.95 vs 1.57 ± 1.13). For all four involved values, except the dimension 'interesting' regarding the experiments, the variation was however quite big and the resulting interval overlaps with a neutral feedback.

Insights regarding emergence of life	No.
Extreme robustness of life	3
High requirements for intelligent life	3
Everything	1
High level of differentiation	1

Insights regarding stars	No.
Consist of hydrogen	2
Everything	2
Relevance of mass	2

Table 3: Summarized free text answers regarding the participants new and/or exciting insights into the emergence of life (left) and stars (right).

All these findings are strongly supported by the free text feedback listed in tab. 3, 4 and 5: The workshop was predominantly perceived as *'interesting'*, *'comprehensive'* and *'complex'* (nine out of 13 total responses, tab. 5 left) with almost twice as many explicit highlights mentioned as lowlights (seven compared to four). The most controversial content clearly is the simulation software WTTS (both highlight and lowlight three times, predominantly described as *'interesting'*, *'boring'/'monotononous'* and *'complex'*). The experiments were seen as highlight by four out of seven participants, while the only other lowlight mentioned is a partial lack of stringency. It is interesting to point out, that the insights regarding stars (tab. 3, right) are dominated by their basic features (*'consist of hydrogen'* and *'everything'*), hinting on the absense of stars as a topic in general classes in school and a predominantly fragmentary knowledge in this matter prior to the workshop.

Insights regarding other worlds	No.
Possibly even on Venus	2
Depends on many factors	1
Everything	1
Info about Titan	1

Impression of WTTS	No.
Interesting	4
Boring/monotonous	3
Complex	3
Adaptation for Windows problematic	1
Well explained	1

Table 4: Summarized free text answers regarding the participants new/exciting insights about extraterrestrial life (left) and their impression of the simulation software WTTS (right).

The workshop was:	No.
Interesting	4
Comprehensive	3
Complex	2
Good materials	1
Partly lacking context	1
Too long	1
Too short	1

Highlight	No.
Experiments	4
WTTS	3

Lowlight	No.
WTTS	3
lack of stringency	1

Table 5: Summarized free text answers regarding overall impression of the workshop (left), its highlight (right, top) and lowlight (right, bottom).

Exploratory Interview

A full transcription of the exploratory interview conducted with two of the participants at the end of their stay at the university is provided in the appendix. The interview was designed to add depth to the feedback by exploring the impression and critique of specific aspects of the workshop - such as the discussion, experiments and simulation - more thoroughly. Additionally it aimed to assess whether these components provided a balanced and engaging workshop experience as an interdisciplinary whole. Finally, it sought to determine, whether the interdisciplinary topic chosen resonated with the participants interests as intended and the initially raised question regarding possibilities of intelligent life, aiming to address topics present in news and media, was answered conclusively. Note, that this interview was taken several hours after the workshop finished, another workshop taking place inbetween. Therefore a reflected impression of features worth remembering can be expected rather than a repetition from short term memory, but also a mixed impression of both workshops can not be fully excluded. The main points raised by the participants are summarized and grouped at the end of the german transcript.
The feedback given in this interview in general greatly supports and outlines the findings given above. Key concepts and methods of the workshop that were strongly and explicitly confirmed were:

- Astrophysics and specifically extraterrestrial life are fascinating topics, well suited to attract interest.
- The presence of those topics in media and news currently is well perceived and lets the topic resonate with a young audience.
- Though very present in media, the scientific perspective is mostly missing, i.e. it is absent in school.
- Also the scientific tool of numerical simulation is barely present in school.

- Integrating varyious methods in form of a group discussion, experiments and the simulation greatly enriches the workshop.
- The simulation immensly adds credibility, authenticity and professionalism.
- With clear and comprehensive guidance the complexity of the software can be handled by highschool students.
- The analysis of stellar life times resulted in a clear and satisfying answer to the initial question raised, what star systems intelligent life can potentially emerge in.

The interviewees also identified challenging aspects, that are essential for the success of the workshop, but were addressed well:

- The complexity of the simulation requires a high ratio of instructors to participants to be able to address questions quickly.
- The interdisciplinary nature and multitude of topics neccessary to discuss requires to emphasize the connections between and goals of the individual parts.

During the interview some suggestions to improve the workshop were stated:

- 'Extraterrestrial life' suggests life outside the solar system. It should be clearly stated, why planets outside the solar system can not realistically be topic of the group discussion.
- The discussion of planets inside the solar system in plenum got repetitive and could be shortened.
- A less open and more guided approach to the simulation of stellar formation shown and the experiments beforehand would improve the understanding and increase the gains from using these media.

My personal impression gained during the workshop was very positive. All participants engaged actively in the discussions and input sessions and showed great interest in the topics presented. The participants showed prior knowledge and understanding of basic biology and especially the ability to quickly transfer biology related information during the group discussion phase and with their open questions at the very end of the workshop. Especially the experiments and the final interpretation of life times along with the discussion of the connections between the different topics before the simulation appeared to gain great interest. This impression aligned with the impression of my supervisor and the teacher, as was given in private communication after the workshop. The concept to connect astrophysics with biology to address the question of extraterrestrial intelligent life, presenting a variety of different methods including experiment and simulation appeared to be very successful overall.

6.2 The Workshop as Intervention for 9th Graders

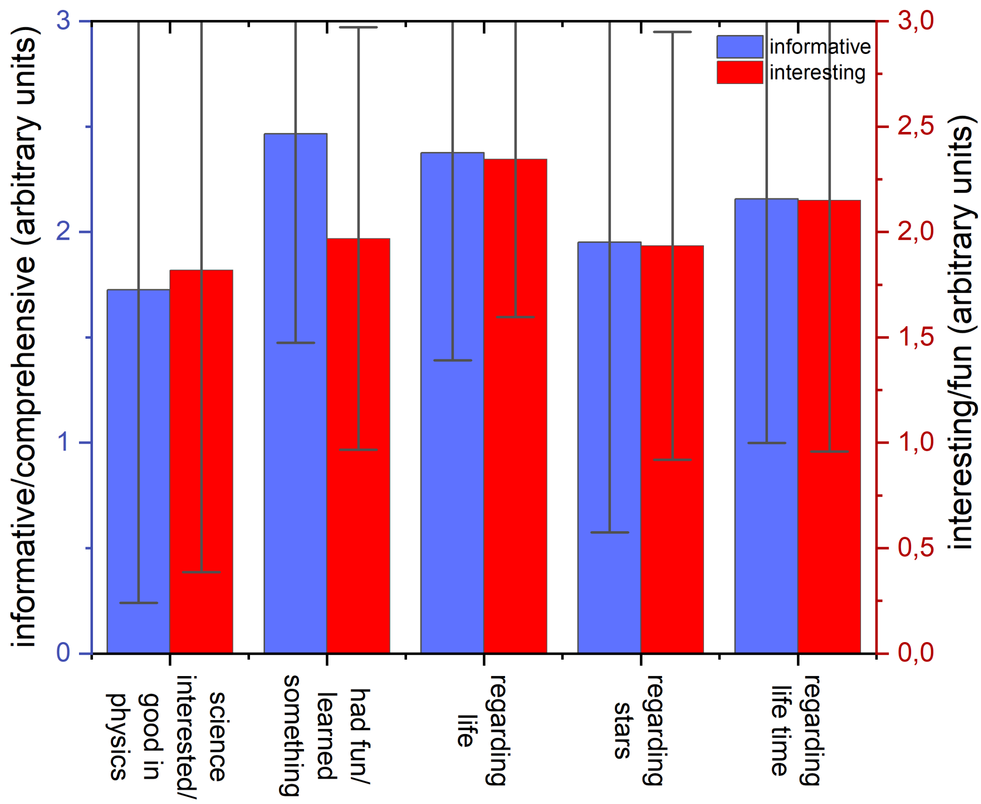

Figure 10: Results of the reduced questionnaire as shown in tab. 10 regarding the dimensions informative/comprehensive (blue) and interesting/fun (red) obtained by a total of 25 participants (9th grade) of a 90 minute intervention.

As stated above, a shortened and simplified version of the workshop was developed and held twice with two regular 9th-grade classes at the same local Gymnasium, both thaught by the same physics teacher. The workshop conditions were kept as similar as possible (same school, same day, etc.). At the end of the intervention, the students were asked to fill out a significantly shortend version of the anonymous and voluntary feedback form. The mean agreements and its variations are shown in tab. 9 and plotted in fig. 48 for both groups, labelled Group A and Group B. While 16 out of 25 students from Group A completed the feedback form, only nine out of 27 did so in Group B. This lower response rate in Group B makes this data more susceptible for biases and reduces its representativeness. Notably, the feedback regarding the questions about students' interest in science and self-assessment in physics and the general impression of the workshop were highly consistent between the two groups, while slight variations in responses to specific workshop parts were observed. Since this evaluation is exploratory in nature and neither aims for a quantitative analysis nor a qualitative comparison of the groups, it appears well motivated to combine the data into one data set with feedback from a total of 25 students. The combined results are presented in tab. 10 and fig. 10. The students slightly tend to describe themselves as good in physics and interested in science. The workshop as a whole was perceived as informative and fun leaning towards the first. Also all parts individually were considered informative/comprehensive and interesting/fun by the students. This result is very promising given the challenges of a big group, complex topic and limited time. Especially the fact, that the agreement with the workshop being informative/comprehensive and interesting/fun exceeds the agreement with being good in physics/interested in science is remarkable. The variance of all results given is however high, as was expected for the low number of total feedback responses considering the inhomogenity present amongst students.

My personal impression during the workshop was, that a clear general interest in the topic of stars and especially extraterrestrial life is given also for most 9th graders. The complex topic is more challenging to address in this format, primarily due to the huge group, short time and strict time schedule, not allowing a flexible approach and rendering experiments and individual engagement more demanding. All these challenges are however dominated by external conditions and the setting, not by the students interest or willingness to engage with the topic itself. The intervention as outlined above in general appeared to address the difficulties well and seemed to provide a comprehensive and interesting addition to the physics course. This impression was also shared by my supervisor and the 9th graders teacher, as was stated in private communication after the workshop.

6.3 Conclusions and Outlook

The theory of stellar structure and evolution covers a multitude of topics and concepts of different disciplines within physics, including a variety of theoretical framework (see section 2, nuclear physics, particle physics, thermodynamics, fluid dynamics, atomic physics, quantum mechanics, etc.). Also the involved numerical procedures are not trivial and the overall number of parameters and inputs is immense, rendering the overall topic rather challenging (see section 3 and 4). Given a working simulation software this multitude of different fields within physics however also provides a very interesting application which trains and provides a deep understanding of the interplay of the different concepts and effects involved. Thus numerical simulation of stellar structure and evolution can be a very interesting example for an application that connects numerical methods with various theoretical approaches and emphasizes a deep understanding of the impact of the numerous aspects given the limited direct observational accessibility of the models and parameters involved.

The simulation output can provide rich insights on a variety of levels of depth already based on a rather limited set of simulations, that can be gained with limited hardware in reasonable time, as was presented in section 4. Therefore a robust theoretical and conceptual understanding and good graphical analysing tools are required, as are provided by WTTS. Thereby the simulation can provide a sophisticated and deep insight into the phenomenologically fascinating topic of stellar evolution to interested people with a background in physics. The conceptual and didactical reduction of this complex topic is challenging but however

possible and conclusive and fascinating insight can be obtained on very different levels of physical understanding. While the simulations presented in section 4 certainly require a physics background both, for conducting, but also comprehending them, in this thesis reductions of the topic for young adults/teenagers down to 9th grade were presented. Certainly adaptations for all age groups and interest groups within this spectrum can be conducted.
Based on the feedback described and summarized above I can conclude that the workshop presented in section 5 seems well suited and very promising for outreach with teenagers, that

(i) are in the age group shortly before finishing highschool, with an above average affinity to sciences (especially physics or biology) as part of a workshop as described in 5.1-5.3 based on the evaluation conducted in 6.1,

(ii) are 9th grade students with an average affinity to physics/sciences as an intervention as described in 5.4 and evaluated in 6.2.

Additional to the formats tested and presented in this thesis, workshops aiming at undergraduate students in form of a longer lab course would be a very interesting format, emphasizing the simulation of stellar structure and its evolution more explicitly. A deeper background knowledge especially in nuclear physics and thermodynamics would greatly benefit the possibilities for an active engagement with the simulation software and sophisticated discussion of the obtained output. Such a lab course could be given optionally or as part of a lecture at an university. This suggestion is supported by the experience of the author of WTTS as gained over years of teaching at universities in the UK and Germany, as was expressed in private communication on 15.03.2024 in a video call.
The very positive and detailed feedback obtained for the workshop in its full form is very promising and highlighted the immense success of the interdisciplinary approach, addressing topics of great general interest and significant media presence. The free text answers in the questionnaire as well as the exploratory interview clearly showed, that the topics astrophysics, life on distant worlds and space travel attract strong interest. Thereby a sophisticated discussion or robust knowledge are mostly absent, especially from media and only in case of fundamentals regarding life and its emergence partly provided by school. This enables the workshop to fill exactly this huge gap between existing interest and fascination on one hand and sophisticated, profound information and knowledge on the other hand. The feedback clearly stated expectations fueld by the way the topics are presented in media and stressed the scientific, authentic and professional approach the workshop took as immense gain. This has to be seen as a valueable chance to draw attention to the fields of astrophysics and astrobiology in a scientific, rather than lurid sense combining modern methods, i.e. numerical simulation with an interactive, engaging experience and a very clear, satisfying final result for the participants. Thus the workshop proved to be capable of covering the immensely deep topic of stellar structure and evolution in a well suited, didactically reduced form to advertise for the fascinating field of physics and specifically astrophysics. This very positive

result is the more outstanding given the average affinity for physics among the participants and renders the workshop very promising for science communication among teenagers with no specific interest in physics.
Additionally several optimizations to the full version of the workshop can be conducted, especially based on the detailed and specific exploratory interview. Those could include adopting the structure from the reduced workshop of presenting a selection of life forms as part of the input session instead of as handouts. An explicit group discussion phase about possibilities of life in extraterrestial worlds rather than a discussion in plenum should be considered. As pointed out by the interviewees, the essential factor water is constantly present in those considerations and discussing this perspective in smaller groups might be more engaging, than in plenum. Also one of the examples given (i.e. Titan) could be replaced by an exoplanet to reduce the focus on the known or assumed existence of water and address the expectations that were mentioned in the interview, as are motivated from media coverage regarding habitable exoplanets outside the solar system. Thereby the highly speculative character of such considerations could be actively developed by the participants themselves and actively discussed. Also a compromise of presenting input like the experiments or simulation of stellar formation rather openly, without framing the intended observation a priori, and a more guided approach as familiar to the participants from school could be considered.
A possible and very interesting extension of the workshop would be to make the implicitly presented scientific methodology explicit, by addressing the speculative character of many topics present in astrophysics due to the lack of measurements directly at the objects themselves. The problematic factor of testability could be openly discussed and a short reflective phase might be added, to compare the participants expectations with the impressions gained during the workshop. Thereby the sensational and mostly inconclusive way of presenting astrophysics and -biology in media and news could be contrasted with the more scientific approach of this workshop involving simple definitions, basic theoretical input and professional software, to reach a very clear and conclusive answer to the initial question for possibilities for intelligent life.
The workshop in general proved to be very promising and offers many chances to extend the topics presented, for example by adressing also the problematic nature of low mass stars, as briefly outlined, when a participant raised this question. Also a more specific discussion of topics presented in media could be added (multi star systems, exoplanets currently found by telescopes, manned mission to Mars, etc.). Thus the workshop could fill an entire project day at the university, as is the concept of so-called Masterclasses, to maximize visibility and attract immense interest in physics in teenagers.
Based on my subjective impression obtained during the workshop I would suggest, that especially the last part of the intervention for 9th graders, regarding the simulation of luminosities and calculation of stellar life times was not fully conclusive and interesting for some in this younger audience. This is not reflected by the feedback obtained, which might be due

to a positive confirmation bias leading to individuals with below average interest in physics being underrepresented in the feedback. To clarify this open question, follow up studies are needed, to gather the necessary statistics to adress more specific questions regarding the suitability of the workshop in the form as described here. Also the intervention could be adjusted by avoiding the explicit calculation of stellar life times from simulated luminosities and instead presenting and interpreting the results directly, labeled as taken from simulations. The gained time should be used to stress the students engagement in the workshop further, for example by extending the experiments conducted or the discussion of possibilities for life on Mars as a group work. Given these potential adjustments the workshop seems capable to communicate the complex field of astrophysics and advertise for physics and sciences in general among teenagers of this younger age, without any specific affinity for physics or sciences. Thereby the workshop seems promising for attracting attention, enhancing interest and showcasing the fascinating nature of physics to young teenagers.

The conclusion clearly has to be, that especially the full version of the workshop but also its reduced version have great potential for science communication and outreach in young audiences, which should be further investigated in consecutive testing.

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Appendix

Constants

Solar radius:	$R_\odot = 6.9598 \times 10^{8}\,\mathrm{m}$
Solar mass:	$M_\odot = 1.9891 \times 10^{30}\,\mathrm{kg}$
Solar luminosity:	$L_\odot = 3.8515 \times 10^{26}\,\mathrm{J\ s^{-1}}$
Speed of light:	$R_\odot = 2.99792458 \times 10^{8}\,\mathrm{m\ s^{-1}}$
Gravitation constant:	$G = 6.67259 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}}$
Atomic mass unit:	$m_\mathrm{u} = 1.6605390 \times 10^{-27}\,\mathrm{kg}$
Planck constant:	$h = 6.6260755 \times 10^{-34}\,\mathrm{J\ s}$
Boltzmann constant:	$k = 1.380658 \times 10^{-23}\,\mathrm{J\ K^{-1}}$
Gas constant:	$\Re = 8.314510\,\mathrm{J\ K^{-1}\,mol^{-1}}$
Radiation constant:	$a = 7.5646 \times 10^{-16}\,\mathrm{J\ m^{-3}\,K^{-4}}$
Stefan-Boltzmann constant:	$4ac = 5.67051 \times 10^{-8}\,\mathrm{J\ m^{-2}\,s^{-1}\,K^{-4}}$

Mass Fractions, Thermodynamics, Equation of State

As mentioned before, the stellar material is a mixture of different species of atoms, which are partly or fully ionized. With ρ_i the mass density of species i, μ_i its molecular weight in units of the atomic mass unit (amu) m_u and $X_i = \rho_i/\rho$ the mass fraction of component i, its particle density is

$$n_i = \frac{\rho_i}{\mu_i m_\mathrm{u}} = \frac{\rho X_i}{\mu_i m_\mathrm{u}}\,, \tag{A.1}$$

where the mass of the electron was neglected. Also a mean particle weight can be introduced:

$$\mu = \frac{\rho}{m_\mathrm{u} n}\,. \tag{A.2}$$

For a fully non-ionized gas one can define:

$$\mu_0 = \left(\sum_i \frac{X_i}{\mu_i}\right)^{-1}\,. \tag{A.3}$$

The first law of thermodynamics gives a relation between variations of heat q, internal energy u and specific volume $v = 1/\rho$ for the case of constant chemical potential and mean molecular weight with the pressure P, temperature T and volume V:

$$dq = du + PdV\,, \tag{A.4}$$

for a general equations of state $\rho = \rho(P, T)$ and $u = u(\rho, T)$.

$$\alpha := \left.\frac{\partial \ln\rho}{\partial \ln P}\right|_T = -\frac{P}{v}\left.\frac{\partial v}{\partial P}\right|_T \tag{A.5}$$

$$\delta := -\left.\frac{\partial \ln\rho}{\partial \ln T}\right|_P = \frac{T}{v}\left.\frac{\partial v}{\partial T}\right|_P \tag{A.6}$$

With these definitions the equation of state can be expressed as:

$$\frac{d\rho}{\rho} = \alpha\frac{dP}{P} - \delta\frac{dT}{T}. \tag{A.7}$$

Further more the specific heat can be formally defined now, such, that

$$c_P := \left.\frac{dq}{dT}\right|_P = \left.\frac{\partial u}{\partial T}\right|_P + P\left.\frac{\partial v}{\partial T}\right|_P, \qquad c_v := \left.\frac{dq}{dT}\right|_v = \left.\frac{\partial u}{\partial T}\right|_v. \tag{A.8}$$

With those

$$du = \left.\frac{\partial u}{\partial v}\right|_T dv + \left.\frac{\partial u}{\partial T}\right|_v dT, \tag{A.9}$$

$$ds = \frac{du}{T} = \frac{1}{T}\left(\left.\frac{\partial u}{\partial v}\right|_T + P\right) dv + \frac{1}{T}\left.\frac{\partial u}{\partial T}\right|_v dT \tag{A.10}$$

$$\Rightarrow \left.\frac{\partial u}{\partial v}\right|_T = T\left.\frac{\partial P}{\partial T}\right|_v, \tag{A.11}$$

$$\left.\frac{\partial u}{\partial T}\right|_P = \left.\frac{\partial u}{\partial T}\right|_V + \left.\frac{\partial v}{\partial T}\right|_P\left(T\left.\frac{\partial P}{\partial T}\right|_v - P\right) \tag{A.12}$$

$$\Rightarrow c_P - c_v = \left.\frac{\partial v}{\partial T}\right|_P \left.\frac{\partial P}{\partial T}\right|_v T = \frac{P\delta^2}{\rho T\alpha} \tag{A.13}$$

$$\Rightarrow dq = c_P dT - \frac{\delta}{\rho}dP \tag{A.14}$$

$$\nabla_{\text{ad}} = \left.\frac{\partial \ln T}{\partial \ln P}\right|_s = \frac{P\delta}{T\rho c_P} \tag{A.15}$$

As discussed before, the pressure P in a star has contributions from the gas P_{gas} as well as radiation P_{rad}. Therefore the description given in (A.18) needs to be extended as follows:

$$P = P_{\text{gas}} + P_{\text{rad}} = \frac{\Re}{\mu}\rho T + \frac{a}{3}T^4 \tag{A.16}$$

for a perfect gas and more generally

$$P = \frac{P_{\text{rad}}}{1-\beta}, \quad \beta := \frac{P_{\text{gas}}}{P}, \tag{A.17}$$

including also not perfect gases. The gas pressure P_{gas} for an ideal gas without degeneracy is the sum of the partial pressures of all constituents, that is electrons and all ions:

$$P_{\text{gas}} = P_{\text{e}} + \sum_i P_i = \left(n_{\text{e}} + \sum_i n_i\right) kT = \Re\rho T \sum_i \frac{X_i\,(1+Z_i)}{\mu_i} = \frac{\Re\rho T}{\mu}. \tag{A.18}$$

In the context of stellar structure and evolution usually a description in terms of the free energy however is used, which is sketched in [Sal05] and described in more detail in [Pol95].

Difference Equations

$$r_{k+1}^2 - r_k^2 = \left(\frac{\partial r^2}{\partial k}\right)_{k+1/2} \Delta k = \left(\frac{\partial r^2}{\partial m}\frac{\partial m}{\partial k}\right)_{k+1/2} = \left(2r\frac{\partial r}{\partial m}\frac{\partial m}{\partial k}\right)_{k+1/2} = \left(2r\frac{1}{4\pi r^2 \rho}\frac{\partial m}{\partial k}\right)_{k+1/2}$$
$$= \left(\frac{1}{2\pi r \rho} m'\right)_{k+1/2} \qquad \text{(A.19)}$$

where (2.7) was used.

$$\ln P_{k+1} - \ln P = \left(\frac{\partial \ln P}{\partial k}\right)_{k+1/2} \Delta k = \left(\frac{\partial \ln P}{\partial m}\frac{m}{\partial k}\right)_{k+1/2} = \left(-\frac{Gm}{4\pi r^2 P} m'\right)_{k+1/2}, \qquad \text{(A.20)}$$

where (4.2) was used.

$$\ln T_{k+1} - \ln T_k = \left(\frac{\partial \ln T}{\partial k}\right)_{k+1/2} \Delta k = \left(\frac{\partial \ln T}{\partial \ln P}\frac{\partial \ln P}{\partial k}\right)_{k+1/2} = \left(-\frac{Gm\nabla}{4\pi r^2 P} m'\right)_{k+1/2}, \qquad \text{(A.21)}$$

where (4.3) was used.

$$L_{k+1} - L_k = (m'E_1)_{k+1/2} + (m'E_2)_k\,[\dot{m}_k] - (m'E_2)_{k+1}\,[-\dot{m}_{k+1}] \qquad \text{(A.22)}$$

where E_1 and E_2 are defined according to

$$\epsilon + \epsilon_\nu + T\left.\frac{\partial S}{\partial t}\right|_m = E_1 + E_2 \dot{m}\,. \qquad \text{(A.23)}$$

$$m_{k+1}^{2/3} - m_k^{2/3} = \left(\frac{\partial m^{2/3}}{\partial k}\right)_{k+1/2} \Delta k = \left(\frac{2}{3m^{1/3}}\frac{\partial m}{\partial k}\right)_{k+1/2} = \left(\frac{2}{3m^{1/3}} m'\right)_{k+1/2} \qquad \text{(A.24)}$$

WTTS and Simulations

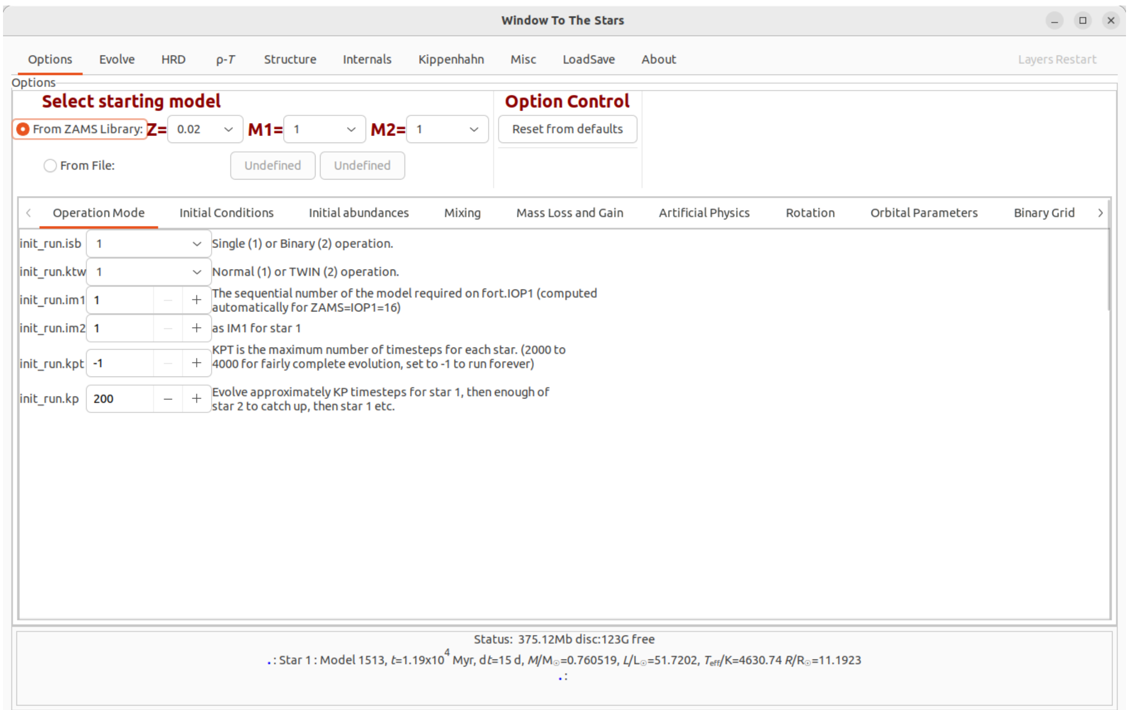

Figure 11: WTTS software, main window for selecting stellar masses, metallicity and mode of simulation for a stellar model sequence of a star with $M = 1\,M_\odot$ and metallicity $Z = 0.02$.

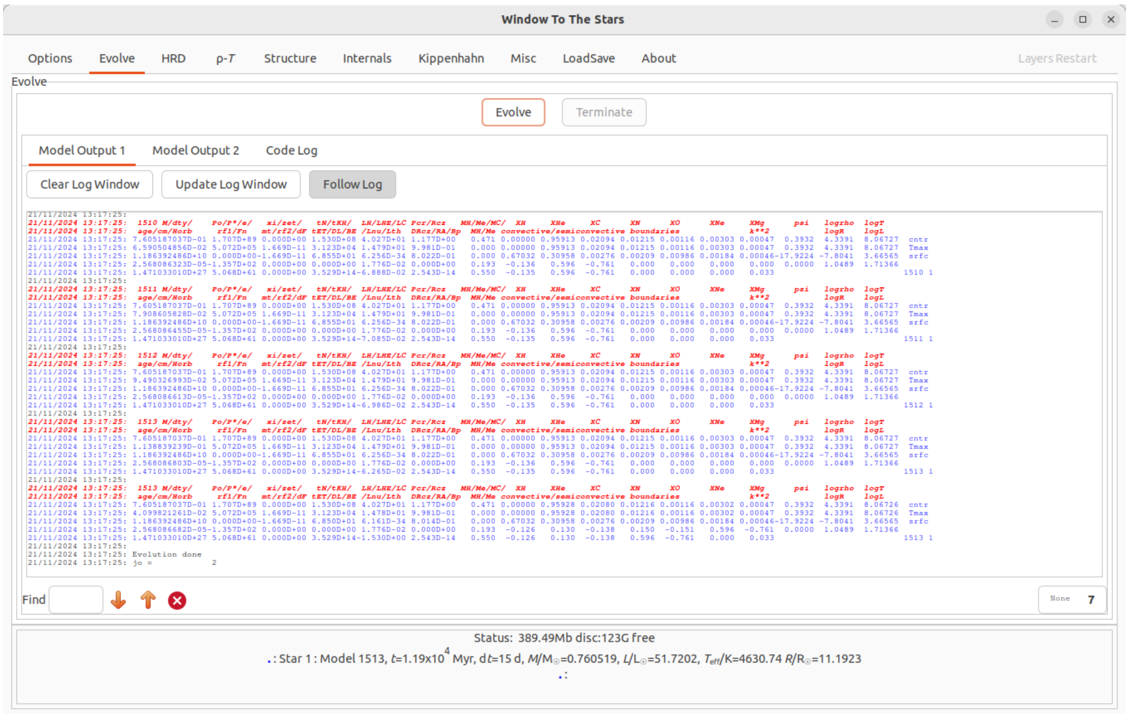

Figure 12: WTTS software, evolve tab showing the TWIN code status output during simulation for a stellar model sequence of a star with $M = 1\,M_\odot$ and metallicity $Z = 0.02$.

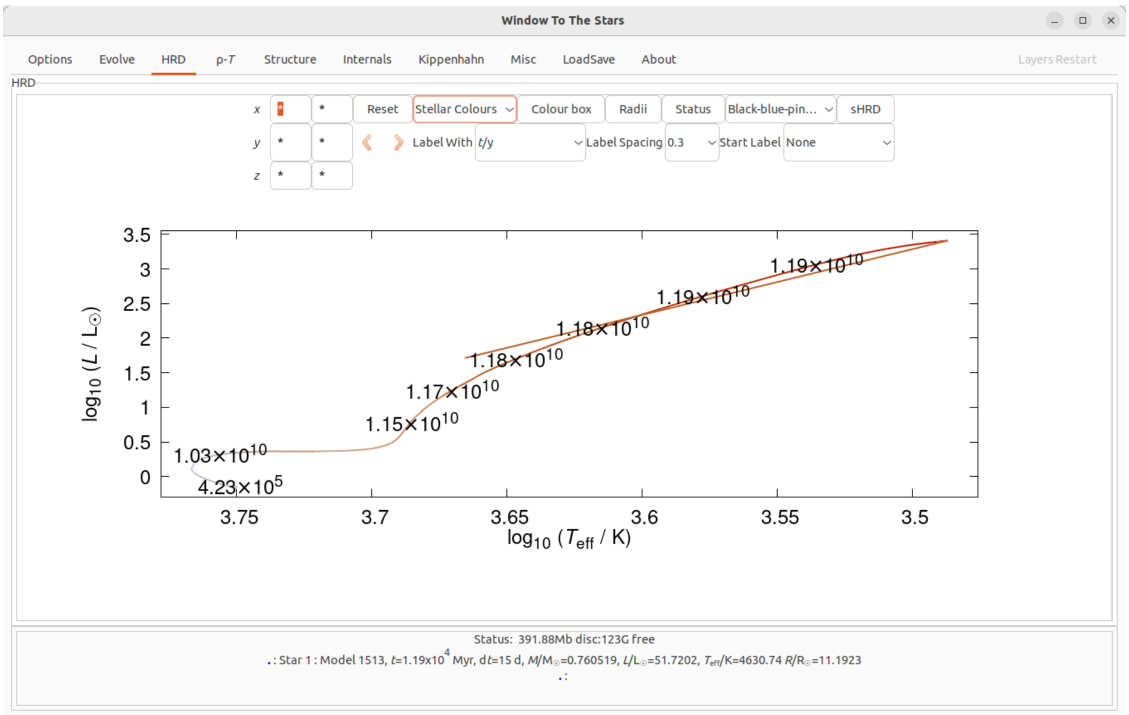

Figure 13: WTTS software, HRD tab: HR diagram with stellar age labels for a stellar model sequence of a star with $M = 1\,M_\odot$ and $Z = 0.02$.

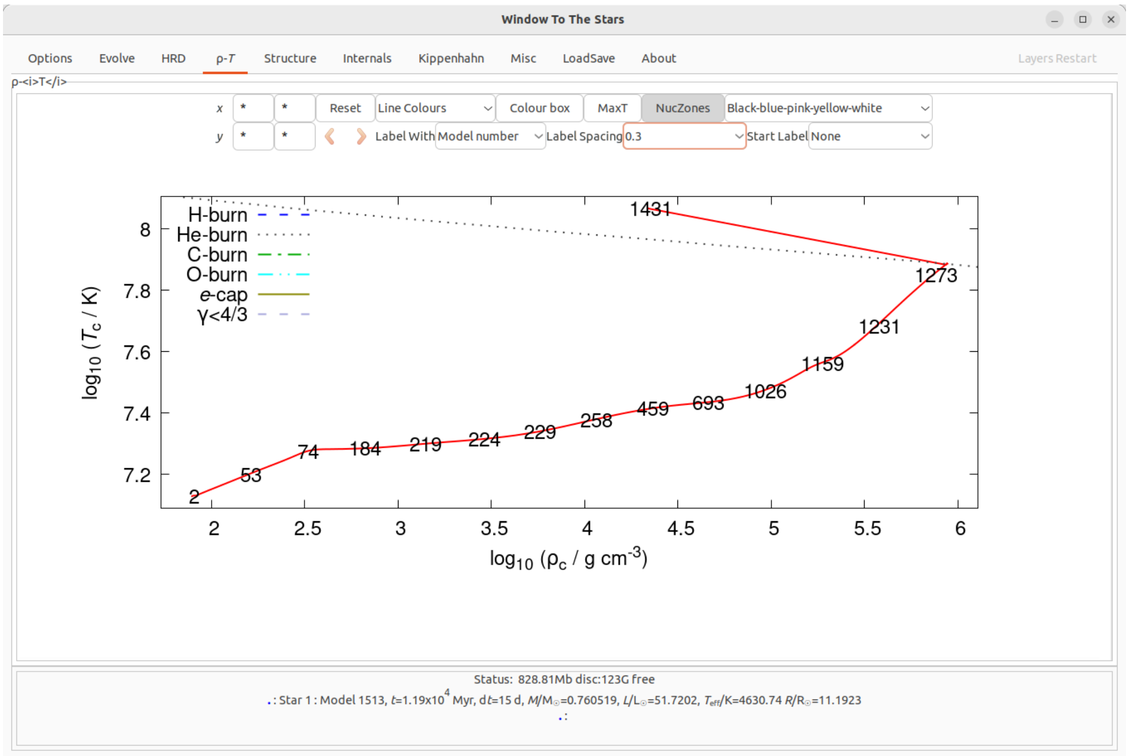

Figure 14: WTTS software, $\rho - T$ tab: double logarithmic plot of central temperature $T_c/[\mathrm{K}]$ against central density $\rho_c/[\mathrm{g\ cm}^3]$ with model number labels for a stellar model sequence of a star with $M = 1\,M_\odot$ and metallicity $Z = 0.02$.

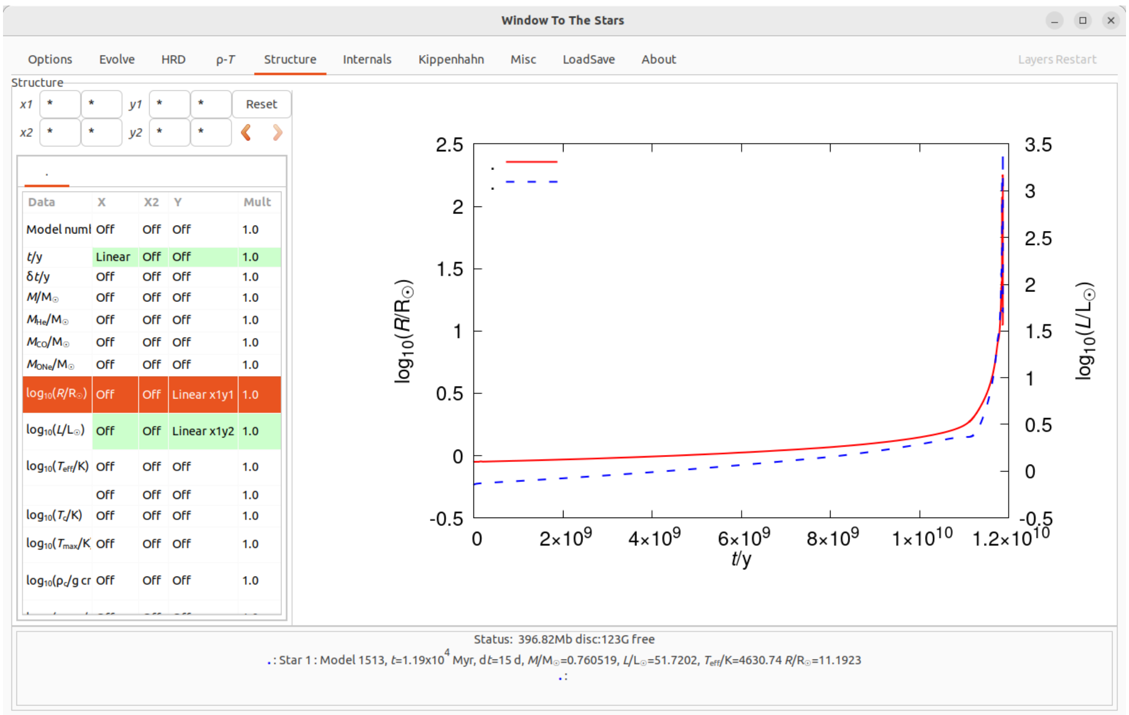

Figure 15: WTTS software, structure tab: 2D plots of various quantities related to stellar structure showing $\lg R/R_\odot$ and $\lg L/L_\odot$ plotted against time in years for ever 500th model of a stellar model sequence of a star with $M = 1\,M_\odot$ and metallicity $Z = 0.02$.

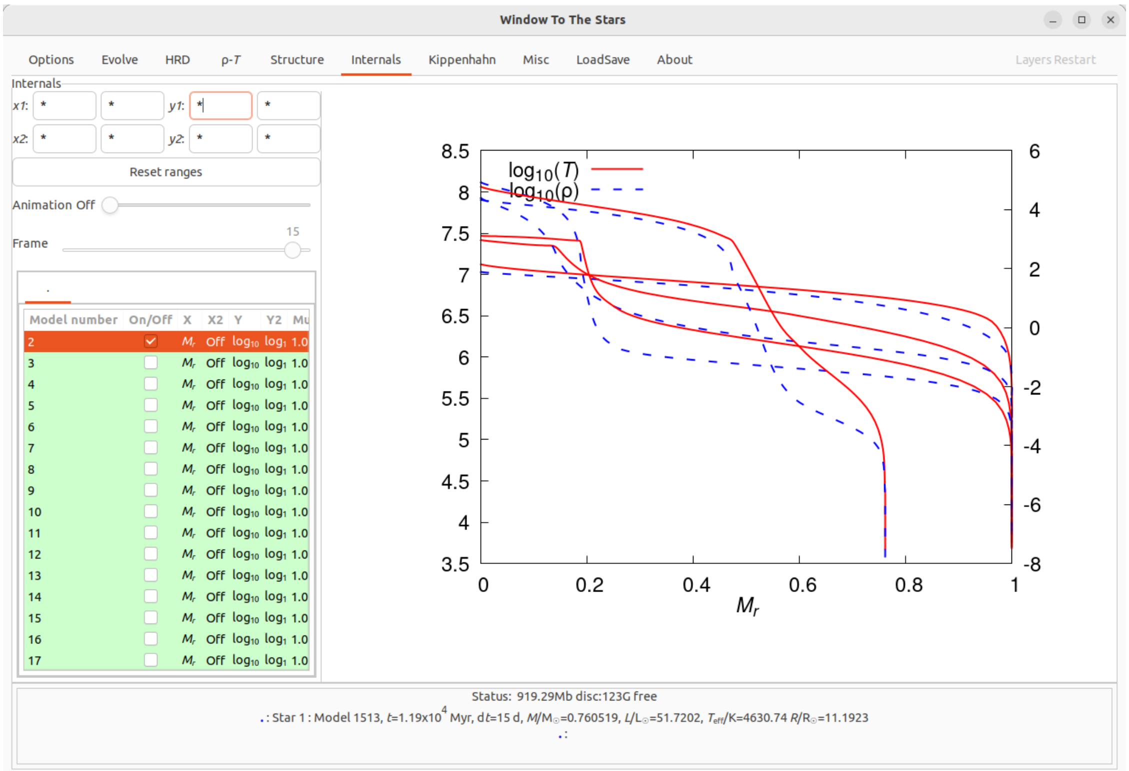

Figure 16: WTTS software, internals tab: series of 2D plots showing $\lg T/[\mathrm{K}]$ and $\lg \rho/[\mathrm{g\ cm}^3]$ against $m(r)/M_\odot$ for ever 500th model of a stellar model sequence of a star with $M = 1\,M_\odot$ and metallicity $Z = 0.02$.

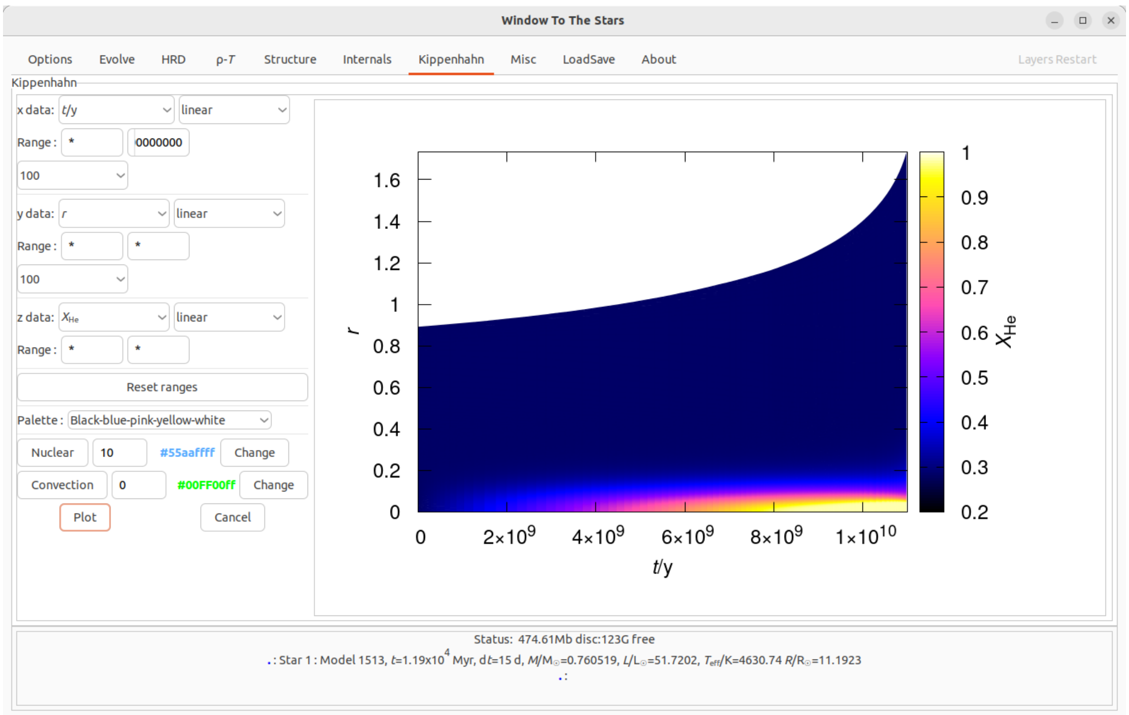

Figure 17: WTTS software, Kippenhahn tab: mass fraction of helium X_{He} plotted against stellar radius $r/R_\odot$ and time $t/[\mathrm{y}]$ for a stellar model sequence with $M = 1\,M_\odot$, $Z = 0.02$. As the radius expands significantly during the red giant phase t is restricted to $t < 1.1\,10^{10}$ y.

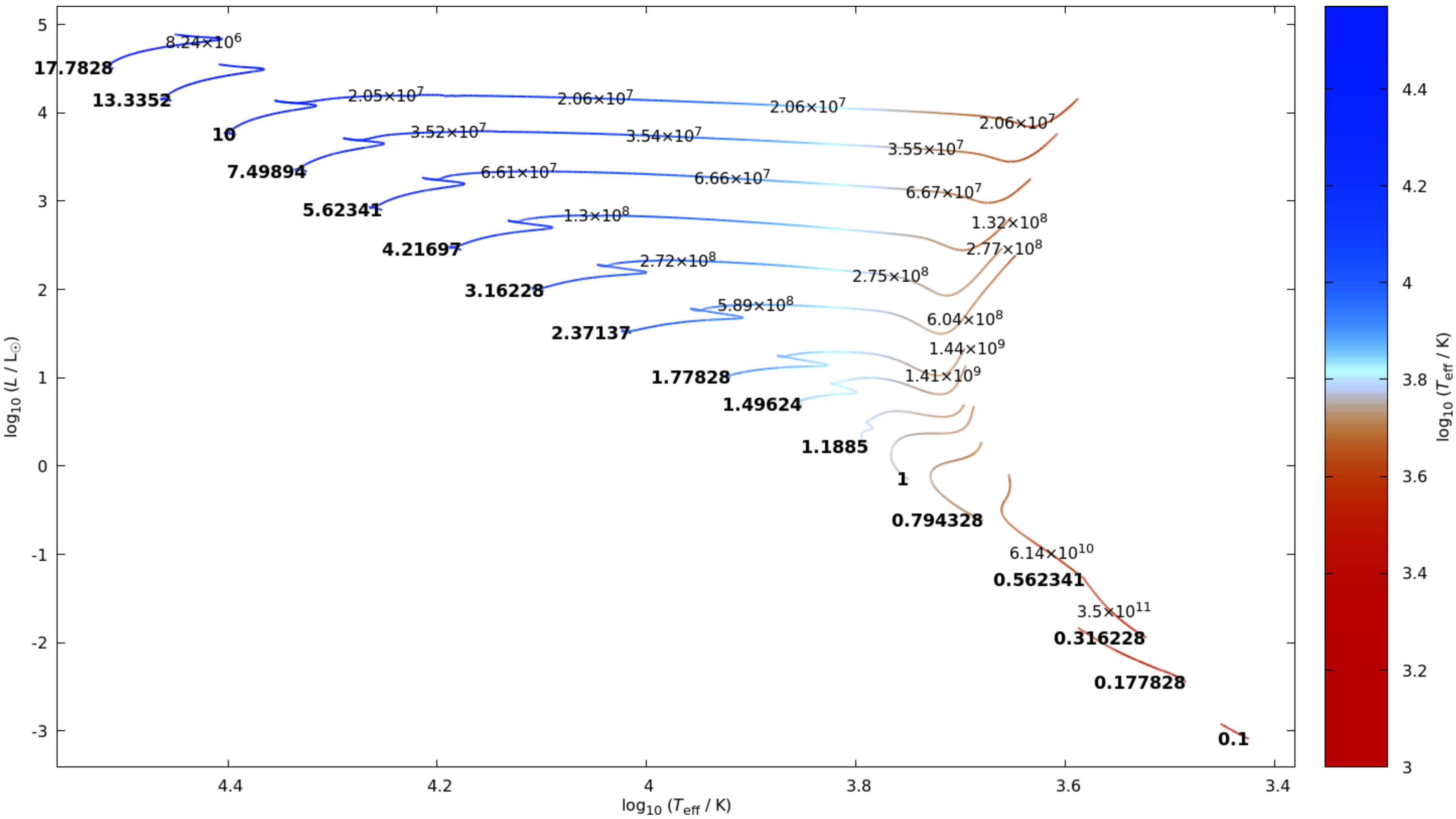

Figure 18: Double logarithmic plot of relativ luminosity $\lg L/L_{\odot}$ against surface temperature $\lg T_{\text{eff}}/[\text{K}]$ with inverted temperature axis showing a series of stellar model sequences for solar metallicity $Z = 0.02$ and different masses. Labels indicate the stellar mass at the begin of the sequence. Stellar ages are indicated along the sequences. The line colour qualitatively represents the observed colour of the stellar models.

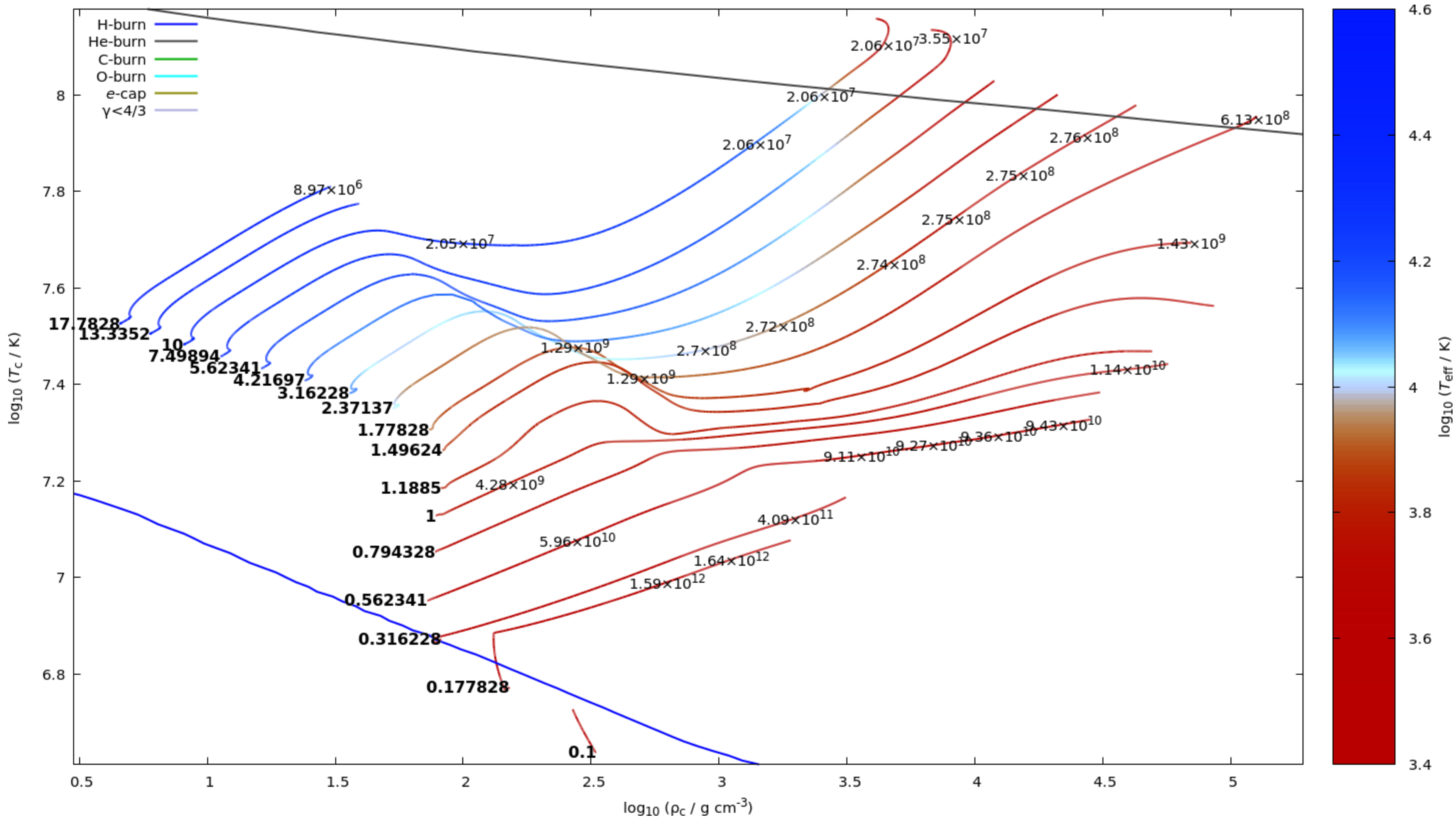

Figure 19: Double logarithmic plot of central temperature $\lg T_c/[\mathrm{K}]$ against central density $\lg \rho_c/[\mathrm{g\ cm}^3]$ showing a series of stellar model sequences for solar metallicity $Z = 0.02$ and different masses. Labels indicate the stellar mass at the begin of the sequence. Stellar ages are indicated along the sequences. The line colour qualitatively represents the observed colour of the stellar models.

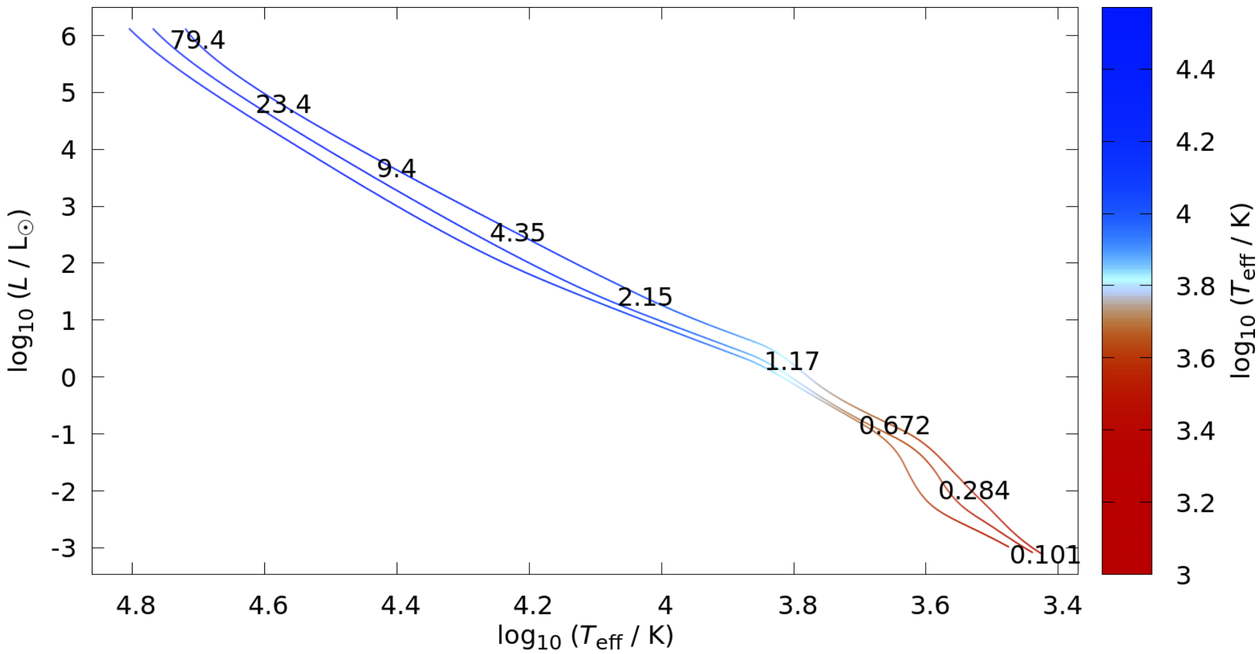

Figure 20: Double logarithmic plot of relativ luminosity $\lg L/L_\odot$ against surface temperature $\lg T_{\mathrm{eff}}/[\mathrm{K}]$ with inverted temperature axis showing ZAMS runs for different metallicities ($Z = 10^{-5}$, $Z = 10^{-3}$, $Z = 10^{-2}$ from left to right). Labels indicate the stellar mass for $Z = 0.01$. The line colour qualitatively represents the observed colour of the stellar models.

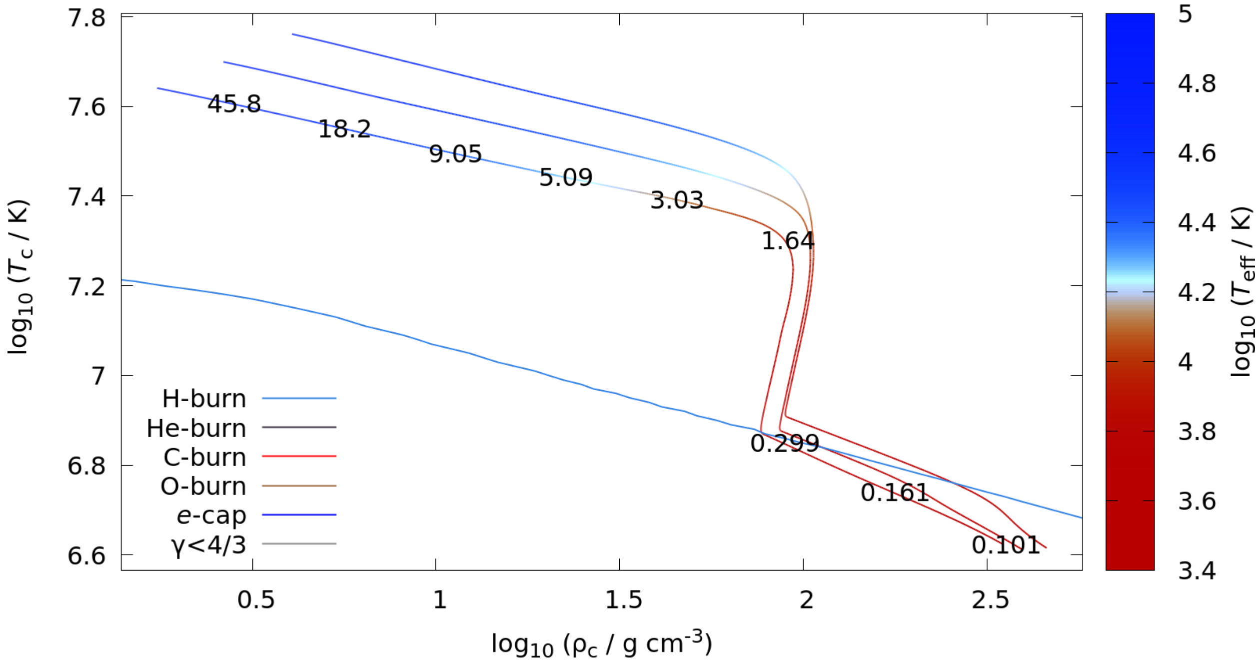

Figure 21: Double logarithmic plot of central temperature $\lg T_{\mathrm{c}}/[\mathrm{K}]$ against central density $\lg \rho_{\mathrm{c}}/[\mathrm{g\ cm}^3]$ showing ZAMS runs for different metallicities ($Z = 10^{-2}$, $Z = 10^{-3}$, $Z = 10^{-5}$ from left to right). Labels indicate the stellar mass for $Z = 0.01$. The line colour qualitatively represents the observed colour of the stellar models.

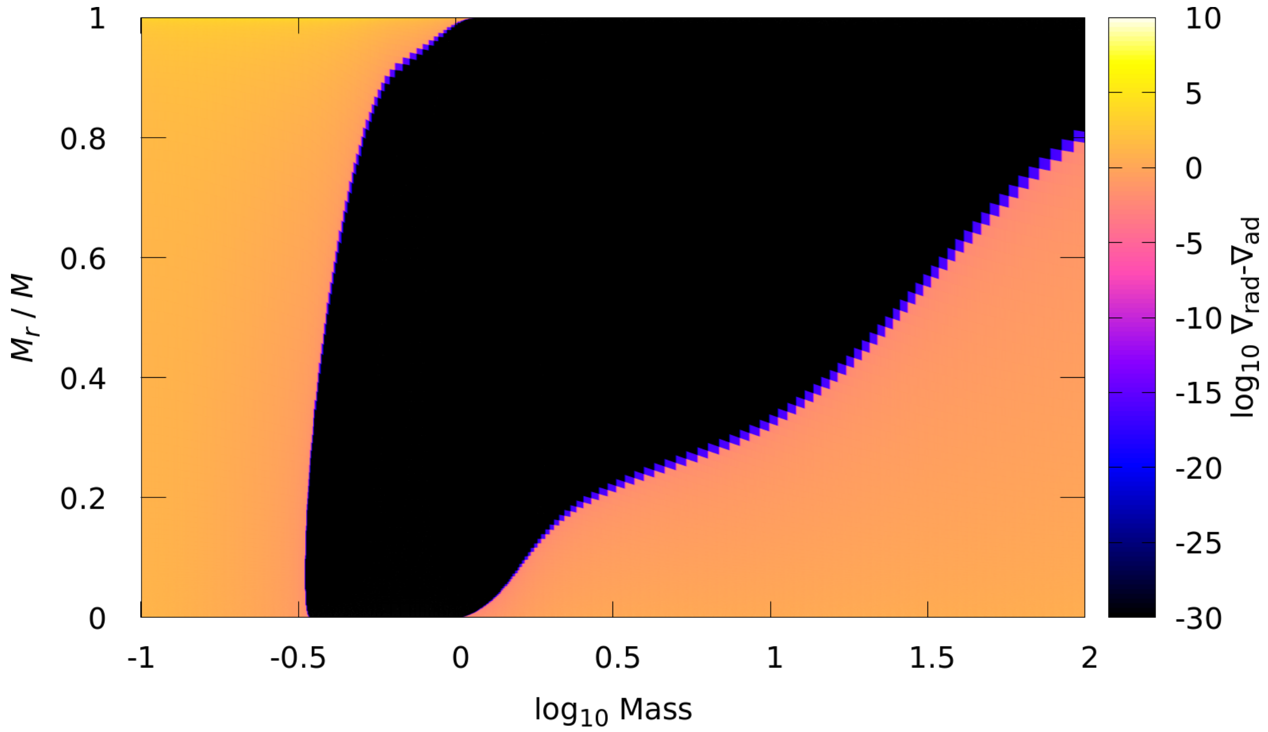

Figure 22: $\lg\left(\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}\right)$ plotted against $m(r)/M$ and $\lg M/M_{\odot}$ for a ZAMS run with $Z = 10^{-2}$. Black regions indicate stable energy transport by radiation.

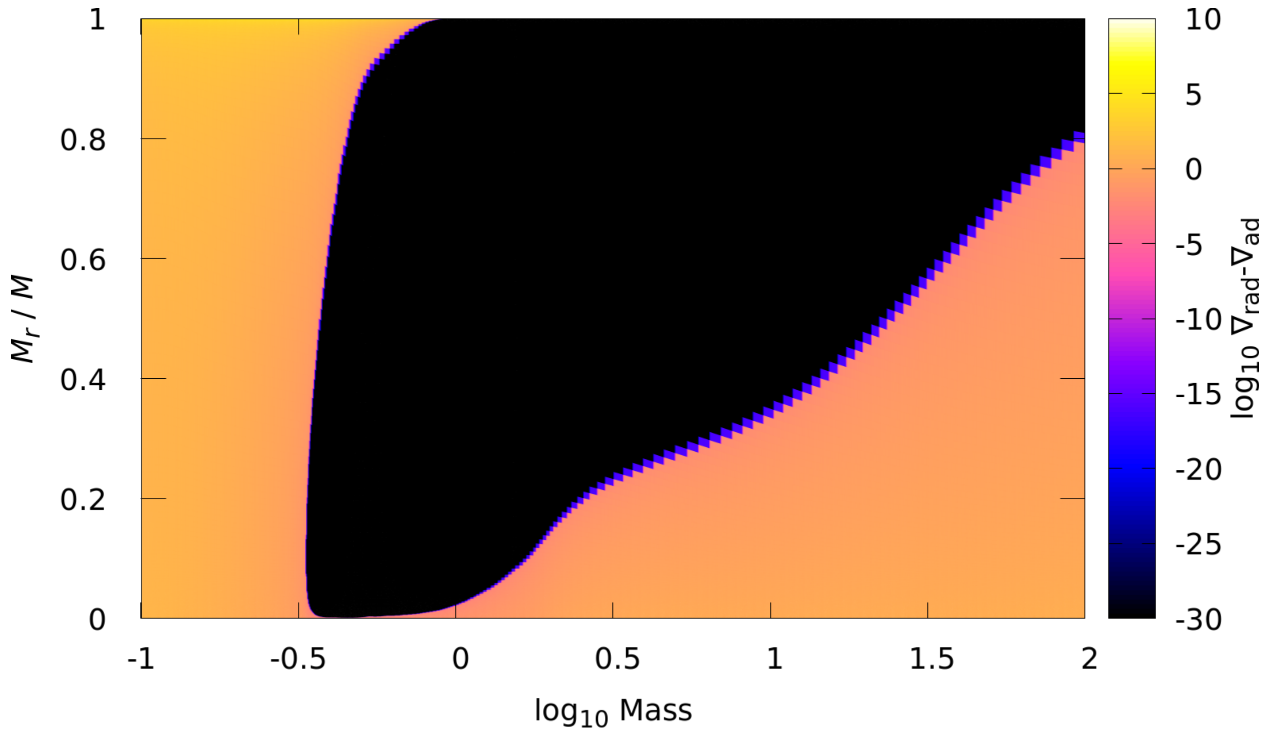

Figure 23: $\lg\left(\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}\right)$ plotted against $m(r)/M$ and $\lg M/M_{\odot}$ for a ZAMS run with $Z = 10^{-3}$. Black regions indicate stable energy transport by radiation.

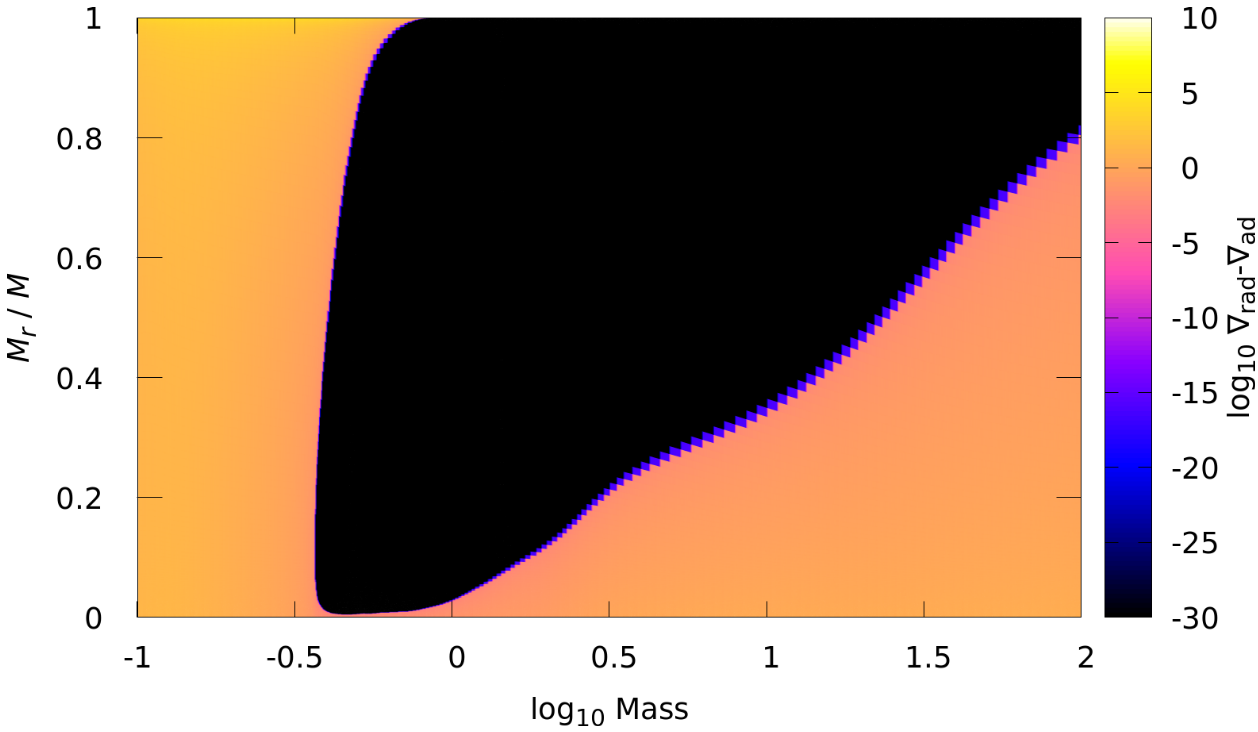

Figure 24: $\lg\left(\nabla_{\rm rad} - \nabla_{\rm ad}\right)$ plotted against $m(r)/M$ and $\lg M/M_\odot$ for a ZAMS run with $Z = 10^{-5}$. Black regions indicate stable energy transport by radiation.

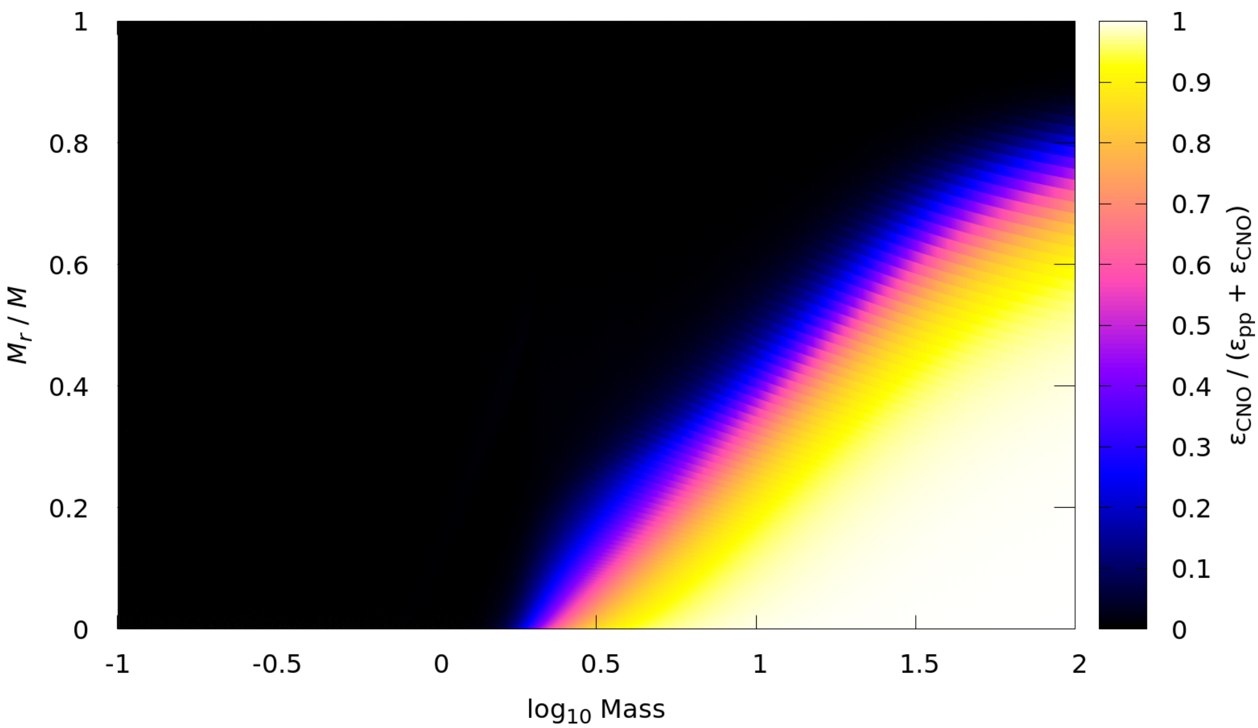

Figure 25: $\epsilon_{\rm CNO}/(\epsilon_{\rm pp} + \epsilon_{\rm CNO})$ plotted against $m(r)/M$ and $\lg M/M_\odot$ for a ZAMS with $Z = 10^{-3}$.

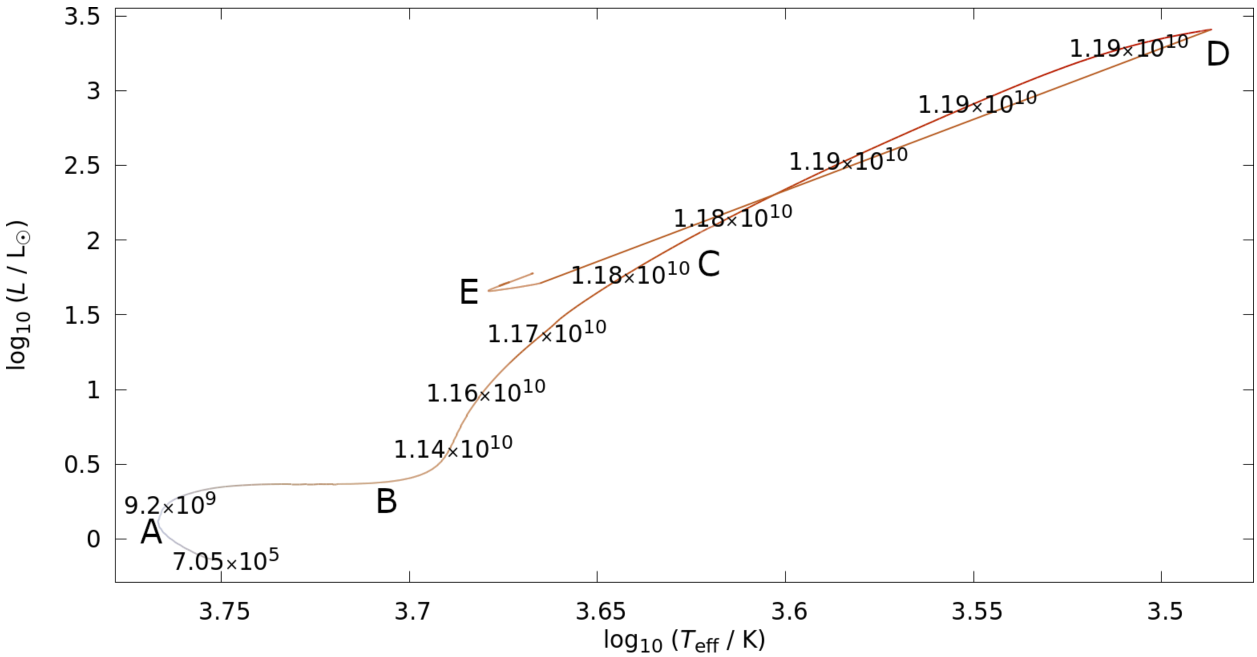

Figure 26: Double logarithmic plot of relativ luminosity $\lg L/L_{\odot}$ plotted against surface temperature $\lg T_{\rm eff}/[{\rm K}]$ with inverted temperature axis showing the evolution for a stellar model sequence with $M = 1\,M_{\odot}$, $Z = 0.02$. Labels indicate the stellar age. The line colour qualitatively represents the observed colour of the stellar models. Annotations show specific stages of evolution.

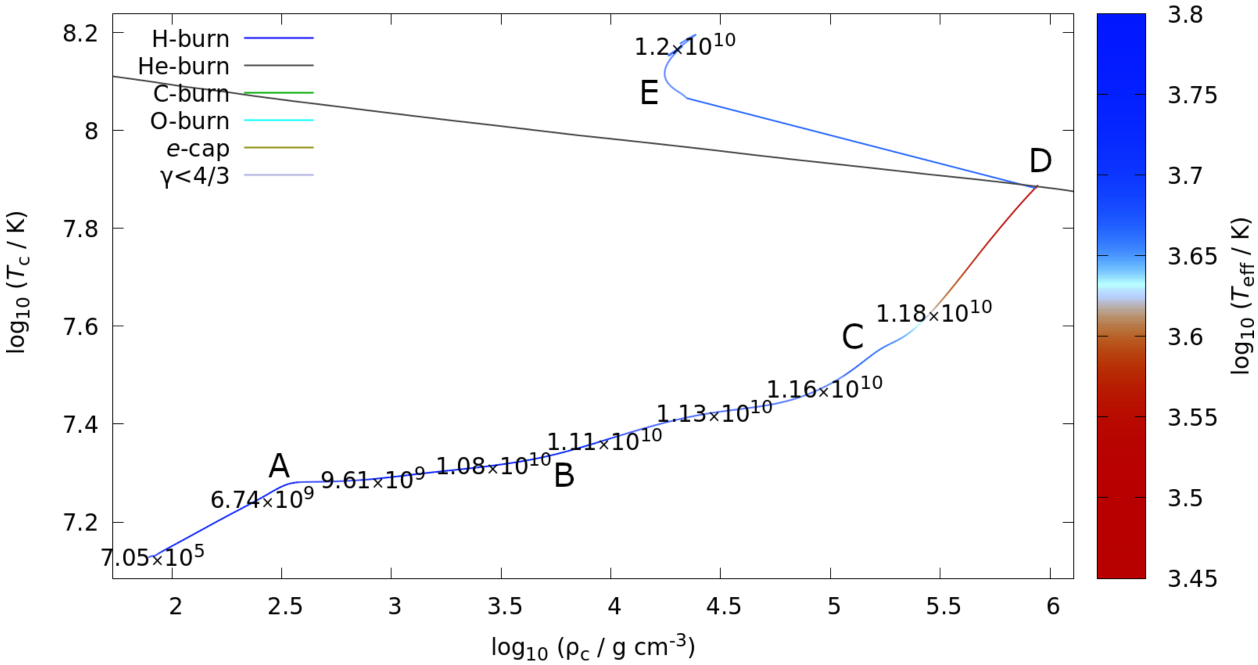

Figure 27: Double logarithmic plot of central temperature $\lg T_{\rm c}/[{\rm K}]$ against central density $\lg \rho_{\rm c}/[{\rm g\ cm}^{3}]$ for a stellar model sequence with $M = 1\,M_{\odot}$, $Z = 0.02$. Labels indicate the stellar age. The line colour qualitatively represents changes in the observed colour of the stellar models. Annotations show specific stages of evolution.

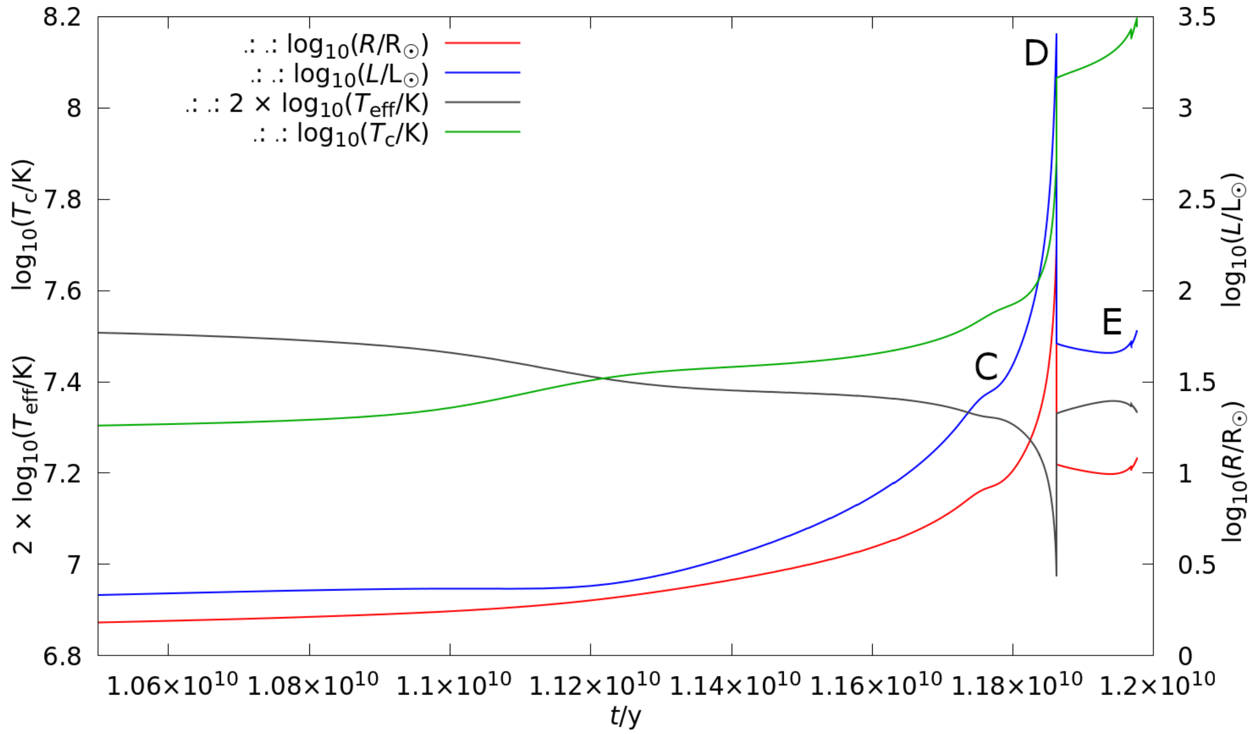

Figure 28: $\lg R/R_{\odot}$ (red, right y-axis), $\lg L/L_{\odot}$ (blue, right y-axis), $2 \lg T_{\text{eff}}/[\text{K}]$ (black, left y-axis) and $\lg T_{\text{c}}/[\text{K}]$ plotted against time $t/[\text{y}]$ for a stellar model sequence with $M = 1\,M_{\odot}$, $Z = 0.02$. Annotations show specific stages of evolution.

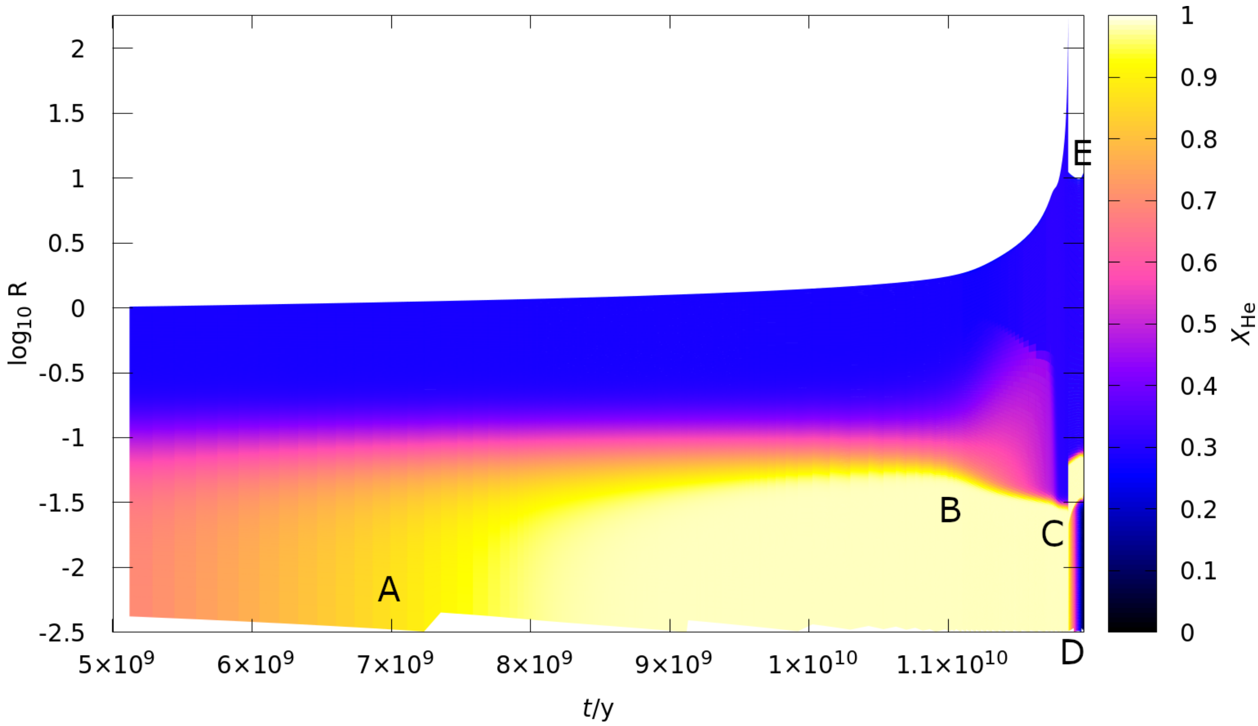

Figure 29: Mass fraction X_{He} plotted against $\lg R/R_{\odot}$ and time $t/[\text{y}]$ for a stellar model sequence with $M = 1\,M_{\odot}$, $Z = 0.02$. Annotations show specific stages of evolution.

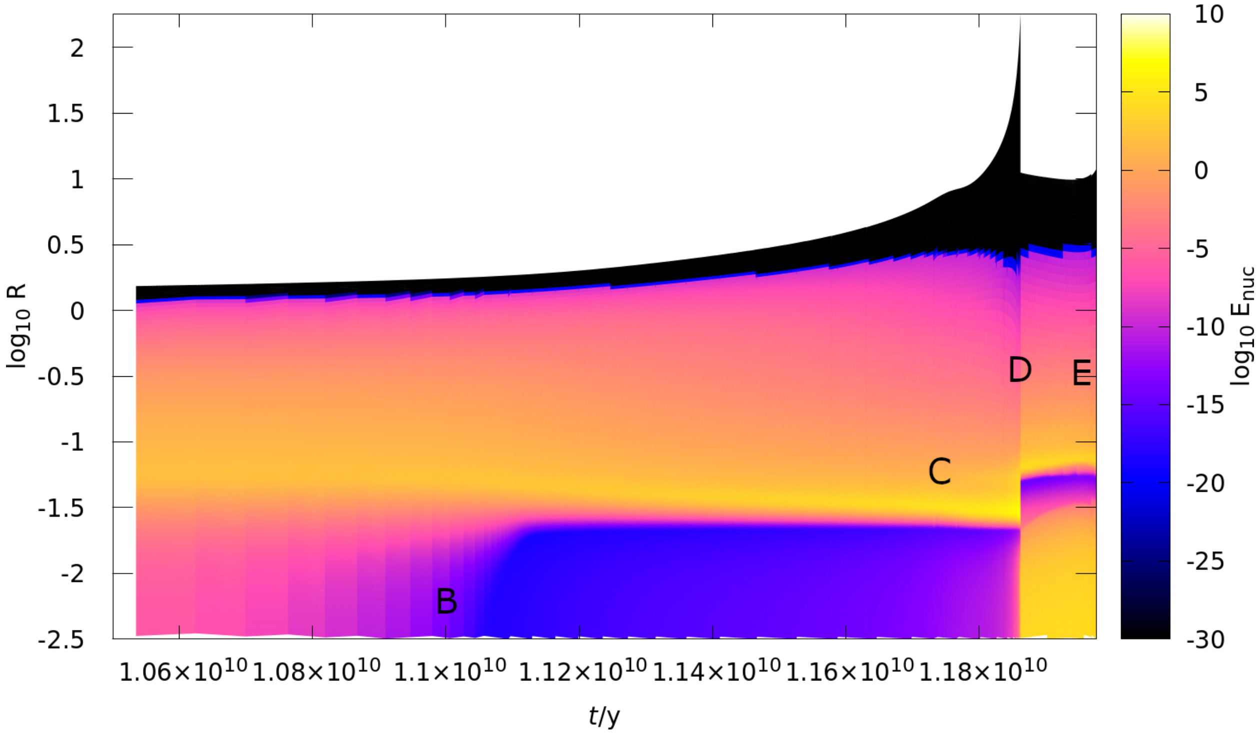

Figure 30: Energy release rate per unit mass ϵ in code units plotted against $\lg R/R_\odot$ and time $t/[\mathrm{y}]$ for a stellar model sequence with $M = 1\,M_\odot$, $Z = 0.02$. Annotations show specific stages of evolution.

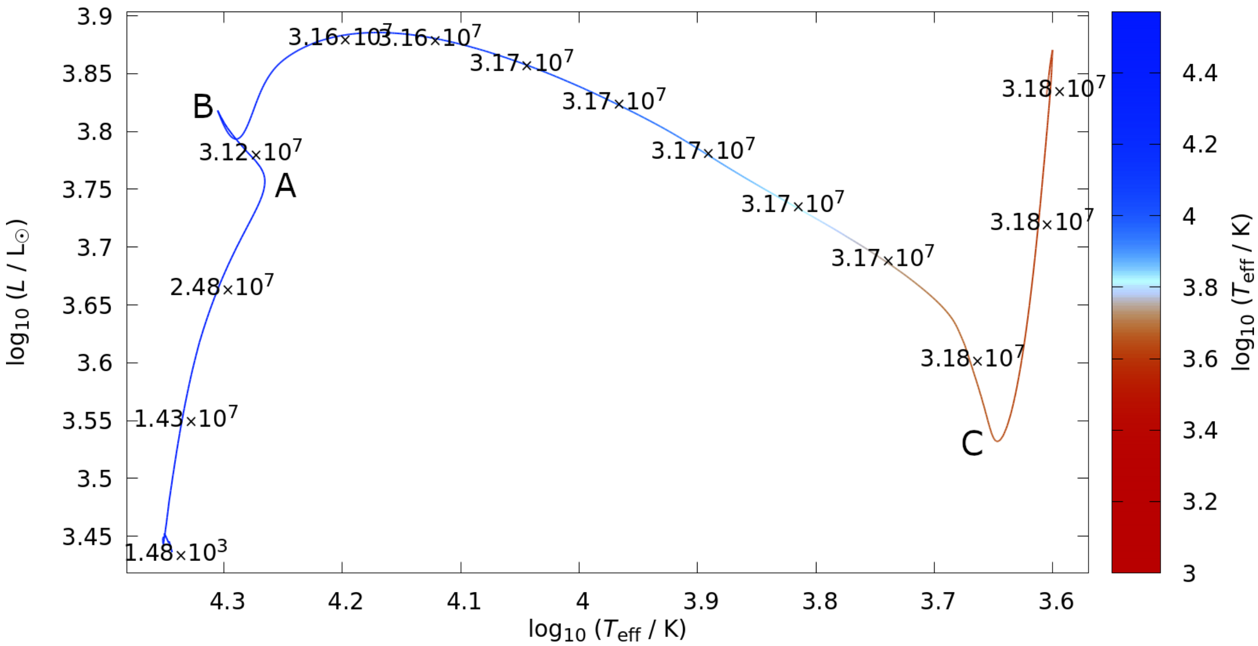

Figure 31: Double logarithmic plot of relativ luminosity $\lg L/L_\odot$ plotted against surface temperature $\lg T_{\text{eff}}/[\mathrm{K}]$ with inverted temperature axis showing the evolution of a stellar model sequence with $M = 8\,M_\odot$, $Z = 0.02$. Labels indicate the stellar age. The line colour qualitatively represents the observed colour of the stellar models.

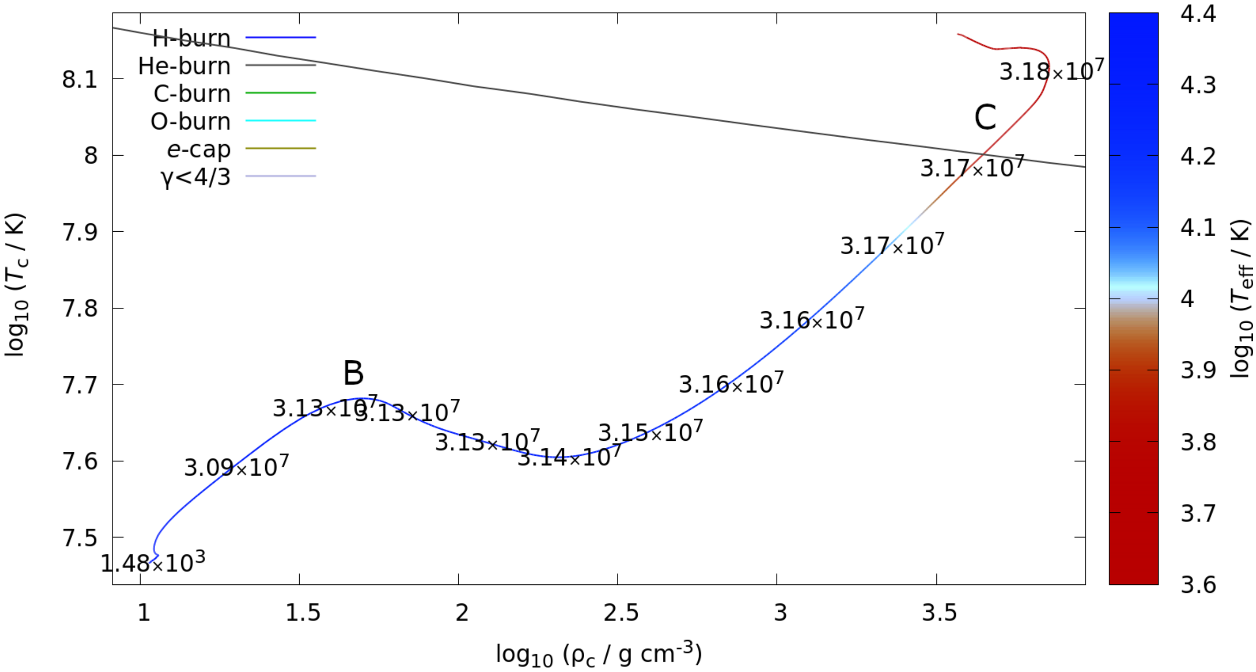

Figure 32: $\lg R/R_\odot$ (red, left y-axis), $\lg L/L_\odot$ (blue, left y-axis) and $\lg T_{\rm eff}/[{\rm K}]$ (black, right y-axis) plotted against time $t/[{\rm y}]$ for a stellar model sequence with $M = 8\,M_\odot$, $Z = 0.02$.

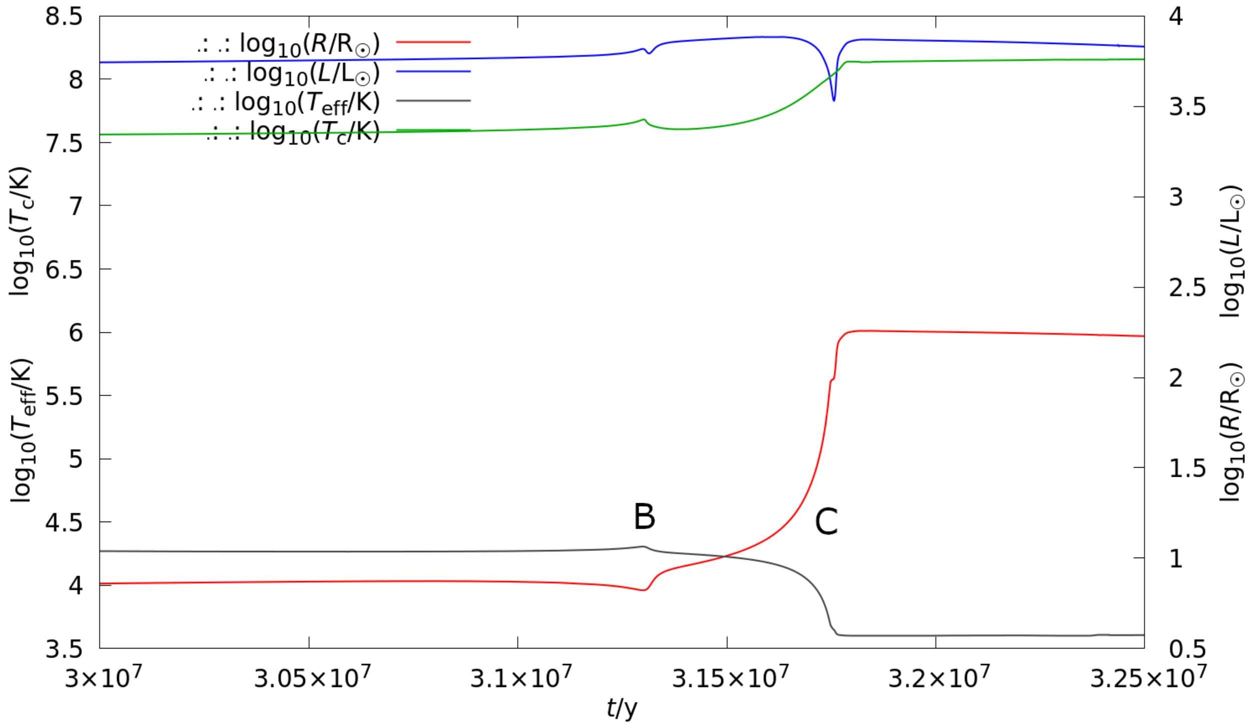

Figure 33: $\lg R/R_\odot$ (red, right y-axis), $\lg L/L_\odot$ (blue, right y-axis), $\lg T_{\rm eff}/[{\rm K}]$ (black, left y-axis) and $\lg T_{\rm c}/[{\rm K}]$ plotted against time $t/[{\rm y}]$ for a stellar model sequence with $M = 8\,M_\odot$, $Z = 0.02$. Annotations show specific stages of evolution.

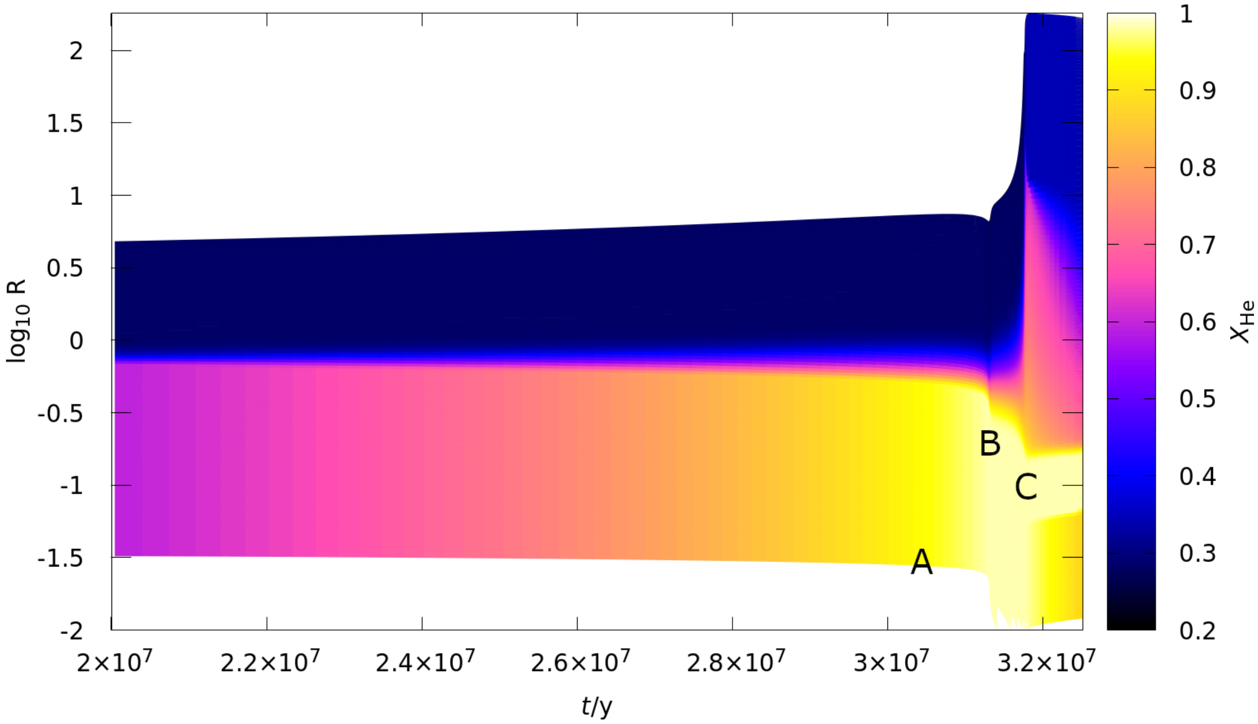

Figure 34: Mass fraction X_{He} plotted against $\lg R/R_{\odot}$ and time $t/[\mathrm{y}]$ for a stellar model sequence with $M = 8\,M_{\odot}$, $Z = 0.02$. Annotations show specific stages of evolution.

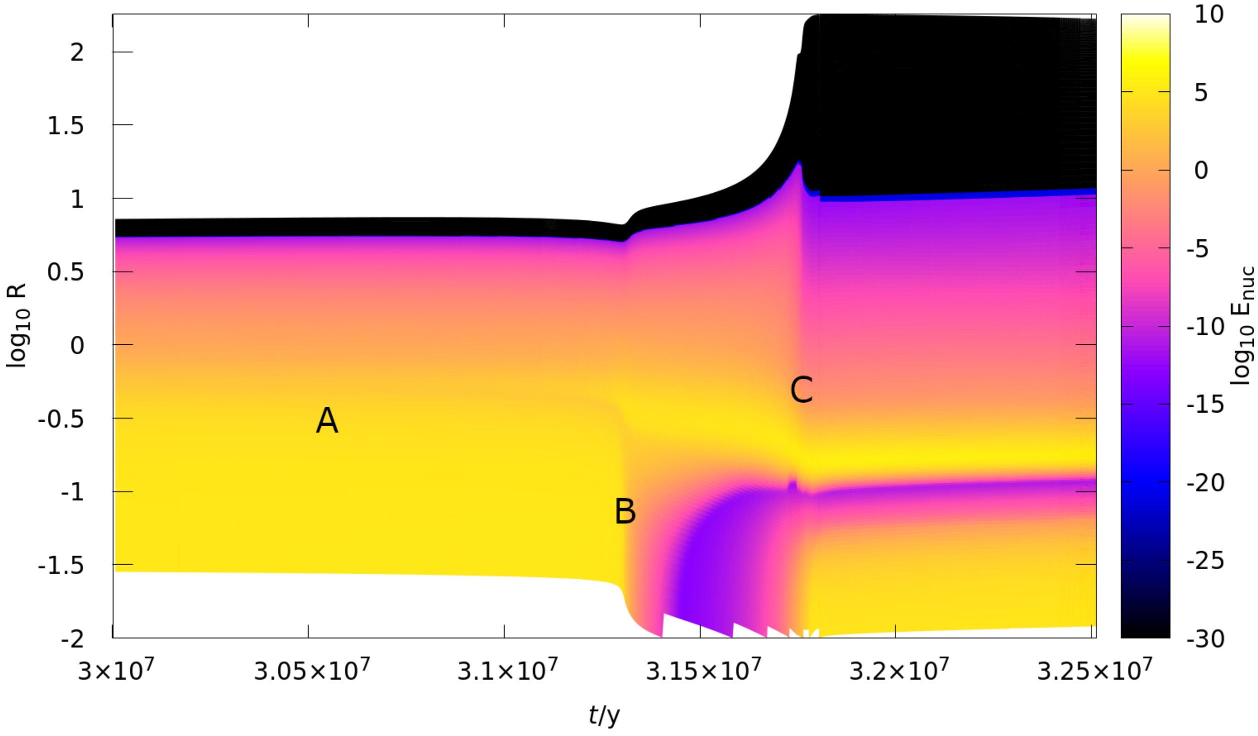

Figure 35: Energy release rate E_{nuc} in code units plotted against $\lg R/R_{\odot}$ and time $t/[\mathrm{y}]$ for a stellar model sequence with $M = 8\,M_{\odot}$, $Z = 0.02$. Annotations show specific stages of evolution.

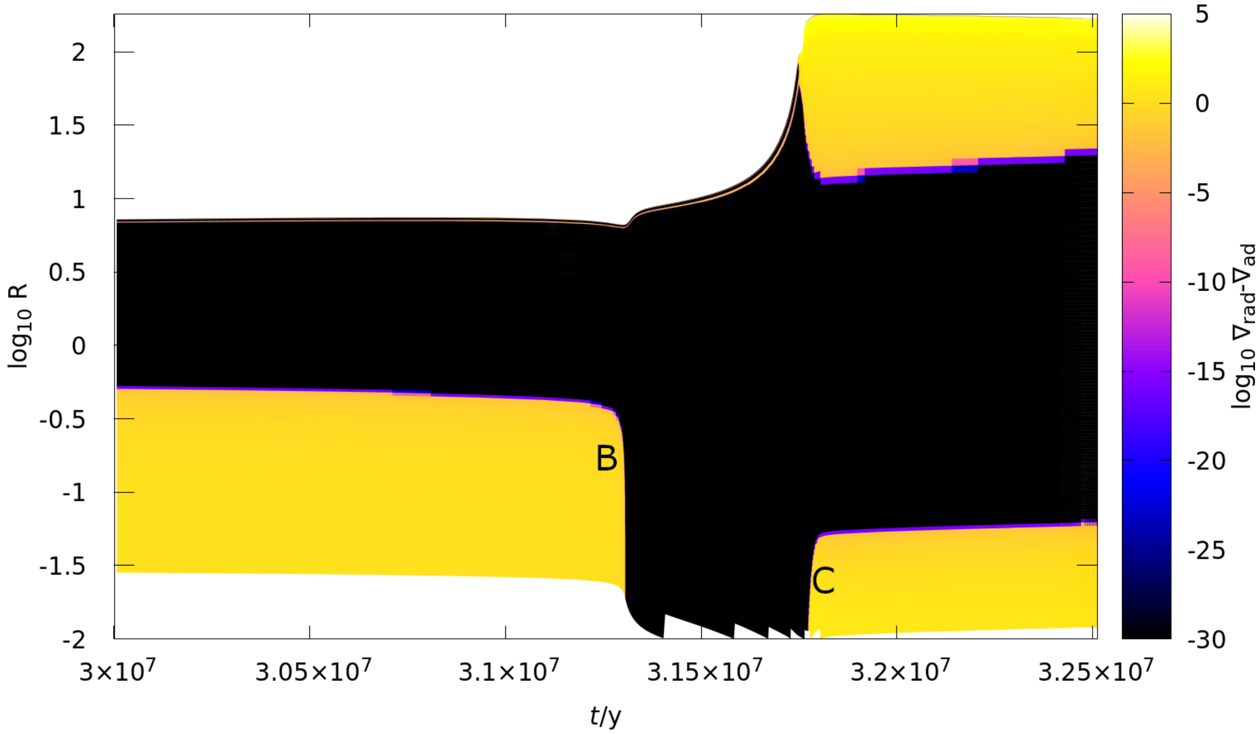

Figure 36: $\lg\left(\nabla_{\mathrm{rad}} - \nabla_{\mathrm{rad}}\right)$ plotted against $\lg R/R_\odot$ and time $t/[\mathrm{y}]$ for a stellar model sequence with $M = 8\,M_\odot$, $Z = 0.02$. Annotations show specific stages of evolution.

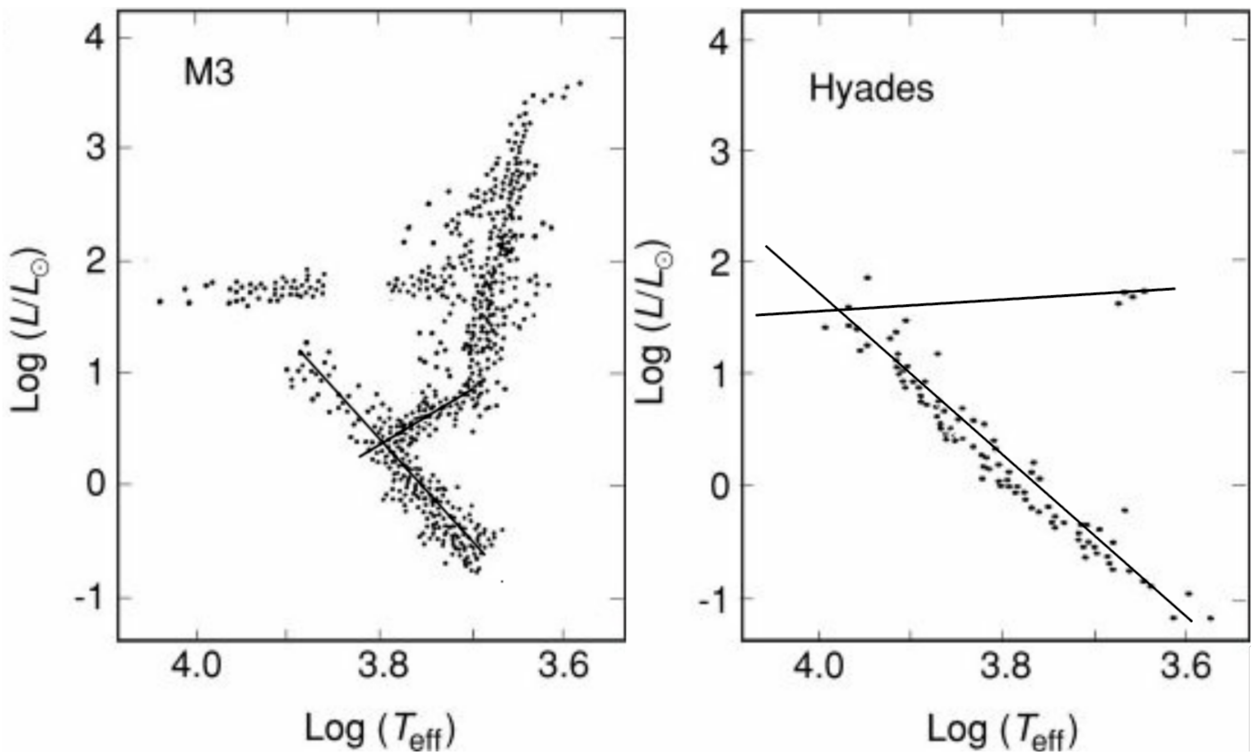

Figure 37: HR diagram for two stellar clusters: M3 and the Hyades, taken from [Pri10]. Lines are drawn as guide to the eye to indicate the turn-off points from the MS, from which an age estimate can be derived.

Workshop Materials

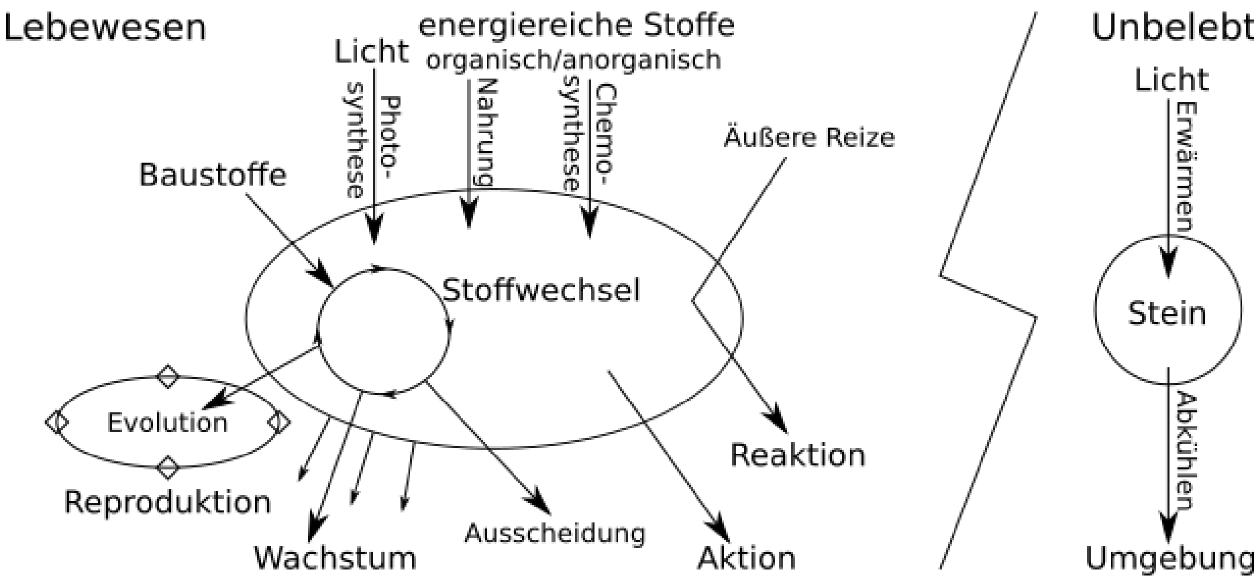

Figure 38: A simple illustration of typical processes and abilities for life forms in contrast to those for non-living objects.

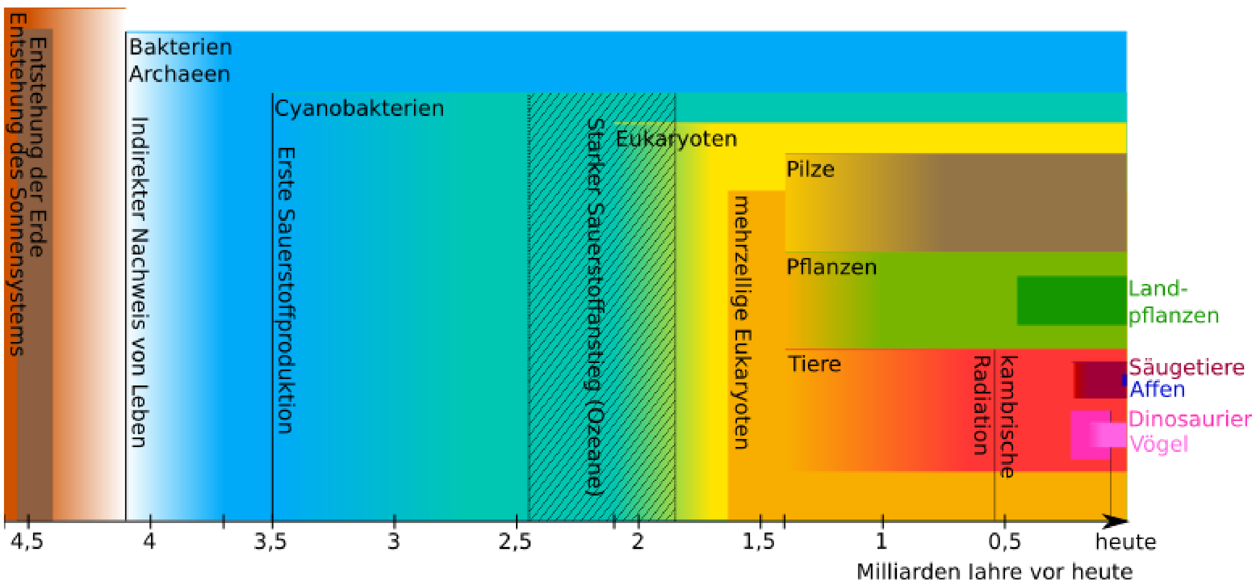

Figure 39: Evolution of different life forms from simplemost life to apes with time in billions of years before today is shown; several very relevant events are included, such as the formation of the solar system, rise of oxygen concentrations in oceans, etc.; shaded regions indicate regions of uncertainty originating from different forms of evidence, typically the onset refering to indirect evidence/methods, while the end off the shaded area refers to direct forms of evidence; uncertainties within the individual methods and findings are not included.[Wan99] [New23] [Kut04]

Verschiedene Lebensformen: Gruppe A

Deinococcus Radiodurans (Radiophil)

Bei Experimenten zur Sterilisierung von Konserven mittels hoher Dosen radioaktiver Strahlung wurde 1956 in den USA ein extrem strahlungsresistentes Bakterium entdeckt, welches diesen Prozess überstand. Es erhielt später die Bezeichnung Deinococcus Radiodurans und ist ein recht großes Bakterium, welches das 2000-fache einer für Menschen 100/50/und Säuren und hat in einem Versuch 3 Jahre an der Außenseite der ISS überdauert. Deinococcus Radiodurans wurde in Staub, organischen Proben, in der Antarktis, in Kühlwasserbecken von AKWs, aber auch im menschlichen Darm nachgewiesen.[Jin19]
Als Energiequelle benötigt Deinococcus Radiodurans organisches Material ("Nahrung") und Sauerstoff. Der genaue Entstehungszeitpunkt dieser Bakterien ist nicht bekannt, es wird aber vermutet, dass sich verwandte Bakterien vor mehreren hundert Millionen Jahren in Umgebungen mit hoher natürlicher Strahlung gebildet haben.[Jin19]

Physarum Polycephalum (Schleimpilz)

In den 1970er Jahren stellte man fest, dass der bereits seit Jahrhunderten bekannte Schleimpilz Physarum Polycephalum einen Verbund aus einer Vielzahl von Einzellern bilden und in dieser Form wie ein einziger Organismus agieren und komplexe Probleme (z.B. das Finden des kürzesten Weges durch ein Labyrinth) lösen kann, ohne überhaupt ein Nervensystem zu besitzen. Physarum Polycephalum kann sich kriechend fortbewegen und gewinnt dabei Energie aus organischer Nahrung. Wissenschaftler ließen den Schleimpilz das U-Bahn-Netz Tokios, eines der komplexesten der Welt, und auch die Vernetzung von Galaxien nachbilden, um seine Fähigkeit, möglichst effektive Vernetzungen zu bilden, zu demonstrieren. Der Organismus überlebt vor allem unter feuchten Bedingungen bei mäßiger Temperatur, z.B. in Wäldern oder verrottendem Holz und entstand bereits vor mehr als einer Millarde Jahren.[Wik24a]

Schleimpilz[Tim21]

Krähen (Rabenvögel)

Krähen findet man in städtischen, ländlichen und wäldlichen Gebieten. Sie zeigen außergewöhnliche Problemlösungsstrategien, komplexes Sozialverhalten, den Gebrauch von Werkzeugen und können durch reine Beobachtung lernen und abstrahieren. Beispielsweise wurden Krähen dabei beobachtet, Nüsse auf Straßen fallen zu lassen, wo diese von Autos geknackt werden, um die Nuss während einer roten Ampelphase dann aufzusammeln und zu verspeisen. Krähen erkennen sich außerdem im Spiegel selbst und haben also eine Art Bewusstsein von sich selbst. Rabenvögel sind Allesfresser und ernähren sich von Samen, Früchten, Insekten und sogar Aas. Sie benötigen ausreichend Nahrungs- und Trinkwasserangebot, Temperaturen zwischen 0 und 35 °C und sind recht anpassungsfähig. Rabenvögel entwickelten sich vor etwa 17 Millionen Jahren.[Wik24b]

Verschiedene Lebensformen: Gruppe B

Pyrolobus Fumarii (Hyperthermophil)

1997 wurden an einem sogenannten schwarzen Raucher (eine Quelle am Meeresboden, ähnlich einem kleinen Vulkan, aus der heisses Wasser austritt, welches reich an Eisenverbindungen ist) sogenannte Archaeen isoliert. Das sind einzellige Organismen ähnlich Bakterien, aber mit einer anderen Zellmembran. Diese wachsen bei Temperaturen um 100 Â°C, bei 200-300 bar Druck in völliger Dunkelheit unter Sauerstoffauschluss und wurden Pyrolobus Fumarii genannt. Pyrolobus Fumarii gehört damit zu den hitzeresistentesten bekannten Lebensformen. Später wurde diese Art auch an schwarzen Rauchern in anderen Gebieten gefunden.
Als Energiequelle dient energiereicher Wasserstoff (H2), der aus schwarzen Rauchern austritt und den es mit Nitrat oder Schwefel reduzieren kann (Chemosynthese). Der Entstehungszeitraum liegt vermutlich etwa 3,5 Millarden Jahre zurück, als die Erdoberfläche noch sehr heiss und hydrothermale Umgebungen sehr verbreitet waren. Es gab lange die Vermutung, dass Lebensformen um schwarze Raucher den Ursprung des Lebens auf der Erde bilden.[Wik24c]

Bakteriophagen

Zu Beginn des 20. Jahrhunderts fanden Wissenschaftler in Stuhlproben einen Organismus, der in der Lage war bestimmte Bakterien effektiv zu zerstören. Sie isolierten und identifizierten diesen als ein Virus, welches Bakterien befällt und nannten es Bakteriophage. Es gibt viele Arten von Bakteriophagen, die jeweils auf eine bestimmte Bakterienart spezialisiert sind. Sie wurden während des zweiten Weltkriegs insbesondere seitens der Sowjetunion zur Behandlung von bakteriellen Infekten eingesetzt. Heutzutage werden Bakteriophagen in der Lebensmittelindustrie verwendet und aktuell als Alternative zu Antibiotika wegen zunehmender Resistenzbildung erwogen und erforscht.
Bakteriophagen haben keine eigene Energiegewinnung, sondern nutzen dafür die Bakterien, die sie Befallen. Sie entstanden vermutlich bereits kurz nach dem Aufkommen der Bakterien vor über 3 Millarden Jahren. Bakteriophagen kommen in allen Lebensräumen vor, in denen Bakterien vorkommen.[Wik24d]

Oktopoden (Kraken)

Oktopoden leben in marinen Umgebungen wie Korallenriffen, aber auch Tiefseegräben. Sie zeigen ein bemerkenswertes Problemlösungsvermögen, inklusive Verwenden von Werkzeugen und Manipulieren von Objekten (z.B. öffnen von Schraubverschlüssen) und können durch Beobachtung lernen und abstrahieren. Oktopoden sind Fleischfresser und Jäger und benötigen neben sauberem, sauerstoffreichem Wasser je nach Art Temperaturen zwischen 0 und 30 Â°C. Ihre wirbellosen Körper sind sehr flexibel und ermöglichen es ihnen kleinste Verstecke und Passagen zu nutzen, sowie großem Außendruck standzuhalten (Tiefsee). Oktopoden entwickelten sich vor etwa 300 Millionen Jahren.[Wik24e]

Verschiedene Lebensformen: Gruppe C

Halobacterium Salinarum (Halophil)

Zu Beginn des 20. Jahrhunderts wurden Forscher auf die intensive, rote Färbung einiger Salzseen in der Küstenregion des San Francisco Bay aufmerksam und vermuteten einen bakteriellen Ursprung, trotz lebensfeindlich hoher Salzkonzentrationen von 20-30(einfache Lebensform, ähnlich Bakterien, aber mit anderer Zellwand) isolieren, welches Halobacterium Salinarum genannt wurde und nur bei entsprechend hohen Salzkonzentrationen überleben kann. Das auffällige, rote Pigment dient dem Organismus zur Energiegewinnung aus Licht, wobei energiereiche, organische Verbindungen gebildet werden. Dieser Prozess ist aber von der Photosynthese der Pflanzen zu unterscheiden. Halobacterium Salinarum entstand vermutlich vor etwa 2 bis 3 Millarden Jahren.[Wik24f]

Tardigraden (Bärtierchen)

Tardigrade[Gar20]

Im späten 18ten Jahrhundert entdeckte ein deutscher Zoologe in Wasserproben kleine Mehrzeller, die sich mit 8 winzigen Beinchen laufend fortbewegen. Wegen ihrer sehr unbeholfenen Gangweise nannte er sie Bärtierchen. Bärtierchen leben in einer Vielzahl von Umgebungen, wie Moosen, Salz- und Süßwasser. In den 1950er Jahren fiel auf, dass Bärtierchen ihren Stoffwechsel massiv herunterfahren und spezifische Proteine bilden können, was es ihnen erlaubt völlige Austrocknung, große Hitze und extreme Kälte zu überstehen. So konnten Bärtierchen, die in einer Probe in Japan über 30 Jahre lang eingefroren waren, wieder aktiviert werden und waren noch fortpflanzungsfähig. 2007 schickte man sie außerdem zu Forschungszwecken ins All, wo sie Vakuum, extremen Temperaturen und Strahlung trotzten und überlebten. Bärtierchen gibt es seit mindestens 500 Millionen Jahren. Ihren Energiebedarf decken sie, indem sie sich von Pflanzenzellen, Algen und Mikroorganismen ernähren.[Wik24g]

Ratten (Nagetiere)

Ratten sind sehr anpassungsfähig und kommen in städtischen Gebieten, Wäldern, Feldern und ähnlichem vor - insbesondere auch überall, wo Menschen leben. Sie sind sozial interaktiv, zeigen bemerkenswerte Lern- und Gedächtnisfähigkeit, Sozialstrukturen und insbesondere sehr gute Orientierung (z.B. in Labyrinthen). So können Ratten lernen, bestimmte Standorte oder Objekte mit Belohnungen zu verknüpfen (episodisches Gedächtnis) und aus Erfahrungen lernen, auf Gegenseitigkeit basierende soziale Interaktionen aufbauen und haben Strategien entwickelt, vergiftete Nahrungsquellen zu identifizieren und zu meiden. Sie sind Allesfresser, benötigen moderate Temperaturen und entstanden vor etwa 12 Millionen Jahren.[Wik24h]

Verschiedene Lebensformen: Gruppe D

Acidithiobacillus Ferrooxidans (Acidophil)

Zu Beginn des 20. Jahrhunderts wurde die Umweltbelastung durch den Bergbau zunehmends problematisch. Aus Bergwerken traten große Mengen stark saurer Grubenabwässer aus, welche die Umgebung verseuchten und das Grundwasser gefährdeten. Bei Untersuchungen der Grubenabwässer in den 1940ger Jahren fiel auf, dass trotz des hohen pH-Wertes von 1-2 noch Bakterien in diesen vorkamen. Im Weiteren stellte man fest, dass diese nicht nur in sauren Gewässern überleben können, sondern die Übersäuerung der Grubenabwässer mitbedingen, indem sie Eisen- und Schwefelverbindungen oxidieren, wobei Schwefelsäure entsteht. Man nannte diese Bakterien Acidithiobacillus Ferrooxidans und ihre Entdeckung trug dazu bei, die Umweltbelastung durch den Bergbau zu verstehen und zu begrenzen.
Acidithiobacillus Ferrooxidans gewinnt Energie aus energiereichen, anorganischen Verbindungen und damit über Chemosynthese. Es wird vermutet, dass es vor etwa 2,5 Millarden Jahren entstand, als die Erde noch von sauerstoffarmen, sauren Bedingungen bestimmt wurde.[Wik24i]

Nematoden (Fadenwürmer)

Im 17. Jahrhundert entdeckte man unter anderem kleine, wurmartige Lebewesen in Wasserproben, die sich aktiv bewegten und nannte sie Nematoden. Im 18. und 19. Jahrhundert stellte sich heraus, dass es unzählige Arten dieser Spezies gibt und sie in diversen Umgebungen zu finden sind - von Tiefseegräben, über die Arktis, Böden bis hin zum Inneren von Pflanzen und Tieren. Mittlerweile ist klar, dass dies der artenreichste Tierstamm überhaupt ist und etwa 80/Permafrostboden Nematoden isoliert und durch Auftauen aktiviert werden. Sie waren nach 30-40.000 Jahren im gefrorenen Zustand noch lebens- und reproduktionsfähig. Der Nematode C. Elegans wurde seit dem späten 20. Jahrhundert zu einem der best untersuchten Modellorganismen im Bereich der Genetik.

Nematode[Gro02]

Nematoden ernähren sich meist von Pilzen, Algen und anderen Mikroorganismen, es gibt aber auch zahlreiche parasitäre Arten, die Nährstoffe von einem Wirtstier/Wirtspflanze beziehen. Vorfahren heutiger Nematoden entstanden vor etwa 400 Mio. Jahren.[Wik24j]

Delfine (Zahnwale)

Delfine findet man weltweit in Ozeanen, Flüssen und Küstengewässern. Sie sind bekannt für komplexe soziale Strukturen, sprachähnliche Kommunikation, zeigen komplexe Problemlösungsstrategien und erkennen sich selbst in einem Spiegel - haben also eine Art Bewusstsein von sich selbst. Sie scheinen außerdem etwas wie Empathie zu besitzen. Delphine sind Fleischfresser, Jäger und benötigen moderate Temperaturen (10-25 Â°C) sowie sauberes, sauerstoffreiches Wasser. Die modernen Zahnwale, zu denen Delfine gehören, entstanden vor etwa 10 Millionen Jahren.[Wik24k]

Figure 40: Mars surface, taken by Perseverance rover (2021); additional information given was: atmosphere: very thin, mostly CO_2, traces of water; surface: high level of radiation, temperature −125 to +20 °C, no liquid water; underground: frozen water detected, temperature above 0 °C expected in several km depth; history: billions of years ago probably significantly more moderate and warm, liquid water in rivers and oceans speculated.[Bea89]

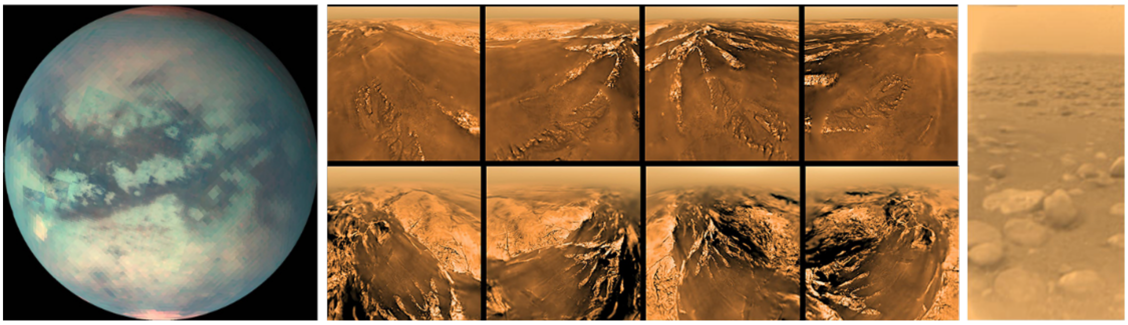

Figure 41: Titan as seen from the Cassini spacecraft (left) and its surface as seen while decent of Huygens lander (middle) and after landing (right); additional information given was: atmosphere: thick, mostly N_2 with methan/ethan clouds and methan cycle including rain, complex organic nitrogen-/carbon molecules detected; surface: temperature around −179 °C, covered in ice, lakes and rivers of methan/ethan; underground: probably ocean of water/ammonium, active kryo vulcanos speculated.[Bea89]

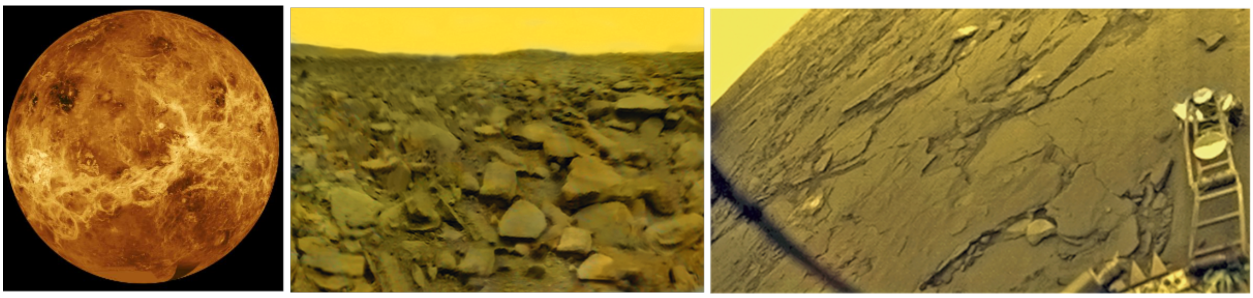

Figure 42: Venus surface as detected by Magellan probe by radar (left) and seen on ground by Venera 9 (middle) and Venera 14 landers (right); additional information given was: atmosphere: very thick, mostly CO_2, clouds of sulfric acid, traces of water; surface: high level of radiation, temperature around +460 °C, pressure around 90 bar, no liquid water; underground: extremly hot and dry, active vulcanism; history: billions of years ago possibly significantly more moderate, liquid water and plate tectonics speculated.[Bea89]

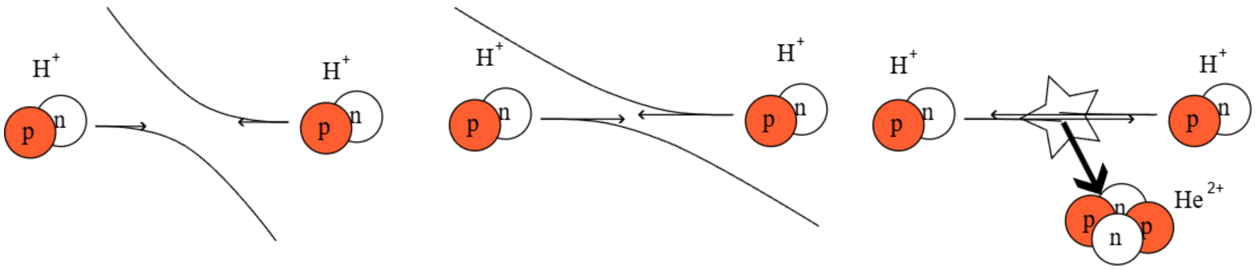

Figure 43: A simplified sketch of the fusion of 2 deuteron to helium: ${}^2_1$d + ${}^2_1$d → ${}^4_2$He to illustrate density and temperature dependence of fusion processes.

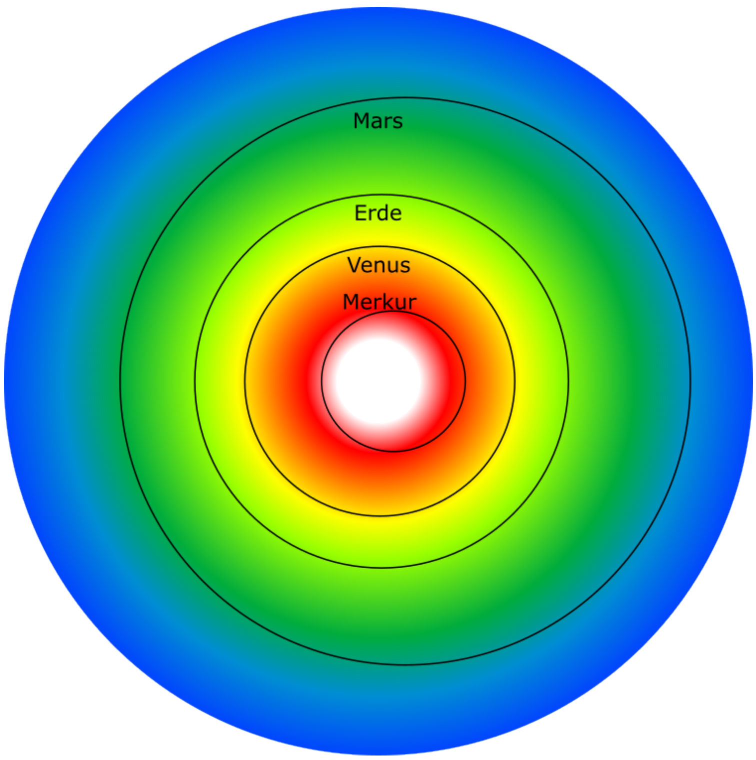

Figure 44: Habitable zone of the Sun with orbits of Mercury, Venus, Earth and Mars.[PFS24] For illustrative purpose the aphels are aligned. The different shades follow estimates for the habitable zone as follows: generally habitable zone (green), possibly habitable zone for specific atmosphere composition and density (dark green/yellow), excluded by all models are the blue and red zone. The colour gradients areodot chosen such, that 50% colour adjustment occurs from the most conservative estimate to the least conservative for a given scenario.[Kop13][Spi10][Vla13][Zso13]

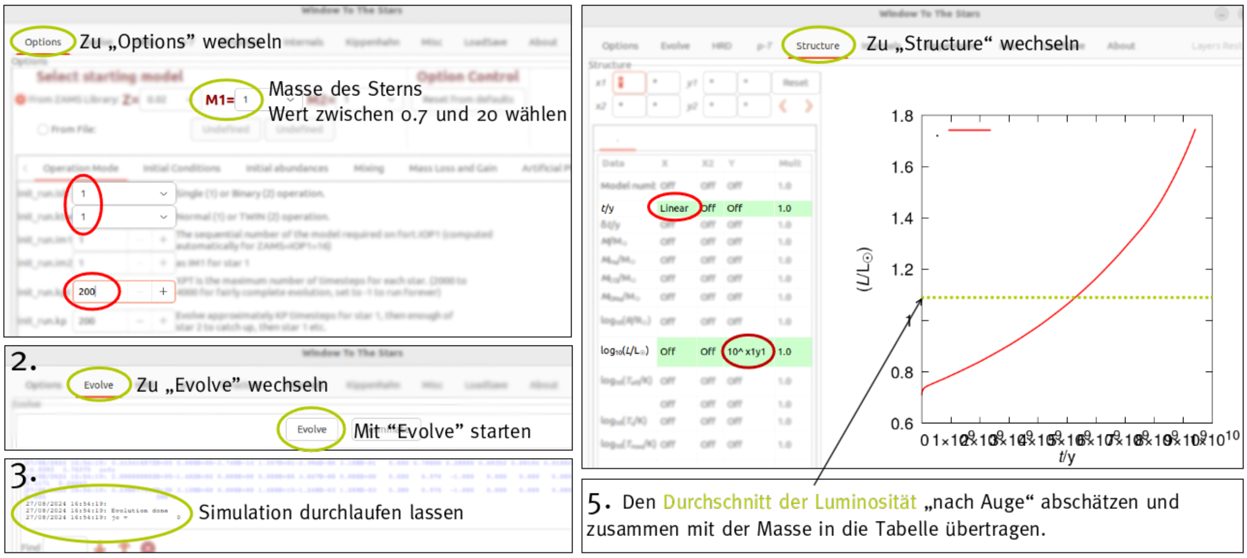

Figure 45: Work flow when using WTTS to simulate stars with different masses M to find an estimate for their time averaged luminosities $\bar{L}$ calculate the life times. The steps are: i) choose a M, ii) start the simulation, iii) let the simulation finish, iv) average the luminosity 'by eye', v) document mass M and averaged luminosity $\bar{L}$ in a table.

Experiments

Figure 46: A beaver with tap water is partly placed on a heat source. Food dye indicating a convective current of warm water rising above the heat source (left) and a circular flow is seen from the side (right).

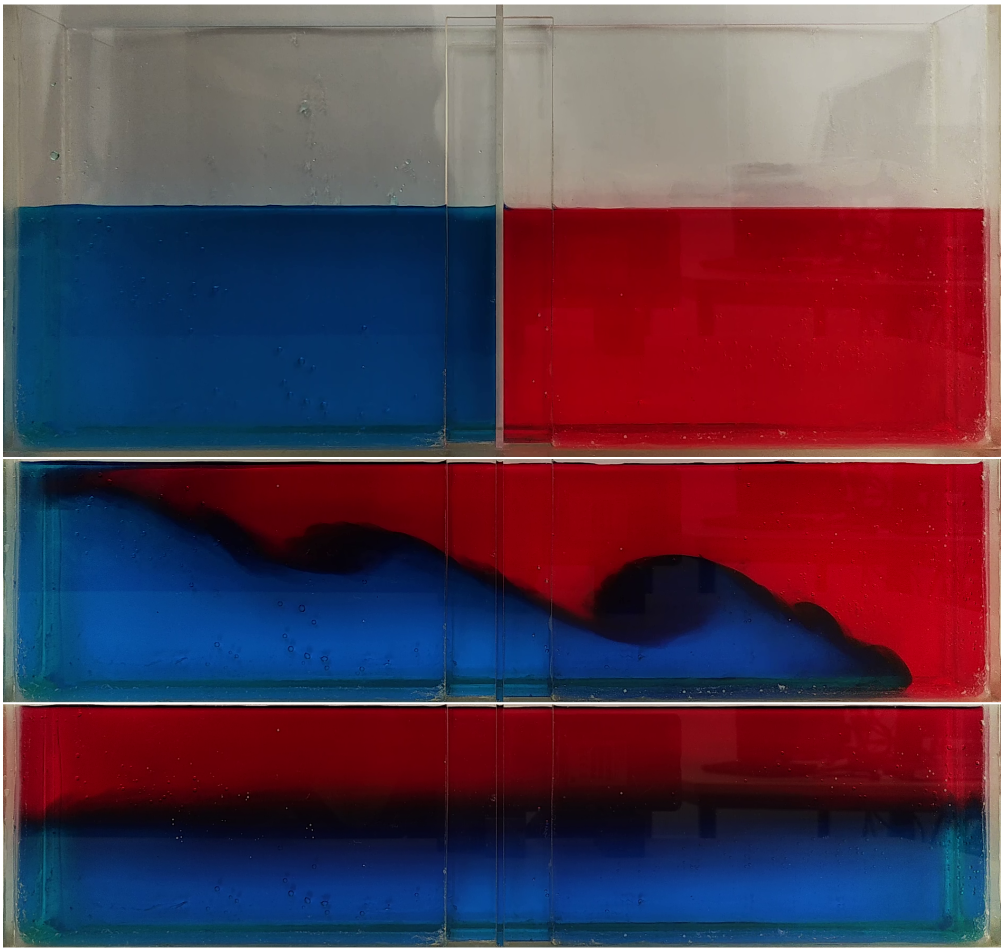

Figure 47: Hot tap water (red) and cold tap water (blue) mixing in a container at 0 s (top), 5 s (middle) and 2 min 20 s (bottom), resulting in a rearrangement by convection.

Google Forms Questionnaire

Feedback: Workshop Sterne und Leben

Dies ist eine Umfrage, um Teilnehmende des Workshops “Sterne und Leben" die Möglichkeit zu geben, Feedback, Kritik, Anregungen etc. zu geben. Die Teilnahme an der Umfrage ist vollständig freiwillig und anonym. Die Ergebnisse werden ausschließlich zur Evaluation und Verbesserung des Workshops verwendet und könnten als solches in der Masterarbeit von Jona Dreier verwendet werden. Bei Fragen kontaktiert mich gerne über jona.dreier@uni-muenster.de

Ich habe etwas Neues gelernt.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Die Inhalte des Workshops konnte ich gut verstehen.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Der Workshop hat mir Spaß gemacht.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Ich habe NEUE Einblicke in die Wissenschaft bekommen.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Ich glaube, dass ich gut in Physik bin.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Naturwissenschaftliche Themen begeistern mich.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Ich glaube, ich bin für Physik völlig ungeeignet.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Nenne ein Highlight und ein Lowlight von heute. Hier hast du auch Platz für sonstige Anmerkungen.

. . . .

Der einleitende Vortrag und die Handouts zu Leben und Lebewesen sind verständlich und nachvollziehbar.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Der einleitende Vortrag und die Handouts zu Leben und Lebewesen sind interessant.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Was hast du Neues oder Spannendes über Leben und Lebewesen erfahren.

. . . .

Ich wurde für die Diskussion über Leben auf anderen Planeten/Himmelkörpern gut und ausreichend informiert.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Die Diskussion über Leben auf anderen Planeten/Himmelkörpern war interessant.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Was hast du Neues oder Spannendes über mögliches Leben auf anderen Planeten /Himmelkörpern erfahren?

. . . .

Der Vortrag zu Entstehung und Eigenschaften von Sternen ist verständlich und nachvollziehbar.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Der Vortrag zu Entstehung und Eigenschaften von Sternen ist interessant.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Was hast du Neues oder Spannendes über die Entstehung und Eigenschaften von Sternen erfahren?

. . . .

Die Experimente zu Konvektion haben zu meinem Verständnis von Sternen beigetragen.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Die Experimente zu Konvektion sind interessant.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Es ist nachvollziehbar und klar, warum eine Simulationssoftware verwendet wurde.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Das Bedienen der Simulationssoftware hat mir Probleme bereitet.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Das Arbeiten mit der Simulationssoftware ist interessant.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Bitte beschreibe Deinen Gesamteindruck von der Simulationssoftware.

. . . .

Die abschließende Auswertung der Simulationsergebnisse ist verständlich und nachvollziehbar.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Die abschließende Auswertung der Simulationsergebnisse ist interessant.

◯ Trifft zu. ◯ Trifft eher zu. ◯ Trifft eher nicht zu. ◯ Trifft nicht zu.

Bitte fasse das Fazit des Workshops mit Deinen eigenen Worten zusammen.

. . . .

Vielen Dank für Deine Teilnahme!

Grouping of Statements

	informative	interesting
The workshop was	informative/comprehensive	fun
I learned/gained	something new	insights into science
Presentation regarding life was	informative/comprehensive	interesting
The discussion (other worlds) was	well prepared & informed	interesting
Presentation regarding life was	informative/comprehensive	interesting
The convection experiments were	informative/comprehensive	interesting
The use of WTTS was	well motivated/comprehensive	interesting
Analysing the results was	informative/comprehensive	interesting

Table 6: Schematics of the statements used in the questionnaire for evaluation; the phrases given in the top row are in the following used to refer to the attributes as specified here.

Results for Full Workshop

unsuited for physics	1.00 ± 0.58
interested in science	2.29 ± 0.49
good in physics	1.71 ± 0.76

Table 7: Participants self perception of their physics and science skills and interest based on the google forms feedback for the full version of the workshop (students in their last year of highschool, 8 participants, 7 responses).

Content	informative	interesting
Total Workshop	2.29 ± 0.49	2.14 ± 0.38
I learned/gained	2.86 ± 0.38	2.71 ± 0.49
Presentation regarding life	2.57 ± 0.54	2.57 ± 0.54
Discussion (other worlds)	2.14 ± 0.38	2.14 ± 0.69
Presentation regarding life	2.29 ± 0.76	2.00 ± 0.82
Convection experiments	2.00 ± 0.82	2.71 ± 0.49
Use of WTTS	2.29 ± 0.95	1.57 ± 1.13
Analysing the results	2.43 ± 0.79	2.71 ± 0.49
Problems using software	0.87 ± 1.22	

Table 8: Participants impression of the workshop and its parts regarding the dimensions 'informative' and 'interesting' as explained in tab. 6 for the full version of the workshop (students in their last year of highschool, 8 participants, 7 responses).

Free Text Answers

Nenne ein Highlight und ein Lowlight von heute. Hier hast du auch Platz für sonstige Anmerkungen. (6 Antworten)
Highlight: Experimente mit Farben, Lowlight: Berechnung der Luminosität

Highlight waren die Experimente!

Highlight: Wasserexperiment, Lowlight: die Durchführung der Simulation der Lebensdauer.

Highlight: Experimente, Lowlight: Sonnenmassen berechnen

Highlight: Kalkulieren mit dem "Windiw to the Stars Programm", Lowlight: Temperaturen, teils fehlende Stringenz

WTTS2 simulation (highlight), es gab kein lowlight, alles war suprr interessant :))

Was hast Du Neues oder Spannendes uber Leben und Lebewesen erfahren? (7 Antworten)
Differenzierung von Lebewesen

Das sehr viele Aspekte ideal sein müsen, um inteligentes Leben möglich zu machen

Das es sehr anpassungsfähig Bakterien und sehr schlaue Lebewesen gibt, aber das man lange braucht um zu intelligenten Lebewesen zu kommen.

Das Leben auch in extremen Umgebungen wie auf Venus eventuell leben kann.

Wie selten unsere Sonne ist und dass Leben auf der Erde überhaupt möglich ist

So ziemlich alles war neu, da ich vorher noch keinen Kontakt zu dem Thema hatte.

Überlebunfschancen auf den Planeten, mathematische Berechnungen

Was hast Du Neues über mögliches Leben auf anderen Planeten/Himmelskörpern erfahren? (6 Antworten)
Hängt von vielen verschiedenen Faktoren ab

das es auf anderen Planeten auch Leben geben kann und das es vor Milliarden von Jahren dort vermutlich schon Leben gab.

-

Dass kleine Organismen sich selbst so extremen Bedingungen wie die, die auf der Venus herrschen anpassen können

Alles war neu.

informationen über den titan waren mir neu

Was hast Du Neues oder Spannendes über die Entstehung und Eigenschaften von Sternen erfahren? (6 Antworten)
Eigentlich alles darüber

Das je kleiner sie sind desto länger die Lebensdauer ist.

Das sie aus Wasserstoff entstehen.

Dass sie aus Wasserstoff bestehen

So ziemlich alles war neu, aber besonders interessant war, wie flexibel Leben in Bezug auf die Umweltlichen Bedingungen ist.

Simulation (wusste nicht, dass die Masse wichtig ist für die Berechnung der Luminosität)

Bitte beschreibe Deinen Gesamteindruck von der Simulationssoftware. (7 Antworten)
Interessant aber sehr komplex

Ein bisschen eintönig

Sie war gut erklärt und dadurch verständlich.

Interessant, es hat aber alles ein wenig lang gedauert.

Die Ergebnisse daraus ergeben im Kontext Sinn, aber das Bedienen und Anwenden der Software hätte man weg lassen können

Die Software ist an sich gut, allerdings hat die Adaption des für Linux gedachten Programms auf Windows teils erwas schlecht funktioniert.

sehr kompliziert im detail, aber mega interessant

Bitte fasse das Fazit des Workshops mit Deinen eigenen Worten zusammen. (7 Antworten)
Ich fande den Workshop gut auch wenn er sehr komplex war. Ich fande das Thema und das gelernte sehr interessant

Meiner Meinung nach sehr gut verständlich und interessant. Teilweise bestimmt noch ausbaufähig aber an sich schon sehr gut

Sehr interessant, viel Neues gelernt und ein gutes Level an Komplexität. Offen für Fragen! Und Sehr gute Materialien.

Es hat mir gefallen, vielleicht hätte man aber ein wenig mehr machen können.

Man lernt etwas über Sterne und Leben auf Planeten und es wird verständlich erklärt

An sich ein guter Workshop, allerdings könnten die Zusammenhänge zwischen Experimenten und dem theoretischen Inhalt besser beleuchtet werden. Zwar habe ich verstanden, was es mit den nicht nicht vermischenden und zirkulierenden auf sich hat, allerdings war ein bisschen unklar, was das jetzt in Bezug auf Sonnen aussagen soll.

Results for Workshop as Intervention

	Group A		Group B	
Content	informative	interesting	informative	interesting
Good/interested in physics/science	1.56 ± 1.00	1.75 ± 1.03	1.89 ± 1.10	1.89 ± 0.99
Total Workshop	2.38 ± 0.86	1.94 ± 0.75	2.56 ± 0.50	2.00 ± 0.67
Presentation 'Life'	2.38 ± 0.70	2.44 ± 0.61	2.38 ± 0.70	2.26 ± 0.43
Presentation 'stars'&experiments	2.13 ± 0.78	2.31 ± 0.58	1.78 ± 1.13	1.56 ± 0.83
Analysing the results	2.31 ± 0.77	2.19 ± 0.81	2.00 ± 0.87	2.11 ± 0.87

Table 9: Participants impression of the workshop and its parts regarding the dimensions 'informative' and 'interesting' as explained in tab. 6 for the shortened workshop (9th grade, 16 responses for 25 participants for group A, 9 responses for 27 participants for group B).

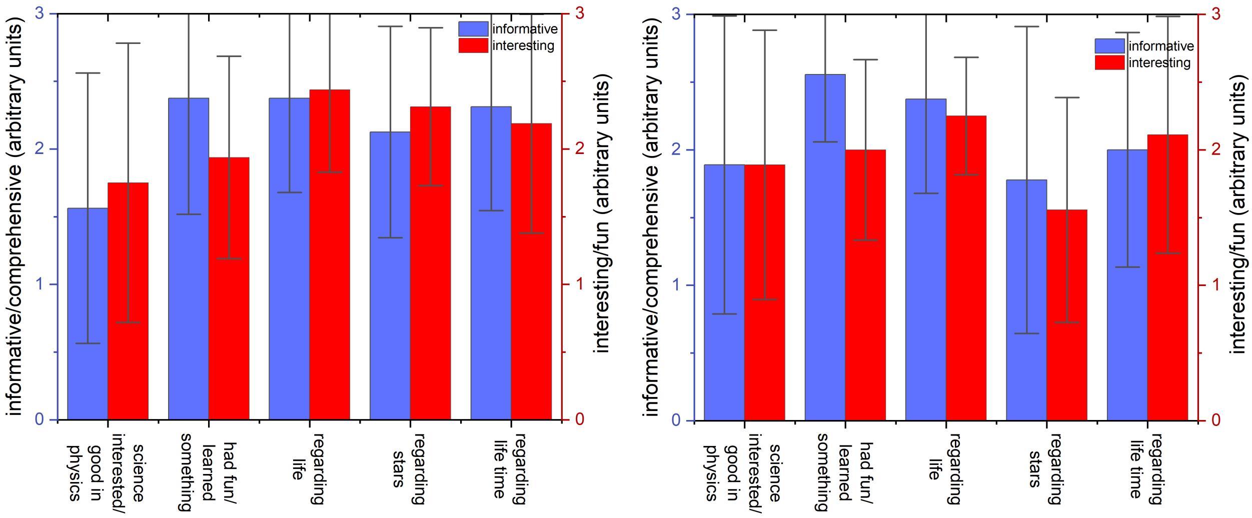

Figure 48: Results of the reduced questionnaire as shown in tab. 10 regarding the dimensions informative/comprehensive (blue) and interesting/fun (red) obtained 16 participants (9th grade, Group A) and 9 participants (9th grade, Group B) of a 90 minute intervention.

Content	informative	interesting
Good/interested in physics/science	1.73 ± 1.49	1.82 ± 1.43
Total Workshop	2.47 ± 0.99	1.97 ± 1.00
Presentation 'Life'	2.38 ± 0.98	2.34 ± 0.75
Presentation 'stars'&experiments	1.95 ± 1.38	1.93 ± 1.02
Analysing the results	2.16 ± 1.16	2.15 ± 1.19

Table 10: Participants impression of the workshop and its parts regarding the dimensions 'informative' and 'interesting' as explained in tab. 6 for the shortened workshop (9th grade, 16 responses for 25 participants for group A, 9 responses for 27 participants for group B).

Exploratory Interview Transcription

Ich: Dann machen wir das jetzt einfach einmal. Also ich habe jetzt eine Aufnahme gestartet und zeichne eure Sprache auf. Könnt ihr mir einmal sagen, ob das für euch okay ist?
A: Ja.
B: Für mich ist es auch okay.
Ich: Okay, wunderbar. Genau. Dann würde ich einmal mit euch ganz kurz die verschiedenen Punkte aus dem Workshop einfach chronologisch durchgehen, dass ihr mir da nochmal ein bisschen spezifischeres Feedback geben könnt. Sprich erstmal das Thema insgesamt: Ist das was, wo ihr sagen würdet: "das ist interessant" oder eher: "pff, joa"
B: Ich find's.. Also es geht jetzt um das Thema Strahlen und Sterne, oder...?
Ich: Also um den ersten Part, also nicht den zweiten mit (...)
B: Das astrophysische (...)
Ich: Genau.
B: Also ich finde, prinzipiell ist das sehr interessant, weil man in der Schule darüber nicht viel lernt und das ja an sich was faszinierendes ist, so Weltall und so und wie das alles abläuft, weil das ist ja nochmal ein bisschen anders alles und unerklärlicher ist, als so das ganz abstrakte, was wir auf der Welt hier haben. Und deshalb finde ich das eigentlich prinzipiell sehr spannend, gerade weil man eben sonst kaum was drüber lernt, aber eben es die ganze Zeit vor sich hat und so Halbwissen hat eben.
Ich: Ist das was, was du jetzt quasi rückblickend sagst, nachdem du es gehört hast, oder hättest du das auch ähnlich (...)
B: Hmn, nö, also ich hatte schon im Vorhinein Lust darauf, weil das einfach so... Gerade, man hört ja viel immer, auch Nachrichten, über Leben, auf anderen Planeten irgendwie - auch Elon Musk und so. Aber wirklich Hintergrund hat man halt nicht dazu. Deshalb fand ich das schon im Vorhinein eigentlich eine sehr spannende Sache so.
Ich: Okay, wie sieht das bei dir aus?
A: Würde ich ehrlich gesagt auch sagen. Ich habe jetzt zu Physik, Astrophysik, Chemie eigentlich gar keinen Bezug gehabt, aber ich finde man konnte sich das trotzdem gut geben. Und das war auch nicht wirklich... Oder sagen wir, fachlich anspruchsvoll, im Sinne von, dass man das auch ohne Vorwissen leicht verstanden hat. Also anspruchsvoll war es schon, aber es war wirklich relativ leicht verständlich für jeden, würde ich sagen.
Ich: Okay, sehr schön!
A: Es war wirklich relativ angenehm.
Ich: Dann, konkreter zu diesem ersten Vortragspart, wo es quasi um "Was ist Leben überhaupt, was sind die Bedingungen" geht. Wie konntet ihr dem folgen? Fühltet ihr euch da überfordert? War es vielleicht langweilig, war unklar, warum das relevant ist...?
B: Ne, ich fand, es war eigentlich sehr klar, warum das relevant ist, also so ein bisschen Vorarbeit zu machen auf das, was wesentlich dann ist. Das finde ich eigentlich auch prinzipiell wichtig, weil da eben auch nicht nur Sachen vorkamen, wo man... Die man ganz klar schon wusste. Ich finde man konnte ihm gut folgen. Das fand ich war auch nicht sehr anspruchsvoll, weil es ja auch prinzipiell wieder um das ging, was man im Biounterricht lernt. Deshalb fand ich das eigentlich

wichtig...
A: Spannend fand ich das auch, das war relativ gut zugänglicher Stoff, aber ich hätte mir vielleicht - ich weiß nicht wie, oder ob man das überhaupt umsetzen kann - gewünscht, dass man vielleicht nicht nur auf Planeten aus dem Sonnensystem eingeht, sonder... Also ich hab das irgendwann mal in einem Podcast gehört, dass es auch relativ weit entfernte Planeten gibt, auf denen es höchst wahrscheinlich, oder... Weit entwickeltes Leben existieren sollte, aber vielleicht ist das auch etwas hypothetischer.
B: Das fand ich auch, also das war so, was ich im Vorhinein mir überlegt hatte, was dran kam. Dass man ein bisschen darüber redet, dass es ja noch viel mehr gibt. Also man hört ja in den Nachrichten immer sehr viel von sehr anderen, sehr weiten Planeten - das man da so ein bisschen...
Ich: Geht natürlich primär um eure Meinung, aber ganz kurz, um da was zu zu sagen: Grundsätzlich sobald Planeten weit weg sind, ist es sehr vage, was man darüber weiß, es geht eher so in Richtung man kennt ungefähre Abstände, was das genau bedeutet ist schwer zu interpretieren.
B: Man kommt natürlich nicht hin.
Ich: Genau. Also die Infos, die man über diese Planeten bzw. Diesen Mond hat, den ich da präsentiert habe sind viel, viel konkreter, als bei allem, was außerhalb des Sonnensystems ist.
B: Das ist was, was ich vielleicht so als Verbesserung, also wenn du das nochmal machen solltest, vielleicht erklären würde, weil man das ein bisschen immer im Hintergrund finde ich, hat. (Anm. zu A gewandt:) *Ich weiß nicht, bei dir war es vielleicht auch so. Das man auch immer in diesem Hinterkopf (hat), was so in den Nachrichten läuft von sehr weit entfernten Planeten...* (Anm.: A nickte.) *Das man da halt erklärt, dass das eben nicht realistisch ist.*
Ich: Wie fandet ihr das Format, dann quasi diese Fakten über eben diese Planeten zu präsentieren und so eine leichte Diskussion da anzustoßen?
A: Also an sich fand ich das relativ gut, das Ding ist, so ab dem 2ten, 3ten, 4ten Beispiel war das vielleicht etwas zu viel. Einfach aus dem Grund - also das ist natürlich alles hypothetisch und so, alles mit verschiedenen Ursachen - aber, dass ist dann - also ja gut, eine Diskussion war es in der Hinsicht nicht, als dass das Ende vom Lied mehr oder weniger immer war "Wir wissen es halt nicht ganz genau, aber es hat mal höchstwahrscheinlich Wasser gegeben." Also ich hätte das jetzt auf ein, zwei Beispiele verkürzt, also zum Beispiel die Venus, weil ich das persönlich sehr interessant finde und der eine Saturn oder Jupitermond. Auf die zwei hätte ich das beschränkt, aber ansonsten war das relativ gut.
B: Ich fand das auch, also ich finde das eine gute Idee, deshalb würde ich das auch so machen. Ich fand, manche Informationen waren so ein bisschen schwer, wie man damit umgeht. Also bei 460 ° ist klar so, da wird's kein Leben geben. So was Minusgraden und so angeht, weiß man halt immer, dass ein Mensch und ein Löwe da nicht überleben kann, aber so, man hat nicht Ahnung, was so an Bakterien oder sonst was, was es noch gibt, ob das da realistisch ist. Deshalb ist es für so sehr Unwissende schwer zu beurteilen in dem Moment.
Ich: Dann gab es ja einen Vortrag quasi, der sich den Sternen gewidmet hat und den Vorgängen darin. Wie seid ihr damit klargekommen? Was ist da euer Eindruck?
A: War das das Experiment, oder...?

Ich: Das war noch quasi der Teil davor, von diesem Video, was ich euch gezeigt habe, bis zu den Experimenten.

B: Das Video fand ich ein bisschen... Das hatte ich in dem Moment nicht ganz verstanden, was das genau... Also es wurde irgendwann, aber das war mir lange Zeit sehr schwer zu verstehen, was genau abgebildet wird, in was für einem Jahr wir uns quasi befinden, also ob das von Anfang an ist, oder die letzten Jahre, oder so. Da hatte ich ein bisschen Probleme, das zu verstehen. Der Rest, die Erklärungen fand ich prinzipiell sehr logisch, so...

A: Hätte ich auch gesagt. Also eine grobe Datierung, von wegen in welchem Stadium befindet man sich. Ich hatte auch am Anfang nicht ganz auf dem Schirm, ich dachte das wäre dann am Ende einfach so eine Sonne oder so. Beziehungsweise, dass man am Anfang sagt so, okay, das ist das, so bildet sich quasi eine Sonne, so wie das jetzt unsere ist und die Punkte die ihr seht ist dann das und das. Und dass man das einmal größenmäßig einordnen kann, weil ich dachte dann zwischenzeitlich: "Ist das gerade ein schwarzes Loch, ist das ein Sternennebel.." Also...

B: Ja, weil das einfach diese Form... Die war nicht so klar so einer normalen optischen Sonne zuzuordnen.

A: Ja, so eine kleine Datierung, dann ging das eigentlich.

Ich: Okay, dann, der nächste Punkt wären die Experimente. Einmal, fandet ihr die interessant, waren die vielleicht nach der ganzen Theorie auch eine Abwechslung und auf der anderen Seite, war für euch ein Zusammenhang erkennbar zwischen Sternen und diesen Experimenten, also welchen Zweck quasi die Experimente in dem Workshop haben?

B: Also prinzipiell finde ich, Experimente sind immer wichtig, weil die so ein bisschen Interaktion reinbringen, dass man selber was machen kann oder selber was eben visualisiert bekommt und nicht immer nur zuhört oder sich Folien anschaut. Ich finde das eigentlich auch sehr... Ich fand, also das erste fand ich sehr interessant, weil das einfach nicht so, weil das schon irgendwo auch eine coole Aktion ist, die man da sieht, dass sich Wasser voneinander trennt, was mir jetzt noch nicht so klar war. Beim zweiten muss ich gestehen, dass ich nicht ganz sehen konnte, was da passiert ist und nicht ganz verstanden habe. Und ich glaube deshalb mir dann der Bezug von diesem Experiment hin zu diesen Sternen dann gefehlt hat, diesen Teil. Aber sonst habe ich verstanden von den Sternen, also der danach kam.

A: Also an sich würde ich mich dem relativ anschließen. Also Experimente sind sehr sehr wichtig, vor allem bei einem Vortrag der etwas länger geht und so und wir als Schüler, gut - wir sind auch gewohnt, dass wir alle 45 Minuten einmal kurz aufstehen können und so. Deshalb sind Experimente, allein dass man mal wieder selber ein bisschen beansprucht wird, auf JEDEN Fall wichtig. Das würde ich definitiv beibehalten. Aber ich würde die vorher vielleicht noch einmal ganz ganz kurz einordnen von wegen "Das ist Phase und so steht das quasi in Bezug". Also die Experimente waren gut, ich habÂ´s dann auch am Ende verstanden, aber zwischenzeitlich dachte ich mir "okay" so, aber eigentlich waren die echt ganz gut. Würde ich beibehalten.

Ich: Meint ihr, es hätte dem ganzen wesentlich geholfen, wenn man die ausgiebiger nach besprochen hätte, auch?

A: Also nach besprochen würde ich nicht sagen, eher im Sinne von "Also wir machen jetzt ein Experiment, das ist das und so könnt ihr euch das dann vorstellen, das symbolisiert das." Das man

das noch einmal klarer am Anfang verknüpft in ein, zwei Sätzen. Danach wenn man das dann gemacht hat, hat man es schon verstanden gehabt irgendwie. Aber während dem Experiment war man da ein bisschen ratlos. Hätte man das am Anfang einmal ein bisschen verknüpft, dann...
B: Das finde ich - also ich finde, man sollte einmal klar feststellen was passiert, was da los ist und warum, aber prinzipiell bin ich kein Freund von sehr langem vertiefen, gerade wenn man irgendwann es begriffen hat schon und so. Das man dann Dinge, die man sieht und eigentlich auch versteht 10 Minuten wiederholt. Und ich glaube dafür ist es ja auch... es ist ja letztendlich schon sehr viel Stoff für so eine relativ kurze Zeit. Ich glaube da könnte man sich sicherlich ein halbes Jahr lang mit beschäftigen und deshalb ist es dann vielleicht auch nicht nötig, dass man solche Sachen, wenn jeder sie versteht, dass man sie dann solange noch bespricht.
Ich: Okay, also eher im Sinne von einer Einordnung vorweg, was der Kontext und der Zweck ist, als eine vertiefende...
B: Genau, ja, oder, dass man später einmal feststellt, dass jeder verstanden hat, was da passiert und warum das so ist, aber man muss das nicht auf sehr detaillierter Ebene wirklich so besprechen.
Ich: Okay, dann der nächste Schritt, das war dann der Step von der ganzen Vorarbeit zu “Okay, wir machen jetzt eine Simulation". Und zwar erst mal der Schritt “Warum machen wir eine Simulation?". War euch das halbwegs klar, oder fiel das eher vom Himmel, dass man jetzt auf einmal eine Simulationssoftware auspackt?
B: Nö, ich finde, dass Simulation prinzipiell ja einen guten Sinn haben. Weil das ja, also wir sind ja in einem sehr fortschrittlichen Zeitalter und haben eben solche Möglichkeiten wie Computer und so und ich glaube, dass (das) in fast allen Bereichen wo wir heutzutage agieren eine wirklich sehr gute Möglichkeit sind, um Dinge zu veranschaulichen, zu bearbeiten und ich finde das auch sinnvoll. Also es gibt ja nicht wirklich, also auf der und der Form, wirklich eine Alternative zu dieser Simulation und die war ja auch sehr... Ich finde die war auch sehr anschaulich.
A: Also ich würde sagen, dass die Simulation in erster Linie eins bewirkt hat und zwar Authentizität, einfach dadurch, dass das ja wohl auch eine Software ist, die man quasi als Profi benutzt zur Kalkulation. Das ist ja nicht irgendwie etwas wie ein Kindergartenspielzeug. Das einzige, was nur wäre, ist, dass ich zwischenzeitlich manchmal nicht verstanden habe “Okay, und das heißt jetzt was genau?". Beziehungsweise, also, das ist halt ein Profi-Tool, das ist schwierig - ein bisschen schwierig zu bedienen. Wir haben es dann hinbekommen, aber so eine kleine Erklärung da noch am besten.
Ich: Könntest du ein Beispiel nennen für...
Ein wildes David betritt den Raum...
A: Also zum Beispiel: Dieser Wert, den ihr da eingeben müsst, das ist quasi das und das, die Brenndauer, blablabla... Und joa, keine Ahnung, also ansonsten fand ich das eigentlich ganz gut. Und joa, also es geht ja um Mikroprozesse, also die hat man danach auch verstanden. Joa.
Ich: Okay. Hattet ihr vorher schon einmal was mit Simulationen zu tun? Sind die in der Schule schon einmal irgendwie motiviert/eingeführt worden?
B: Ne, ich glaube das ist ja, in der Schule arbeitet man nicht an so Laptops und so. Deshalb also, das ist auch... Das ist natürlich sehr viel auch, bei der letzten jetzt also sehr viel, wenn du da so einen Computer hast und nicht so viel mit Informatik und Computer am Hut hast, dann sind das

sehr viele Lücken und sehr viele Felder mit Zahlen und Buchstaben und so. Was es... Ein bisschen unübersichtlich macht, aber ich finde ihr habt das ja gut erklärt auf dieser Seite, die ihr da gemacht habt. Das man da prinzipiell eigentlich, wenn man sich da so ein bisschen dran gehalten hat, was auf dieser Seite da, finde ich, sehr übersichtlich gemacht war - das man dann eigentlich auch zum Ziel gekommen ist.
A: Ja, also in der Schule benutzen wir die aus gutem Grund nicht, einfach weil, wir haben da Ipads und wenn man da für 26 Leute einen Laptop hinstellen muss, dann gibt's ein paar, die machen Faxen, dann versteht der und der das nicht - Also gut, das war jetzt relativ gut aufgeteilt, einfach weil wir 8 Leute waren und dann noch zwei dabei waren, die helfen konnten. Das war ein gutes Verhältnis, aber wenn das dann eine größere Gruppe ist und am Ende nur eine Person da steht, das könnte ein bisschen knifflig werden, aber ansonsten würde ich das beibehalten.
Ich: Okay. Wie fandet ihr diese Simulationssoftware an sich? War die einigermaßen okay zu bedienen, war die völlig ungeeignet, wie fandet...
B: Ich fand, die war... Also man musste natürlich ein bisschen verstehen so, das schon sehr viel so da ist und wir beide brauchten - wir haben das ja zusammen gemacht - ein bisschen Zeit auch, aber Zeit die finde ich auch noch voll okay ist, wenn man 5 Minuten braucht, um das zu einzustellen.
A: Man ist wirklich überraschend schnell reingekommen, also das ging eigentlich klar.
B: Und ich finde 3 von 4 habens geschafft, so, dass es irgendwen gibt, bei dem es nicht klappt ist ja meistens so.
Ich: Ist auch manchmal technisch kaum vermeidbar.
B: Deshalb find ich das - also wenn es jetzt bei niemandem funktioniert hätte oder das für alle schwer geworden wäre, dann ist das natürlich blöd und das ist natürlich auch prinzipiell gut, wenn man in so einer Gruppe ist mit 8 Leuten, das man da eine relativ - wir hatten heute eine relativ persönliche Betreuung für alles, wenn man zwei Leute, noch ne Lehrerin, zwei Lehrerinnen eigentlich auf 8 Personen hat, dass das natürlich alles sehr kompakt ist, das macht's natürlich auch für Fragen einfacher, das muss man natürlich zu so etwas wie heute sagen. Aber dann finde ich, ging das eigentlich gut. Also ich glaube bei 20, 25 ist das immer schwierig mit Fragen und so, wenn man da nicht die nötigen Personen hat, die bei so einer komplizierten Software dann einem da helfen können. Aber für Gruppen wie heute und sicher auch noch etwas größere Gruppen ist das auf jeden Fall eine gute Lösung.
Ich: Okay, dann der vorletzte Schritt im Grunde: Die Auswertung, die wir mit diesen Paaren von Masse und Luminosität dann gemacht haben. War da der Kontext klar, warum man das überhaupt so rechnen kann und was quasi der Sinn und Zweck davon war?
A: Also für mich nur so halb, aber ich bin auch total schlecht in Mathe, Physik, in allem, also das sagt nicht wirklich viel aus.
B: Also mir fehlte dieser Begriff Luminosität - ich hatte mal in England auch eben genau das - das war damals schon für mich immer so ein bisschen schwierig zu verstehen. Ich glaube es ist auch eine sehr schwierige Sache eigentlich, also es ist nicht so simple, wenn man sich nicht viel damit befasst. Aber das hat so ein bisschen mir Probleme bereitet.
Ich: Und als aller letzten Schritt, die abschließende Einordnung dann, quasi also diese Auswertung mit dem Diagramm und dieser letzte Schritt zu sagen "Okay, diese und jene Sterne, die sind jetzt

relevant wenn man an hochentwickeltes Leben denkt, die anderen Sterne nicht" - war das für euch ein, ich sag mal, zufriedenstellender Abschluss, hat das quasi die Fragen, die am Anfang in dem gesamten Workshop aufgeworfen wurden, okay beantwortet?

B: Ja, ich finde ein bisschen hat man sich teilweise überlegt, warum man das alles gerade macht und wohin uns das bringt so, wo wir gerade noch bei Leben auf Erden war und jetzt über so etwas reden. Aber das hat das natürlich abgerundet und ganz klar gezeigt, warum man das macht und warum das wichtig ist. Und, dass das auch ein wichtiger Aspekt ist, dass man eben Zeit braucht, dem Stern- oder... dem Planeten Zeit bieten, um sich zu entwickeln. Also das ist ja letztendlich auch ein Teil von dieser Frage, die wir da heute bearbeiten wollten. Und deshalb, ja, würde ich sagen: zufriedenstellend.

Ich (zu A): Für dich auch?

A: Dem kann ich nichts hinzufügen.

Ich: Super - das wären auch alle Fragen, die ich hatte.

The main statements can be summarized and sorted as follows.

Interest and General Impression

- astrophysics and especially extreterrestrial life is a fascinating, exciting topic, that is very present in news and media, but barely discussed in school, if at all
- the expectation was to discuss rather alien life outside of the solar system
- for topics included in courses at school, the aspects presented were significantly different from prior knowledge
- the content of the workshop was complex, diverse and challenging, but well explained and put into context and thus comprehensive
- the general structure and course of the workshop was clear and perspicuous

Feedback to the Introducing Parts

- three examples of other worlds were perceived as somewhat repetitive (essential point was always water)
- some of the provided information was hard to interpret (i.e. very low temperature)
- the simulation of star formation shown in a video was challenging to understand at first - clearer guidance and a specific explanation what will be seen would be benefitial
- the experiments were a very important part, providing engagement, interaction and amazement
- again, a clearer guidance/more previous explanation would benefit the understanding

Feedback regarding WTTS/Simulations

- simulations are intuitively considered a very powerful, modern tool in general
- the use of WTTS thus seemed well motivated
- simulation and individual computers were not used at school at all and require clear and detailed instructions
- the simulation provided credibility, authenticity and put the results in a professional context
- the software itself was very complex and somewhat overwhelming
- the explanation provided was very intelligibly, well illustrated and thus the complexity was manageable
- the structure of WTTS was perceived as confusing and requires a high degree of supervision
- the formula for calculating life times was somewhat challenging, mostly because of the scientific term of luminosity
- the final result and its interpretation for the emergence of intelligent life around distant stars was clear, very convincing, lucid
- this final result made the previous parts of the workshop perspicuous and puts them into context, leading to a very satisfying conclusion

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