

Laser spectroscopy of lithium-like carbon with a novel XUV detection system at the ESR and construction of a precision high voltage divider for the CRYRING@ESR electron cooler

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Laser spectroscopy of lithium-like carbon with a novel XUV detection system at the ESR and construction of a precision high voltage divider for the CRYRING@ESR electron cooler

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"[Die Quantentheorie] ist das geheimnisvolle Organon, auf dem die Natur die Spektralmusik spielt und nach dessen Rhythmus sie den Bau der Atome und der Kerne regelt."

- Arnold Sommerfeld, Atombau und Spektrallinien, 1919

Abstract

As of today, quantum electrodynamics (QED) is regarded as the most precisely tested theory in physics. Still, particularly concerning bound-state QED, there are many open issues to be addressed. Two prominent cases being the *proton size puzzle* [1, 2] and the *hyperfine puzzle* [3], which were resolved in 2019 [4, 5]. Due to their simple structure, highly charged ions (HCI) are ideal candidates to reveal or resolve such issues as well as further the understanding of the atomic structure with regard to QED corrections in high electromagnetic fields, electron correlation, and relativistic effects. The Facility for Antiproton and Ion Research (FAIR), currently under construction, will provide one of the largest infrastructures of storage rings and trap facilities to conduct experiments on HCI.

In order to investigate effects of electron correlation and relativity a laser spectroscopy measurement of the $(1s^22s2p)^3P_0-^3P_1$ level splitting in beryllium-like krypton $(^{84}\text{Kr}^{32+})$ has been proposed by Winters *et al.* [6] to be conducted at the Experimental Storage Ring (ESR) as part of the "FAIR-Phase-0" program. To determine the transition energy, the ${}^{3}P_{1}$ - ${}^{1}S_{0}$ de-excitation photons with wavelengths around 10 nm have to be detected. In the scope of this thesis, an extreme ultraviolet (XUV) detector system has been further developed and subsequently commissioned in a test beamtime at the ESR. The setup is similar to the visible light detector used in the Lithium like Bismuth Experiment with Laser Light at the ESR (LiBELLE), which led to the *hyperfine puzzle* [7]. Fluorescence photons are collected by a movable cesium iodide (CsI) coated cathode plate with a central slit that can be positioned around the ion beam axis, collecting mainly forward emitted photons. Secondary electrons created on the cathode are guided via electric and magnetic fields, generated by a system of ring electrodes and two solenoid coils, to a microchannel plate detector (MCP) placed inside the ultra-high vacuum of the ESR. During the beamtime, the detector performance was tested by measuring the $(1s^{2}2s)^{2}S_{1/2}-^{2}P_{1/2}$ and $(1s^{2}2s)^{2}S_{1/2}-^{2}P_{3/2}$ level splittings of lithium-like carbon $(^{12}C^{3+})$. In conclusion, it was demonstrated that the detection system showed high sensitivity to the fluorescence photons at a laboratory wavelength of approx. 93 nm, leading to both transitions being measured with a relative uncertainty of approximately 8 ppm.

The second part of this thesis focuses on the construction of a precision high voltage divider for the electron cooler of the CRYRING@ESR storage ring also at the GSI/FAIR facility. In past experiments at the ESR (e.g., LiBELLE), the limiting uncertainty arose from the knowledge of the ion velocity, which is determined by the electron cooler voltage measurement [8, 9]. This could only be improved by using a precision high voltage divider from the Physikalisch-Technische Bundesanstalt (PTB) [3, 10], thus motivating the construction of an HV divider for the CRYRING@ESR electron cooler. The design of the high voltage divider is based on the design of the KATRIN (**Ka**rlsruhe **Tr**itium **N**eutrino experiment) dividers K35 [11] and K65 [12]. By utilizing a new absolute calibration method developed in Münster [13], the relative precision of the divider scale factors was demonstrated to be in the order of 1 ppm. The divider precision is therefore close to the world's most precise dividers of PTB [14] and the KATRIN experiment.

Zusammenfassung

Die Quantenelektrodynamik (QED) gilt heute als die am genauesten getestete Theorie in der Physik. Dennoch gibt es insbesondere im Hinblick auf die "bound state QED" noch viele offene Fragestellungen. Zwei prominente Fälle sind das *proton size puzzle* [1, 2] und das *hyperfine puzzle* [3], welche 2019 gelöst werden konnten [4, 5]. Aufgrund ihrer simplen Struktur sind hochgeladenen Ionen (HCI) ideale Kandidaten, um solche Probleme aufzudecken oder zu lösen sowie das Verständnis der atomaren Struktur in Bezug auf QED-korrekturen in hohen elektromagnetischen Feldern, Elektronenkorrelationen und relativistische Effekte zu erweitern. Die im Aufbau befindliche Facility for Antiproton and Ion Research (FAIR) wird eine der größten Infrastrukturen von Speicherringen und Falleneinrichtungen zum Experimentieren mit HCI bereitstellen.

Um die Auswirkungen von Elektronenkorrelation und realtivistischen Effekten zu untersuchen, wurde eine Laserspektroskopiemessung der $(1s^22s2p)^3P_0-^3P_1$ Aufspaltung in berylliumähnlichen Krypton (⁸⁴Kr³²⁺) von Winters et al. am Experimentierspeicherring (ESR) als Teil des "FAIR-Phase-0"-Programms vorgeschlagen. Um die Übergangsenergie zu bestimmen, müssen die ³P₁-¹S₀ Abregungsphotonen mit Wellenlängen von ca. 10 nm gemessen werden. Im Rahmen dieser Arbeit wurde ein Detektorsystem für Photonen im extremen UV-Bereich (XUV) weiterentwickelt und anschließend bei einer Teststrahlzeit am ESR in Betrieb genommen. Der Aufbau ähnelt dem im Lithium like Bismuth Experiment with Laser Light at the ESR (LiBELLE) verwendeten Detektor für sichtbares Licht, welches zum *hyperfine puzzle* geführt hat [7]. Dabei wird eine einfahrbare und mit Cäsiumiodid (CsI) beschichtete Kathode um die Ionenstrahlachse im Ultrahochvakuum des ESR positioniert, um vorwärts emittierte Fluoreszensphotonen zu sammeln. In der Kathode werden Sekundärelektronen erzeugt, die über elektromagnetische Felder zu einer Mikrokanalpatte (MCP) geführt werden. Während der Teststrahlzeit wurde die Leistungsfähigkeit des Detektors durch Messung der $(1s^22s)^2S_{1/2}$ - $^2P_{1/2}$ und $(1s^22s)^2S_{1/2}$ - $^2P_{3/2}$ Aufspaltungen von lithiumähnlichem Kohlenstoff $({}^{12}C^{3+})$ getested. Der Detektor zeigte dabei eine hohe Empfindlichkeit für Fluoreszenzphotonen bei einer Laborwellenlänge von ca. 93 nm, wodurch beide Übergänge mit einer relativen Unsicherheit von ca. 8 ppm gemessen wurden.

Der zweite Teil dieser Arbeit befasst sich mit dem Aufbau eines Präzisions-Hochspannungsteilers für den Elektronenkühler des CRYRING@ESR Speicherrings ebenfalls an der GSI/FAIR-Anlage. In früheren Experimenten am ESR (z.B. LiBELLE) ergab sich die dominierende Unsicherheit aus der ungenauen Kenntnis der Ionengeschwindigkeit, welche durch Messung der Kühlerspannung bestimmt wird. [8, 9]. Die Präzision konnte durch den Einsatz eines Präzisions-Hochspannungsteilers der Physikalisch-Technischen Bundesanstalt (PTB) verbessert werden, wodurch das Projekt zum Bau eines HV-Teilers für den CRYRING@ESR motiviert wurde. Das Design des Hochspannungsteilers basiert auf dem Design der KATRIN-Teiler (Karlsruhe Tritium Neutrino Experiment) K35 [11] und K65 [12]. Durch die Anwendung einer neuen Absolutkalibrierungsmethode, die in Münster entwickelt wurde [13], konnte gezeigt werden, dass die relative Genauigkeit der Maßstabsfaktoren des Teilers in der Größenordnung von 1 ppm liegt, womit die Teilerpräzision nahe an der Präzision der weltweit genauesten Teiler der PTB [14] und des KATRIN-Experiments ist.

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Chapter 1

Introduction

Over the last decades, particle accelerators became an important tool to create extreme experimental conditions to search for new physics or test well-established theories. Especially with regard to advancements concerning the standard model of particle physics, such as the discovery of the W- and the Z-Bosons in 1983 [15, 16] or the Higgs Boson in 2012 [17], did the accelerator facilities become a cornerstone in modern physics. Nowadays, the spectrum of applications has extended to applied sciences like materials research, radiation biophysics, space science, and cancer therapy. The Facility for Antiproton and Ion Research (FAIR), which is currently under construction, will feature one of the largest accelerator structures worldwide at the GSI Helmholtzzentrum für Schwerionenforschung.

A central research pillar of FAIR is pursued by the **S**tored Particles Atomic Physics Research Collaboration (SPARC). One of the topics pursued by the collaboration consists of precision tests of bound-state quantum electrodynamics (QED) precision tests in bound states and fundamental interactions in extremely strong electromagnetic fields. Despite being considered as the most precisely tested theory in physics, there are still many open issues regarding QED, as can be deduced from past experiments that led to the *proton size puzzle* [1, 2] and the *hyperfine puzzle* [3] which both have been resolved in 2019 [4, 5]. The *proton size puzzle* refers to a measured deviation of 7 σ between the proton charge radius derived from the Lamb shift in muonic hydrogen and the average of all measurements in normal hydrogen. It has been solved by a direct measurement of the n = 2 Lamb shift in atomic hydrogen [4] that agrees with the muon-based Lamb shift measurement. The *hyperfine puzzle* emanated after a measurement of the specific difference between the hyperfine splittings in hydrogen-like and lithium-like bismuth (²⁰⁹Bi⁸²⁺, ²⁰⁹Bi⁸⁰⁺) within the scope of the LiBELLE experiment (Lithium like Bismuth Experiment with Laser Light at the ESR) [3]. In conclusion, a 7 σ discrepancy between the measured values and the theoretical predictions [18] was found. In search for a reason for this discrepancy, the magnetic moment of ²⁰⁹Bi was remeasured since the value is a critical quantity for the theoretical predictions of the investigated transitions. With the newly measured value, the observed discrepancy disappeared, leading to an agreement between the experimental and theoretical values within the respective uncertainties [5]. In summary, LiBELLE constitutes a first sensitive test of bound-state QED in extreme magnetic fields and stresses the importance of high precision measurements under these conditions for a deeper understanding of QED.

Large solid angle fluorescence detection at the Experimental Storage Ring (ESR) has in the past been limited to ultraviolet (UV), visible and infrared (IR) spectral regions. However, to further study the electronic structure of highly charged ions (HCI), the ability to efficiently detect fluorescence photons in the extreme ultraviolet (XUV) region is mandatory since only a fraction of the transition energies are accessible from the UV to the visible region. In 2011 Winters et al. proposed an experiment to study effects of electron correlation and relativity in a laser spectroscopy measurement of the $(1s^22s2p)$ ³P₀-³P₁ level splitting in beryllium-like krypton (⁸⁴Kr³²⁺) at the ESR as part of the "FAIR-Phase-0" program [6]. As the transition energy of the ${}^{3}P_{1}$ - ${}^{1}S_{0}$ de-excitation photons is expected to be at around 10 nm in the laboratory system, a new detection system for this region had to be developed. Due to the success of the novel detector design applied in the laser spectroscopy measurements of LiBELLE [7], the concept was adapted to the XUV region in the context of the Ph.D. thesis of J. Vollbrecht [19]. The design features a collector for fluorescence light that can be positioned around the ion beam to maximize the fluorescence light collection yield in forward direction. In the case of the LiBELLE detector, the fluorescence light is directly reflected via a copper mirror onto a photomultiplier outside the vacuum where the photons are counted. The mirror system limits the detector to the optical region ($\sim 400 - 850$ nm) because the copper absorbs fluorescence photons with shorter wavelengths. Hence, in the case of the XUV detector, the fluorescence photons are converted to secondary photoelectrons when impinging on a cesium iodide (CsI) coated cathode plate, which are then electromagnetically guided to a microchannel plate detector (MCP) where the electrons are counted. In this work, the XUV detection system has been further developed and successfully commissioned in a beamtime with lithium-like carbon ions $({}^{12}C^{3+})$ at the Experimental Storage Ring (ESR). In conclusion of this beamtime, the $(1s^22s)$ ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ and $(1s^{2}2s){}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splittings at approximately 93 nm in the laboratory rest frame were determined with a relative uncertainty of approximately 8 ppm demonstrating the high sensitivity to XUV photons of the new detection system.

1.1. Thesis outline

Besides its successful detector design, another takeaway from the LiBELLE experiment was the importance of the electron cooler voltage measurement. For laser spectroscopy measurements in storage rings, the ion velocity is a critical quantity due to the resulting Doppler shift of the investigated transitions compared to the laboratory system. In the case of the ESR, the velocity is determined by an electron cooler that creates a low momentum spread inside the ion beam. Due to an imprecise voltage measurement in a first LiBELLE beamtime in 2011 [8, 9] the experiment was repeated in 2014 with a precision high voltage divider provided by the Physikalisch-Technische Bundesanstalt (PTB) [3, 10]. In this manner, the accuracy of the LiBELLE result was improved by more than an order of magnitude to the 10^{-5} -level. The experience of the laser spectroscopy measurements at the ESR lead to the development of a 35 kV precision high voltage divider for the electron cooler of the CRYRING@ESR storage ring. Its design is based on the design of the KATRIN (Karlsruhe Tritium Neutrino experiment) precision high voltage dividers K35 [11] and K65 [12] that belong to the worlds most precise dividers together with the MT100 from PTB [14]. The new divider, constructed in the scope of this thesis, can compete with the dividers of KATRIN and PTB in terms of precision. Measurements that demonstrate the precision of the respective scale factors to be on the order of 1 ppm were achieved by utilizing a new absolute calibration method developed in Münster [13, 20].

1.1. Thesis outline

The main focus of this thesis lies on the XUV detector commissioning and the associated measurement results, as well as the construction and characterization of the precision high voltage divider for the electron cooler of the CRYRING@ESR storage ring. The thesis structure is presented in the following paragraphs:

Chapter 2 introduces the fundamentals of atomic structure calculations. It gives an overview of modern bound-state QED calculation methods that are needed to numerically evaluate the many-body atomic structures of highly charged ions. The chapter concludes with a motivation as to why HCI are such interesting candidates for QED tests and an overview of calculation results for the beryllium-like krypton and lithium-like carbon transitions that are relevant for this work.

Chapter 3 gives an overview of the science goals and infrastructure of FAIR. Additionally, an introduction to beam cooling techniques is given since these are essential to produce high precision results when conducting experiments at ion storage rings. Another focus will be on the measurement technique of laser spectroscopy at storage rings, which was applied in the commissioning beamtime of the XUV detection system.

Chapter 4 describes the detector setup and detection principle. An overview of the detector optimization process and results based on the SIMION[®] 8.1 [21] field and particle trajectory simulator is presented. Especially the optimization of the electromagnetic field layout and the cathode design is emphasized as both mostly determine the detector efficiency.

Chapter 5 is focused on the commissioning of the XUV detection system and the related beamtime with lithium-like carbon ions $({}^{12}C^{3+})$ at the ESR. The chapter begins with the experimental setup followed by an detailed description of the $(1s^22s) {}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ and $(1s^22s) {}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting analysis. In the context of these results, the detector performance is discussed.

Chapter 6 introduces the new precision high voltage divider for the electron cooler at CRYRING@ESR, starting with a motivation as to why a dedicated divider was constructed for the storage ring. Subsequently, the fundamental concept of precision high voltage divider setups is reviewed, which is followed up with a presentation of the new divider setup and design. The chapter is finalized by a description of the different calibration methods used to characterize the HV divider and a discussion of the calibration results determining the device's precision.

Chapter 7 closes this thesis by summarizing the main results and concluding with an outlook towards future developments.

Chapter 2

Fundamentals of modern atomic structure calculations

The foundation of modern atomic structure calculations was laid in the early 20th century when Niels Bohr¹ presented his atomic model in 1913 [22]. Bohr was the first to express the angular momentum (known as principal quantum number *n*) of the revolving electron as an integer multiple of \hbar , which he deduced from the Balmer formula found by J.J. Balmer in 1885 [23]. However, Michelson² and Morley uncovered a fine structure in the hydrogen spectrum already in 1887 [24], which could not be explained with Bohr's atomic model. Following the fine structure, the hyperfine structure was subsequently measured again by Michelson in 1891 [25] and Fabry/Perot in 1897 [26], also with no means of deriving it with the Bohr model. Through the extension by Sommerfeld, a theoretical basis to calculate the fine structure was found in 1916. First, Sommerfeld introduced a second quantum number (known as orbital angular momentum quantum number *l*) [27, 28] by quantizing the z-component of the angular momentum, thus allowing the electron orbits to be ellipses instead of circles. Then he calculated the orbital degeneracy by deriving the relativistic solution of the energy levels in the process introducing the fine structure constant α . The third quantum number (magnetic quantum number *m*) was introduced by Schwarzschild and Epstein in the same year [29, 30], completing the set of quantum numbers that define atomic orbitals.

Due to the success in describing the spectrum of the hydrogen atom with at this point unprecedented precision, further advancements leading to the theory of quantum mechanics were initiated. In 1924 de Broglie³ established wave-particle duality characterizing electrons as particle waves [31]. In this manner, Bohr's condition that the electron's angular momentum is an integer multiple of \hbar could be interpreted as a

¹ Nobel Prize in Physics, 1922 ² Nobel Prize in Physics, 1907 ³ Nobel Prize in Physics, 1929

standing wave condition. Erwin Schrödinger⁴ incorporated de Broglie's particle waves in his linear partial differential equation (known as Schrödinger equation) to describe the state function of a quantum-mechanical system [32]. Together with Heisenberg's uncertainty principle [33], it explained for the first time why bound electrons do not lose energy by synchrotron radiation, thereby having stable orbits at discrete distances from the nucleus.

The next significant advancements for the understanding of the atomic structure were carried out by Wolfgang Pauli⁵ in the mid-20s of the 19th century. In 1924 Pauli found the cause of the hyperfine structure by proposing the existence of nuclear magnetic moments. One year later, he contributed a fourth quantum number (called spin quantum number *s*) and the exclusion principle [34] thus finalizing the full set of quantum numbers that characterize the atomic structure. In a next step, Paul Dirac⁶ merged the principles of quantum mechanics and special relativity, being the first to fully account for both in a single equation [35]. Especially the prediction of the existence of the positron was an important result of Dirac's theoretical approach. Schrödinger's equation, the equivalent matrix mechanics created by Born⁷, Heisenberg⁸ and Jordan [36, 37, 38], as well as the generalization by Dirac, constitute a major step for the understanding of the atomic structure and its respective calculation.

With the possibility of higher precision measurements, more effects with impact on the atomic structure could be revealed. Especially the Lamb shift measured on the hydrogen microwave spectrum by Lamb⁹ and Retherford in 1947 [39] gave rise to new developments leading to the relativistic quantum field theory of electrodynamics (QED). These theoretical developments were led in particular by Bethe¹⁰, Kramers, Tomonaga¹¹, Schwinger¹¹, Feynman¹¹ and Dyson in the subsequent years. QED was able to provide an understanding of the Lamb shift and is viewed as the most precisely tested theory in physics amongst other things because of the extremely precise measurements of the electron magnetic moment in Bohr magnetons g/2 (relative uncertainty of 0.28 ppt) [40] that nowadays coincide with theoretical values [41] to the 100 ppt level. In order to include all QED effects and handle more complex systems in the context of bound-state QED (systems with multiple electrons), that cannot be solved analytically anymore, numerical methods had to be established. One of the first numerical methods was developed by D. R. Hartree in 1928 [42, 43] with a subsequent generalization by Fock in 1930 [44], which was reformulated and published again by

⁴ Nobel Prize in Physics, 1933 ⁵ Nobel Prize in Physics, 1945 ⁶ Nobel Prize in Physics, 1933

⁷ Nobel Prize in Physics, 1954 ⁸ Nobel Prize in Physics, 1932 ⁹ Nobel Prize in Physics, 1955

¹⁰ Nobel Prize in Physics, 1967 ¹¹ Nobel Prize in Physics, 1965

Hartree in 1935 as the nowadays known Hartree-Fock method [45]. With the basic idea to trace the multi-electron problem back to a single-electron problem, this method was the first to provide the means to derive the structure for complex many-body atomic systems. Most of the numerical methods in use today are based on this method, with the extension of taking electron correlation effects into account that the Hartree-Fock method neglects. A more detailed discussion of these modern methods is presented in section 2.2.

In the first paragraphs of this chapter, 13 physics Nobel Prize awardees are mentioned for their breakthrough discoveries in atomic and particle physics. Experiments with increased precision repeatedly lead to new developments in theoretical atomic and particle physics and, therefore, to an advanced understanding of the fundamental structure of matter. With more sophisticated calculations and higher precision in theoretical predictions, new measurements are constantly needed to successively refine the theoretical models. Therefore, despite its high precision predictions, this science field is still pursued with great effort at facilities such as FAIR, which provides an ideal testing ground for such research.

This chapter gives an overview of atomic structure calculation fundamentals and methods. It has to be noted that it would go beyond the scope of this thesis to present detailed calculations. They can be looked up in the original references given or in a selection of textbook sources used as a basis for this chapter [46, 47, 48, 49, 50]. The first part focuses on hydrogen-like atoms to discuss the different effects that explain the level splittings in the atomic structure in detail. From this point on, an overview of current methods to deal with many-body systems is given. Additionally, a motivation is presented as to why highly charged ions are ideal systems to study the different techniques and corrections presented in the first two parts of this chapter. In conclusion, a summary of current structure calculation results for the two relevant ion species in this work is given. Lithium-like carbon ($^{12}C^{3+}$) is of particular interest as this system was investigated with the XUV detection system commissioned in the scope of this thesis (see chapter 5).

2.1. Hydrogen and hydrogen-like atoms

For all of the developments mentioned in the introduction of this chapter, the hydrogen atom played an essential role due to its simple structure and easy accessibility in spectroscopy measurements. This is why nowadays hydrogen-like systems still play an important role in the understanding of the atomic structure. Hence, this section introduces the fundamentals of atomic structure calculations and the different corrections on the example of these systems.

2.1.1. Non-relativistic solution

The Schrödinger equation is the typical starting point for non-relativistic unperturbed one-electron systems. In such a case, the general one-body time-independent Schrödinger equation must be solved

$$\hat{H}\Psi(\mathbf{r}) = E\Psi(\mathbf{r}), \qquad (2.1)$$

where \hat{H} is the Hamilton operator, E the particle energy, Ψ the state vector of the quantum system, and r the position vector. With the Hamilton operator for an electron in a Coulomb potential $V_{\text{nuc}}(r)$ of an atomic nucleus in the center of mass frame, the equation becomes

$$\begin{bmatrix} -\frac{\hbar}{2\mu} \nabla^2 - \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{r} \end{bmatrix} \Psi(\mathbf{r}) = E\Psi(\mathbf{r}), \qquad (2.2)$$

where \hbar is the reduced Planck constant, μ the reduced mass of the electron and the nucleus $\mu = \frac{m_e M}{m_e + M}$, ∇ the differential operator Nabla, ε_0 the electric constant, Z the atomic number and e the elementary charge. By solving the eigenvalue equation for the relative motion using spherical coordinates, discrete energy levels are obtained

$$E_n = -R_y \frac{Z^2}{n^2} \,. \tag{2.3}$$

Energy levels E_n are, in this case, only dependent on the atomic number Z and the principal quantum number n. The Rydberg energy R_y is constant and defined as $R_y = Rhc$ with the Rydberg constant R, the Planck constant h and the speed of light c.

This solution indicates that every state with identical quantum number n is degenerate. Hence effects like the fine structure cannot be derived from the Hamiltonian. One issue is that relativistic corrections are not an integral part of the Schrödinger equation (it is not Lorentz invariant). For hydrogen and hydrogen-like systems with low Z, it is possible to apply perturbation theory so that the Schrödinger equation can be used to obtain approximate solutions since in these systems, electron speeds are low with regard to the speed of light (on the order of a few percent). A more sophisticated approach is to use the more general Dirac equation described in the next section, which intrinsically includes the theory of relativity as well as the particle spin.

2.1.2. Relativistic theory and fine structure

The Dirac equation includes the theory of special relativity and, by that, lifts the degeneracy of electrons with the same principal quantum number n for the hydrogen atom. The source of the degeneracy is the magnetic interaction of the electron spin with its orbital motion inside the nuclear potential J = L + S, where J is the total angular momentum, L the total angular momentum of all electrons and S the total spin of all electrons.

Dirac developed his equation by incorporating the energy–momentum relation $E = p^2c^2 + m_0^2c^4$ into his equations. For a particle in a central Coulomb potential $V_{\text{nuc}}(\mathbf{r})$ (as in equation (2.2)) this leads to the time-independent relativistic analogon of the Schrödinger equation

$$\left[\underbrace{c\boldsymbol{\alpha}\cdot\boldsymbol{p}-\beta m_e c^2+V_{\rm nuc}(\boldsymbol{r})}_{\hat{H}}\right]\Psi(\boldsymbol{r})=E\Psi(\boldsymbol{r}), \qquad (2.4)$$

with *c* for the speed of light, m_e for the electron rest mass and $\mathbf{p} = -i\hbar \nabla$ as the momentum operator. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are 4×4 matrices which are defined as follows

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \qquad \boldsymbol{\beta} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad (2.5)$$

while *I* represents the unit 2 × 2 matrix, the three components of σ are the Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$
(2.6)

Since α and β are 4 × 4 matrices, they must act on a four component vector $\Psi(\mathbf{r})$ with components $\Psi_{1,...,4}$. In Dirac-theory, these four-component wave functions appear because, besides the two spin orientations, positive and negative energy values for free

particles are allowed. The negative energy solutions are nowadays interpreted as positive energy states of anti-particles, in this case of the positron. With the assumption of an infinitely massive nucleus, the Dirac equation can be solved analytically. Solving equation (2.4) for the energy eigenstates $E_{n,j}$ yields

$$E_{n,j} = m_e c^2 \left\{ \left[1 + \left(\frac{Z\alpha}{n - j - \frac{1}{2} + \sqrt{\left(j + \frac{1}{2}\right)^2 - Z^2 \alpha^2}} \right)^2 \right]^{-1/2} - 1 \right\},$$
(2.7)

with the fine structure constant $\alpha = \frac{e^2}{2c\varepsilon_0 h}$. Compared to the Schrödinger solution equation (2.7) not only depends on the principal quantum number *n* but also on the total angular momentum quantum number j = |l - s|, ..., |l + s| where *l* is the orbital angular momentum quantum number and $s = +\frac{1}{2}, -\frac{1}{2}$ the spin quantum number.

The *j*-dependence means that the degeneracy of states with identical principal quantum number *n* is removed, which is called fine structure splitting. Only the degeneracy with regard to the magnetic quantum number m_l remains. The Dirac equation quantifies the fine structure correctly and is, therefore, an integral part of most atomic structure calculations. Further splittings such as the hyperfine structure or quantum electrodynamics effects like the Lamb shift can be incorporated by extending the Dirac equation as shown in the following subchapters.

2.1.3. Hyperfine structure

The hyperfine effects in atomic spectra arise from interactions between the nucleus and the electron shell. These interactions are handled as a perturbation to the Dirac Hamiltonian, so that [49, p. 494]

$$\hat{H} = \hat{H}_{\rm D} + \underbrace{\hat{H}_{\rm MD} + \hat{H}_{\rm Q} + \hat{H}_{\rm vol}}_{\hat{H}_{\rm HFS}},\tag{2.8}$$

where $\hat{H}_{\rm D}$ is the unperturbed Dirac Hamilton operator, including the Coulomb potential $V_{\rm nuc}(\mathbf{r})$. $\hat{H}_{\rm HFS}$ is the total hyperfine structure Hamiltonian, with $\hat{H}_{\rm MD}$ as the perturbative term due to the nuclear magnetic dipole moment $\boldsymbol{\mu}_{I}$, \hat{H}_{Q} as the electric quadrupole operator and $\hat{H}_{\rm vol}$ as the volume shift operator. In the following, the different contributions to the hyperfine structure are discussed.

Magnetic dipole hyperfine structure

The main contribution to the hyperfine splitting originates from the interaction between the nuclear magnetic dipole moment μ_I with the average magnetic field B_J the electron shell creates at the nucleus

$$\ddot{H}_{\rm MD} = -\boldsymbol{\mu}_{\boldsymbol{I}} \cdot \boldsymbol{B}_{\boldsymbol{J}} \,. \tag{2.9}$$

The nuclear magnetic dipole moment μ_I is related to the nuclear spin I via the g-factor of the nucleus g_I and the nuclear magneton μ_N

$$\mu_I = \frac{g_I \mu_N}{\hbar} I \,. \tag{2.10}$$

For the magnetic field of the electron shell B_I , the relation to J can be written as

$$B_{J} = -\frac{JB_{J}}{|J|} = -\frac{JB_{J}}{\sqrt{J(J+1)}\hbar},$$
(2.11)

with the magnetic field strength B_J and the magnitude of the electron shell's total angular momentum magnitude $|J| = \sqrt{J(J+1)}\hbar$. Using equations (2.10) and (2.11) to rewrite equation (2.9) then yields

$$\hat{H}_{\rm MD} = \frac{g_I \mu_{\rm N} B_J}{\sqrt{J(J+1)}} \cdot \frac{I \cdot J}{\hbar^2} = a \cdot \frac{I \cdot J}{\hbar^2}, \qquad (2.12)$$

with the hyperfine structure constant *a*. Analog to the spin–orbit coupling J = L + S in the atomic fine structure, the total angular momentum of the electron shell *J* and the nuclear spin *I* couple to a total angular momentum of the atom F = I + J. Together with the law of cosines, the coupling of *I* and *J* from equation (2.12) becomes

$$I \cdot J = \frac{1}{2}(F^2 - I^2 - J^2) = \frac{1}{2}(F(F+1) - I(I+1) - J(J+1)).$$
(2.13)

The general result for the energy shifts ΔE with respect to the fine structure solution is then [48, p. 378]

$$\Delta E = \frac{a}{2\hbar^2} [F(F+1) - I(I+1) - J(J+1)]. \qquad (2.14)$$

Possible values of quantum number *F* are given by F = |J - I, J - I + 1, ..., J + I - 1, J + I|. To calculate the hyperfine splittings for an individual isotope, the magnetic dipole hyperfine structure constant *a* has to be evaluated. According to equation (2.12), the relevant contributions are g_I , μ_N and B_J . The g-factor g_I and the nuclear magnetic moment μ_N are input parameters to the calculations and have to be determined experimentally by nuclear magnetic resonance (NMR) measurements. If these values are known, an analytical solution is possible for hydrogen and hydrogen-like atoms since exact solutions of the magnetic field strength created by the electron B_J can be obtained. For the described systems, *a* is then given by [49, p. 502]

$$a = \frac{\mu_0}{\pi} g_I \mu_{\rm N} \mu_B \frac{1}{J(J+1)(2l+1)} \frac{Z^3}{a_0^3 n^3},$$
(2.15)

where μ_0 is the magnetic constant, μ_B the Bohr magneton and a_0 the Bohr radius. Especially the Z^3 -dependence is of interest for spectroscopy measurements, as transition frequencies for heavier elements are shifted from the radio frequency spectrum towards the optical region.

It has to be noted that measurements of magnetic moments are not trivial and can be the dominant source of uncertainty when determining the hyperfine structure. This uncertainty was the reason that the hyperfine puzzle of LiBELLE occurred and was resolved by a re-measurement of the magnetic moment of ²⁰⁹Bi [5].

Electric quadrupole hyperfine structure

Similar to the magnetic dipole interaction, the electric quadrupole hyperfine splitting is caused by interactions between the electric quadrupole moment Q_0 of the nucleus and the electrical field of the electron shell. The splitting occurs in atoms with nuclei that have a non-spherical charge distribution, so the electric field shows no spherical symmetry. It is common to define the magnitude Q of the electric quadrupole moment as the average value of Q_0 [46]. In the following, only the result for the total hyperfine energy shift is given for completeness, since for both ion systems emphasized in this thesis, the electric quadrupole contribution is negligible and can therefore be disregarded. The total hyperfine structure yields

$$\Delta E = \frac{a}{2}C + \frac{b}{2} \frac{\frac{3}{4}C(C+1) - I(I+1)J(J+1)}{I(2I-1)J(2J-1)},$$
(2.16)

where C = F(F + 1) - I(I + 1) - J(J + 1) and *b* is the quadrupole coupling constant which has, similar to *a*, to be evaluated for each investigated system.

2.1.4. Nuclear structure effects

Besides the level splitting introduced by the Dirac treatment and the hyperfine structure, further splittings arise from nuclear structure effects. These shifts were not accounted for in previously shown calculations due to the basic assumption of a pointlike and infinitely-massive nucleus. Both assumptions lead to significant offsets from the true energy levels and have to be accounted for by applying more realistic models for the nuclear mass and charge distributions [51, 52]. To incorporate the effects into structure calculations, energy corrections to the Dirac energy levels E_D are considered

$$E_{\text{bind}} = E_{\text{D}} + E_{\text{M}} \,, \tag{2.17}$$

where $E_{\rm M}$ is the leading nuclear mass recoil correction (see equation (2.19)).

Nuclear charge and magnetic moment distribution

For calculations to be competitive with spectroscopic precision, the assumptions of a point-like nucleus and an infinitely-massive nucleus have to be abandoned. The part of the electron wave function overlapping with the nuclear volume thus experiences a modified Coulomb potential leading to a weaker binding of the electron. Resulting corrections of the binding energy, called **finite nuclear size (fns)** corrections, are part of the Lamb shift and depend on the charge distribution in the nucleus. In [51], three different charge distribution models were investigated. The model to be applied is chosen in dependence of the nuclear size. For example in nuclei with charge numbers Z < 10 (as in the investigated ${}^{12}C^{3+}$) a Gaussian distribution $\rho_{\text{Gaus}}(r)$ of the nuclear charge is assumed

$$\rho_{\text{Gaus}}(r) = \left(\frac{3}{2\pi R^2}\right)^{3/2} \exp\left(-\frac{3r^2}{2R^2}\right),$$
(2.18)

where *R* is the root mean square charge radius of the nucleus. Applying a given charge distribution yields a modified Coulomb potential, which is then input to the Dirac equation to calculate the modified energy levels. The finite nuclear size corrections scale with *Z* and even become the dominant 1*s* energy shift contributions for *Z* > 100. Furthermore, nuclear charge distributions can be affected by the motion of the electron, leading to additional modifications of the Coulomb potential. This effect is called **nu**-

clear polarization and also has to be taken into account for high precision predictions of atomic energy levels. Due to the complexity of such calculations, this effect will not be presented in the scope of this thesis. The modified Coulomb potential also affects QED contributions like the self-energy and vacuum polarization introduced in section 2.1.5, which also have to be accounted for in advanced calculations. Coverage of this aspect would again go beyond the scope of this thesis and can, e.g., be looked up in [51]. Analog to the fns correction due to the nuclear charge distribution, the **Bohr-Weisskopf (BW) effect** takes the spatial distribution of the nuclear magnetic moment into account. A. Bohr and V. Weisskopf first described the effect in [53]. As this effect is associated with the nuclear moments, it does not contribute to the Lamb shift but to the hyperfine structure, especially in heavy elements.

The introduction of a finite mass of the nucleus to atomic structure calculations leads to a center of mass movement as introduced in section 2.1.1. This is taken into account by the reduced mass μ for the non-relativistic case. However, additional relativistic **nuclear recoil corrections** arise from the relativistic treatment and the leading corrections $E_{\rm M}$ are given by [51]

$$E_{\rm M} = \frac{m_e^2 c^4 - (m_e c^2 + E_{\rm D})^2}{2Mc^2} - \left(\mu - m_e + \frac{m_e^2}{M}\right) c^2 \frac{(Z\alpha)^2}{2n^2} \,. \tag{2.19}$$

Higher orders of these corrections have to be calculated numerically. Since the size of this effect is strongly related to the ratio between electron mass and nuclear mass $\frac{m_e}{M}$, it is especially important for light ions and almost negligible for heavy nuclei.

2.1.5. Lamb-shift

In atomic physics, the Lamb shift refers to all further energy splittings besides the previously introduced (some definitions even assign every shift from the Dirac value to the Lamb shift). The most significant part of these splittings is attributed to QED effects consisting of so-called self-energy and vacuum polarization. As in section 2.1.4, the resulting Lamb shift energy corrections $E_{\rm L}$ are added to the Dirac energy + $E_{\rm D}$

$$E_{\rm bind} = E_{\rm D} + E_{\rm M} + E_{\rm L} \,,$$
 (2.20)

with the leading nuclear mass recoil correction $E_{\rm M}$ introduced in equation (2.17). In this section, a short overview of the most relevant QED-corrections is given. This summary

is mainly based on overviews published in [51, 52].

Quantum electrodynamics (QED)

The theory of quantum electrodynamics describes the interactions between electrically charged particles via the exchange of photons. In bound-state QED, the interactions of electrons with the quantized electromagnetic fields inside atoms/ions in terms of virtual photon exchanges give rise to the dominant parts of the Lamb shift. The term virtual accounts for the fact that virtual particles cannot be measured, and only their impact on a physical system can be observed. Two main contributions, called self-energy and vacuum polarization, account for the necessary corrections to the energy levels with regard to QED. Calculations of these contributions are based on perturbative series expansion in powers of the fine structure constant α [51].

Within this framework, the lowest-order terms contribute most to the total energy shift, and the order directly indicates the number of involved interacting virtual photons. The impact of higher orders rises with the nuclear charge number Z and thus determines the perturbative approach to calculate the different contributions. For small Z, the electron is regarded as a quasi-free particle with the addition of a perturbation in the form of the nuclear potential. To obtain the energy levels, the perturbation is expanded in terms of $(Z\alpha)^o$. The orders o represent the number of virtual photons involved in the respective processes and are referred to as o – loop contributions (e.g., one-loop for one involved virtual photon).

For higher Z and thereby stronger electromagnetic nuclear fields, higher orders become more important. For such systems, so-called all-order calculations are performed, where the nuclear potential is already included in the electron wave functions in the first part of the calculations. This approach incorporates the external fields exactly, and yields bound states that are used as a starting point for the self-energy and vacuum polarization corrections (called Furry picture). All-order calculations, therefore, yield much higher precision, especially for higher nuclear charges. Figure 2.1 demonstrates the necessity for the application of the all-order approach, especially for nuclear charges Z > 20. Shown are the higher-order contributions to the self-energy as a function of Z, which become dominant in the high Z region. In this section, a short overview of the relevant effects is given. As in the previous section, detailed calculations are not presented, and only qualitative results for first-order contributions are given as an indication of the total quantitative impact on electron binding energies in



Figure 2.1: Ratios between higher-order self-energy contributions and total self-energy correction as a function of nuclear charge number *Z*. Figure adapted from [54].

the atomic structure.

Self-energy The term self-energy refers to a process where a bound electron constantly emits and re-absorbs virtual photons. In consequence, does the electron not act as a point-like charge but as a smeared out spatial charge. The resulting finite electron radius leads to a correction of the Coulomb field proportional to Z^4 , which was first calculated by Bethe in 1947 [55]. A practical way to describe such effects is via Feynman diagrams as depicted in figure 2.2a. Feynman diagrams visualize interaction processes schematically while at the same time representing the underlying mathematical formalism. To observe the self-energy effect in experiments, it is generally desirable to use ion systems with a high nuclear charge, as the Z^4 indicates. In total, the self-energy accounts for the largest part of the Lamb shift up to nuclear charge numbers $Z \leq 100$.

Vacuum polarization The Feynman diagram for the vacuum polarization effect can be found in figure 2.2b. In this process, the virtual photons (wave-like lines) produce virtual electron-positron pairs (solid circle), which subsequently decay. The electronpositron pairs are aligned by the positive nuclear charge, which leads to a change in the Coulomb potential. Through this alignment, the total binding energy of the electron is raised and therefore leads to a stronger binding. The first theoretical description was given by Uehling in 1935 [56], predating the experimental discovery through Lamb



Figure 2.2: Feynman diagrams of one-loop QED contributions to the Lamb shift. Straight double lines illustrate electrons bound to the atomic nucleus, and wave-like lines represent photons. The black dots denote vertices and indicate interactions between the involved particles. **(a)** One-loop self-energy: A virtual photon is emitted and reabsorbed by a bound electron. **(b)** One-loop vacuum polarization: A virtual electron-positron pair (double-lined circle) is created by a photon.

and Retherford of the effect by several years [39]. In the first order, the scaling of the vacuum polarization is proportional to Z^4 , same as for the self-energy scaling.

Higher-order corrections In principle, combinations of both previously described effects are possible and have been observed in several Lamb shift experiments (overview given in [51]). Especially in QED calculations of the anomalous magnetic moment, it has been shown that by accounting for higher-order effects, more precise results can be achieved. The most recent values were obtained by calculations taking up to tenth-order contributions into account [41]. These higher-order terms are represented by all possible combinations of self-energy and vacuum polarization effects concerning the number of orders *o*.

Total Lamb shift

The total Lamb shift contains all contributions described in this section. For each atomic or ion system in question, an individual approach with respect to the nuclear size and specific electron configuration has to be pursued. A formula that highlights the general (analytical) dependencies of the Lamb shift in units of the individual contributions $F(Z\alpha)$ is given by

$$E_{\rm L} = m_e c^2 \frac{\alpha}{\pi} \frac{(Z\alpha)^4}{n^3} F(Z\alpha).$$
(2.21)

 $F(Z\alpha)$ contains the individual properties of the atomic/ion system regarded and has i.a. been calculated via all-order calculations by Yerokhin and Shabaev for the n = 1and n = 2 states in hydrogen-like atoms up to Z = 110 [51]. Precision is usually limited by imprecise knowledge of the atomic properties, such as the nuclear size, or by computational limits in calculations of higher-order QED contributions. An overview of the size of the different contributions is shown in figure 2.3, which in this case also includes nuclear structure effects described in section 2.1.4.



Figure 2.3: Contributions to the Lamb shift of 1s-levels for hydrogen-like ions in dependence of the nuclear charge Z. The dominant contribution is provided by the self-energy (SE). VP denotes the vacuum polarization and is the second-highest contribution for charge numbers Z < 80. The finite nuclear size contribution becomes more important with rising nuclear charge until the effect size is of the same order as the self-energy for charge numbers of $Z \approx 100$. Graphic taken from [57].

In conclusion of section 2.1, an overview of the main contributions for detailed calculations of electron binding energies in hydrogen-like atomic systems was presented. A graphical overview of the impact of these contributions on the hydrogen atom is given in figure 2.4. However, to understand and calculate structures with more than one electron, electron correlations have to be taken into account while still including all of the effects presented in the previous sections. Calculations, therefore, become much more complicated and numerically demanding. Depending on the complexity of the system under study, different approaches have been developed and will be presented in the



Figure 2.4: Schematic of energy splittings for the two lowest levels in hydrogen. Depicted is the impact of different contributions on the binding energies. The energy levels are not to scale. This schematic is based on [47].

next section.

2.2. Many-body systems

The fundamental problem when treating many-electron systems is that the Schrödinger- (non-relativistic) or Dirac- (relativistic) equation cannot be solved analytically anymore. Over the last decades, different kinds of approximation methods were established that can be applied to calculate transition energies for atomic many-body systems. In the following, four different methods are presented that were used in previous works for transition energy calculations of the investigated lithium-like carbon ions (${}^{12}C^{3+}$).

Many-body Dirac-Coulomb-Breit Hamiltonian

In order to find approximate solutions for atoms/ions with several electrons, one has to take different interactions between the electrons into account. By introducing several

negative charges, an electrostatic repulsion will occur between the negative charge carriers. Furthermore, will these particles interact magnetically through their intrinsic spins and thus affect their orbital motion. Additionally, all effects considered in the previous sections are still valid and have to be included in the calculations. Hence, same as for hydrogen-like systems, the eigenvalue problem $\hat{H}\Psi = E\Psi$ has to be solved for the complete atomic wave function Ψ . In order to demonstrate the strategy of many-body calculations, QED- and hyperfine-effects will be neglected in this section, as they can be included by introducing perturbations to the single-electron Hamiltonian according to sections 2.1.3 and 2.1.5.

In many-body theory, wave functions depend on the spatial coordinates r_i and the spins σ_i of all electrons

$$\ddot{H}\Psi(\mathbf{r}_{1}\sigma_{1},\mathbf{r}_{2}\sigma_{2},\ldots,\mathbf{r}_{N}\sigma_{N}) = E\Psi(\mathbf{r}_{1}\sigma_{1},\mathbf{r}_{2}\sigma_{2},\ldots,\mathbf{r}_{N}\sigma_{N}), \qquad (2.22)$$

where \hat{H} now represents the many-body Hamiltonian. The Hamiltonian most commonly used for the presented numerical methods is the so-called Dirac-Coulomb-Breit Hamiltonian \hat{H}_{DCB} . It consists of the sum over all single-particle Dirac-Hamiltonians (see equation (2.4)), the sum over the Coulomb repulsion between electrons $V_{el}(\mathbf{r}_{ij})$ and the Breit operator B_{ij} that partially accounts for retardation effects and magnetic interactions between the electrons [46, 58, 59]

$$\hat{H}_{\text{DCB}} = \sum_{i=1}^{N} \underbrace{c \boldsymbol{\alpha}_{i} \cdot \boldsymbol{p}_{i} - \beta_{i} m_{e} c^{2} + V_{\text{nuc}}(\boldsymbol{r}_{i})}_{\hat{h}_{\text{D}}(i)} + \sum_{i$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ denotes the distance between electrons *i* and *j*. In general, this Hamiltonian has to be solved to obtain the energy levels for a many-body system (with possible extensions of QED- and hyperfine-effects). However, the two-electron terms are not separable without further assumptions or approximations.

Central field approximation

In order to address the inseparability of the many-body Hamiltonian, the so-called central field approximation can be utilized. Instead of introducing two additional terms for the two-electron potentials $V_{el}(\mathbf{r}_{ij})$ and the Breit operator B_{ij} , an effective spherically symmetric potential $V_i(\mathbf{r}_i)$ acting on electron *i* is applied together with the nuclear potential $V_{\rm nuc}(\mathbf{r}_i)$

$$V_i(\mathbf{r}_i) = V_{\text{nuc}}(\mathbf{r}_i) + u_i(\mathbf{r}_i), \qquad (2.24)$$

where $u_i(r_i)$ represents the average effect of the repulsive interactions between electron *i* and all other N - 1 electrons. This leads to an independent particle model where each electron moves in an effective potential so that the N-electron Hamiltonian can be approximated as

$$\hat{H}_{\text{DCB}} \approx \sum_{i=1}^{N} \underbrace{c \boldsymbol{\alpha}_{i} \cdot \boldsymbol{p}_{i} - \beta_{i} m_{e} c^{2} + V_{i}(\boldsymbol{r}_{i})}_{\hat{h}_{\text{D}}^{\prime}(i)}, \qquad (2.25)$$

with $\hat{h}'_{\rm D}(i)$ as the effective one-particle Dirac-Coulomb Hamiltonian. In this way, the solution to $\hat{h}'_{\rm D}(i)$ factorizes into one-electron functions and can therefore be separated into radial and spin-angular parts. The individual one-electron orbitals $\phi_{n\kappa m}(\mathbf{r},\sigma)$ then have the following four-spinor form in a spherical coordinate system (r, ϑ, φ) [60]

$$\phi_{n\kappa m}(\boldsymbol{r},\sigma) = \frac{1}{r} \begin{bmatrix} G_{n\kappa}(r) \, \chi_{\kappa m}(\vartheta,\varphi,\sigma) \\ iF_{n\kappa}(r) \, \chi_{-\kappa m}(\vartheta,\varphi,\sigma) \end{bmatrix}, \qquad (2.26)$$

where $G_{n\kappa}(r)$ and $F_{n\kappa}(r)$ denote the radial eigenfunctions, $\chi_{\kappa m}$ is the spin-angular spinor, κ is the relativistic angular parameter

$$\kappa = \begin{cases} -(l+1) & \text{for } j = l + \frac{1}{2} \\ +l & \text{for } j = l - \frac{1}{2} \end{cases}$$
(2.27)

and *m* is the angular momentum projection. The central field approximation has proved successful in the past as a starting point for more sophisticated numerical methods, as shown in the following paragraphs. It has to be noted, however, that finding the effective potential $V_i(\mathbf{r}_i)$ is not a trivial task and has to be considered individually for each many-body system to be investigated.

2.2.1. Relativistic configuration interaction method (RCI)

One of the standard methods in modern atomic structure calculations is the relativistic configuration interaction method (RCI). The fundamental idea of this method is to describe the specific state of a multi-electron system by a linear combination of configuration-state functions (CSF). The atomic state is then represented by a multiconfiguration atomic state function Ψ (ASF) with parity *P*, angular momentum quantum number *J* and magnetic quantum number *M* [60]

$$\Psi(P, J, M) = \sum_{i=1}^{N_{\rm CSF}} c_i \Phi_i(\gamma_i, P, J, M), \qquad (2.28)$$

where the coefficients c_i are variational parameters, $\Phi_i(\gamma_i, P, J, M)$ are the various CSFs and γ_i denotes all further quantum numbers needed to specify the individual CSF. Since electrons are fermions, the wave functions must be antisymmetric under coordinate exchange of two particles.

Therefore, a CSF is constructed from linear combinations of so-called Slater determinants that satisfy the Pauli exclusion principle due to their antisymmetric shape. The Slater determinants Φ_S are built from antisymmetrized products of one-electron orbitals as in equation (2.26) [46, p. 296]

$$\Phi_{\mathrm{S}}(\boldsymbol{r}_{1}\sigma_{1},\boldsymbol{r}_{2}\sigma_{2},\ldots,\boldsymbol{r}_{N}\sigma_{N}) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_{1}(\boldsymbol{r}_{1}\sigma_{1}) & \phi_{2}(\boldsymbol{r}_{1}\sigma_{1}) & \cdots & \phi_{N}(\boldsymbol{r}_{N}\sigma_{1}) \\ \phi_{1}(\boldsymbol{r}_{2}\sigma_{2}) & \phi_{2}(\boldsymbol{r}_{2}\sigma_{2}) & \cdots & \phi_{N}(\boldsymbol{r}_{2}\sigma_{2}) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1}(\boldsymbol{r}_{N}\sigma_{N}) & \phi_{2}(\boldsymbol{r}_{N}\sigma_{N}) & \cdots & \phi_{N}(\boldsymbol{r}_{N}\sigma_{N}) \end{vmatrix}, \quad (2.29)$$

with $\phi_i(\mathbf{r}_i\sigma_i)$ as the one-electron orbitals depending on the spatial coordinates \mathbf{r}_i and the spins σ_i of the electrons. By using this basis, a CSF $\Phi_i(\gamma, P, J, M)$ is a normalized $(\langle \Phi_i(\gamma_i, P, J, M) | \Phi_i(\gamma_i, P, J, M) \rangle = 1)$ eigenstate of the squared total angular momentum operator \hat{J}^2 , the squared total spin operator \hat{S}^2 and their respective z-projections \hat{J}_Z and \hat{S}_Z .

To find solutions to equation (2.28), the coefficients c_i have to be determined by applying variational methods such as the Rayleigh-Ritz method [61, 62]. This leads to the matrix eigenvalue problem

$$Hc = ESc, \qquad (2.30)$$

where c is the coefficient vector that includes all expansion coefficients c_i of equation (2.28), E is the total energy of the desired state and S is the overlap matrix, which becomes the identity matrix if Φ_i are constructed to be orthonormal. H is an $N \times N$ matrix called interaction matrix and its elements H_{ij} are obtained by applying the Hamil-
tonian (e.g. \hat{H}_{DCB}) to the basis functions Φ_i

$$H_{ij} = \left\langle \Phi_i \middle| \hat{H}_{\text{DCB}} \middle| \Phi_j \right\rangle \,. \tag{2.31}$$

Electronic wave functions can in principle be exactly described by equation (2.28) if all possible combinations of Slater determinants/CSFs are constructed from a complete set of one-electron functions $\phi_i(\mathbf{r}_i\sigma_i)$. This procedure is called full configuration interaction and yields exact results for the eigenfunctions and eigenvalues. However, this technique can only be applied to relatively small systems due to the high number of N-electron basis functions required, whose amount is limited by computation power. Therefore, it is common to use further approximations to reduce the number of basis functions, as discussed in [62].

2.2.2. Multiconfiguration Dirac-Hartree-Fock method (MCDHF)

The multiconfiguration Dirac-Hartree-Fock method (MCDHF) can be viewed as an expansion of the RCI method. It is also based on the atomic state function $\Psi(P, J, M)$ (see equation (2.28)) expanded in a basis set of *i* configuration state functions (CSFs). The main difference to RCI is that besides varying the expansion coefficients c_i , also the radial functions ($G_i(r), F_i(r)$) are varied for a stationary energy. In essence, the solution process is then divided into two phases [61]:

- 1. determination of expansion coefficients c_i for the given set of CSFs, which is identical to the RCI process of section 2.2.1
- 2. determination of the radial wave functions that define the orbitals ϕ_i for the construction basis of CSFs for a given set of expansion coefficients c_i

Applying the variational principle to the so-called *energy functional* yields the full set of MCDHF equations. The energy functional is defined as the total energy for the normalized wave function or ASF $\Psi(P, J, M)$

$$E\Psi = \langle \Psi | \hat{H}_{\text{DCB}} | \Psi \rangle \quad \text{with } \langle \Psi | \Psi \rangle = 1,$$
 (2.32)

where Ψ includes the radial functions as explained in section 2.2. Variation of the energy functional in terms of the expansion coefficients c_i again leads to the matrix eigenvalue problem (equation (2.30)) and thus to the solution of the RCI phase. For the second phase, the energy functional is now varied with respect to the radial wave

functions, which yields the radial MCDHF equations [61]

$$w_{i} \begin{bmatrix} V_{i}(r) & -c \left[\frac{d}{dr} - \frac{\kappa_{i}}{r}\right] \\ c \left[\frac{d}{dr} - \frac{\kappa_{i}}{r}\right] & V_{i}(r) - 2c^{2} \end{bmatrix} \begin{bmatrix} G_{i}(r) \\ F_{i}(r) \end{bmatrix} = \sum_{j} \epsilon_{ij} \,\delta_{\kappa_{i}\kappa_{j}} \begin{bmatrix} G_{i}(r) \\ F_{i}(r) \end{bmatrix}, \qquad (2.33)$$

where w_i is the occupation number of orbital *i*, $V_i(r)$ is the sum of the nuclear potential and the direct part of the electron-electron potential, *c* is the vacuum speed of light, $\kappa_{i,j}$ are the relativistic angular quantum numbers of the orbitals, δ is the *Kronecker delta*, and $\epsilon_{i,j}$ are so-called *Lagrange multipliers* to impose the orthonormality for each set of radial eigenfunctions ($G_i(r)$, $F_i(r)$) throughout the whole MCDHF process.

The general procedure to solve the MCDHF equations if only the radial wave functions are to be varied is to take an initial estimate for the radial functions and solve equation (2.33) which yields a more accurate set of orbitals. This procedure is applied iteratively until the derived solutions for the radial wave functions converge and is referred to as *self-consistent field method* (SCF). However, in the MCDHF approach, the expansion coefficients c_i and the radial functions ($G_i(r)$, $F_i(r)$) are coupled so that the RCI and the SCF processes are iterated simultaneously until the solutions of the radial wave functions and the energies both converge. This combined process is called multiconfigurational self-consistent field (MC-SCF) method.

2.2.3. Relativistic many-body perturbation theory (RMBPT)

With the relativistic many-body perturbation theory (RMBPT), effects like electron correlation are treated as a perturbation V to the unperturbed Hamilton operator H_0

$$H = H_0 + V$$
. (2.34)

As a prerequisite, the energy eigenvalues E_0 and the eigenfunctions Ψ_0 of the undisturbed system have to be evaluated by, e.g., Dirac-Hartree-Fock (DHF) calculations. The general undisturbed solution is denoted as:

$$H_0 \Psi_0 = E_0 \Psi_0 \,. \tag{2.35}$$

The application of the perturbative approach requires that the perturbation of the system (e.g., the contribution of the correlation energy) is small. In this case, Ψ_0 and E_0 are close to the exact wave function Ψ and the energy *E*. If this requirement is met, the

disturbed system can be described by the Hamilton operator H_{λ}

$$H_{\lambda} = H_0 + \lambda V \,, \tag{2.36}$$

where λ denotes a variable real parameter that determines the size of the perturbation. The wave function Ψ_{λ} and the energy eigenvalues E_{λ} of the disturbed system can each be represented by Taylor series

$$\Psi_{\lambda} = \Psi_0 + \lambda \Psi_1 + \lambda^2 \Psi_2 \dots = \sum_{i=0}^m \lambda^i \Psi_i , \qquad (2.37)$$

$$E_{\lambda} = E_0 + \lambda E_1 + \lambda^2 E_2 \dots = \sum_{i=0}^{m} \lambda^i E_i .$$
 (2.38)

Both series are then substituted into the time-independent Schrödinger equation (see equation (2.1)) yielding

$$(H_0 + \lambda V) \left(\sum_{i=0}^m \lambda^i \Psi_i \right) = \left(\sum_{i=0}^m \lambda^i E_i \right) \left(\sum_{i=0}^m \lambda^i \Psi^{(i)} \right).$$
(2.39)

Expanding the products and collecting like terms gives (exemplary for m = 2)

$$E_0 = \langle \Psi_0 | H_0 | \Psi_0 \rangle , \qquad (2.40)$$

$$E_1 = \langle \Psi_0 | V | \Psi_0 \rangle , \qquad (2.41)$$

$$E_2 = \langle \Psi_0 | V | \Psi_1 \rangle . \tag{2.42}$$

These equations are represented by the known eigenfunctions Ψ_0 and energy eigenvalues E_0 , where Ψ_0 is the DHF Slater determinant. Therefore, the sum of the energies E_0 and E_1 corresponds exactly to the DHF energy E_{DHF} , which at the same time is the first-order result of the perturbation energy E_{MP1}

$$E_{\rm MP1} = E_{\rm DHF} = E_0 + E_1 \,. \tag{2.43}$$

The effects of electron correlation are, therefore, only taken into account from the second-order energy E_2 onward. The desired precision of the result then determines the orders to be taken into account. However, in many cases, the series development is stopped after the second-order since the computational effort increases significantly for higher orders. This method is then referred to as Möller-Plesset (MP2) perturbation

theory for which the energy is

$$E_{\rm MP2} = E_0 + E_1 + E_2 = E_{DHF} + E_2.$$
(2.44)

2.2.4. Fock space coupled-cluster method (FSCC)

The basis for the Fock space coupled-cluster method (FSCC) is the general coupledcluster method (CC). In the CC approach, an alternative description of the wave function Ψ is chosen [63]

$$\Psi_{\rm CC} = e^T \Phi_0 \,, \tag{2.45}$$

where Φ_0 is the configuration state function (CSF) for the system's ground state, and *T* is the so-called cluster or excitation operator. This operator is written as

$$T = T_1 + T_2 + T_3 + \dots = \sum_i T_i, \qquad (2.46)$$

where the operators T_i generate the determinants in the *i*-times excited states. In its general form, the coupled cluster operator is given by

$$T_n = \frac{1}{(n!)^2} \sum_{h_1 \dots h_n} \sum_{p_1 \dots p_n} t_{p_1 \dots p_n}^{h_1 \dots h_n} a^{p_1} \dots a^{p_n} a_{h_1} \dots a_{h_n}, \qquad (2.47)$$

where a^p and a_h are the creation and annihilation operators for the occupied orbitals h and the unoccupied orbitals p. To fully determine T, the unknown coefficients t have to be found. This is achieved by inserting the Dirac-Coulomb-Breit Hamiltonian H_{DCB} (see equation (2.23)) and the wave function Ψ_{CC} into the Schrödinger equation and solving it for t [63].

Instead of simply applying the operators T_n to the ground state function Φ_0 , the exponential cluster operator e^T is used as seen in equation (2.45). This has the advantage that due to the different powers of the different T_i that appear in the resulting expressions (such as below expansion), more than twice the excitation determinants can be generated. The exponential cluster operator itself is usually expanded as a Taylor series, which yields

$$e^{T} = 1 + T + \frac{1}{2!}T^{2} + \dots = 1 + T_{1} + T_{2} + \frac{1}{2}T_{1}^{2} + \frac{1}{2}T_{1}T_{2} + \frac{1}{2}T_{2}T_{1} + \frac{1}{2}T_{2}^{2} + \dots$$
(2.48)

In the above expansion, only operators up to T_2 are considered to illustrate the occurrence of mixed terms such as $\frac{1}{2}T_1T_2$. Through these mixed terms, higher-order excitations are also partly included, which is why the CC method is especially suited for calculations of highly correlated systems. The full method is referred to as Fock-space coupled-cluster method (FSCC) if the ground state function Φ_0 is determined by a specialized method called *Dirac-Fock-Breit self-consistent-field method* [64].

2.3. Conclusion

In total, four different methods of computing electron correlation energies in manybody systems were presented in the previous section. However, these represent only a part of the existing methods and were selected representatively since many of the ion systems examined at GSI are calculated with these methods (see sections 4.1 and 5.3.5). Each method has its advantages and disadvantages in terms of accuracy in contrast to calculation complexity and the associated computing effort. At the same time, all presented many-body methods have in common that the QED effects also presented in this chapter are not taken into account intrinsically and, therefore, have to be included in the calculations as additional perturbation or correction terms, which leads to inherent inaccuracies. One of the major tasks of theory is to integrate QED-, relativistic, and electron correlation effects into one cohesive theory. Nowadays, this is researched via dedicated investigations of the individual effects by comparing high-precision measurement results to the different theoretical approaches and thus refining the available structure calculation methods.

Especially the advent of improved spectroscopic measurement techniques and machinery at heavy-ion accelerator facilities such as GSI/FAIR allowed substantial progress in the understanding of various (higher-order) effects in relativistic many-body calculations and QED over the past years [65]. Highly charged ions are ideal test subjects since they provide some of the most extreme conditions that can be produced in a laboratory. As emphasized in this chapter, relativistic, correlation, and QED effects become more relevant with higher atomic charge numbers and stronger electromagnetic fields in general. Figure 2.5 illustrates the achievable electric and magnetic field strengths that hydrogen-like ions can provide compared to the strongest laser and magnetic pulse systems. The field strengths provided by the ions surpass the field strengths generated via other techniques by several orders of magnitude in both domains. Another advantage of using highly charged ions is the possibility to prepare them in config-



Figure 2.5: Expectation values of electric and magnetic field strengths in hydrogen-like highly charged ions depending on the nuclear charge *Z*. **Left:** Electric field strength *E* for ${}^{1}S_{1/2}$ state (blue) calculated according to [66]. The dashed red line marks the upper limit achieved with the strongest lasers available ($I_{\text{max}} = 5.5 \cdot 10^{22} \text{ W/cm}^2$) [67]. **Right:** Magnetic field strength *B* for ${}^{1}S_{1/2}$ state (blue) calculated according to [68]. The dashed red line marks the upper limit achieved with the strongest pulsed magnets available ($B_{\text{max}} = 1200 \text{ T}$) [69].

urations, which allow a detailed investigation of individual effects. For the study of electron correlations, the configurations are chosen in such a way that energy differences between electron energy levels remain almost unaffected by QED-effects. Such a transition was, for example, found in be-like krypton (84 Kr³²⁺) for which the XUV detection system described in chapter 4 was constructed. Here, the transition energy of the ($1s^22s2p$) $^{3}P_{0}$ - $^{3}P_{1}$ level splitting is mainly dominated by electron correlation and relativistic effects [6].

In conclusion, a large part of the atomic research conducted at GSI is committed to refining available theoretical atomic structure calculations or even finding new underlying physics to create more sophisticated models. To reach this aim, it is essential to enhance the precision of the obtained experimental results used to compare to theoretical predictions. This work is therefore dedicated to the development and commissioning of two new instruments that extend the experimental capabilities (detection of forward emitted XUV photons at the ESR) and improve the knowledge of experimental parameters (ultra-precise HV measurement at the CRYRING@ESR electron cooler) to increase the precision of spectroscopy experiments with highly charged ions at FAIR.

Chapter 3

The experimental framework of FAIR

The facility for antiproton and ion research (FAIR) will be one of the largest and most complex accelerator facilities worldwide. Its accelerator system will deliver particle beams from hydrogen to uranium as well as antiprotons available for experiments covering a variety of different scientific fields such as heavy ion physics, atomic physics, materials research, radiation biophysics, and nuclear astrophysics. The particles are accelerated to almost speed of light in the FAIR accelerator system and fed to the different experiments. FAIRs centerpiece is an underground ring accelerator called SIS100 with a circumference of 1100 m, which is currently under construction. Experiments are then conducted at different storage rings and experimental stations that add up to several kilometers of beamlines in total. An overview of the planned FAIR facility is shown in figure 3.1.

Fair will be an extension to the already existing accelerator infrastructure at GSI (blue) so that the existing accelerators UNILAC and SIS18 serve as injectors for the new FAIR facility (red). The 120 m long linear accelerator UNILAC can accelerate particles up to 20 % speed of light. It is used as a pre-accelerator for the SIS18 ring accelerator, which has a circumference of 216 m and can further accelerate ions to speeds as high as 90 % speed of light. Connected to the SIS18 will be the new SIS100 accelerator able to accelerate all the natural elements in the periodic table up to 99 % speed of light [71].

Directly connected to the SIS100 are multiple storage rings and experimental stations, all serving different research opportunities by being able to handle different energy ranges or particle types such as ions, antiprotons, or exotic isotopes. As the XUV detection system and the precision high-voltage divider presented in this thesis are instruments for the storage rings ESR and CRYRING@ESR, the following overviews



Figure 3.1: Overview of the planned FAIR facility. The existing structures of GSI are depicted in blue, whereas the structures under construction are depicted in red. Figure taken from [70].

are focused on these facilities.

3.1. The Experimental Storage Ring (ESR)

The ESR is an ion storage ring that allows the storage of charged particles at relativistic speeds between 10% and 90% speed of light and nuclear charge numbers from 2 to 92 (helium to uranium). It has a circumference of approximately 108 m in which an injected ion beam is kept on a fixed trajectory by a total of six dipole magnets, which generate magnetic fields of up to 1.6 T resulting in a bending power of up to 10 Tm (see figure 3.2). In addition, it contains quadrupole and sextupole magnets to focus and correct the ion beam. Before ions are stored in the ESR, they are first accelerated in the heavy ion synchrotron SIS18 to the desired velocities/energies and then transferred to the ESR. An essential feature of the ESR is the possibility to improve the beam quality by many orders of magnitude through phase space cooling techniques for which electron cooling and stochastic cooling are available. The electron cooling process occurs at a dedicated electron cooler (see section 3.3.1) in the eastern section of the ESR. On the opposite side of the electron cooler, the ESR features an experimental section with a



Figure 3.2: Schematic of the ESR with the most important ion-optical elements. The ESR features a dedicated electron cooler on the opposite side of the optical detection/experimental section. Figure taken from [72].

gas-jet target and slots for optical detectors used in laser spectroscopy experiments, where the XUV detection system described in chapter 4 is located.

Another feature of the ESR is the ability to decelerate ion beams produced at high energies to lower energies. The decelerated beams can be used for experiments in the storage ring itself or extracted from the storage ring. For this purpose, two extraction processes are available. A *slow extraction* for experiments at external targets and a *fast extraction* with which the low-energy ions can be forwarded to CRYRING@ESR, where they are further decelerated and stored. The storage of highly-charged ions, especially at low energies, requires an excellent vacuum, which is why the ESR uses a UHV-system that allows a pressure in the range of $1 \cdot 10^{-11}$ mbar.

3.2. The CRYRING@ESR storage ring

CRYRING@ESR is a storage ring for heavy ions formerly based at the Manne Siegbahn Laboratory of Stockholm University. In 2012/2013. it was transferred to GSI to be used as low energy storage ring in the FAIR infrastructure. It is located downstream of the



Figure 3.3: Top view schematic of the CRYRING@ESR storage ring. Figure taken from [73].

ESR and can decelerate and store ions in an energy range of 0.1 - 30 MeV/u. It has a circumference of approximately 54 m featuring twelve dipole magnets with a maximum bending power of up to 1.44 Tm to keep the stored ions on a fixed trajectory (see figure 3.3). Same as the ESR, it features an electron cooler to improve the beam quality (see section 3.3.2). It also features numerous quadrupole and sextupole magnets to focus and correct the ion beam. One of the key components of the CRYRING@ESR is its electron cooler used to optimize the ion beam quality in terms of the ions' momentum spread (see section 3.3.2). Further sections contain an injection system, an extraction system, multiple sections for experiments, and an RF cavity for (de-) acceleration. Through the injection system, ions can be injected from the ESR or alternatively from a local ion injector that allows a standalone operation of the CRYRING@ESR.

3.3. Electron cooling

One of the most important techniques for cooling ion beams in storage rings is electron cooling. By cooling the ions, an improved beam quality with regard to the ion moment spread can be achieved. In this manner, the uncertainty of the ion velocity, which is often a limiting factor in storage ring experiments, is decreased significantly.

In electron cooling, a beam of cold electrons (low momentum spread) is superimposed with the ion beam to reduce the ion beam's velocity distribution, size, and divergence. The electrons are first generated in an electron gun and accelerated electrostatically to match the ions' speed in the storage ring. Due to the electrostatic acceleration, the electrons have a narrow momentum distribution. Via magnetic fields, the electrons are then superimposed with the ion beam and, after leaving the interaction section, are then guided to a dedicated collector.

Ions with large velocity differences compared to the electron beam transfer energy to the electrons via Coulomb interactions. Because every time they pass the electron cooler, the ions come into contact with new, cold electrons, the momentum distribution of the ion beam is reduced. Collisions between the ions (intrabeam scattering) act against the cooling force, which creates a balance between both effects after a few seconds of cooling. The absolute speed of the electrons β_{electron} is mainly determined by the acceleration voltage U_{ecool} of the electron gun that produces the electrons. This velocity is given by

$$\beta_{\rm ion} \equiv \beta_{\rm electron} = \sqrt{1 - \left(1 + \frac{e \cdot U_{\rm ecool}}{m_e \cdot c^2}\right)^{-2}}.$$
(3.1)

Therefore, the ion velocity β_{ion} is determined by the electron cooler voltage U_{ecool} , whose knowledge is typically one of the main sources of uncertainty in ion storage ring experiments.

The great advantage of electron cooling is that it works with every ion species. It can further counteract typical heating mechanisms in the ion beam, for example, in the event of collisions with gas targets or when the beam is decelerated. This typically increases the lifespan of the beam significantly. Furthermore, the electrons of the electron cooler can also be utilized as electron targets on which collision experiments are carried out.



3.3.1. The ESR electron cooler

Figure 3.4: Schematic view of the electron cooler at the ESR. In the electron gun, an indirectly heated cathode generates an electron beam and accelerates it towards ground potential. A bent magnetic field is superimposed with the ion and electron beams and guides both beams through drift tubes, where the cooling takes place. The beams are then separated again via the magnetic field, and the electrons are caught in the collector. Figure taken from [74].

The electron cooler of the ESR is shown schematically in figure 3.4. A barium coated tungsten cathode inside an electron gun is supplied with the acceleration voltage by a *Heinzinger HNC 320000-10 neg* high-voltage power supply (max. voltage $U_{ecool, max.} =$ 320000). The cathode is heated to 1300 K to extract electrons via thermionic emission. Heating is applied indirectly so that the heating current does not affect the acceleration potential. This electrostatic acceleration has the effect that the original Maxwell-Boltzmann velocity distribution of the electrons are guided along magnetic field lines created by solenoid coils towards grounded drift tubes, where they are overlapped with the ion beam. At the drift tube's exit, the electrons are again extracted via the magnetic field and guided towards the collector.

It has to be noted that the materials used for the cathode (barium coated tungsten) and

drift tubes (stainless steel) of the electron cooler also play a significant role, as materialdependent work functions lead to a shift of the electron cooler potential. This shift has to be accounted for, which is discussed in section 5.2.9.

3.3.2. The CRYRING@ESR electron cooler

The CRYRING@ESR electron cooler features a similar setup as to the ESR electron cooler. Figure 3.5 shows a cross-section of the cooler. The cooler's main components are normal conducting steering coils and correction coils, a superconducting magnet coil for the electron gun, a vacuum system with NEG7- and cryopumps, a reservoir for liquid helium, an electron collector, and the electron gun itself. The working principle is identical to the ESR electron cooler described in the previous section. The cooler can also be utilized as an electron target for ion–electron recombination measurements. For this cooler, the maximum allowed voltage is limited to 20 kV.



Figure 3.5: Schematic view of the CRYRING@ESR electron cooler. Figure taken from [75].

3.4. Laser spectroscopy at storage rings

The two storage rings, ESR and CRYRING@ESR, each offer the possibility to carry out laser spectroscopy experiments. This is a proven technique for measuring transition energies / wavelengths of the atomic systems to be investigated. When applying this technique, an atomic transition is excited using a laser system, and the resulting fluorescence light of the subsequent de-excitation processes is measured. Whenever a fluorescence signal is measured, the transition wavelength can be deduced from the laser wavelength that was set during the excitation process. Since the resonance wavelength is usually unknown in advance, a variable laser source must be available to tune the laser system to different excitation wavelengths until the transition is found. Because tunable laser systems are generally expensive to purchase and do not always provide the desired energies / wavelengths, one can take advantage of the Doppler shift between the laboratory frame and the ion's rest frame in storage ring experiments. It is then possible to scan the ion velocity instead of the laser wavelength by changing the electron cooler voltage according to equation (3.1). The varying velocity translates into a varying wavelength of the laser photons observed in the rest frame of the ions. As the ions at GSI are usually stored at relativistic velocities, the Doppler shift can also be utilized to excite transitions with commercial laser systems, which would otherwise not be accessible.

The laser spectroscopy measurements are distinguished between collinear and anticollinear laser spectroscopy. In collinear laser spectroscopy, the laser is directed in the same direction as the ion beam, and in the anti-collinear case, the laser is directed in the opposite direction to the ion beam. The Doppler shift of the wavelength between laboratory and ion rest frame is given by

$$\lambda_{0} = \frac{\lambda_{\text{lab}}}{\gamma \left[1 - \beta \cos(\theta)\right]} \qquad \begin{cases} \text{collinear} : & \theta = 0\\ \text{anti-collinear} : & \theta = \pi \end{cases}, \tag{3.2}$$

where λ_0 is the wavelength in the ion rest frame, λ_{lab} is the measured laser wavelength in the laboratory system, $\gamma = (\sqrt{(1 - \beta^2)^{-1}})$ is the Lorentz factor, β is the ratio ν/c , and θ is the angle between ion and laser beam. Here, $\theta = 0$ refers to collinear laser spectroscopy and $\theta = \pi$ to anti-collinear laser spectroscopy. The next chapter describes an XUV detection system designed for such a laser spectroscopy experiment, where the Doppler shift is utilized to shift the laser wavelength from 276 nm to 118 nm in the ion's rest frame.

Chapter 4

Detection system for forward emitted XUV photons from relativistic ion beams

In this chapter, the XUV detection system for extreme ultraviolet fluorescence photon detection in laser-spectroscopy measurements at the ESR is presented. This device was initially developed and constructed in the scope of the Ph.D. thesis by J. Vollbrecht [19]. The detector development was taken over in 2014, and the optimizations presented in this chapter were investigated in close collaboration with F. Tritmaak and C. Egelkamp, who already published the results in their respective bachelor and master theses [76, 77]. This chapter gives an overview of the detector setup, the working principle, and a summary of the optimizations, which mainly resulted from SIMION 8.1 particle tracking simulations [21].

4.1. Anti-collinear laser spectroscopy of beryllium-like krypton (⁸⁴Kr³²⁺)

As described in section 2.3, highly charged heavy ions (HCI) are ideal systems to investigate individual contributions of atomic structure calculations, such as QED-, relativistic- or electron correlation effects. In many cases are the systems chosen so that only one of the effects mainly contributes to the transition energy that is to be investigated. In 2011 an experiment to investigate electron correlation effects by means of anticollinear laser spectroscopy was proposed by Winters *et al.* [6]. The $(1s^22s2p)^3P_0^{-3}P_1$ fine-structure transition in beryllium-like krypton (⁸⁴Kr³²⁺) fulfills multiple requirements that allow for a precise measurement of electron correlation effects. The main advantage of this system is that the transition energy is mainly determined by electron correlation effects, as effects of QED almost do not contribute. A detailed break down of the individual contributions is given in [6] which was calculated with the multi-configuration Dirac-Hartree-Fock method (MCDHF) described in section 2.2.2. Besides the dominating electron correlation, this transition has further advantages that make it feasible to measure in a laser-spectroscopy setup at the ESR. One advantage is that the transition with an approximate wavelength of 118 nm can be excited by a commercial laser system in an anti-collinear laser spectroscopy setup by utilizing the large Doppler shift when ions are stored at relativistic energies according to equation (3.2). In this proposed experiment, the ions are stored at a velocity of $\beta \approx 0.69$. In this manner, the transition can be excited with laser systems operating at approximately 276 nm (see figure 4.1). The left figure shows the two-step process in which the ions are excited



Figure 4.1: Beryllium-like krypton (⁸⁴Kr³²⁺) level scheme and Lorentz boost. **Left:** The level scheme shows the two-step process of exciting the M1 transition and de-excitation via the E1 transition. This figure was taken form [77]. **Right:** Emission characteristic of ⁸⁴Kr³²⁺ fluorescence photons in the laboratory rest frame when stored at $\beta \approx 0.69$ in the ESR. The wavelengths of the photons emitted in the E1 transition are Doppler shifted down to approximately 7 nm and Lorentz boosted in forward direction.

from the metastable ${}^{3}P_{0}$ -state via an M1 transition to the ${}^{3}P_{1}$ -state. The metastable state has the advantage of a long lifetime in the order of $1 \cdot 10^{7}$ s which allows for enough time between preparation of the ions in the UNILAC and excitation in the ESR. A downside of this transition is that after the de-excitation via the E1 transition, the ions cannot be excited anymore, which leads to comparatively low signal rates (see section 5.3.6). Further, does the system need to be excited in the same section where the detector is located since decay times are in the order of 1 ns. This has the downside that laser stray light in the beam pipe can cause high background rates in the detector. Therefore, background reduction methods become necessary to guarantee a feasible signal-to-noise ratio, as described in chapter 5.

For this experiment, a suitable detection system needs to be able to measure fluores-

cence photons down to 7 nm, as the E1 de-excitation photons of approximately 17 nm in the ion rest frame are also Doppler-shifted according to equation (3.2). The ion emission characteristic is also altered from an isotropic emission in the ion rest frame to a narrow forward cone with regard to the laboratory frame due to a Lorentz boost. This boost is described by

$$\theta_{\text{lab}} = \arccos\left(\frac{\cos(\theta_{\text{ion}}) + \beta_{\text{ion}}}{1 + \beta_{\text{ion}}\cos(\theta_{\text{ion}})}\right),\tag{4.1}$$

where θ_{lab} is the emission angle in the laboratory rest frame, θ_{ion} is the fluorescence photon emission angle in the ion rest frame, and β_{ion} is the ratio v_{ion}/c . The resulting emission characteristic is shown in figure 4.1. Therefore, the detection system has to be positioned in the vicinity of the ion beam inside the vacuum, in order to collect the majority of the emitted fluorescence photons.

4.2. Detector setup

The design of the detection system is similar to the detection system used for the measurements of optical photons in the LiBELLE experiment [7]. The mechanical setup of the detection system before the optimization is shown in figure 4.2. To account for the Lorentz boost described in the previous section, the detector's design features a movable cathode with a central slit that can be positioned in the vicinity of the ion beam. This is where the majority of the photons are expected due to the described emission characteristics. The cathode can be moved into the ESR via a linear feedthrough that is connected to an air pressure motor. Both positions are shown in figure 4.2 top and bottom, respectively. Since the collected photons have wavelengths down to 10 nm, a reflection of the photons from the cathode onto a photomultiplier as used for LiBELLE is not possible since photons in this wavelength region are rarely reflected anymore. Instead, a suitable metal is used as cathode material that allows impinging photons to create photoelectrons inside the cathode, which create secondary electrons (see section 4.4) that leave the cathode (see figure 4.3). In case of the old design by J. Vollbrecht this was chosen to be stainless steel. The created secondary electrons are then guided electromagnetically onto a microchannel plate detector (MCP) in the vacuum, located in position c of figure 4.2. One part of the electromagnetic guiding is realized via an electrode system, also shown in figure 4.2, that creates a static electric field to focus the secondary electrons towards the central axis of the MCP detector. Further (and more



Figure 4.2: Mechanical setup of the XUV detection system as constructed in the Ph.D. thesis of J. Vollbrecht [19]. **Top** shows the fully extended system and **bottom** the contracted system. The movable parts consist of the photocathode (a) and four ring electrodes (b). A static electrode (\tilde{b}) is mounted in front of the MCP detector (c), which is connected to the flange. Two electrical feedthrough flanges (d) and (\tilde{d}) are implemented to supply the photocathode, the electrodes, and the MCP with the desired voltage. The linear feedthrough (e) can be adjusted with micrometer screws (f) and is moved by an air-pressure motor (g). Two end switches (h) limit the movement of the motor. This figure is taken from [19].

important) guidance is provided by a magnetic field, which is created by two magnet coils mounted concentrically around the MCP (not shown in the figure). The cathode plate is operated on a negative potential to accelerate the secondary electron away from the cathode surface. The MCP is operated on a positive potential so that electrons are accelerated towards the MCP. Subsequently, the MCP amplifies the incoming electron signal, which can then be further processed with dedicated measurement electronics described in section 5.1.5.

To maximize the efficiency of the XUV detection system, all the electric and magnetic field settings were optimized. All relevant components and their optimizations are described in the following sections, leading to an optimized detector configuration used for the lithium-like carbon laser spectroscopy measurement presented in chapter 5.



Figure 4.3: Schematic depiction of secondary electron generation with the cathode plate. The laser beam (red) is overlapped anti-collinearly with the ion beam (black). Fluorescence photons are created (blue lines) that impinge on the cathode plate, where they create secondary electrons (green dots) that are guided onto an MCP detector (not shown). Figure taken from [77].

4.3. MCP

The microchannel plate detector (MCP) used for secondary electron detection is of type Roentdek DET40 and has an active area diameter of 40 mm. It features two channel plates in Chevron configuration and an additional anode. The Chevron configuration is used to minimize possible ion feedback inside the device. Figure 4.4 shows the detection principle on the example of a single microchannel plate. As seen in the figure, an electron cascade is created induced by incident electrons, thus amplifying the incoming signal. The electron cascade is then detected as a negative charge pulse on the anode behind the Chevron stack. Two main factors determine the MCP performance. One is the conversion efficiency of the microchannel walls, which are coated with a semiconductor material. This efficiency depends on the incoming particle species and energy. It is given by the manufacturer in [79]. The second criterion that determines the MCP performance is its gain. Besides the material properties fixed by the manufacturer's design, the gain is determined by the voltage difference between the front- and backplate of the Chevron stack. The manufacturer states a gain of approximately $1 \cdot 10^7$ for a voltage difference of 2400 V between front- and backplate. However, the gain can also be affected by the incident rate. Since each MCP channel has to provide the electrons for the electron cascade, it is possible that for high incident rates, the MCP channel cannot recharge in time before the next particle(s) hit the regarded channel. This leads



Figure 4.4: Schematic working principle of MCP detector. An incident electron impinges in the wall of a microchannel. Secondary electrons are created that are accelerated according to the applied positive potential (see right). The accelerated electrons collide with the channel wall due to the channel's tilted angle with respect to the central axis, and thus creating a new wave of electrons. As a result, an electron cascade is created, which can be measured as a voltage drop on the anode behind the Chevron stack. Figure taken from [78].

to saturation effects inside the MCP and, therefore, a nonlinear gain dependence on the incident particle rate. During the commissioning beamtime of the XUV detector, rates up to ca. $1 \cdot 10^6$ cps were measured, leading to MCP saturation effects affecting the MCP gain. This behavior was accounted for by a dedicated gain characterization based on theoretical MCP gain models, further discussed in section 5.3.2.

Concerning the mechanical setup, the MCP stack is housed inside a grounded shielding since incoming electrons could otherwise be influenced by the voltage connections of the plates (also see figure 4.2). MCP backplate and anode are connected to the same high voltage supply via a voltage divider. The voltage difference between the two is set to 300 V as proposed by the manufacturer. To detect electrons, the MCP is operated on positive potential. A typical voltage configuration during the XUV test beamtime is given by

MCP frontplate : +300 V MCP backplate : +2100 V MCP anode : +2400 V



Figure 4.5: MCP decoupling schematic according to [80]. Each HV connection of the respective plate features a $10 \text{ k}\Omega$ resistor to limit the currents. Signal decoupling is realized by a 4.7 nF capacitor where the signals can be picked up, e.g., with a fast amplifier. Signal reflections can be minimized by adjusting a 250Ω potentiometer.

However, these settings can vary depending on the desired gain. To decouple the signal from the applied high voltages, an RC circuit is connected to the frontplate, backplate, and anode of the MCP detector. The circuit with its individual components is shown in figure 4.5.

4.4. Cathode

One central component of the detector is the cathode plate, where the fluorescence photons are converted to secondary electrons. Since the first interaction with the fluorescence photons occurs at the cathode, the XUV detector efficiency strongly depends on the optimization of this component. An extensive discussion on the underlying physics, the optimization process, and the resulting design decisions is given in [77], and only a summary of this discussion is presented in the following. The optimizations mainly focused on three different properties of the cathode. The first property is the choice of cathode material, which determines the photoelectron yield and, therefore, the secondary electron yield. The second is the cathode shape and size, and the third property is the cathode angle relative to the ion beam axis and the MCP. The detector geometry was optimized in simulations with the particle tracking software *SIMION*[®] 8.1 [21]. In parallel, the simulation results were cross-checked at a test setup at the University of Münster. A detailed discussion of the simulations and test measurements is also given in [77].

Cathode material The cathode material directly determines the photoelectron yield through its quantum efficiency. It is furthermore desirable that photoelectrons created inside the cathode create multiple secondary electrons that can escape the material and be guided onto the MCP. Maximizing the number of secondary electrons per incident



Figure 4.6: Cesium iodide secondary electron properties. **Left:** Measured (dotted line) and calculated (solid line) secondary electron spectrum of a cesium iodide coated gold cathode (300 nm thick CsI layer) in response to irradiation with 1487 keV photons. The spectrum shows mostly low energy secondary electrons escaping from the surface, which was found to be favorable for the electromagnetic guiding towards the MCP of the XUV detector [77]. The figure was taken from [81] **Right:** Comparison of secondary electron yield of aluminum, 300 nm thick CuI coated, and 300 nm thick CsI coated surfaces. CsI shows an increased electron yield by approximately a factor 20 in the relevant 10 nm region. The figure was taken from [82].

photon directly increases the overall sensitivity to photons of the whole detection system. The old detector setup described in [19] featured a stainless steel cathode, which was found to have inferior electron emission characteristics compared to other available materials. An in-depth discussion on the secondary electron yield and possible material choices is given in [77].

In conclusion of these investigations, cesium iodide (CsI) was chosen as cathode material in the form of a 300 nm thick coating evaporated onto the polished stainless steel cathode. The most important characteristic of this material for the XUV detector is the high secondary electron yield when irradiated with photons in the 10 nm range. The right plot of figure 4.6 shows the measured secondary electron spectra of different materials as a function of the incident photon wavelength. While metal surfaces (in this case aluminum) show a photo efficiency of approximately 0.1 electrons/photon in the desired wavelength region, the photo efficiency of cesium iodide is larger by approximately a factor of 20 (up to 2.5 electrons/photon), leading to a significantly larger electron yield.

Cathode shape and angle Design considerations for the cathode shape are comparatively simple. The larger the cathode is, the more fluorescence photons can be



Figure 4.7: Picture of cathode plate used for the test beamtime described in chapter 5. The final geometry features a larger plate than the old design. The cathode is additionally coated with a 300 nm CsI layer, which shows in the rainbow-colored reflections.

collected. The size is constrained by the beam pipe's diameter when the detection system is in retracted position. Similar to the shape and size of the cathode, does the cathode angle affect the collection efficiency as the effective area irradiated by the fluorescence photons is maximized when the cathode is positioned orthogonal relative to the ion beam. However, the angle also has implications for the electromagnetic guidance of the secondary electrons. The SIMION simulations showed that the direction of the electrons when leaving the cathode relative to the MCP plays an important role for the electromagnetic guiding properties of the detection system. In this case, flat angles with respect to the ion beam yield optimum guiding properties. However, since a zero-angle would eliminate the area irradiated by the fluorescence photons, a trade-off between optimal guidance and irradiated cathode area had to be found. In conclusion of the simulations performed by C. Egelkamp, an angle of 35° relative to the ion beam axis was found as the optimum setting [77].

The cathode that was constructed based on the previous considerations and used for the test beamtime described in chapter 5 is shown in figure 4.7.

4.5. New magnet coil design

The *old* detector design described in the Ph.D. thesis of J. Vollbrecht featured two magnet coils in Helmholtz configuration to guide the electrons towards the MCP. As sim-



Figure 4.8: SIMION 8.1 simulation results of the XUV detector setup with the new magnet coils. **Left:** SIMION 8.1 detector geometry with exemplary electron trajectories (red). The electrons are guided in cyclotron motion along the magnetic field lines and are thus imaged onto the MCP detector housed in the central shielding. **Right:** CAD representation of the inner XUV detector setup overlaid with the calculated field lines of the new magnet coil setup. The magnetic field lines cover the whole cathode area and are focused on the MCP detector. The magnetic field strength in the vicinity of the ions (located at the center of the slit) stays well below the maximum allowed 0.8 mT, thus not affecting the ion beam.

ulations in the thesis revealed, this configuration was not suitable to fully image the cathode area onto the much smaller active MCP area [19]. Therefore, a new coil design was developed to optimize the detector efficiency.

Based on an idea by V. Hannen, the new coils are operated with counteracting magnetic fields. This configuration serves two purposes. Firstly, the flux tube geometry is shaped in such a way that the magnetic field images almost the complete cathode surface area onto the MCP detector front plate. The electron trajectories follow the magnetic field lines in a cyclotron motion. Figure 4.8 shows exemplary electron trajectories (red lines in left image) and the magnetic field lines created by the magnet coils (right image) that emphasize the optimized detector configuration. The second purpose of applying a counteracting magnetic field with the bottom coil is to lower the magnetic field strength at the ion beam position. The maximum allowed field strength in the vicinity of the beam is 0.8 mT, which could be easily exceeded if the upper coil was operated alone (see legend in figure 4.8). With the counteracting field, the field strength at the ion beam position stays well below 0.8 mT.

The optimized coil parameters determined in the thesis of C. Egelkamp are presented in table 4.1. With the application of the new magnet coil design, the simulations also revealed that the electrostatic focusing realized via the ring electrodes becomes negligible, which is why they were operated on ground potential from then on [77]. However, the cathode potential is crucial for the electron guiding process, as the electrons are accelerated towards the MCP when applying a negative potential to the cathode. The optimum cathode potential setting was determined to be -5 V [77]. The detector configuration described in this chapter was subsequently tested in an anti-collinear laser spectroscopy measurement with lithium-like carbon ions at the ESR for the first time, demonstrating the successful adaptation of the LiBELLE detector concept to the XUV energy range. This beamtime and the analysis of the lithium-like carbon transitions are presented in the following chapter.

Table 4.1.: Optimized magnet coil settings determined with SIMION 8.1 particle tracking simulations. This table is taken from [77].

parameter	value
upper coil current	8 A
lower coil current	-2.4 A
upper coil radius	140 mm
lower coil radius	140 mm
upper coil position	50 mm above MCP center
lower coil position	-30 mm below MCP center

Chapter 5

Anti-collinear laser spectroscopy of lithium-like carbon (¹²C³⁺)

This chapter gives a detailed description of the beamtime at the ESR in which the XUV detector was commissioned. The measurement had two objectives: a test of laser cooling techniques at the ESR and to prove the sensitivity of the detection system to XUV fluorescence photons in anti-collinear laser spectroscopy measurements. For this purpose, the $(1s^22s)$ $^2S_{1/2}$ - $^2P_{1/2}$ and $(1s^22s)$ $^2S_{1/2}$ - $^2P_{3/2}$ transitions of lithium-like carbon $(^{12}C^{3+})$ that were already successfully measured in previous beamtimes for laser cooling tests [83, 84], were investigated. Two laser systems were utilized: firstly, a continuous wave (CW) laser system from TU Darmstadt, which was also used in the laser cooling tests presented in [84]. This system was tested with the aim to increase the output power to a level that makes it feasible for laser cooling at the future accelerator SIS100 (see [85]).

Secondly, a pulsed laser system from Helmholtz-Zentrum Dresden–Rossendorf (HZ-DR), also developed for laser cooling, amongst other things [86]. Both systems operate at $\lambda_{\text{laser,lab}} \approx 257 \text{ nm}$ and are thus able to excite transitions in the XUV region when used in an anti-collinear laser spectroscopy setup. In the case of ${}^{12}\text{C}^{3+}$ stored at $\beta \approx 0.47$, the laser wavelength is Doppler shifted to $\lambda_{\text{laser,ion}} \approx 155 \text{ nm}$ in the ion rest frame. By varying the electron cooler voltage and, therefore, the ion velocity, both investigated transitions could be excited. Since both excited states of ${}^{12}\text{C}^{3+}$ de-excite to the ground state, the fluorescence photons have the same wavelengths as the excitation photons in the ion rest frame (see figure 5.1a). However, in the laboratory frame, the photons experience a Doppler shift and Lorentz boost in forward direction, resulting in wavelengths down to $\lambda_{\text{fluo,lab}} \approx 93 \text{ nm}$ (see figure 5.1b). Compared to the minimum wavelength expected in the Be-like krypton experiment ($\lambda_{\text{min}} = 10 \text{ nm}$, see section 4.1), the detection system could, therefore, only be tested in the 93 nm region, for which it



Figure 5.1: Energy level scheme and fluorescence emission characteristic of lithium like carbon at the ESR. (a) Level scheme with the excitation and de-excitation processes of interest. (b) Emission characteristic of ${}^{12}C^{3+}$ fluorescence photons in the laboratory frame when stored at $\beta \approx 0.47$ in the ESR. The photon wavelengths are Doppler shifted down to $\lambda_{\text{fluo,lab}} \approx 93$ nm and Lorentz boosted in forward direction.

was not optimized. Despite the different wavelength region, the results of this chapter still prove a high sensitivity of the detection system to UV- and XUV photons, which led to successful measurements for both investigated transitions.

The chapter is divided into three parts, starting with an overview of the experimental setup. In the second part, the necessary device calibrations and synchronizations are presented, as they determine the precision of the final result. The last part consists of the transition wavelength analysis, concluding with the results and a discussion of the uncertainties with a focus on future optimizations for similar measurements using the XUV detection system.

It has to be noted that the essential parts of this chapter were submitted to the journal *Scientific Reports* [87] and published in the open-access repository *arXiv* [88] under first authorship of D. Winzen and V. Hannen. The chapter of this thesis was completely written by myself. Additions and comments of the co-authors regarding the paper submissions are also included in this version of the chapter.

5.1. The experimental setup

In order to conduct laser spectroscopy experiments at the ESR, multiple facilities of GSI are utilized to produce the ions and store them for measurements. A general overview of the respective facilities is shown in figure 5.2 (for a full overview of the FAIR facility, refer to chapter 3). For this beamtime, the carbon ions were directly produced with the desired charged state of 3+, in the electron cyclotron resonance ion source (ECRIS),



Figure 5.2: Overview of the facilities available at GSI during the 2016 beamtime. For the lithium-like carbon experiment, facilities 2, 3, 4, 6, and 10 were used. The CAPRICE-type ECR ion source first produces and then delivers ${}^{12}C^{3+}$ -ions to the high charge state injector (HLI), which are both located in (2). In (10) at the UNILAC, the carbon ions are accelerated to energies of approximately 7 MeV/u and subsequently guided through the transfer channel (3) to the SIS18 (4). Here, the ions are accelerated to energies of approximately 122 MeV/u and are then injected into the ESR (6) for the laser spectroscopy measurements. This image was taken from [89].

which was also used in the previous laser cooling beamtime with lithium-like carbon in 2012 [84, 90]. An ECRIS produces the desired ion species from the plasma of a suitable carrier gas (in this case, CH_4). Through irradiation with microwaves, the electron cyclotron resonance (whose frequency is determined by an applied magnetic field) is excited. If the microwave frequency is matched to the gyration period of the free electrons inside the plasma, their kinetic energies are increased up to a point where they cause further ionization of the gas constituents through collisions. For the production of ${}^{12}C^{3+}$, the kinetic energy was tuned such that only the three least bound electrons of carbon could be extracted from the atoms. The ${}^{12}C^{3+}$ -ions were subsequently separated from the also produced H⁺-ions in an analyzing dipole following the ion source.

From the ECRIS, the ions were then guided through the high charge state injector (HLI) to the UNILAC, where they were pre-accelerated to $E_{\rm pre} \approx 7 \,\text{MeV/u}$ by applying high-frequency electric fields. Through a transfer channel, the ions are further guided to the SIS18, where they are accelerated to their final velocity of $\beta \approx 0.47$, which corresponds to $E_{\rm fin} \approx 122 \,\text{MeV/u}$. As the last step, the ions are injected into the ESR

for the measurements. Due to the production scheme of the $^{12}\mathrm{C}^{3+}$ -ions, no stripper targets were needed to achieve the desired charge state. However, utilizing stripper targets can also help to remove possible contaminations created in the ion sources. In this case, it was not possible to separate $^{12}\mathrm{C}^{3+}$ -ions from contaminations of $^{16}\mathrm{O}^{4+}$ -ions, as both have the same mass-to-charge ratio and are therefore both able to pass the analyzing dipole behind the ion source. The resulting admixture has been estimated to be between 4 % and 11 % during the 2012 measurement campaign [90]. Since $^{16}\mathrm{O}^{4+}$ has no transitions in the same energy regime as the investigated $^{12}\mathrm{C}^{3+}$ -transitions, no measurable effect on the transition analysis is to be expected from this contamination. With this production scheme, ion currents up to $I_{\rm ion} \approx 1.5$ mA in coasting beam mode¹ were achieved together with storage times in the order of several minutes in the ESR.

5.1.1. Laser spectroscopy setup at the ESR

After injecting the ${}^{12}C^{3+}$ -ions into the ESR, the ions were stored for approximately five minutes for a single run in order to scan over a resonance. The transition measurements were conducted in an anti-collinear setup, as illustrated in figure 5.3. Throughout each measurement, the ions were cooled by the electron cooler to reduce the momentum spread of the ion beam, as well as to modify the ion velocity, as described in section 3.3. The voltage applied to the electron cooler amounted to approximately -67 kV corresponding to an ion velocity of $\beta \approx 0.47$. To precisely measure the cooler voltage, which determines the ion velocity (see equation (3.1)), a voltage divider in combination with a 6.5 digit multimeter was used (detailed setup shown in section 5.1.4).

On the opposite side of the ring is the optical detection region, where the XUV detector was mounted. In this section of the storage ring, the counter-clockwise propagating ions were overlapped with the laser beam of $\lambda_{\text{laser}} \approx 257 \text{ nm}$ in an anti-collinear geometry. When the Doppler-shifted laser photons are in resonance with the transition under investigation, fluorescence photons are created that can be detected by the XUV detector. To scan the wavelength of the photons over the resonance region, the velocity of the ions was varied by changing the electron cooler voltage, thereby obtaining a different amount of Doppler shift. As depicted in the magnified part of figure 5.3, generated fluorescence photons are forward boosted towards the cathode plate of the detector, where secondary electrons are created that are subsequently detected by an

¹ Coasting beam refers to a continuous ion beam with turned off high voltage resonators (bunchers).



Figure 5.3: Experimental setup at the ESR. Carbon ions revolve counter-clockwise in the ESR at approximately 47% of the vacuum speed of light. The ion velocity spread is reduced by electron cooling, and the ion velocity is determined by the acceleration voltage applied to the cooler, which is measured with a high-voltage divider system. In the detection region, the ions are excited by an anti-collinear laser beam. The resulting forward boosted fluorescence photons impinge on the cathode plate of the XUV detector and are converted to secondary electrons that are measured by the MCP. This schematic is based on the illustration given in [3].

in-vacuum MCP (see chapter 4 for more details). In the following subsections, the different subsystems of figure 5.3 are discussed in detail.

Additional components of the experiment that will be discussed were located in separate sections of the GSI complex. These components were located in areas such as the main control room (HKR), a dedicated laser lab, and a dedicated room for the DAQ devices. This separation is necessary since, during beam operation, it is forbidden to be in the ESR hall for radiation protection.

5.1.2. CW laser system

The laser system used for the transition measurements is shown in figure 5.4. This system was originally developed in the scope of the Ph.D. thesis by T. Beck [74, 91] and further developed by D. Kiefer and S. Klammes [85, 92]. It can be operated in three dif-



Figure 5.4: Schematic overview of the CW-laser system used for the transition measurements. The external cavity diode laser (ECDL) is used as master oscillator, generating the source laser beam of approximately 1028 nm. Further power enhancement is achieved by guiding the beam through a fiber amplifier. Subsequently, the beam is frequency-doubled twice. The first frequency doubling (second-harmonic generation) is realized via a magnesium-doped periodically poled lithium niobate (MgO:PPLN) crystal. The adjoining UV-cavity utilizes a beta-barium borate (BBO) crystal to achieve fourth-harmonic generation and, therefore, the desired wavelength of 257 nm. To guarantee a stable output wavelength, the laser master oscillator is stabilized via a wavelength meter. This illustration is a modified version from [91] and was taken from [87].

ferent wavelength regimes, being at 1028 nm, 514 nm and 257 nm. The setup is divided into five main components. A diode laser with an external resonator (external cavity diode laser, ECDL) serves as master oscillator operating at 1028 nm. The ECDL setup features a blazed grating that can be tilted via a piezoelectric motor so that the output frequency of the laser system is tunable. In the next step, the laser light is amplified in an Ytterbium based fiber amplification system to achieve higher output powers for more efficient frequency doubling of the 1028 nm light. In order to generate the desired wavelength of 257 nm, the light is frequency-doubled twice. The first frequency doubling is achieved by guiding the light through a magnesium-doped periodically poled lithium niobate (MgO:PPLN) crystal. From there, the emerging green light of approximately 514 nm is guided into the UV cavity, where it is again frequency-doubled with a beta-barium borate (BBO) crystal.

The resulting laser beam of approximately 257 nm can then be utilized for the desired experiments. During the beamtime, a fraction of the laser beam was diverted by a beam

splitter at the output of cavity 2 in order to measure the output power. As seen in figure 5.4 it was measured by an XLP12-1S-H2-D0 Gentec-EO power detector, and the data was stored on a local laptop. To achieve absolute stability of the laser wavelength and also determine the exact value, the setup features a wavelength meter (WLM) from HighFinesse of type WS7-60 [93]. For that purpose, a small fraction of the beam from the ECDL is coupled into a single-mode fiber and guided onto the WLM, where the wavelength is measured.

5.1.3. Laser beam transport and position stability

For high-intensity laser beams in the UV region, as used in the experiment, beam transport via optical fibers is not suitable due to fast degradation of the fibers. Therefore, a system for laser beam transport over a distance of over 50 m has been constructed in the scope of the master thesis by J. Ullmann [94] (see figure 5.5). Via periscope A, the beam is sent through a passage between the laser laboratory and the ESR hall. Subsequently, the beam is guided to the other side of the ESR through an approximately 36 meter long tube that is mounted to the ceiling of the ESR hall. The beam is then aligned to the height of the entrance window of the ESR by a second periscope (periscope B). Located inside periscope B is a telescope that is used to adjust the beam focus in the interaction region of the ESR. The focus position can then be adjusted to the desired interaction region, which in case of this beamtime was situated in front of the XUV detection system. With a total distance of approximately 50 meter from the laser to the focus point, and the necessity to overlap the laser beam with the ion beam that has a cross-sectional area of a few mm², high demands on the beam guidance have to be met. The general alignment was performed by utilizing the scrapers of the ESR. Scrapers are metal plates that can be moved into the ion or laser beam. In total, two vertical and two horizontal scrapers are available at the ESR. For the alignment, each scraper is moved into the ion beam until it blocks half of the beam, and the corresponding scraper position is recorded. The laser alignment is subsequently conducted without the ion beam. For that purpose, the scrapers are positioned at the recorded ion beam position so that the laser beam can be aligned according to the scrapers.

A beamtime usually consists of several shifts over the course of a few days or more. On such a time scale, it cannot be guaranteed that all optical components stay in the exact state as they are when the alignment is finished. Therefore, several mirrors throughout the laser beamline are motorized in order to connect them to an automated beam sta-



Figure 5.5: Laser beam transport setup at the ESR (horizontal and vertical views). The laser beam is transported from the laser laboratory to the entrance of the ESR via a complex arrangement of mirrors. Several mirrors can be moved via motors in order to be steered by a beam stabilization system. For this beamtime, a new stabilization system was implemented but did not function during the three days of measurement. In consequence, the laser beam had to be adjusted manually several times between the measurements. This schematic was taken from [94] and modified to fit the format of this thesis.

bilization system. The system continually stabilizes the beam to a reference position, in this case, measured by a Quad-photodiode at the exit window of the ESR. As part of

the beamtime, a new beam stabilization system was tested. After unsuccessfully trying to get the system into a working state within the first day, it was shut down, and the beam had to be readjusted manually each time the overlap between laser and ion beam was not sufficient anymore. The lack of stabilization of the beam gives rise to systematic uncertainties for the transition analysis due to a varying fluorescence rate caused by changes in the overlap between the ion beam and the laser beam throughout the measurements. This uncertainty will be discussed in section 5.2.2.

5.1.4. Electron cooler voltage supply and measurement

The electron cooler voltage is a critical quantity for the determination of the ion velocity and was the source of the largest systematic contribution in some previous experiments. Figure 5.6 shows the setup of the electron cooler voltage supply and measurement. A high voltage supply of type Heinzinger HNC 320 000 - 10 neg generated the negative voltage of approximately -67 kV, which was applied to the electron cooler and a high-voltage divider. The voltage is set in the main control room (HKR) and passed to the control unit of the Heinzinger high voltage supply via a fiber optic cable in the form of an 18-bit digital value $U_{\text{set, dig.}}$. The HNC voltage source internally transforms the digital set voltage to an analog set voltage $U_{\text{set, ana.}}$ using a digital-to-analog converter (DAC) in the range of 0-10 V, representing the possible output voltage range up to $U_{\text{HV, max.}} = -320$ kV. At the same time, the actual high voltage at the output of the device U_{HV} is scaled to the same range using an internal voltage divider and inverted to $-U_{\text{actual, ana.}}$. This allows a comparison of $U_{\text{set, ana.}}$ to $-U_{\text{actual, ana.}}$ at a differential amplifier that creates a correction signal for the high voltage regulator.

The second part of the setup is dedicated to the high voltage measurement. The general setup consists of a precision high-voltage divider that scales down the cooler voltage by a known scale factor M. In this case, a Julie Research Labs HV divider type HVA-100 from TU Darmstadt [10, 95] was used to scale down the high voltage by a scale factor $M_{\text{HVA}-100}$ of approximately 10000:1. The voltage was then measured in the 10 voltrange of the Keysight 34465A DVM, which is the most precise range for this multimeter type. The total precision of the voltage measurement resulting from the calibration of the HV divider and the multimeter is discussed in sections 5.2.6 and 5.2.7.



Figure 5.6: Schematic overview of high voltage supply and measurement chain at the ESR in 2016. The voltage to be applied to the electron cooler is set in the main control room (HKR) and passed to the voltage regulation of the Heinzinger high voltage supply. The produced high voltage $U_{\rm HV}$ is divided by an internal voltage divider and compared to the analog representation of the set voltage $U_{\rm set, ana.}$. If the signals deviate, a new voltage is set by a regulator. The output voltage is measured by scaling down the high voltage $U_{\rm HV}$ with a precision high-voltage divider with a known scale factor and a measurement of the scaled-down voltage with a 6.5 digit precision digital multimeter.

5.1.5. XUV detection system setup and background

Besides aligning the laser beam to the ion beam, the cathode of the XUV detector also had to be aligned vertically with the ion beam. This was also done by utilizing the ESR scrapers, as described in section 5.1.3. A graphical illustration of the alignment process can be found in appendix A.1. The XUV detection system was operated according to the optimized parameters determined in the scope of the master thesis by C. Egelkamp [77], as presented in table 4.1. Only the MCP was operated at a potential 100 V lower than proposed to reduce gain saturation effects that occur at high signal rates. The connection scheme for the beamtime setup is shown in figure 5.7. All current and voltage supplies used for the XUV detector were previously used at the test setup in Münster. As the optimized parameter table gives a range for possible magnetic coil currents, several settings have been tested in advance of the fluorescence measurements, with optimal currents found at 8 A for the upper coil and -2 A for the lower coil.


Figure 5.7: XUV detector connection scheme. Voltages and currents applied to the respective parts of the XUV detector were chosen according to the optimization result from C. Egelkamp shown in table 4.1. As proposed by Roentdek in the user manual, the signal and HV supply lines of the MCP parts were shared and decoupled outside the vacuum chamber. All three signals from the MCP were amplified by a factor of 100 using a fast amplifier and then transformed to a logic signal using a constant fraction discriminator (CFD). The resulting logical signals were then processed further in the DAQ setup described in section 5.1.6.

The connection of the MCP to the high voltage and the DAQ was set up according to the manufacturer. As proposed by Roentdek, the HV the signals of anode, backplate, and frontplate are decoupled from the HV lines according to figure 4.5. The signals were then amplified by a factor of 100 using a C.A.E.N. N979 fast amplifier. Subsequently, the signals were converted to a logic signal using a C.A.E.N. N840 constant fraction discriminator set to a threshold of 40 mV and a pulse width of approximately 5.4 ns. Further processing within the data acquisition (DAQ) system is presented in section 5.1.6.

Laser background measurements with CW laser in the beamline and the cathode positioned around the beam axis revealed a laser-induced background rate in the order of several hundred kcps, with a maximum measured normalized background of $BKG_{las., max} = 37.38 \text{ kcps/mW}$. To minimize the risk of damaging the MCP detector due to high rates, the cathode was not positioned close to the main beam axis but at the side of the beam tube (see figure 5.8) whenever ions were injected into the ESR. In this configuration, the normalized laser background rate could be reduced by almost a



(a) Laser background at $BKG_{las, max} \approx 37.38 \text{ kcps/mw.}$ (b) Laser background at $BKG_{laser} \approx 0.11 \text{ kcps/mw.}$

Figure 5.8: Illustration of cathode positions for two different views, respectively. Both positions were tested during beamtime with regard to laser and ion beam induced background. (a) Nominal cathode position in direct vicinity of the ion beam (depicted as red line/dot). In this configuration, the background induced by the laser beam (depicted by blue halo) is in the order of 37.38 kcps/mw. (b) Cathode position as used for the fluorescence scans analyzed in this chapter. The laser background reduces to approximately 0.11 kcps/mw at the cost of a lower total fluorescence rate due to lower collection efficiency.

factor 370 to approximately $BKG_{laser} \approx 0.11 \text{ kcps/mW}$.

When injecting ions into the ESR, the MCP background is increased by ion-induced background. Due to the previously mentioned risk of damaging the MCP, this background component was only measured for the detector configuration shown in figure 5.8b in a dedicated background measurement. For this measurement, the electron cooler voltage and, therefore, the ion velocity was tuned to a setting at which the resonance condition was not fulfilled so that the MCP only measured background events. Figure 5.9 displays the results of this measurement. While the top graph shows the ion current and laser power as a function of time during the measurement, the lower graph shows the measured background rate in blue. It can be observed that this rate follows the exponentially falling beam current plus an offset caused by the approximately constant laser background. An exponential fit according to the following equation was applied:

$$f(t) = A \cdot \exp\left(-\frac{t}{\tau_{\text{ion}}}\right) + c, \qquad (5.1)$$



Figure 5.9: Determination of ion and laser background components. **Top:** Ions are injected into the ESR (green) and stored until most ions are lost due to rest gas interactions (for further details see section 5.2.4). At the same time, the CW laser beam with a power of approximately 25 mW (purple) is overlapped with the ion beam. **Bottom:** MCP background rate over time (blue). The electron cooler was operated at a constant voltage, at which the resonance condition is not fulfilled so that the signal is only determined by ion and laser beam background. An exponential fit with an offset component according to equation (5.1) was applied to determine both background components (red line). The legend shows the normalized components for the ion (BKG_{lon}) and the laser beam (BKG_{laser}) background respectively.

where *A* is the amplitude of the ion-induced background rate, *t* the time since ion beam injection, τ_{ion} the mean lifetime of the ion beam, and *c* an offset. The offset *c* is directly connected to the laser-induced background assumed to be constant over the course of a single run (interval between two ion injections; approximately 5 – 10 min). Dividing offset *c* by the measured laser power shown as the purple line in the top part

of figure 5.9 and taking the average of the result yields the normalized laser-induced background BKG_{laser} shown in the bottom legend of figure 5.9. The normalized ion-induced background BKG_{ion} is determined dividing $A \cdot \exp(-t/\tau_{ion})$ by the measured ion current depicted as the green line in the top part of figure 5.9. Again the averaged result is shown in the bottom legend of figure 5.9.

As a consequence of the background investigations, all following measurements presented in this chapter were conducted in the configuration shown in figure 5.8b if not stated otherwise. Even with the reduced background in this configuration, the total signal rates during the fluorescence measurements sometimes exceeded 1 Mcps, which led to significant gain saturation effects of the MCP further described in section 5.3.2. The lowered detection efficiency compared to the intended configuration was compensated by the fact that all injected carbon ions could potentially be excited to the desired states and, therefore, the fluorescence yield was sufficient to get clear signals as presented in section 5.3. However, this may not be the case for configurations in other ion species to be investigated, such as the $(1s^22s2p)^3P_0^{-3}P_1$ level splitting in beryllium-like krypton (⁸⁴Kr³²⁺) for which the detector was designed. In these configurations, a significantly lower fluorescence photon yield is to be expected (see section 4.1), which is why the cathode should be positioned in the vicinity of the ion beam in order to maximize the fluorescence photon detection efficiency of the XUV detector. In consequence of the background observations, a shielding component was developed and integrated into the detector setup following the beamtime described in this thesis. This new system was first tested at the ESR in 2020, and the results will be published in the scope of the Ph.D. thesis by A. Buß [96].

5.1.6. Data acquisition (DAQ)

The data acquisition setup of this beamtime was similar to the setup used during the LiBELLE measurements in 2014, as most of the old hardware setup was still in place and ready for use. The DAQ system was developed in the scope of the dissertation by M. Lochmann, where a complete description can be found [9]. The core of this system is the so-called VUPROM (VME Universal Processing Module) [97] which has been developed at GSI and which features an integrated FPGA (Field Programmable Gate Array) that can process signals with a timing resolution of 3.33 ns. A selection of specifications is shown in table 5.1. The module has 192 scaler channels, of which 16 can be used as time-to-digital-converter (TDC) channels in parallel. While the scaler

channels only count incoming signals, the TDC channels register the signals with an individual time stamp.

Table 5.1.: Overview of VUPROM specifications. The table is a modified version taken from [9].

	Specification	Comment
TDC channels	16	every TDC channel is used as scaler channel in parallel
Stop channels	1	all TDC / scaler channels are stopped simultaneously
Scaler channels	total of 192	only 32 used
Timing resolution	$^{10}/_{3}$ ns	results from 300 MHz clock
Dynamic range	218 µs	maximum time between start and stop
Multihit	16	maximum registered signals per stop per channel
Common-stop trigger	$^{4 MHz}/_{200} = 20 \text{ kHz}$	common stop from Master oscillator divided by 200
MBS Readout rate	200 Hz	LiBELLE default; can be set by user

Each TDC has a multi-hit capacity of 16 and cannot record any more signals until the data is transferred into the VUPROM buffer after a so-called common-stop signal is sent. During the beamtime, the common stop signal was sent with a frequency of 20 kHz. The VUPROM data readout was conducted with a frequency of 200 Hz by the data acquisition software MBS (Multi Branch System) running on a RIO4-computer with a LynxOS real-time operating system. To minimize dead times during the data readout process, two buffer memories are used. While the scalers/TDCs write their data to one buffer, the other buffer is read out until the common stop signal triggers a new event readout, and the tasks of the buffers are interchanged. At the highest rates of approximately 1 MHz measured by the MCP, the TDCs reached their maximum capacity after approximately one-third of the time before the next common stop was sent. Therefore, the TDC channels were neglected in the analysis, and only the scaler channels were used.

Since an extensive preparation of the DAQ setup was not possible in 2016, not all the data sources could be integrated into the VUPROM data stream and had to be synchronized later in the analysis. An overview of all data streams used in the analysis is given in table 5.2. A detailed discussion of the synchronization is given in section 5.2.1. The VUPROM data was written to so-called lmd-files (list mode data). File sizes are capped at approximately 100 MB upon which data is written to a new file to guarantee simple data handling. The files were saved on GSIs tape storage system Gstore, which has been developed to archive raw data from experiments. In order to use the stored data for the transition analysis, every data channel has to be calibrated, which is the subject of the following section.

Quantity	Last data processing device	Data storage
Electron cooler voltage	Keysight 34465A	USB mass storage
Electron cooler voltage	HKR set voltage	HKR data storage
Laser power	XLP12-1S-H2-D0	SSD storage on laptop
Laser frequency	WS7-60 / WS6-200	Logbook entries
MCP frontplate	VUPROM TDC / scaler	Gstore MBS
MCP backplate	VUPROM TDC / scaler	Gstore MBS
MCP anode	VUPROM TDC / scaler	Gstore MBS
Ion current	VUPROM scaler	Gstore MBS
Ion injection signal	VUPROM scaler	Gstore MBS
Electron cooler current	VUPROM scaler / HKR set current	Gstore MBS / HKR data storage
4 MHz clock / 200	VUPROM scaler	Gstore MBS
1 MHz clock	VUPROM scaler	Gstore MBS

Table 5.2.: Overview of all data streams relevant to the analysis in this chapter. In total, five different data streams had to be synchronized (see section 5.2.1).

5.2. Calibrations and corrections

Before an analysis of the measured transitions, calibrations of the relevant data channels have to be performed. These calibrations determine the systematic uncertainty of the results presented in section 5.3.5 and are, therefore, a fundamental part of the analysis. A detailed description of the individual calibrations and corrections is provided in the following subsections. In conclusion of each calibration, the resulting 1σ systematic uncertainty is given.

5.2.1. Time synchronicity calibration

As described in section 5.1.6, the different data streams were stored on separate storage devices. In total, four data storage devices that provide absolute timestamps were synchronized to the global GSI time in preparation of the beamtime. These were the MBS (VUPROM TDCs/scalers), the main control room data devices (HKR), the laptop storing the laser power data, and the Keysight 34465A DVM storing data on the USB mass storage. The fifth data source was the logbook with the laser frequency data, where entries were also recorded with the GSI timestamp. The global GSI time itself refers to an hourly synchronization with an atomic clock based timeserver [98] and has a resolution of approximately 1 ms. To synchronize the data, each sample stored has to be saved with an absolute timestamp. This was true for all data streams, except for the VUPROM TDC and scaler data. For the VUPROM based data, timestamps (from now on called "MBS time") were not given for every sample, but only with every new sub-header of the lmd-files (which occurred approximately every 150 ms). The individual time of a data sample has thus to be determined using a combination of the MBS timestamps and a 1 MHz reference clock recorded using one of the VUPROM scaler channels and included in the data stream. The latter is subject to deadtime effects due to the time required for readout and clearing of the scalers, which will be determined in the following. For that purpose, the MBS time t_{MBS} passed since the beginning of the beamtime up to the *i*th timestamp is compared to the time derived from the 1 MHz clock t_{clock} , taking into account a deadtime τ_{dead}

$$t_{\text{MBS}} - t_{\text{clock}} = (t_{\text{MBS}, i} - t_{\text{MBS}, 0}) - \sum_{j=1}^{n_i} (t_{\text{clock}, j} + \tau_{\text{dead}}),$$
(5.2)

where $t_{\text{MBS},0}$ is the first MBS timestamp, n_i the number of readout cycles since this timestamp and $t_{\text{clock},j}$ the corresponding time intervals determined from the clock scaler for each readout cycle.

The top of figure 5.10 shows the progression of the difference $t_{\text{MBS}} - t_{\text{clock}}$ over the full extent of the beamtime when assuming no deadtime (black) and for an average deadtime $\tau_{\text{dead}} = 6.43 \,\mu\text{s}$ (red). For the uncorrected data, the MBS time t_{MBS} develops a discrepancy of more than 150 s compared to the 1 MHz clock time t_{clock} during 40 hours of beamtime. Taking into account an average dead time of $\tau_{\text{dead}} = 6.43 \,\mu\text{s}$ keeps this difference down to a level of a few 100 ms (see bottom zoom).

The zoomed part of figure 5.10 also shows some unexpected structures in the timing. Firstly the time difference shows a sawtooth-like structure with a periodicity of one hour. This hints to a clock drift of the MBS time, which is reset by an hourly synchronization to the GSI clock. Further substructures appear at the marked time intervals, where the two transitions have been investigated (blue and green data points). Within these intervals, the average dead time seems to change to higher values, resulting in a positive slope. This can be explained with the increased data flow compared to intervals where mainly hardware calibrations and maintenance were conducted without introducing ions into the ESR, yielding much lower MCP-rates.

To account for the dead time being different during transition investigations and the rest of the beamtime, the optimization of the average dead time has been repeated for the respective time intervals yielding a value of $\tau_{\text{dead}} = 6.51 \,\mu\text{s}$ that keeps the time



Figure 5.10: Comparison between MBS time t_{MBS} and 1 MHz clock time t_{clock} over the course of the beamtime. The black curve shows the difference between t_{MBS} and t_{clock} when the deadtime is neglected. After approximately 40 h, both times differ by more than 150 s. By accounting for an average deadtime for each readout cycle, a synchronization with a precision of a few 100 ms is possible. The red curve shows a correction with $\tau_{\text{dead}} = 6.43 \,\mu\text{s}$ according to equation (5.2). Despite the correction, substructures appear in the corrected signal (see bottom zoom), which are further discussed in the text.

difference $t_{\text{MBS}} - t_{\text{clock}}$ during the transmission measurements below ±100 ms.

Due to the observed structures in the MBS time, it was decided to use for the following analysis the time t_{clock} reconstructed as described above with an estimated uncertainty $\sigma_{clock} = 100$ ms.

5.2.2. Laser wavelength calibration

The laser wavelength is a critical quantity for the determination of the transition wavelength. Two different wavelength meters were used to determine and monitor the CW-laser frequency f_{laser} . A HighFinesse WS6-200 wavelength meter [99] that was calibrated to a HeNe-laser system was used to measure the absolute frequency of the external cavity diode laser (see section 5.1.3) before and after the transition investigations.

For continuous monitoring of the laser stability, a HighFinesse WS7-60 wavelength meter [100] was utilized. Both wavelength meters operate in a range of 330 – 1180 nm and mainly differ in terms of accuracy. The WS6-200 has an uncertainty of 67 MHz and the WS7-60 has an uncertainty of 20 MHz. Throughout both transition measurements, the frequency measured by the WS7-60 remained stable to the last digit. The measurement of the ECDL frequency with the calibrated WS6-200 amounted to $f_{\text{laser}} = 291.433 \, 40(7)$ THz. The frequency is quadrupled in a two-step process (see section 5.1.2), and thus, the resulting laser wavelength in the laboratory system is given by

$$\lambda_{\text{laser}} = \frac{c}{4 \cdot f_{\text{laser}}} = 257.170\,642(62)\,\text{nm}\,.$$
(5.3)

Besides the frequency measurement uncertainty, an additional uncertainty arises from the unknown angle ϕ between laser and ion beam. The effective laser wavelength the ion beam experiences directly depends on this angle, according to equation (3.2). Because the laser beam could not be position stabilized, the angle ϕ is unknown, and a systematic uncertainty is estimated and taken into account in the total systematic uncertainty of the result. The position of the ion beam in the vacuum beam-pipe of the ESR can be determined using the position-calibrated scrapers inside the vacuum shown in appendix A.1, which are a length of $l \approx 6.33$ m apart. By moving the scrapers from the outside towards the center of the beam-pipe, the radius and the central position of the ion beam can be determined in the same way described for the laser beam alignment in section 5.1.3. Since the center points of laser and ion beam at the positions of the scrapers were at maximum a distance of $d \approx 0.002$ m apart, the misalignment is given by

$$\phi_{\max} = \arctan\left(\frac{2d}{l}\right) \approx 0.036^{\circ}.$$
 (5.4)

According to the equation, an angle ϕ can only reduce the Doppler shift for an anticollinear setup, and therefore, the uncertainty only acts in one direction. The resulting systematic uncertainty then amounts to

$$\sigma_{\lambda_{\phi}} = \lambda_0 - \lambda_{\phi, max} = +0.000\ 027\ \mathrm{nm}\,,\tag{5.5}$$

for both transitions, where λ_0 is the Doppler shifted wavelength for an angle of 0°

5.2.3. Laser power calibration

The output power of the laser system is needed to normalize the MCP rate of the XUV detector (see section 5.3.1). Ideally, the power of the laser beam in the ion-laser-interaction region is desired as the normalization quantity. Since the data of the closest photodiode at the ESR exit window has not been stored during the beamtime, the laser power at the exit of cavity 2 of the CW-laser system (see figure 5.4) measured by a power meter of type Gentec-EO XLP12-1S-H2-D0 [101] was utilized for monitoring the laser stability.

The relative uncertainty of the laser power measurement was estimated conservatively to be [102]

$$\sigma_{\text{power, rel.}} = 10\%. \tag{5.6}$$

It has to be noted that the angular drift caused by the missing laser beam stabilization (see section 5.2.2) does not only lead to a varying effective laser wavelength in the ion rest frame. It also affects the length of the overlap between laser and ion beam and thus also affects the effective laser power, which is delivered to the ions. As these drifts can be assumed to be slow compared to the duration of a resonance scan, they are neglected in the analysis.

5.2.4. Ion current calibration

The total fluorescence rate detected by the XUV detection system directly depends on the number of ions stored in the ESR. The stored ion current decreases exponentially over time due to the interactions with residual gas molecules. The total amount of ions injected additionally varies for each injection due to varying conditions during the ion production process. Both variations have to be taken into account by normalizing the detector rate to the ion current in the analysis.

Ion currents are measured by a DC-transformer installed at the ESR [103]. The current signal is then converted to a frequency and fed into the VUPROM scaler. The relation

between the observed scaler rate r_{ion} and the ion current I_{ion} is described by

$$r_{\rm ion} = a_{\rm DC} \cdot I_{\rm ion} + b_{\rm DC} \,. \tag{5.7}$$

The value for $a_{\rm DC} = 100 \, {\rm kcps}/{\rm mA}$ was determined in a previous beamtime [104] with a conservative uncertainty estimate of $10 \, {\rm kcps}/{\rm mA}$. The offset $b_{\rm DC}$ is determined by an ion current measurement over approximately 20 minutes. To derive the offset from the measurement, an exponential fit is applied to the ion current signal (see figure 5.11)

$$r_{\rm ion} = A \cdot \exp\left(-\frac{t}{\tau_{\rm ion}}\right) + b_{\rm DC} \,, \tag{5.8}$$

where *A* is the initial value, *t* is the ion storage time and τ_{ion} is the mean lifetime of the ion beam. The offset b_{DC} from the fit equals approximately 1.4 kcps. The fit only



Figure 5.11: Ion current signal offset determination. To determine the offset $b_{\rm DC}$, ions were injected into the ESR and stored for approximately 20 minutes. An exponential fit to the ion current signal (green) yields the offset parameter, amongst others. The mean lifetime of the ion beam is $\tau_{\rm ion} \approx 56$ s. Since the uncertainty of the data point is unknown, an uncertainty estimate was obtained by scaling the individual error bars so that the $\chi_{\rm r}^2$ (reduced chi-square) equals one.

gives a measure of the statistical uncertainty for b_{DC} . A systematic uncertainty was estimated by cross-validation of the offset parameter with shorter ion current measurements conducted during the beamtime. The standard deviation of the collected offset measurements is taken as uncertainty for the offset. The total calibration, based

on equation (5.7) then amounts to

$$I_{\rm ion} = \frac{r_{\rm ion} - 1.4(7)\,\rm kcps}{100(10)\,\frac{\rm kcps}{\rm mA}}\,.$$
(5.9)

Typical beam lifetimes were in the order of $\tau_{\text{ion}} \approx 56 \text{ s}$ as the fit result shows. Data below $I_{\text{ion}} < 0.025 \text{ mA}$ was discarded due to noise effects from the current signal becoming dominant in that range.

5.2.5. Electron cooler current calibration

The electron cooler current directly depends on the acceleration voltage applied to the anode of the cooler's internal electron gun (see section 3.3.1). Cooler currents are therefore set in terms of anode voltages via the HKR and have to be derived from the stored anode voltage set values using a conversion formula [105]

$$I_{\text{ecool, set}} = U_{\text{anode, set}}^{3/2} \cdot 1.9 \cdot 10^{-3} \, \frac{\text{mA}}{\text{V}^{3/2}} \,, \tag{5.10}$$

with $U_{\text{anode, set}}$ as the set value of the cooler anode voltage, $I_{\text{ecool, set}}$ as the resulting current set value and a proportionality factor. For inclusion in the data stream of the experiment, the set values were converted to a variable frequency and fed into one of the scaler channels of the VUPROM. To derive the conversion between current set values $I_{\text{ecool, set}}$ and rate measured in the scaler, the latter has been recorded for a number of current set values and fitted with a linear relation (see fig. figure 5.12). From the fit, the following conversion formula between the rate r_{ecool} and the current setpoint $I_{\text{ecool, set}}$ is obtained.

$$I_{\text{ecool, set}} = \frac{r_{\text{ecool}} - b}{a} = \frac{r_{\text{ecool}} + 4.89 \,\text{kcps}}{0.499 \,\frac{\text{kcps}}{\text{mA}}}.$$
(5.11)

As there is no measurement of the actual electron current flowing in the cooler, the determined current set values were used for the further analysis of the data. As a conservative estimate for the resulting systematic uncertainty of the actual cooler current I_{ecool} a value of $\sigma_{I_{\text{ecool}}} = 2 \text{ mA}$ was specified [105].



Figure 5.12: Conversion from scaler rate to electron cooler current set point. A linear regression is performed (red line) to describe the scaler data r_{ecool} with respect to the electron cooler set current $I_{ecool, set}$ (blue points).

5.2.6. Keysight 34465A calibration

The calibration of the Keysight 34465A DVM has been performed 76 days in advance of the beamtime by D. Winters of GSI at TU Darmstadt. Before both, the calibration measurements and the beamtime measurements, the auto-calibration function (ACAL) of the DVM was used, which brings the device into a defined state [106]. The calibration was performed for the most precise range of the DVM (10 V range), which was also used throughout the beamtime. DVM calibrations aim to determine two parameters, the offset U_{off} and the gain G of the instrument. The offset is determined by short-circuiting the *Hi* and the *Lo* input of the instrument with a subsequent voltage measurement. In this case, the offset was determined to

$$U_{\rm off} = 0.00000(15)_{24\rm h}(20)_{90\rm d}\,\rm V\,, \tag{5.12}$$

with systematic uncertainties for 24 hours and 90 days, according to the Keysight operating guide. The gain is then determined utilizing a Fluke 732B 10 V reference source. The reference source provides a known stable voltage that is applied to the DVM, from which the gain *G* can be deduced according to

$$G = \frac{U_{\text{ref}}}{U_{\text{DVM}} - U_{\text{off}}} = 1.0000006(131)_{90\text{d}} \text{V}, \qquad (5.13)$$

where $U_{\text{ref}} = 10.000\ 076(50)$ V is the last calibrated value of the Fluke voltage reference. The uncertainty is the combined uncertainty of the calibration by PTB (2 ppm) and the uncertainty from the drift rate according to the Fluke 732B manual (2 ppm/year). To determine the measured beamtime voltages, the gain *G* and the offset U_{off} were used with the 90 day uncertainties according to

$$U_{\text{DVM, cal.}} = (U_{\text{DVM}} - U_{\text{off}}) \cdot G.$$
(5.14)

The integration times of the DVM during the beamtime were set to approximately 0.3 s and thus substantially larger than the VUPROM readout intervals of approximately 0.005 s. A measure to deal with the limited DVM timing resolution is the application of cuts. The transition measurement concept is based on the determination of the fluorescence rate measured with the XUV detector for different electron cooler voltages and, therefore, different ion velocities. During the beamtime, for each voltage step, the voltage was kept constant for a few seconds before it was changed again. This allows to cut away data at the beginning and the end of each interval to allow the cooler voltage to stabilize. In consequence, 120 data buffers at the beginning and 120 data buffers at the end of each set voltage were discarded (ca. 1.2 s of data in total), which amounts to a cut of two DVM measurements at the beginning and two at the end of each set voltage interval.

5.2.7. HV divider JRL HVA-100 calibration

The precision of the JRL HVA-100 scale factor $M_{\rm HVA-100}$ is one of the main systematic uncertainties of the results presented in section 5.3.5. It is determined by several calibration measurements performed in the years before and after the beamtime. The actual scale factor for the time of the measurements is, therefore, derived from the divider calibration history. High-voltage dividers typically show a temperature and a voltage dependency of the scale factor, which mostly depend on the type of precision resistors used in the divider chain (for details see chapter 6). In case of the JRL HVA-100, the voltage dependency shows a quadratic behavior, and the scale factor has to be extracted for the input voltage of -67 kV as this was the approximate input voltage during the beamtime. Therefore, data from all calibration measurements in that voltage range was gathered and analyzed, which were conducted before and after the beamtime. In total, data from three calibration measurement campaigns was available covering the necessary input voltage range, as well as featuring similar ambient conditions for the high-voltage divider. Since the divider did not have this kind of temperature regulation during the beamtime 2016, only calibration measurements without temperature regulation were taken into account as the divider characteristics change significantly with any kind of regulation [107]. The general procedure for all calibrations was to input a high voltage into the JRL HVA-100 and, at the same time, into a reference highvoltage divider with a known scale factor, similar to the setup described in detail in section 6.5. The output voltages were then measured by precision digital multimeters for both dividers. Subsequently, the scale factor of the device to be tested (in this case $M_{\rm HVA-100}$) was determined from the reference divider scale factor $M_{\rm ref.}$, the measured output voltage of the reference divider $U_{\rm ref.}$ and the measured output voltage of the divider to be calibrated, in this case $U_{\rm HVA-100}$, so that

$$M_{\rm HVA-100} = \frac{U_{\rm in}}{U_{\rm out}} = \frac{U_{\rm ref.} \cdot M_{\rm ref.}}{U_{\rm HVA-100}},$$
 (5.15)

where U_{in} is the input voltage of the unit under test and U_{out} the output voltage of the unit under test. The results of all three calibration campaigns are presented in figure 5.13.

The turquoise data shows a calibration measurement performed at the University of Münster in 2013 with an input voltage range of -5 kV to -60 kV for which the KATRIN K65 HV divider [12] was used as a reference. The data was fit with a polynomial of second order (due to the quadratic voltage dependency of the scale factor) according to the following equation

$$M_{\rm HVA-100} = a \cdot U_{\rm in}^2 + b \cdot U_{\rm in} + c, \qquad (5.16)$$

so that the scale factor can be extrapolated from the resulting curve (for result see top legend of figure 5.13).

The blue data shows a calibration measurement conducted in 2014 at GSI with an input voltage range of -10 kV to -80 kV (data provided by J. Ullmann [108]). The scale factor for -67 kV was determined in a similar manner as the 2013 scale factor by fitting the data with a second-order polynomial. One major difference between the two measurements is the high-voltage divider that was used as reference. For this measurement, the PTB HVDC2.1 high-voltage divider was used, which has a substantially larger scale factor uncertainty (approximately 14 ppm) compared to the KATRIN divider (approximately 1 ppm) leading to larger uncertainties for the JRL HVA-100 scale factor determination. This difference alone cannot explain the different behaviors for lower



Figure 5.13: JRL HVA-100 scale factor analysis for three available calibration measurements. The measurements conducted in Münster (turquoise) and Darmstadt (blue) were both carried out with a negative input voltage over a range of $-5 \dots -60$ kV and $-10 \dots -80$ kV respectively. To extrapolate the scale factor at -67 kV a second-order polynomial according to equation (5.16) was fit to the data (results shown in top legend). From the 2017 calibration, only two measurements at positive polarity are available without temperature regulation. In the latter case, a linear interpolation was used, and the scale factor was assumed to be polarity independent as supported by dedicated measurements during the same campaign presented in [107]. The uncertainties were estimated by scaling the initial statistical uncertainties of the 2013 and 2014 data sets such that respective the reduced chi-square χ_r^2 equals one (for further explanation see text).

voltages at approximately -10 kV observed between the 2013 and 2014 calibration measurements. As details of the individual setups like the exact cabling and measurement procedures cannot be reproduced anymore, the discrepancy remains unresolved. However, for voltages in the relevant region of -67 kV, both curves show a similar shape, so that the impact of the low voltage discrepancy in that range is negligible.

The red points show the relevant data resulting from a calibration measurement campaign at PTB in 2017, where the MT100 [14] was used as reference divider. Here, only

Table 5.3.: Results from the three calibration measurement campaigns for the HVA-100 HV divider. To account for the temperature dependency of the HVA-100, the scale factors were extrapolated to a common temperature value given by the first calibration campaign using the temperature coefficient given in eq. 5.17.

year	$M_{\rm HVA-100}(-67{\rm kV})$	$T_{\text{cal.}}$ [°C]	<i>M</i> _{HVA-100} (-67 kV) @ 22.54 °С
2013	9999.822(14) _{sys} (7) _{fit}	22.54(15)	9999.822(16)
2014	9999.789(152) _{sys} (14) _{fit}	18 - 27	9999.789(196)
2017	9999.693(3) _{sys} (2) _{fit}	21.56(12)	9999.667(45)

two measurements at 60 kV and 70 kV were conducted with similar ambient conditions as were present during the beamtime 2016. Hence, a linear interpolation between both data points was performed to determine the scale factor at 67 kV. Despite being measured only at positive polarity, the data is still valid for this analysis, as the polarity dependence of the measured scale factors was shown to be negligible during the campaign in 2017 [107]. Results of the three measurements are tabulated in the left two columns of table 5.3. The individual uncertainties of the data points in figure 5.13 are based on the statistical uncertainties, which were used as initial weights for the presented fits. Since details of the analyzed measurements (such as stabilities of the used measurement devices) are not part of the data, it is probable that some systematics are unknown and thus cannot be directly included in the error bars. To get an estimate of the true uncertainty, the fit was performed twice, while the second time the error bars were scaled so that the reduced chi-square χ^2_r becomes one. The uncertainties resulting from the fit are given in the brackets following the values and are labeled with the word fit. Additionally, the systematic uncertainties are given in the bracket labeled sys.

The systematic uncertainties are determined from the known systematics of the measurement devices given in manuals (in case of DVMs) and by calibration histories (in case of reference dividers). An additional uncertainty arises from the ambient temperature at which the calibrations were performed and which has a significant influence on the measured scale factors. Ideally, all calibrations should be extrapolated to the temperature conditions present in the electron cooler cage during the 2016 beamtime. However, as exact temperatures are only known for the 2013 and 2017 calibration measurements, the former is set as central temperature to which the 2017 scale factor is extrapolated. For the scale factor of the 2014 calibration and for the interpolated value of the 2016 beamtime, an uncertainty is added corresponding to a possible temperature range inside the cooler cage (where the divider was placed in both cases) from 18 °C



Figure 5.14: HV divider JRL HVA-100 scale factor determination for an input voltage of -67 kV. The calibrated scale factor $M_{\text{HVA}-100}(-67 \text{ kV})$ used for the transition analysis is shown as black square. The uncertainty of the scale factor is a combination of the uncertainty resulting from the fit (black part of error bar) and an additional uncertainty resulting from the missing temperature measurement in 2016 (green part of error bar).

to 27 $^{\circ}$ C which is plausible from available temperature measurements taken inside the cage at other time slots.

The relative temperature coefficient $\alpha_{\text{HVA}-100}$ of the HVA-100 scale factor was determined in a temperature range from 16 °C to 28 °C during the calibration measurements at PTB in 2017 to be [107]

$$\alpha_{\rm HVA-100} = -2.75(25) \,\frac{\rm ppm}{\rm K} \,. \tag{5.17}$$

We, therefore, obtain a temperature-related systematic uncertainty on the scale factors determined for 2014 and 2016 of $\sigma_T \approx 12.4$ ppm. To deduce a common scale factor from the available calibration measurements presented in table 5.3, a linear fit was applied to all measured values (see figure 5.14). This approach is used to account for the ongoing aging process of the resistors inside the HV divider. Besides the uncertainties from the individual calibration measurements taken into account for the fit, the uncertainty σ_T arising from the missing temperature measurement of 2016 was added linearly to the uncertainty of the fit result for a conservative estimate of the 2016 scale factor uncertainty. In conclusion, the scale factor used for the transition wavelength analysis

(displayed as a black square in figure 5.14) amounts to

$$M_{\rm HVA-100}(-67 \,\rm kV, \, 2016) = 9999.698(155),$$
 (5.18)

where the combined statistical and systematic uncertainty estimate is given in the brackets.

5.2.8. Set voltage calibration

Due to problems with the software of the Keysight DVM, the measured electron cooler voltages were not recorded for long intervals of the beamtime. This includes nearly the complete measurement of the ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ transition and data acquired for electron cooler currents $I_{ecool} > 220$ mA of the ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ investigation. During these intervals, only the set voltage data of the main control room (HKR) is available to determine the ion velocities.

As described in section 5.1.4, the actual electron cooler voltage is measured by scaling down the supply voltage with an HV-divider and measuring the scaled-down voltage U_{DVM} with the Keysight voltmeter. A prescription for reconstructing the missing DVM measurements, yielding reconstructed values $U_{\text{DVM}, \text{reconstr.}}$, can be obtained from those intervals, where both the HKR set voltages U_{set} and corresponding DVM voltages U_{DVM} were recorded. Two additional corrections have to be taken into account when doing so. One refers to shifts of the supply voltage due to the delivered cooler current, and another arises from nonlinearities of the DAC from the Heinzinger voltage supply. Both effects were investigated in dedicated measurements performed by D. Winters at GSI approximately two weeks after the beamtime. The latter measurements exhibit a constant offset of the voltages delivered by the Heinzinger power supply of about 4 V compared to the beamtime. This shift is attributed to the two-week interruption in the operation of the power supply but can be accounted for using the available beamtime data.

An overview of the calibration process, both for runs with valid DVM data and for runs where the DVM measurements need to be reconstructed from the HKR set values, is shown in figure 5.15. For runs with valid DVM data, only the right (green) panel of the process is relevant, where measured DVM voltages are corrected for the voltmeter gain and scaled up to the actual electron cooler voltage U_{ecool} (see sections sections 5.2.6 and 5.2.7).



Figure 5.15: Flow chart of the complete voltage calibration process. The central and the left parts in blue show the set voltage calibration process, which yields DVM-like data $U_{\text{DVM, reconstr.}}$ for runs where this data was not recorded. The green part on the right shows how actual or reconstructed DVM measurements have subsequently to be corrected for the DVM gain and multiplied with the divider scale factor to obtain the electron cooler voltage U_{ecool} .

For runs with missing DVM data, a relation between HKR set voltages and measured DVM values (see left, light blue panel) first has to be established. To do so, runs where both voltages have been recorded were used and, after correcting for the voltage shift due to the cooler current (resulting in DVM values at zero cooler current $U_{\text{DVM},0\text{mA}}$) and for the DAC nonlinearity (resulting in corrected set values $U_{\text{set},\text{DAC}}$), perform a linear fit of the resulting data. The fitted relation is then used in the reconstruction of missing DVM measurements (shown in the middle panel of figure 5.15), resulting in values $U_{\text{DVM},\text{reconstr.}}$ which are then replacing missing DVM measurements in the determination of the cooler voltage.

In the following paragraphs, a detailed description of the procedure is given.

Calibration of cooler current-induced potential shifts The large electron currents created by the electron cooler leads to a measurable shift of the supply voltage. This effect was investigated by measuring the supply voltage in dependence of the electron cooler current. Measurements were performed for two fixed voltages central to the voltage ranges of the two transitions investigated during the beamtime (see figure 5.16). The voltage of $U_{\text{set}} = 66809$ represents the ${}^{2}S_{1/2} - {}^{2}P_{1/2}$ transition measurements and $U_{\text{set}} = 67321$ the ${}^{2}S_{1/2} - {}^{2}P_{3/2}$ transition measurements. It should be noted that HKR set voltages are always given as absolute values and therefore have no negative sign. The correction voltage $\Delta U_{\text{DVM}}(I_{\text{ecool}})$ is defined as the difference between



Figure 5.16: Determination of electron cooler current (I_{ecool}) dependent set voltage correction based on measurements conducted by D. Winters of GSI. To investigate the effect of the cooler current on the measured DVM voltages, the cooler current was varied for two fixed set voltages $U_{set} = 66809 \text{ V}$ (blue) and $U_{set} = 67321 \text{ V}$ (green). The y-axis shows the difference between the corresponding DVM voltages $U_{DVM}(I_{ecool})$ and the average voltage measured at $I_{ecool} =$ 0 mA. The data shows a linear dependency on the cooler current up to $I_{lim} \approx 120 \text{ mA}$ and an approximately constant shift for higher currents. To model the dependency, a two-interval fit (see equation (5.19)) was performed for each set voltage.

the measured DVM voltage $U_{\text{DVM}}(I_{\text{ecool}})$ at a cooler current I_{ecool} and the average of the measured DVM voltages at zero cooler current $\overline{U}_{\text{DVM}}(0)$. The progression of the correction voltage is almost identical for both measurements, leading to a maximum shift of approximately $-80 \,\mu\text{V}$. To model the correction voltage $\Delta U_{\text{DVM}}(I_{\text{ecool}})$ a twointerval fit was applied, with one interval depending linearly on the cooler current I_{ecool} and one interval modeled as a constant shift

$$\Delta U_{\rm DVM}(I_{\rm ecool}) = \begin{cases} a \cdot I_{\rm ecool} + b & \text{for} \quad I_{\rm ecool} < I_{\rm lim} \\ a \cdot I_{\rm lim} + b & \text{for} \quad I_{\rm ecool} \ge I_{\rm lim} \end{cases},$$
(5.19)

where *a* is the slope, *b* is the intercept, and I_{lim} marks the interval limit.

Where necessary in the following analysis, the difference voltage $\Delta U_{\text{DVM}}(I_{\text{ecool}})$ is subtracted from measured DVM values to make it possible to compare measurements taken at different cooler currents. The resulting DVM values are labeled $U_{\text{DVM}, 0 \text{ mA}}$ for clarity.

Correction of DAC nonlinearity The second correction relates to a nonlinear behavior of the Heinzinger voltage supply caused by the finite resolution of the 18-bit DAC used in the control loop of the device. Figure 5.17 shows the effect of this nonlinearity for data gathered during the beamtime and in the series of systematics measurements conducted by D. Winters. Data from the latter measurements exhibit a constant voltage shift caused the power supply not coming back to the same absolute voltage as during the beamtime. This, however, does not affect the following analysis.

It is observed that voltages measured with the DVM sometimes stay constant despite a new set voltage being chosen. This happens about every 5th or 6th voltage step and, being a reproducible effect, can be taken into account in the analysis. By combining the beamtime data and data from the systematics checks, most positions where changing the set voltage has no effect on the measured voltage could be identified (filled arrows). To account for set voltage regions where neither measurements had obtained suitable data, the effect was interpolated and the positions where changing the set voltage is expected to have no effect on the measured voltage marked by hollow arrows. These interpolated positions were cross-validated using the fluorescence rates measured with the XUV detector. Fluorescence rates staying constant when changing the set voltage by 1 V indicate that the electron cooler voltage was not changed despite setting a new value via the HKR.

After identifying the pattern of nonlinearity, a modified version of the set voltages $U_{\text{set, DAC}}$ is obtained by subsequently shifting for each arrow all voltages left of the arrow by 1 V. This modified set voltage then does not directly represent the cooler voltage anymore but allows to eliminate the effect of the finite resolution of the control of the Heinzinger voltage supply when reconstructing DVM values for those runs



Figure 5.17: Measurement of DAC nonlinearities. X-axes show set voltage data U_{set} , which is compared to measured DVM data $U_{DVM,0 \text{ mA}}$ (y-axes) at zero cooler current. The blue axes show beamtime data (filled blue) and systematics measurements (filled cyan) for the ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ transition. The green axes show beamtime data (filled green) and systematics measurements (filled light green) for the ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ transition. The finite resolution of the DAC leads to voltages sometimes not being changed despite changing the set voltage in the HKR. These positions are indicated by arrows. The hollow symbols represent interpolated data and are used for illustration purposes only. The hollow arrows represent interpolated shift positions, where no set voltage data together in combination with DVM data was available.

where these were not recorded.

Calibration of electron cooler set voltage After determining the DVM voltages at zero cooler current $U_{\text{DVM},0 \text{ mA}}$ and the modified set voltages $U_{\text{set, DAC}}$, a linear fit can be applied to describe the relationship between the two. As a result of the DAC correction, this calibration is determined for the two transition voltage ranges individually because the effect of the DAC nonlinearity has not been measured for voltages between 66 880 V and 67 256 V. Figure 5.18 shows calibration curves for the available data sets which were fitted using the following functional shape

$$U_{\text{DVM, fit}}(U_{\text{set, DAC}}) = a \cdot (U_{\text{set, DAC}} - x_{\text{off}}) + b, \qquad (5.20)$$

where *a* is the slope, *b* the intercept and x_{off} an offset parameter that shifts the data to around 0 V to guarantee numerical stability of the fit routine. For the ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ transition data in figure 5.18b (blue), where DVM data for only two set voltages was



(b) Set voltage calibration for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ transition.

Figure 5.18: Calibration of modified set voltages for (a) ${}^{2}S_{1/2} - {}^{2}P_{3/2}$ and (b) ${}^{2}S_{1/2} - {}^{2}P_{1/2}$ transitions. Linear fits were performed for each dataset according to equation (5.20). The dark green (figure a) and blue (figure b) calibration curves are used for the further analysis. The calibrations for the systematics data in light green (figure a) and cyan (figure b) are used to get an estimate for the fixed slope a_{fix} parameter of the ${}^{2}S_{1/2} - {}^{2}P_{1/2}$ calibration, where insufficient DVM data is available from the beamtime.

available, the slope *a* of the fit was fixed to the slope of the ${}^{2}S_{1/2}-{}^{2}P_{1/2}$ systematics data (cyan). This approach is justified by a comparison of the slopes of ${}^{2}S_{1/2}-{}^{2}P_{3/2}$

transition data (green) and systematics data (light green) in figure 5.18a which only differ by approximately 0.008 V/v. The comparison shows the stability of the Heinzinger voltage supply over a time interval of approximately 12 days (time interval between beamtime and systematics investigations) in terms of linearity. An apparent offset between both calibration curves is visible, which amounts to a constant shift in the absolute electron cooler voltage U_{ecool} of approximately 4 V for a given set voltage. This, however, does not affect the further analysis. As an estimate for the uncertainty of the slope assumed for the ${}^{2}\text{S}_{1/2}$ - ${}^{2}\text{P}_{1/2}$ transition, the difference between fitted slopes shown in figure 5.18a is taken, which also reflects in the visible confidence band of the blue line in figure 5.18b.

In conclusion of this section, the conversion of set voltage data to reconstructed DVM values is given by the sum of the two individual calibrations from equations (5.19) and (5.20)

$$U_{\text{DVM, reconstr.}}(U_{\text{set, DAC}}, I_{\text{ecool}}) = \Delta U_{\text{DVM}}(I_{\text{ecool}}) + U_{\text{DVM, fit}}(U_{\text{set, DAC}}), \quad (5.21)$$

which yields DVM-like data that is subsequently calibrated in the same way as actual DVM measurements according to sections 5.2.6 and 5.2.7. Since the relative deviation of measured DVM voltages U_{DVM} and reconstructed values $U_{\text{DVM}, \text{reconstr.}}$ stays below 10 ppm for all available data points, the 10 ppm deviation is taken as a conservative measure for the systematic uncertainty of the reconstructed DVM voltages $\sigma_{U_{\text{DVM}, \text{reconstr.}}}$.

5.2.9. Work function impact on electron cooler potential

The work function W of a material has a direct impact on the electrostatic potential ϕ produced in the vacuum when a voltage U is applied to the material

$$\phi = U - \frac{W}{e} \,. \tag{5.22}$$

For the ESR electron cooler, this means that potential shifts have to be accounted for resulting from different electrode materials.

Electrons in the ESR electron cooler are generated in a dispenser cathode of an electron gun (see section 3.3.1). It consists of a tungsten filament coated with barium, having a nominal work function of $W_{t,b} \approx 1.66 \text{ eV}$ [47]. The beampipe of the electron cooler, where the ion beam is overlayed with the electron beam, consists of stainless steel

with a nominal work function of $W_{\text{steel}} \approx 4.5 \text{ eV}$ [109]. Work functions are typically determined under ideal conditions, where surfaces are pre-cleaned and materials are ultrapure. This is done because work functions are influenced by effects such as impurities, adsorbates, surface roughness and electrical fields. This can lead to substantial deviations from the literature values, as observed in the Ph.D. thesis of J. Behrens [110] in the framework of the KATRIN experiment. KATRIN features a large vacuum vessel (called main spectrometer) on which a high voltage is applied that has to be known to the ppm-level. As work functions can have an effect on electric potentials in the order of a few Volts, a detailed investigation of the spectrometer work function was undertaken. The KATRIN main spectrometer also consists of stainless steel and, as a result of J. Behrens' thesis, its work function has been determined to be in the range of 3.39 - 3.65 V. The literature values of the work functions used for the correction are, therefore, awarded with an uncertainty of 1 eV each. The acceleration voltage felt by the electrons is then determined by the difference of the electrode potentials of the cathode and the beampipe $\Delta \phi$ which, according to equation (5.22), is given by

$$\Delta \phi = \phi_{\text{cath.}} - \phi_{\text{pipe}} = \left(U_{\text{cath.}} - \frac{W_{\text{t,b}}}{e} \right) - \left(U_{\text{pipe}} - \frac{W_{\text{steel}}}{e} \right)$$
(5.23)

$$= (U_{\text{cath.}} - U_{\text{pipe}}) - \left(\frac{W_{\text{t,b}}}{e} - \frac{W_{\text{steel}}}{e}\right)$$
(5.24)

$$= U_{\rm ecool} + 2.84(142) \,\mathrm{V}\,, \tag{5.25}$$

where $U_{\text{cath.}}$ is the voltage applied to the cathode, U_{pipe} the voltage of the beampipe and U_{ecool} is the voltage applied between the two and thereby the difference of $U_{\text{cath.}}$ and U_{pipe} . In conclusion, the potential of the electron cooler is shifted by +2.84(142) V, effectively lowering the electron energies by 2.84 eV. Considering an electron cooler voltage of approximately -67 kV, the relative effect of the work function on the cooler potential amounts to 42 ppm, leading to one of the major sources of uncertainty on the total result (see section 5.3.5).

5.2.10. Ion space charge correction

Up to this point, it was assumed that measuring the electron cooler acceleration voltage would yield the electron velocities and, therefore, the ion velocities through equation (3.1). This assumption does not take space charges into account that are caused by the high fluxes of charged particles in the electron-ion interaction area. When ion- and electron beams are concentrically overlapped, and the electron beam has a much larger diameter than the ion beam, only the electrons in the vicinity of the ions determine the ion velocity. Assuming a cylindrical shape of both beams, the center particles of both beams are shielded by outer particles from the potential of the surrounding drift tube. Electrons and ions create electrostatic potentials ϕ_{e^-} and ϕ_{ion} that have to be taken into account when calculating the electron and ion velocities. The electrostatic potential shift caused by the electrons can be up to +50 V depending on the applied electron current I_{ecool} . The electrostatic potential shift caused by the ions is only in the order of up to -1 V due to the much lower ion currents I_{ion} stored in the ESR ($I_{ion, max} \approx 1.5$ mA). The total potential ϕ_{tot} experienced by the inner ions and electrons is a superposition of all relevant potentials

$$\phi_{\text{tot}} = \phi_{\text{ecool}} + \phi_{\text{e}^-} + \phi_{\text{ion}} \,, \tag{5.26}$$

where ϕ_{ecool} is given by equation (5.25). The space charges only affect the ion velocity indirectly as the kinetic energies of the electrons are reduced, and thus, the electron velocities change. Via the electrons, the ion velocities are then altered. In consequence, ion space charges lead to an acceleration of the electrons, and hence the potential ϕ_{ion} is negative, and electron space charges decelerate the electrons, which is why ϕ_{ecool} is positive.

While the space charge effect caused by the electron currents can be measured and thereby corrected (see section 5.3.4), this cannot be easily done for the ion-induced space charges. Hence an analytical approach is applied as has been done in previous works [9, 111, 112, 113]. In the analytical approach, the potential created by the ions is assumed to be cylindrical and can be calculated using [9]

$$\phi_{\text{ion}}(r) = \begin{cases} \frac{\rho_{\text{ion}}}{4\varepsilon_0} r_{\text{ion}}^2 \left[\frac{r^2}{r_{\text{ion}}^2} - 1 - 2\ln\left(\frac{r_{\text{drift}}}{r_{\text{ion}}}\right) \right] & \text{for } r < r_{\text{ion}} \\ \frac{\rho_{\text{ion}}}{2\varepsilon_0} r_{\text{ion}}^2 \ln\left(\frac{r}{r_{\text{drift}}}\right) & \text{for } r \ge r_{\text{ion}} \end{cases},$$
(5.27)

where *r* is the distance from the center beam axis, ε_0 the electric constant, r_{ion} the radius of the ion beam, r_{drift} the radius of the electron cooler drift tube and ρ_{ion} is the ion space charge, which is assumed to be constant over the whole beam diameter

$$\rho_{\rm ion} = \frac{I_{\rm ion}}{\pi r_{\rm ion}^2 \beta c} \,, \tag{5.28}$$

with I_{ion} as the ion current, β as the ion/electron velocity and c as the vacuum speed



Figure 5.19: Ion space charge correction for ${}^{2}S_{1/2}{}^{-2}P_{1/2}$ measurement at $I_{ecool} \approx 208$ mA. The left y-axis shows the correction potential ϕ_{ion} (orange) derived with equation (5.29). The maximum correction is at $I_{max} \approx 1.4$ mA. The resulting effect on the electron cooler voltage is shown on the right y-axis in red. The voltage measurement does not show any fluctuations because at that time, the Keysight DVM was not recording data and only set voltage data was available (see section 5.2.8). The electron cooler voltage U_{ecool} was determined according to all previously presented calibrations with regard to voltages.

of light. With the assumption of an ideal alignment of the ion beam to the center axis of the electron beam, equations (5.27) and (5.28) yield

$$\phi_{\rm ion}(r=0) = -\frac{I_{\rm ion}}{\pi r_{\rm ion}^2 \beta c} \left[1 + 2 \ln \left(\frac{r_{\rm drift}}{r_{\rm ion}} \right) \right].$$
(5.29)

Besides the measured ion current I_{ion} (see section 5.2.4), the parameters r_{drift} and r_{ion} have to be known. The drift tube radius of the ESR electron cooler is $r_{drift} \approx 0.1 \text{ m}$ [9, 111] and the ion beam radius during the beamtime was $r_{ion} \approx 3 \text{ mm}$ [114]. As the effect of the electron and ion space charges on the ion velocity β is on the $1 \cdot 10^{-4}$ level, β can, for this purpose, simply be calculated from the cooler voltage. The resulting potential correction as a function of the ion current is shown in figure 5.19. To get a conservative measure of the uncertainty arising from the ion space-charge correction, the whole analysis was conducted with and without applying the correction, and the difference of both results was taken as systematic uncertainty.

5.3. Transition wavelength analysis

In this section, the transition wavelength analysis for the ${}^{2}S_{1/2} - {}^{2}P_{1/2}$ - and the ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ - transitions of lithium-like carbon (${}^{12}C^{3+}$) is presented. All calibrations provided in the previous section are applied to interpret the fluorescence signal measured with the MCP response of the XUV detection system. In order to determine the final result for each respective transition, two further systematic corrections have to be accounted for. The first correction is related to the unknown gain of the MCP, which depends on the total particle flux onto the MCP. A method to reconstruct the unknown gain is implemented based on gain saturation models by Giudicotti *et al.* [115] and Gershman *et al.* [116]. The second correction concerns shifts in the effective cooler potential caused by electron-induced space charges in the electron cooler (similar to the ion space charge effect presented in section 5.2.10).

This section is divided into six parts. In the first part, the determination of the fluorescence signal without the inclusion of the MCP gain is presented. Subsequently, the MCP gain is analyzed and incorporated into the analysis, yielding the corrected fluorescence signal that is used for the wavelength analysis. In the third part, the wavelength of each transition is analyzed for the different electron cooler currents that were set throughout the measurements. The fourth part comprises the electron space charge correction, which yields the corrected transition wavelength for each transition. In the fifth part, the results are presented with a discussion of the uncertainties and a conclusion with regard to future detector applications and developments. The section closes with a conclusion and a proposal for possible improvements with regard to future measurements.

5.3.1. Determination of fluorescence signal

In order to analyze the transitions, the fluorescence signals have to be extracted from the MCP response of each individual scan. Figure 5.20 shows a representative fluorescence scan at an electron cooler current of $I_{ecool} \approx 218$ mA. The figure demonstrates the high sensitivity of the detection system to the fluorescence photons, indicated by a very high signal-to-noise ratio. The minimum step size of 1 V utilized for the set voltages is resolved in terms of fluorescence photon rates. To clear the MCP signal of background for quantitative analysis, the individual components of the signal have to be identified. With the laser wavelength λ_{laser} fixed throughout all shown fluorescence



Figure 5.20: Exemplary fluorescence scan for ${}^{2}S_{1/2} {}^{2}P_{3/2}$ level splitting at an electron cooler current of approximately 218 mA. **Top:** Ions injected into the storage ring are typically stored for 200 s to 500 s (green). The ions are overlapped with the laser beam (purple) to excite the investigated transition. **Bottom:** To find the transition energy, a scan over the transition is performed by varying the ion velocity via the electron cooler voltage (red). The occurring maxima in the MCP signal (blue) indicate when the ion velocity and the laser wavelength are matched to the energy of the excited transition. The red band indicates the voltage range in which the maximum of the resonance is located.

measurements (see section 5.2.2) and the electron cooler current I_{ecool} fixed for the single scans, the MCP rate r_{MCP} is mainly determined by the electron cooler voltage U_{ecool} , the ion current I_{ion} and the laser power P_{laser} .

The measured MCP rate r_{MCP} shown in figure 5.20 is proportional to the total fluorescence rate R_{fluo} , the background from the ion beam BKG_{ion} and the background from the laser beam BKG_{laser}. The fluorescence rate $R_{\rm fluo}$ can be parameterized by a normalized fluorescence rate $r_{\rm fluo}$ (unit: kcps/(mA · mW)) scaled with the ion current $I_{\rm ion}$ and the laser power $P_{\rm laser}$

$$r_{\rm MCP} \sim \underbrace{r_{\rm fluo} \cdot I_{\rm ion} \cdot P_{\rm laser}}_{R_{\rm fluo}} + BKG_{\rm ion} + BKG_{\rm laser}.$$
 (5.30)

By solving equation (5.30) for the normalized fluorescence rate r_{fluo} , the effects of backgrounds, varying ion current, and laser power can be accounted for, and the maxima in the resulting signal shape can be analyzed to the find transition wavelength.

However, the normalized signal shown in figure 5.21 deviates from the expected shape since a correctly normalized fluorescence signal should have a constant amplitude for a given voltage applied to the electron cooler. The points of maximum fluorescence (at U_{ecool} between $-67\,272.8\ldots -67\,271.5$ V) are pointed out by the black arrows in the figure and comparing the rates at these positions shows an increase in rate over time instead of a constant rate. This indicates that further corrections have to be applied,



Figure 5.21: Uncorrected fluorescence signal (blue y-axis) and electron cooler voltage (red y-axis) for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at an electron cooler current of approximately 218 mA (as in figure 5.20). The uncorrected fluorescence signal is calculated from the MCP rate according to equation (5.30) and is referred to as "uncorrected" due to gain saturation effects and the MCP efficiency not being taken into account. These missing effects are indicated by the signal showing an increase in rate towards a longer run duration. For an ideal normalized fluorescence signal, the rate should be constant for a given voltage (or ion velocity), as pointed out by the black arrows marking maximum fluorescence occurring at voltages between $-67 272.8 \dots -67 271.5 \text{ V}$. The data is binned into 1 s bins for better visibility.

which were not taken into account in the figure. Considering the maximum rates of approximately 1 MHz measured by the MCP, saturation effects can cause significant degradation of the MCP performance and are therefore discussed in the next section.

5.3.2. MCP gain

As stated in section 4.3, a crucial quantity determining the performance of an MCP is the gain and its linearity. According to the manufacturer of the MCP, two main effects can cause saturation of the MCP, leading to significant drops in the gain [117]. The first effect refers to the total incident rate of particles hitting the MCP, because of which the MCP cannot recharge in time before further particles hit the MCP. Secondly, a local saturation of single channels can occur if too many particles hit the same channel before it can recharge. Due to the forward boost of the fluorescence light, it can be assumed that the radiation pattern on the cathode plate and therefore the imaged electron hit pattern on the MCP are not homogeneously spatially distributed. Hence, both gain degradation effects will probably occur.

Since the gain characteristic of the used MCP is unknown, a correction is derived based on analytical models developed by Giudicotti *et al.* [115] and Gershman *et al.* [116]. In [115] an MCP saturation model is described, modeling the MCP channels as a discrete number of dynodes N, where N is related to the length-to-diameter ratio L/d of the MCP. Two different analytical gain models are derived in the publication, with regard to different operating modes of the MCP. The MCP was operated in so-called pulse mode during the beamtime, which is characterized by the input current pulse having a much shorter duration (in this case ns) than the recharging time $\tau_D \approx RC$. Since time constants τ_D of MCPs are typically in the order of a few milliseconds, the pulse mode model applies. In this mode, the actual gain G in terms of undistorted gain G_0 becomes

$$G(\rho_{\rm i}) = \frac{G_0}{1 + \frac{\rho_{\rm i}}{\rho_{\rm 50\,\%}}},\tag{5.31}$$

where the ratio $\rho_i/\rho_{50\%}$ refers to the so-called gain saturation parameter described in [115]. It has to be noted that the nomenclature of [116] is used for consistency. In this case the parameter $\rho_{50\%}$ is defined as the critical number of particles per channel, where the average MCP gain has dropped to 50 % and ρ_i is the ratio between the amount

of incident particles and the number of MCP channels

$$\rho_{\rm i} = \frac{\phi_{\rm i} A_{\rm MCP} \Delta t}{\frac{A_{\rm MCP}}{d^2}} = \phi_{\rm i} \Delta t d^2 \,, \tag{5.32}$$

where ϕ_i is the incident particle flux (unit: $1/\text{cm}^2 \text{s}$), A_{MCP} is the area of the MCP, $\Delta t \ll \tau_{\text{D}}$ is the time interval during which the burst of incident flux occurs and d is the center-to-center spacing of adjacent MCP channels. Since the input flux of the MCP is unknown and has to be reconstructed, a relation between input flux ϕ_i and measured flux ϕ_m has to be established, or equivalently between incident particles per channel ρ_i and measured particles per channel ρ_m . The incident particle flux ϕ_i is directly connected to the current signal I picked up at the anode of the MCP via

$$I = qG\phi_{i}A_{\rm MCP}\varepsilon_{\rm MCP}\,,\tag{5.33}$$

with *q* as the unit charge, *G* the MCP gain and ε_{MCP} as the MCP efficiency. The measured particle flux ϕ_m is proportional to the amount of charge collected by the anode

$$\phi_{\rm m} = \frac{1}{qA_{\rm MCP}\Delta tG_0} \int_0^{\Delta t} I(t)dt \,. \tag{5.34}$$

By combining equations (5.31) to (5.34), the incident flux ϕ_i can be recovered from the measured flux ϕ_m

$$\phi_{\rm m} = \varepsilon_{\rm MCP} \frac{\phi_{\rm i}}{\Delta t} \int_{0}^{\Delta t} \frac{G(t)}{G_0} dt = \varepsilon_{\rm MCP} \frac{\rho_{50\,\%}}{\Delta t d^2} \ln\left(1 + \frac{\phi_{\rm i} \Delta t d^2}{\rho_{50\,\%}}\right). \tag{5.35}$$

Because of equation (5.32), this is equivalent to

$$\rho_{\rm m} = \varepsilon_{\rm MCP} \rho_{50\,\%} \ln \left(1 + \frac{\rho_{\rm i}}{\rho_{50\,\%}} \right). \tag{5.36}$$

By solving equation (5.36) for ρ_i , the incident particle flux per channel can be recovered from the measured flux ρ_m

$$\rho_{\rm i} = \rho_{50\%} \left(\exp\left(\frac{\rho_{\rm m}}{\varepsilon_{\rm MCP} \rho_{50\%}}\right) - 1 \right). \tag{5.37}$$

All three particle flux parameters $\rho_{m/i/50\%}$ in equation (5.37) can be replaced by particle flux rate parameters $r_{MCP/in/50\%}$ in kcps/s (since these were the observables in the measurements), without the previous equations losing their validity. Hence, all flux parameters $\rho_{m/i/50\%}$ are substituted by their respective counterparts $r_{MCP/in/50\%}$ in the following. While the particle flux rate r_{MCP} is known by measurement with the MCP, the quantity $r_{50\%}$ is a characteristic of the MCP (dependent on the MCP stack geometry and the applied voltage) and is not specified by the manufacturer. Therefore, neither the incident particle flux rate r_{in} nor parameter $r_{50\%}$ can be derived from equation (5.37) without further assumptions. However, with the available data, a model is deduced in the following to determine the parameter $r_{50\%}$ which characterizes the MCP gain (see equation (5.31)), which then allows to calculate the incident particle flux rate r_{in} .

The reconstruction of the MCP gain is based on the assumption that the normalized fluorescence rate $r_{\rm fluo}$ (according to equation (5.30)) should be constant for a given fixed parameter set of electron cooler voltage $U_{\rm ecool}$, electron cooler current $I_{\rm ecool}$ and laser wavelength $\lambda_{\rm laser}$. As demonstrated in figure 5.21, this is not the case without taking gain saturation effects into account. The difference to a constant rate can be attributed to the saturation effects affecting the MCP gain in dependence of the incident rate $r_{\rm in}$. The gain model that was applied is parameterized in terms of the relative gain $G_{\rm rel.}$, which is given by the ratio G/G_0 according to equation (5.31)

$$G_{\text{rel.}}(r_{\text{in}}) = \frac{G(r_{\text{in}})}{G_0} = \frac{1}{\left(1 + \frac{r_{\text{in}}}{r_{50\,\%}}\right)}.$$
(5.38)

Since the incident MCP rate is unknown, equation (5.37) is utilized to derive the relative MCP gain in dependence of the measured MCP rate r_{MCP}

$$G_{\text{rel.}}(r_{\text{MCP}}) = \exp\left(-\frac{r_{\text{MCP}}}{\epsilon_{\text{MCP}} \cdot r_{50\,\%}}\right).$$
(5.39)

The fluorescence rate for a given set of U_{ecool} and λ_{laser} can then be modeled by

$$f(r_{\rm MCP}) = r_{\rm fluo, 0} \cdot \exp\left(-\frac{r_{\rm MCP}}{\epsilon_{\rm MCP} \cdot r_{\rm 50\%}}\right),\tag{5.40}$$

where $r_{\text{fluo}, 0}$ represents the fluorescence rate for an undistorted MCP gain for the specific parameter set. With this model, all available fluorescence scan data was fit with the gain parameter $r_{50\%}$ being shared for all parameter sets since the gain characteristic was not expected to change throughout the beamtime. The MCP efficiency is stated by



Figure 5.22: Illustration of MCP gain parameter $r_{50\%}$ fit for ${}^{2}S_{1/2} - {}^{2}P_{3/2}$ level splitting at an electron cooler current of approximately 218 mA. The gain parameter $r_{50\%}$ is determined to 1603(32) kcps meaning that the MCP gain drops to 50 % at an incident rate of $r_{in} = 1603(32)$ kcps. The gain degradation effect becomes especially visible for higher fluorescence rates (top two curves). For lower fluorescence rates, deviations from the model occur for some curves (e.g. see curve for $U_{ecool} = -67274.13$ V) which could for example be caused by changes of the laser background over time.

the manufacturer to be $\epsilon_{\text{MCP}} \approx 0.6$ for electrons in an energy range of 100-1000 eV[79]. During the beamtime, electrons impacting on the MCP had an approximate energy of 300 eV mainly determined by the potential difference of $U_{\text{diff}} = 305 \text{ V}$ between the XUV detector cathode and the MCP front plate. Figure 5.22 shows the fit sequence for the fluorescence scan presented in figures 5.20 and 5.21 where each curve corresponds to a single electron cooler voltage setting. As previously discussed, each different parameter set features an individual parameter $r_{\text{fluo},0}$, while the parameter $r_{50\%}$ is shared for all fluorescence scan fits. The curves in figure 5.22 illustrate the MCP gain degradation effect in dependence of the measured MCP rate r_{MCP} . $r_{50\%}$ is determined to $r_{50\%} = 1603(32) \text{ kcps}$ which means, that at an incident rate $r_{\text{in}} = r_{50\%}$, the relative MCP gain drops to 50 %.



Figure 5.23: Illustration of relative MCP gain in dependence of measured MCP rate r_{MCP} (black curve) and incident rate r_{in} (blue curve). The data points result from the measured fluorescence rate divided by the undistorted fluorescence rate $r_{\text{fluo},0}$ that was determined in the previous fit. The data points are color-coded according to the electron cooler current I_{ecool} and the investigated transition. Here index 1 represents the ${}^{2}\text{S}_{1/2}$ - ${}^{2}\text{P}_{1/2}$ and index 2 represents the ${}^{2}\text{S}_{1/2}$ - ${}^{2}\text{P}_{3/2}$ transition. Especially for higher MCP rate, the data points agree well with the determined MCP gain (black line). For lower rates, statistical fluctuations and unknown systematics have a larger impact because of the relative y-axis scale. The blue curve represents the gain in dependence of the incident rate $G_{\text{rel}}(r_{\text{in}})$, which can be reconstructed from the parameters obtained from the fit. Error bars were omitted for better visibility.

To further illustrate the relative MCP gain, data points of all fluorescence scans were divided by their respective undistorted fluorescence rates $r_{\text{fluo}, 0}$ resulting from the aboveexplained fit. This yields the relative MCP gain according to equations (5.38) and (5.39) and has the advantage of visualizing all data points used for the fit in one graph. Figure 5.23 shows the resulting graph. On the black axes, the gain in dependence of the
measured MCP rate r_{MCP} according to equation (5.39) is presented. For MCP rates above 500 kcps data and model are in good agreement, while for lower rates, the spread of the data points with regard to the relative gain curve gets larger. This effect can be attributed to the relative scaling of the y-axis since statistical fluctuations and unknown systematics that are constant on the absolute scale gain more weight towards lower rates on the relative scale. Additionally, the blue axis with the blue curve shows the relative gain in dependence on the incident particle flux r_{in} , which is the typical form of representation in publications/manuals regarding MCP gain saturation effects. The curve is constructed from the parameters of the fit result applied to equation (5.38) and, amongst other things, emphasizes the gain drop to 50 % at an incident rate of $r_{\text{in}} = r_{50\%}$ which characterizes the MCP of the XUV detection system.

In conclusion, the corrected fluorescence rate $r_{\text{fluo, corr.}}$ is given by (compare to equations (5.30) and (5.39))

$$r_{\text{fluo, corr.}} = \frac{r_{\text{fluo}}}{G_{\text{rel.}}(r_{\text{MCP}})} = \frac{r_{\text{MCP}} - \text{BKG}_{\text{ion}} - \text{BKG}_{\text{laser}}}{I_{\text{ion}} \cdot P_{\text{laser}} \cdot \exp\left(-\frac{r_{\text{MCP}}}{\epsilon_{\text{MCP}} \cdot r_{\text{sor}}}\right)}.$$
(5.41)

which is used to calculate the fluorescence rate for all further analysis steps from this point on.

The effect of the correction is demonstrated in figure 5.24 for the fluorescence scan presented in figure 5.22. After the correction, the plateaus for maximum fluorescence are leveled as indicated by the black arrows that mark the difference between the uncorrected fluorescence rate (transparent blue line) and the corrected fluorescence rate (blue line).

5.3.3. Fluorescence analysis

Taking all previous calibrations and corrections into account, the fluorescence analysis was performed for a total of 44 individual scans (27 for the ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting and 17 for the ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting). The scans were performed for different electron cooler currents to be able to correct for the space charge effects caused by the electrons in the electron cooler (see section 5.3.4). To derive the transition wavelength for an individual scan, the corrected fluorescence rates are plotted as a function of the cooler voltage (see figure 5.25), which in turn, can be used to calculate the laser wavelength in the rest frame of the ions. The shape of the observed resonances can be approxi-



Figure 5.24: Demonstration of the MCP gain correction for the ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at an electron cooler current of approximately 218 mA according to equation (5.41). Compared to the uncorrected fluorescence rate (transparent blue line), the plateaus at maximum fluorescence rate for the corrected signal (blue line) are elevated to a constant level.

mated by a Voigt profile [118], which is defined as the convolution of a Gaussian and a Lorentzian distribution. In this case, the Gaussian part of the resonance signal is related to the velocity distribution of the ions within the beam and the Lorentzian part to the natural linewidth of the investigated transition. The Voigt profile is given by

$$V(x;\mu,\sigma,\gamma) = c + A \cdot \int_{-\infty}^{\infty} G(x-\mu;\sigma) L(x'-(x-\mu);\gamma) \, dx' \,, \tag{5.42}$$

where *c* describes a constant background, *A* the area of the Voigt profile, μ the line center, σ the standard deviation of the Gaussian profile and γ the half-width at half-maximum of the Lorentzian profile. Here, the Gaussian profile *G* is defined as

$$G(x;\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2\sigma^2}\right),\tag{5.43}$$

and the Lorentzian profile L is defined as

$$L(x;\gamma) = \frac{\gamma}{\pi(x^2 + \gamma^2)}.$$
(5.44)

Fitting equation (5.42) to the observed resonance curves, one obtains values for the line centers μ from which the transition wavelength is calculated according to equations (3.1) and (3.2). Figure 5.25 shows the result for a representative fluorescence scan



Figure 5.25: Fluorescence analysis for ${}^{2}S_{1/2} {}^{-2}P_{3/2}$ level splitting at an electron cooler current of approximately 218 mA. The data was fit with a Voigt profile according to equation (5.42). The top x-axis shows the wavelength derived from the electron cooler voltages by applying equations (3.1) and (3.2). γ was fixed to 0.0029 V which is equivalent to the nominal lifetime $\tau_{\text{nat.}} = 3.8 \text{ ns}$ (see equation (5.45)). The longitudinal ion beam temperature $k_B T_{\parallel, beam}$ can be derived from the Gaussian width σ according to equation (5.46) for beam diagnostic purposes (see *derived parameters*). The area A is directly related to the amplitude h_{amp} , which is also given in the legend under the *derived parameters* tag.

(this is again the same scan shown in figures 5.20 and 5.24). Each of the data points was generated from a weighted mean of all measured fluorescence rates for a given set voltage for better visibility. Here, the error bars represent the standard deviation from the weighted mean. The wavelength shown in the top x-axis is not yet corrected for the previously mentioned electron space charge effects. Uncertainty estimates of the fit parameters were again obtained by scaling the error bars with the square root of the reduced χ^2_{org} -value and repeating the fit. The only difference of this method to previous fits shown in this chapter is that for the minimization process, all fluorescence scans were considered simultaneously for a more realistic error scaling. By using this method, the effects of outliers for individual scans, especially with a low amount of data points, are reduced, and the error bars of all fluorescence scans are scaled by the same amount. The results for all fluorescence scans are presented in appendix A.2. In the legend of figure 5.25 the offset parameter *c* is labeled with the subscript *free*, which refers to the parameter not being restrained in any form. This was not the case for all

fluorescence fits presented in appendix A.2, since for some scans, the scan range was not large enough to determine the offset via the fit routine. For these scans, the average offset gained from the other fits with enough data was taken as a fixed parameter (labeled in the respective legends as c_{fix}).

Comparing the width of the measured profiles to literature values of the carbon transitions (e.g., in [119]) reveals that the obtained Gaussian width σ exceeds the natural linewidth γ by almost three orders of magnitude. Leaving both width parameters free during the optimization process led to unrealistically overestimated Lorentzian widths, which is why the parameter γ was fixed according to the natural linewidths obtained from literature [119]. The connection between the lifetime given in the papers and the linewidth in terms of wavelength is expressed by a variant of the time-energy uncertainty relation

$$\tau_{\text{nat}} = \frac{\hbar}{2\gamma_E} = \frac{1}{4\pi c \left(\frac{1}{\lambda_0 + \gamma_\lambda} - \frac{1}{\lambda_0 - \gamma_\lambda}\right)},\tag{5.45}$$

where τ_{nat} is the lifetime, \hbar the reduced Planck constant, γ_E the HWHM of the Lorentz profile in units of eV, λ_0 the center wavelength derived from the expected value μ and γ_{λ} the HWHM of the Lorentz profile in units of nm. From equation (5.45) the parameter γ_{λ} was derived for the lifetime value $\tau_{\text{nat}} = 3.8$ ns, then transformed to the Lorentzian width γ in units of V via equations (3.1) and (3.2) and used as fixed parameter in the fits.

While the line center μ gained from the fit is the relevant quantity for the transition analysis, further physical and diagnostic quantities can be derived from the other fit parameters (presented under *derived parameters* in the legend of figure 5.25). The first quantity is the amplitude h_{amp} which refers to the amplitude of the fluorescence signal and is derived from the area A by calculating the Voigt profile using the fit parameters at the center $x = \mu$. The fluorescence amplitude provides a measure for the signal-tonoise ratio when compared to the background components presented in section 5.1.5. A parameter that can be used for beam diagnostic purposes is the longitudinal ion beam temperature $k_B T_{\parallel, beam}$. It is related to the Gaussian width σ of the Voigt profile, as the ion velocity distribution is the main source of Doppler broadening. According to [48], the beam temperature is calculated by

$$k_B T_{\parallel, beam} = \left(\frac{\Delta\lambda}{2\lambda_0}\right)^2 \cdot \frac{m_0 c^2}{2\ln(2)}, \qquad (5.46)$$

where $k_B T_{\parallel, \text{beam}}$ is the longitudinal ion beam temperature in eV, $\Delta \lambda$ the FWHM in terms of wavelength associated with the FWHM = $2\sqrt{2 \ln(2)}\sigma$ from the fit via equation (3.2), λ_0 the central wavelength derived from the center value μ , m_0 the ion rest mass and cthe vacuum speed of light.

The comparison of the individual scan results for a given transition shows a wavelength shift which is dependent on the electron cooler current I_{ecool} that was set per individual scan. As previously mentioned, these shifts are caused by electron-induced space charge effects, which have to be accounted for in order to deduce the true transition wavelength. The necessary correction is presented in the following section.

5.3.4. Electron space charge correction

Deviations between the different fluorescence scans for a given transition result from the up to now unaccounted electron space charge effects in the cooler section. However, other than for the ion-induced space charges, the electron space charge effects can be measured by varying the electron cooler current in repeated scans of the same transition. In contrast to the theoretical treatment of the ion space charges, the measurement allows a superior precision for the determination of this effect. In order to account for the electron space charges, all center values μ from the fluorescence scans presented in appendix A.2 are plotted against the electron cooler current used in the respective measurement. By applying a linear regression, the voltage/wavelength can be extrapolated to an electron cooler current $I_{ecool} = 0$, where no electron space charge effects occur. This method was successfully used before in a similar measurement by Schramm *et al.* [83]. Figure 5.26 shows the results for both transition investigations. The data points show the expected linear dependency of the space charge effect on the electron cooler current. The extrapolation yields the following results for both transitions

$$(1s^22s)^2S_{1/2} - {}^2P_{1/2} : 155.0779(1) \,\mathrm{nm}\,,$$
 (5.47)

marked by the blue circle/box in figure 5.26a and

$$(1s^22s)^2 S_{1/2} - {}^2P_{3/2} : 154.8211(2) \,\mathrm{nm}\,,$$
 (5.48)

marked by the green circle/box in figure 5.26b. Realistic estimates of the fit uncertainties were again obtained via the reduced chi-square scaling method. However, this does



Figure 5.26: Electron space charge corrections for both investigated ${}^{12}C^{3+}$ transitions. Applying a linear regression and extrapolating the voltage/wavelength to an electron cooler current of 0 mA yields the real transition wavelengths (marked by the blue and green circles).

not yet include the systematics presented in section 5.2. A detailed discussion of the results with respect to the systematic uncertainties is therefore given in the following section.

5.3.5. Results and discussion of systematic uncertainties

To finalize the results, the systematic uncertainties are estimated for both transitions based on the individual systematics discussed in the calibration section 5.2. Subsequently, the results are compared to previous transition wavelength results obtained through other measurements or by theoretical predictions.

The variance including correlations for all results in this chapter was derived via uncertainty propagation according to

$$u_{\lambda}^{2} = J^{T}(\mathbf{x}) \cdot V_{\mathbf{x}} \cdot J(\mathbf{x}), \qquad (5.49)$$

where u_{λ}^2 is the variance associated with the uncertainty of the transition wavelength u_{λ} , $J(\mathbf{x}) = \partial M / \partial x_i(\mathbf{x})$ is the Jacobian matrix, $V_{\mathbf{x}}$ is the covariance matrix, and \mathbf{x} is the parameter vector. In general, correlations were only assumed for uncertainty propagation of the fit results (leading to the presented confidence intervals), whereas the systematics were assumed to be uncorrelated. Table 5.4 shows the resulting uncertainty budget for the ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ transition in order to identify the limitations of the present laser spectroscopy measurement performed with the XUV detection system. The budget for the ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ transition can be found in appendix A.3. Details about the single uncertainty sources can be looked up in the previous sections. Each uncertainty contribution is assumed to have a Gaussian distribution and the uncertainties are given according to a 1σ standard deviation. The impact of each uncertainty is given in terms of its contribution $\partial \lambda / \partial p \cdot \Delta p$ and relative importance $\left(\frac{\partial \lambda}{\partial p} \cdot \Delta p\right)^2 / \left(\Delta \lambda\right)^2$ which allows an ordering according to the importance of the effect. Values *p* in the table that are labeled with var., refer to values that varied throughout the measurement and thus cannot be represented by a single value in the table. Uncertainties labeled with max. mean that the respective effect was fully omitted for the calculation of the uncertainty contribution leading to a maximum uncertainty estimate by taking the difference between the result including the effect and the result omitting the effect. For some contributions, a sensitivity coefficient $\partial \lambda / \partial p$ could not be calculated since the uncertainty for these contributions (e.g., trigger conversion time) could only be derived through variation of value *p* by the amount of uncertainty Δp .

Table 5.4.: Estimated uncertainty budget for ${}^{2}S_{1/2} - {}^{2}P_{1/2}$ transition for all relevant contributions. The uncertainties are ordered by the size of their contribution. For all parameters p given in the table, a Gaussian distribution (1σ) for the respective uncertainty Δp was considered. The contribution of each parameter is the product of the sensitivity coefficient $\partial \lambda / \partial p$ and Δp . The relative importance of each contribution is derived by $(\partial^{\lambda}/\partial p \cdot \Delta p)^{2} / (\Delta \lambda)^{2}$. It has to be noted, that the set voltage calibration uncertainty is given in terms of DVM voltage, due to the nature of the calibration presented in section 5.2.8. Additionally, all values have been rounded to fit the table format.

parameter	value <i>p</i>	abs. unc. Δp	unit	sensitivity coeff.	contribution (nm)	rel. imp. (%)
work function	2.84	1.42	eV	$5.75\cdot 10^{-4}$	0.000 82	44.15
scale factor	9999.698	0.155		$-3.84\cdot10^{-3}$	-0.00060	23.58
DVM gain	$1.000\ 000\ 6$	0.0000131		-30.84	-0.00050	16.72
set voltage cal.	var.	0.000067	V	5.75	0.000 38	9.76
e-cooler current	var.	2	mA	n/a	0.00026	4.59
DVM offset	0.00000	0.00002	V	-5.75	$-0.000\ 12$	0.88
ion space charge	var.	max.	V	n/a	0.000 05	0.17
laser frequency	291.433 40	0.00007	THz	$-5.32 \cdot 10^{-13}$	-0.00004	0.09
angle laser - ion	0.000	+0.036	0	n/a	0.000 03	0.05
MCP gain	var	max.		n/a	0.000 01	0.01
total uncertainty	155.077 92	0.001 23	nm			100.00

The dominating uncertainty factors are related to the voltage determination of the electron cooler voltage U_{ecool} (and therefore the ion velocity), which in total make up 7 out of 11 contributions. Especially the work function difference between the electron cooler cathode and anode, the HV divider scale factor, and the DVM gain, have the highest impact with relation to the voltage determination, contributing a total of 84.45 % of the uncertainty budget. The next important uncertainty source (set voltage calibration) is related to the missing DVM data for this transition, which can easily be avoided in future beamtimes through a status monitoring of the DVM. Due to the long-standing expertise in laser-ion beam alignment techniques at GSI, do the laser-related uncertainties (laser frequency uncertainty and unknown angle between laser- and ion beam) only play a minor role in the uncertainty budget with a combined contribution of 0.14 %. Further, does the time synchronicity uncertainty of $\sigma_{clock} = 100 \text{ ms}$ (see section 5.2.1) become negligible due to the applied cuts explained in section 5.2.6 and thus drops below the threshold of the top contributions. The only uncertainty directly related to the XUV detector listed in the table is the uncertainty arising from the MCP gain determination with a relative importance of 0.01%. This means that for transitions yielding a similar signal-to-noise ratio the XUV detector is by far not the limiting factor.

Table 5.5.: Overview of available ${}^{12}C^{3+}$ data for the ${}^{2}S_{1/2} - {}^{2}P_{1/2}$ and ${}^{2}S_{1/2} - {}^{2}P_{3/2}$ level splittings. The agreement between the values determined in the scope of this thesis and the values measured by Griesmann *et al.* and Bockasten *et al.* are especially noteworthy, as these values were determined with different experimental methods.

	year	method	$\lambda (^{2}S_{1/2} - ^{2}P_{1/2}) [nm]$	$\lambda (^{2}S_{1/2} - ^{2}P_{3/2}) [nm]$
Experiment:				
this work	2020	laser spec.	155.0779(12) _{svs} (1) _{stat}	155.8211(12) _{svs} (2) _{stat}
Schramm et al. [83]	2006	laser spec.	155.0705(39) _{svs} (3) _{stat}	154.8127(39) _{sys} (2) _{stat}
Griesmann et al. [120]	2000	interferometry	155.0781(2)	154.8204(1)
Bockasten et al. [121]	1963	plasma spec.	155.0774(10)	154.8202(10)
Theory:				
Yerokhin et al. [59]	2017	RCI ¹	155.0880(29)	154.827(36)
Borschevsky [122]	2014	FSCC ²	155.075	154.820
Tupitsyn <i>et al.</i> [83]	2006	n/a	155.0739(26)	154.8173(53)
Johnson et al. [119]	1996	RMBPT ³	155.078	155.819
Kim et al. [123]	1991	RMBPT+MCDHF ⁴	155.060	154.804

The final result for both investigated transitions with their respective uncertainties is presented in table 5.5. The table also features an overview of previous results determined by theoretical and experimental methods. Starting with a comparison of experimental results shows an agreement within the uncertainty intervals between this result and the measurements by Griesmann et al. [120] and Bockasten et al. [121]. At the same time, does the result of Schramm *et al.* [83], determined in a similar setup as used for the measurements in this thesis show a significant deviation from all other measurements. A possible explanation could be the lack of an acceleration voltage measurement with a high-voltage divider for the Schramm measurements, which relied on the calibration of the power supply only. In [83] deviations from theoretical values are explained by systematic uncertainties arising from this limitation. The good agreement between the results of Griesmann et al. and Bockasten et al. compared to this work is especially noteworthy, as the other measurements were undertaken with different measurement techniques, where discharge spectra from a high-current Penning discharge lamp (Griesmann) and plasma spectra from a fusion reactor (Bockasten) were analyzed via interferometric methods. Thus, the result of this thesis can confirm both measurements. Also noteworthy is that the uncertainty compared to the Schramm measurements could be improved almost by a factor of 3 through the advanced voltage determination, as well as the usage of the new XUV detection system.

¹ Relativistic Configuration Interaction

² Fock Space Coupled Cluster

³ Relativistic Many Body Perturbation Theory

⁴ MultiConfiguration Dirac-Hartree-Fock

With the collected experimental results, a comparison to theoretical calculations for both transitions is possible. Results from Johnson *et al.* [119], Tupitsyn *et al.* [83] and Borschevsky [122] can be confirmed with the experimental results obtained in this thesis. All three theoretical predictions were obtained with different calculation methods, still yielding similar results. This indicates that ${}^{12}C^{3+}$ is not an ideal candidate to test these advanced methods of structure calculations, as deviations between the models generally become visible for more complex heavier systems. A comparison to the results of Yerokhin *et al.* is inconclusive as the uncertainty of this prediction is comparatively large, which explains the distinct deviation from the other theoretical predictions. Further details on the individual theoretical methods are given in chapter 2.

5.3.6. Conclusion

To summarize this chapter, the successful commissioning of the XUV detection system was presented. The results for the ${}^{2}S_{1/2} {}^{-2}P_{1/2}$ and ${}^{2}S_{1/2} {}^{-2}P_{3/2}$ level splittings in lithiumlike carbon (${}^{12}C^{3+}$) confirm previous measurements conducted with an independent measurement technique. Furthermore, did a detailed analysis of the individual uncertainty contributions show, that the limiting contributions were not related to the XUV detection system itself. Despite difficulties due to lack of preparation time before the measurements, the result can compete with previous experimental results in terms of uncertainty and even improve the accuracy compared to a similar measurement conducted by Schramm *et al.* [83] published in 2006. For the unknown systematics, conservative estimates have been provided, which can be improved in future measurement campaigns, especially with regard to the determination of the electron cooler voltage U_{ecool} .

Potential improvements regarding systematic uncertainties are provided in table 5.6. Here, the previously determined uncertainties $\Delta p_{\text{orig.}}$ are compared to optimized uncertainties $\Delta p_{\text{pot.}}$ that could be realized as discussed in the following. The largest contribution is the imprecise knowledge of the work function difference between the cathode and anode of the ESR electron cooler. For future measurements that demand high precision, this could be the limiting uncertainty. In such cases, the work function difference between the material could be determined by a dedicated laser spectroscopy measurement similar to the setup presented in this thesis. If other sources of uncertainties can be controlled to be much smaller, e.g., by measurement with an ion species with well-known transitions, the effective work function difference could, in principle, be

parameter	value n	abs.	abs.	unit	
parameter	value p	unc. orig. $\Delta p_{\text{orig.}}$	unc. pot. $\Delta p_{\text{pot.}}$	willt	
work function	2.84	1.42	0.50	eV	
scale factor	9999.698	0.155	0.05		
DVM gain	1.0000006	0.000 013 1	0.0000123		
e-cooler current	var.	2	0.2	mA	
set voltage cal.	var.	0.000 067	0.000 000	V	
DVM offset	0.00000	0.000 02	0.000 02	V	
ion space charge	var.	max.	max.	V	
laser frequency	291.433 40	0.000 07	0.00007	THz	
angle laser - ion	0.000	+0.036	+0.036	0	
MCP gain	var	max.	0		
total uncertainty	155.077 92	0.001 23	0.000 60	nm	

Table 5.6.: Comparison of uncertainty estimates between the result of this beamtime $\Delta p_{\text{orig.}}$ and uncertainties $\Delta p_{\text{pot.}}$ including potential optimizations for future measurements. The systematic uncertainty can be reduced by a factor 2 with the optimizations proposed in the text.

determined to the sub-eV level as proposed in the table. Other improvements proposed in the table were already achieved and are therefore easier to realize in future measurements. For the high voltage measurement, a new HV divider was installed at the ESR with a scale factor precision of approximately 5 ppm. The DVM gain can be improved by using a voltage source that is directly calibrated before a beamtime. Additionally, the set voltage inaccuracy can fully be removed since this calibration was only needed due to a failure of the DVM data recording. Lastly, the uncertainty contributions of the electron cooler current and the MCP gain could be lowered or even removed by dedicated calibration measurements. Taking all improvements into account, the uncertainty for similar measurements with the XUV detection system could potentially be improved by a factor of 2.

However, as mentioned in the previous section, all the estimates given are only valid for a similar signal-to-noise ratio for the investigated ion species. This prerequisite does not hold for the beryllium-like krypton measurements proposed by D. Winters *et al.* [6], for which the XUV detection system was originally designed. Here, the ions are prepared in a metastable state and once this state is de-excited, the ions are unavailable for further measurements (see section 4.1). In contrast, the ${}^{12}C^{3+}$ ions were continuously re-excited by the laser beam since the excitation and the de-excitation work via the same energy levels. This leads to significantly higher signal rates and therefore, a much more favorable signal-to-noise ratio, as shown in the fluorescence scan analysis. However, with an expectantly lower signal-to-noise ratio of several orders of magnitude, the measurement procedure has to be altered in comparison to the one used in this beamtime. Due to the large background when positioning the detector close to the ion beam (see section 5.1.5), the detector had to be re-positioned to the border of the vacuum tube. If a much lower fluorescence signal is to be expected, as is the case for beryllium-like krypton, this trade-off may not be favorable anymore. The detector then has to be positioned in the vicinity of the ion beam again, where the majority of the fluorescence photons can be collected with the detector cathode due to the Lorentz boost (see section 4.1). Therefore, a significant background reduction has to be achieved by e.g., changing the detector setup. After the 2016 beamtime, a new shielding was developed and installed, replacing the focusing electrodes. Developments were conducted by A. Buß, V. Hannen, H.W. Ortjohann, and T. Tews. First tests with the new shielding were conducted in early 2020 at the ESR, and results of these measurements will be published in [96, 124].

Chapter 6

Precision HV divider for the CRYRING@ESR electron cooler

The low energy ion storage ring CRYRING@ESR features an electron cooler similar to the ESR electron cooler used for the laser spectroscopy measurements described in the previous chapter. In such experiments, the ion velocity is a critical quantity with regard to the precision of results obtained from the measurements. As presented in chapter 3, the electron coolers of both storage rings determine the ion velocities through a cooling process¹ by superimposing the ion beam with a mono-energetic electron beam. In consequence of this process, the average ion velocity equals the electron velocity, whereby the electron velocity itself is controlled via an acceleration voltage applied to the electron cooler.

The imprecise knowledge of the electron cooler voltage has been limiting the precision of previous laser spectroscopy measurements conducted at the ESR [3, 8, 9, 10], the CRYRING@ESR storage ring, and also for the laser spectroscopy measurements presented in the previous section. With regard to future precision measurements at FAIR, a precision high-voltage divider for the CRYRING@ESR electron cooler was constructed in the scope of this thesis to remove the electron cooler voltage measurement as the limiting uncertainty. The new high-voltage divider (referred to as G35) is able to measure voltages up to 35 kV, achieving a high voltage measurement precision and stability in the low ppm-regime over the whole range. Since various experiments will be performed at CRYRING@ESR, the divider features five scale factors to allow for optimal precision measurements over the full range of 35 kV.

This chapter presents the design and construction of the HV divider, as well as calibration measurements to determine the absolute values and the stability of the divider

¹ Here, cooling refers to achieving a low momentum spread of the stored ion beam.

scale factors. The design is based on the design of the ultrahigh-precision KATRIN voltage dividers K35 [11] and K65 [12], which have been developed and built in Münster in cooperation with the German National Metrology Institute Physikalisch-Technische Bundesanstalt (PTB) and which are two of the three most precise high-voltage dividers in the world besides PTBs own MT100 divider [14].

The second focus of this chapter lies on the development of a new absolute calibration method, which was developed in parallel to the construction of the high-voltage divider and was also utilized to calibrate the divider before delivery to GSI. This calibration method is based on the idea of C. Weinheimer to determine the voltage-dependent scale factor M by measuring a differential scale factor \widetilde{M} . The method constitutes the most precise, fully traceable calibration method to determine precision high-voltage divider scale factors for absolute input voltages greater than 1 kV. Reproducible measurements up to 35 kV with relative uncertainties below $1 \cdot 10^{-6}$ were demonstrated applying the new calibration method [13, 20]. Another key feature is that this calibration can be performed with commercial equipment. Therefore, it is not restricted to metrology institutes offering the possibility to determine the linearity of high-voltage dividers for a wide range of applications.

The development was mainly performed by O. Rest, D. Winzen, and C. Weinheimer, where the former two were responsible for the hardware setup and the measurement execution. A special focus by the author of this thesis was further set on the analysis of all measured data with regard to the determination of the statistical and systematic uncertainties at the ppm-level. The calibration method and the calibration measurement results with the new divider G35 have been subject to previous publications [13, 20] and part of the Ph.D. thesis by O. Rest [125]. Since the author of the thesis here co-authored the publications, several figures in this chapter are taken from the mentioned publications. Furthermore, are parts of section 6.5 and the complete section 6.6 quoted verbatim from [13]. The respective sections, figures, and captions quoted verbatim are additionally labeled with the according reference.

6.1. Basic principle of precision high voltage dividers

As of today, high voltages above 1 kV cannot be directly measured with a precision in the low ppm regime using commercially available devices. To reach the desired precision, the high voltage is scaled down into the range of typically 10 V, which corresponds to the most precise range of precision digital voltmeters (DVM) with measurement precision in the sub-ppm regime. The voltmeters are calibrated with 10 Vreference sources (e.g., Fluke 732A), which in turn are calibrated with a Josephson voltage standard [126] at metrology laboratories such as PTB. This procedure allows for fully traceable measurements if the scaling (determined by the so-called scale factor *M*) of the used high voltage divider is known. The scale factor, therefore, also has to be determined by dedicated calibration measurements, which is a key factor in achieving the desired precision. The calibration process for HV divider scale factors is discussed in sections 6.5 and 6.6.

Figure 6.1 shows a schematic overview of a simple high-voltage divider. It consists of a chain of ohmic resistors connected in series, divided into a high voltage part (resistors $R_{i,...,n}$) and a low voltage part (resistor R_{LV}). The output voltage U_{LV} measured with a device with ultrahigh input impedance over low voltage resistor R_{LV} is proportional to the input voltage U_{HV} applied to the divider. The ratio between the input voltage U_{HV}



Figure 6.1: Principle circuit of a high voltage divider. The high voltage resistance $R_{i,...,n}$ consists of an optional number of resistors connected in series. The scaled-down voltage is measured at the low voltage resistor R_{LV} . The ratio between input voltage U_{HV} and output voltage U_{LV} is given by equation (6.1) and is referred to as scale factor M.

and output voltage $U_{\rm LV}$ is then given by the scale factor M

$$M := \frac{U_{\rm HV}}{U_{\rm LV}} = \frac{\sum_{i=1}^{n} R_i + R_{\rm LV}}{R_{\rm LV}} = \frac{\sum_{i=1}^{n} R_i}{R_{\rm LV}} + 1.$$
(6.1)

As shown in the above equation, the scale factor is determined by the resistor values chosen for the high and low voltage parts of the divider. The total resistance realized in a high-voltage divider is a compromise between different properties such as power dissipation caused by Joule heating, thermal voltage noise, and current flow with regard to leakage currents. Further, depending on the choice of the total resistance, arbitrary and numerous scale factors can be realized by choosing the value of R_{LV} accordingly and with regard to demands of the desired applications. Even multiple scale factors can be implemented if multiple resistors are used for R_{LV} . Such scale factors always have to be determined by considering the maximum measurable voltage in the most precise range of the attached DVM. E.g., for the G35, a Keysight 3458A digital voltmeter (8.5 digits) is used. In its most precise range (10 V-range), voltages up to 12 V can be measured. In comparison, the KATRIN experiment features Fluke 8508A reference multimeters, which can measure up to 20 V in their most precise range. Therefore, the scale factors have to be chosen so that the high voltages to be measured are scaled-down in such a way that the full range of the DVM is used in order to maximize precision. The realized design in consideration of these aspects is discussed in sections 6.3 and 6.4.

A crucial part of a precision divider are the resistors, as they determine the measurement accuracy and stability of the scale factor. Ppm-precise measurements are only possible if the scale factor is stable on the ppm-level during the measurement interval and on a time scale between the latest calibration and the measurement. Therefore, the next section is dedicated to the resistor selection process, where precision resistors were characterized in terms of stability and thermal behavior.

6.2. Precision resistor selection

For the G35 divider, a selection of over 100 resistors of type Vishay VHA-518-11 [127], which were leftover from previous KATRIN HV divider constructions, were available for testing. This resistor type was again chosen for the G35 as it proved to deliver the necessary precision and stability for both KATRIN dividers over measurement periods of several years [125, 128]. In order to meet the high demands in terms of precision and stability of the G35, each resistor was tested and characterized with respect to warm-up behavior and stability (see figure 6.3). Based on these measurements, 65 resistors for the high voltage part of the G35 and 16 resistors for the low voltage part were chosen. Further investigations included measurements of the temperature coefficient of resistance (TCR) for a selection of the available resistors to find the optimal ambient temperature in which the resistors operate (see section 6.2.3). The applied characterization methods described in this section were already used in the selection process for both KATRIN dividers [129, 130, 131].

It has to be noted that the selection process for the G35 divider, as well as the results, were already published in the bachelor thesis of I. Denesjuk [132] and in [20]. The characterization measurements were performed in cooperation with, and by co-supervising, the author of the previously mentioned bachelor thesis, and the results are summarized in the following subsections.

6.2.1. Vishay precision resistors

The most important part of the G35 divider consists of the so-called *primary divider chain* (see section 6.3) for the precision measurements, which features the Vishay precision resistors of type VHA-518-11 [127]. Each resistor consists of eleven 167 k Ω resistor foils glued to a ceramic base, placed in an oil-filled hermetically sealed container (see figure 6.2, left), guaranteeing the long-term stability of the resistors. Further details of the resistor technology are discussed in [129, 130]. A selection of the most important specifications is given in table 6.1. Here, the most important specifications are related to the resistor stability depending on the temperature (TCR), the voltage load (voltage coefficient), and the age of the resistor (stability during storage).

Table 6.1.: Vishay VHA518-11 resistor specifications [127] for the high voltage part of the primary precision resistor chain. Resistors used for the low voltage part of the precision chain are of the same type but with different resistor values (see section 6.3) and partly lower TCR.

parameter	value	unit
resistance	1.84	MΩ
tolerance of resistance	0.001	%
max. load	600	V
TCR between -55 125 °C	$< \pm 2.0$	ppm/K
voltage coefficient	< 0.1	ppm/V
inductance	0.08	μH
capacitance	0.5	pF
stability during storage	± 5	ppm/year

The specification *stability during storage* refers to the aging process of the resistors and is rated as ± 5 ppm/year. However, as the aging process progresses, the stability is expected to improve over time as the resistors show an asymptotic stabilization of the aging effect [133]. This is further emphasized by the calibration histories of the K35 and K65 (see [125]) as well as the already taken calibrations with the G35 (see section 6.5 and appendix A.7) which indicate a stability well below ± 1 ppm/month as no drifts are observed.

The voltage coefficient is specified to < 0.1 ppm/V. It is caused by Joule heating according to

$$P = I^2 \cdot R = \frac{U^2}{R}, \qquad (6.2)$$

where *P* is the power of heating, *I* the electrical current flowing through the resistor, *R* the resistor value, and *U* the voltage drop over the resistor. To minimize the power loss in the resistors and thus the self-heating, the total resistance has to be as high as possible. However, the positive effect of a high resistance is counteracted by the thermal noise rising when the low voltage resistance R_{LV} is increased, as shown by

$$R_{\text{noise}} = \sqrt{4Rk_{\text{B}}T\Delta f}, \qquad (6.3)$$

where $k_{\rm B}$ is the Boltzmann constant, *T* the resistor's absolute temperature, and Δf the bandwidth. A higher resistance also leads to a lower current flow through the divider, raising the potential risk of parasite leakage currents affecting the divider ratio [134]. Therefore, a suitable balance between a high total resistance and a low resistance $R_{\rm LV}$ has to be found (see section 6.3).

Since the Joule heating leads to a temperature change inside the resistor, the voltage coefficient is directly related to the temperature coefficient of resistance. The temperature coefficient of resistance TCR is specified to be $< \pm 2.0$ ppm/K in a temperature range of $-55 \dots 125$ °C. The fact that resistors with positive and negative TCR were available led to the possibility to compensate resistors with opposite TCRs for the KATRIN dividers. With this compensation method, a negligible linearity below $1 \cdot 10^{-6}$ over an input voltage range of 35 kV could be demonstrated for the K65 divider [13]. For the G35, this matching was not possible since only resistors with negative TCR were available (see section 6.2.2). However, the TCR of the resistors is not constant over the temperature range given in table 6.1. As presented in the right plot of figure 6.2, it shows a quadratic behavior indicating an optimum resistor operating temperature at the curve's vertex. In consequence, dedicated measurements were performed to find the optimum operating temperature for the used resistors, thus minimizing the voltage dependency of the G35 divider (see section 6.2.3).

6.2.2. Resistor selection

Every Vishay resistor used in the precision divider chain of the G35 was characterized in terms of the so-called warm-up drift. To determine this drift, the change in resistance over time was measured by applying a load to the resistor under test while keeping a constant ambient temperature. Even if the ambient temperature of a resistor is regulated stably, the inside of the resistor heats up under load. Through this self-heating, the resistance will change according to the resistor's temperature coefficient, measured



Figure 6.2: Insides of Vishay precision resistor (**left**) and temperature coefficient of resistance (TCR, see **right**) [127]. The TCR shows a quadratic behavior, which indicates an optimum resistor operating temperature at the vertex.

with the following setup.

The test setup is shown in figure 6.3. The measuring circuit consisted of a 51:1 voltage divider to measure the 1.84 M Ω resistances $R_{\rm UT}$ to be tested at the maximum permissible voltage of 600 V with a precision voltmeter in its most precise 10 V range. To realize the 51:1 divider ratio, a Vishay VHA-518 resistor as reference $R_{\rm ref}$ with 36.8 k Ω was used for the voltage tap. A Fluke 5720A calibrator that provides voltages up to 1100 V with a stability of 0.5 ppm/24h was utilized as the voltage source. The calibrator voltage was measured with a Fluke 8508A 8.5 digit multimeter. To measure the voltage drop over reference resistor $R_{\rm ref}$, an Agilent (nowadays Keysight) 3458A digital 8.5 digit multimeter was used. Since the voltage drop over the reference resistor did not exceed 12 V, the Agilent DVM could be used in its most precise measurement range. The measuring circuit itself was located inside a housing that was temperature stabilized to $T_{\rm box} = 25.0(1)$ °C (see figure 6.3, right).

When the calibrator voltage $U_{in} = 600$ V is switched on at $t = t_0$, a voltage of 588.2 V drops over the test resistor R_{UT} and $U_{out} = 11.8$ V drops over the reference resistor R_{ref} . According to the voltage drops, a power of 188 mW is consumed in the test resistor and only 0.56 mW in the reference resistor. In consequence, the test resistor warms up, and its resistance value changes, while the resistance value of the reference resistor shows no measurable change due to the low power dissipation. Any change in the test resistance directly leads to a change in the 51:1 divider ratio. This changes the voltage drop over the reference resistor R_{ref} , which is monitored with ppm precision. The change in resistance ΔR_{UT} relative to its initial value $R_{UT,0}$ is given by

$$\frac{\Delta R_{\rm UT}}{R_{\rm UT,\,0}} \approx -\frac{2 \cdot \Delta U_{\rm out}}{U_{\rm out,\,0}} \tag{6.4}$$



Figure 6.3: Setup of precision resistor test stand for warm-up drift measurements. **Left:** Measuring circuit for resistor selection measurements. The arrangement features a 51:1 voltage divider circuit. An input voltage of $U_{in} = 600$ V is applied, effectively loading the resistor under test R_{UT} with a voltage of approximately 588.2 V which corresponds to a deposited power of about 188 mW. The change in total resistance can be measured by monitoring the voltage drop over the reference resistor R_{ref} , since the low power dissipation of approximately 0.56 mW only leads to negligible changes in the reference resistor value. **Right:** Picture of the resistor test stand. Shown are the resistors to be tested R_{UT} as well as the reference resistor R_{ref} . They are placed in a temperature-stabilized box at $T_{box} = 25.0(1)$ °C.

where $U_{\text{out, 0}}$, in this case, refers to the first recorded value after the voltage source is initialized at $t = t_0$. Equation (6.4) can thereby only be applied, if the input voltage U_{in} and the reference resistance R_{ref} stay constant throughout the complete measurement.

A warm-up drift measurement for a single resistor was measured for 30 min. A result of an exemplary measurement is presented in figure 6.4, left. After approximately 15 min, the resistor stabilizes, and the total deviation to the initial resistance is determined by a constant fit over the last 15 min of the measurement interval. In the graph on the right of figure 6.4, the resulting warm-up deviations for the primary high-voltage chain resistors are shown as a histogram. The resistors show similar warm-up drifts between -8.5 ppm and -5.5 ppm, indicating only negative TCRs. Thus, a compensation of resistors with positive and negative warm-up drift, as used for both KATRIN dividers, was impossible. However, since all resistors were characterized at an ambient temperature of 25.0 °C and showed similar warm-up characteristics, further investigations were conducted to minimize the warm-up drifts by finding a shared optimum operating temperature for the resistors (see section 6.2.3).

It has to be noted that the resistors used for the low voltage part were also tested



Figure 6.4: Results of resistor Vishay characterization for high voltage part of the primary divider chain. **Left:** Warm-up characteristic of exemplary Vishay resistor as response to a load of $U_{\text{load}} \approx 588$ V at an ambient temperature of $T_{\text{amb.}} \approx 25$ °C. The y-axis displays the relative change in resistance with regard to the first recorded value in ppm. After 15 min, the resistor settles at a resistance approximately 6.5 ppm lower than the initial value. **Right:** Overview of warm-up deviations after a settling time of 15 min for all 65 selected primary chain high voltage resistors R_{1-65} . Resistors showing similar characteristics were chosen with regard to thermal design considerations presented in section 6.2.3.

with the same measurement technique. This was mainly done to confirm the sub-ppm stability of the resistors under load. Since the maximum load a resistor in the precision tap experiences does not exceed 133 V (maximum value at $U_{\rm HV} = 35$ kV), warm-up drifts are below 1 ppm and are therefore not further discussed.

6.2.3. Thermal characteristics

From figure 6.2, it can be assumed that the resistors can be operated at an optimum ambient temperature so that the warm-up drifts presented in the previous section can be minimized. In consequence, dedicated TCR measurements were conducted to find an optimum operating temperature for the G35 divider.

The test circuit to determine the TCR of an individual resistor was similar to the setup presented in the previous subsection. Only this time, a much lower load of 10 V was applied to the resistor under test R_{UT} so that the Joule heating was negligible (see schematic in figure 6.5). Instead of measuring the change in resistance as a consequence of the applied load, it was measured in dependence of the resistor temperature. For this purpose, the resistor was put into a temperature-controlled aluminum block (see photo in figure 6.5). The block featured a PT1000 temperature sensor to monitor the block temperature and a Peltier element to change the temperature of the block



Figure 6.5: Electrical circuit diagram and temperature regulation block for TCR determination. **Left:** Measuring circuit for TCR measurements. The arrangement features a 2:1 voltage divider circuit. An input voltage of $U_{in} = 20$ V is applied, effectively loading the resistor under test R_{UT} with a voltage of 10 V, leading to a negligible power dissipation in the resistor. By changing the resistor temperature in a range of $T_{UT} = 5 - 30$ °C, the TCR can be determined by measuring the change in resistance in dependence of the temperature. As in figure 6.3, the change in total resistance is again measured by monitoring the voltage drop over the reference resistor R_{ref} . **Right:** Temperature regulation block featuring a Peltier element to cool the resistor under test R_{UT} . For that purpose, the resistor is placed in the cylindrical opening of the aluminum block.

and, therefore, the resistor temperature over time. The block temperature itself was controlled via a NI LabVIEW-based proportional-integral-derivative (PID) controller. Since the Peltier element was only able to cool the aluminum block, the whole setup was again situated in the measurement box presented in the previous section. The ambient temperature of the box was in this case set to $T_{\text{box}} = 30 \text{ °C}$ so that a maximum resistor temperature $T_{\text{UT}} = 30 \text{ °C}$ could be reached.

Like the previous setup, the change in resistance was measured via the voltage drop over a reference resistor R_{ref} . The reference was a resistor of the same type and with the same resistor value $R_{\text{ref}} = 1.84 \text{ M}\Omega$ as the unit under test so that the voltage drop over the reference was $U_{\text{out}} = 10 \text{ V}$. The ratio $\Delta R_{\text{UT}}/R_{\text{UT}}$ is then again given by equation (6.4).

The result of an exemplary measurement is shown in figure 6.6, left. For each resistor, the temperature was varied in an interval between $T_{\text{UT}} = [5 \,^{\circ}\text{C}, 30 \,^{\circ}\text{C}]$. The change in resistance shows the typical parabolic behavior given in figure 6.2. To get a full description of this behavior, a parabolic fit was applied to the data taken for each resistor (red line). The fit result yields the temperature T_0 , where the resistor's temperature dependency is minimal at the vertex of the parabolic curve. At this vertex position, the



Figure 6.6: Evaluation of Vishay resistor thermal characteristics. **Left:** Exemplary TCR measurement for a single resistor. The left y-axis shows the relative change in resistance in dependence of the resistor temperature. A parabolic fit (red line) was applied to the data (blue points), indicating an optimum resistor operating temperature where the TCR is zero. The temperature coefficient is the red curve's derivative and is shown as the green line on the right y-axis. **Right:** Correlation between vertex temperature from TCR measurements and maximum warm-up deviation from warm-up drift measurements of section 6.2.2 (blue points). The linear relation between both parameters indicates that the optimum operating temperature of a resistor can be derived from the warm-up drift results.

TCR becomes zero indicating the optimum operating temperature. The full TCR curve can be derived from the parabolic curve through differentiation, which is represented by the green line. This measurement, therefore, allows deducing the optimum operating temperature for an individual resistor. However, such TCR measurements are very time-consuming, which is why only eight resistors were investigated with this method.

To still determine the optimum temperature for the resistors of the G35, the correlation between the TCR measurements and the warm-up drift measurements from the previous section were investigated (see figure 6.6, right). Here, each of the eight investigated resistors' vertex temperature is plotted against their respective warm-up deviation (blue data points). A linear relationship is observed between both quantities, further emphasized by the linear fit applied to the data (red line). Using the parameters of the linear fit, the optimum operating temperature for each resistor can be estimated. Since resistors with similar thermal characteristics were chosen for the G35 (see figure 6.4), all optimum temperatures are distributed in an interval between 5 °C and 13 °C. In conclusion, to minimize the G35 voltage dependency, it has to be operated at much lower temperatures than the 25 °C used for the KATRIN divider K35 and K65. A divider temperature of 9 °C (center of 5 - 13 °C) should yield a minimal voltage dependency. However, the minimum divider temperature specified for the G35 divider is at 15 °C. Investigations by I. Denesjuk showed that below 15 °C significantly raises the risk condensation, thus risking the dielectric strength of the G35 [132].

6.3. Electrical setup

Like both KATRIN high-voltage dividers, the G35 divider features a *primary divider chain* and a *secondary divider chain*, sometimes also referred to as *control divider chain* (see figure 6.7). The voltage divider is further separated into five different planes (indicated by the red dashed lines). The high voltage parts of both divider chains are distributed over the top four planes, and the tap planes of both chains are located in the bottom-most plane. This separation was made with regard to the electrical field configuration inside the divider and an optimized shielding of the primary chain from the stainless steel vessel's ground potential. A detailed description of the mechanical setup and the electrical field configuration is given in section 6.4. The individual divider chains and their function is presented in the following.

Primary precision chain The *primary chain* (right part of figure 6.7) consists of 81 Vishay resistors separated into a high-voltage part with 65 precision resistors R_{1-65} of type Vishay VHA-518-11 connected in series and the *low voltage taps* with 16 resistors $R_{LV,i}$ featuring a combination of parallel and serial connections to provide multiple scale factors. The maximum input voltage is, in general, limited by the maximum load that the resistors of the precision chain are specified to. In the Vishay resistors' case, a maximum load of 600 V can be applied according to the manufacturer allowing for a maximum input voltage of 39 kV. However, to keep a security margin and thus minimize the risk of damaging the resistors, the G35 divider is specified to a maximum voltage of 35 kV.

The scale factors were determined under three main aspects. Firstly, the scale factor always relies on the available resistor values, which in this case were limited to $0.14 M\Omega$, $0.88 M\Omega$ and $1.84 M\Omega$. Secondly, as stated in section 6.1, the scale factors have to be optimized to make use of the full Keysight 3458A 10 V-range (limited to



Figure 6.7: Electrical setup of the G35 high voltage divider. The divider is sectioned into two main parts, named *primary chain* and *secondary chain*. Vishay precision resistors R_{1-65} are connected in series in the high voltage part of the primary chain (right). The divider is further sectioned into five mechanically separated planes indicated by the red dashed lines, which are connected to the secondary chain. The low voltage part of the primary precision chain consists of different combinations of Vishay precision resistors connected in series and parallel to realize the different scale factors. A total of five different voltage taps is realized in this section. The secondary chain (left) consists of less precise resistors and additional capacitors to protect the primary chain from transients occurring, e.g., in case of power failure. Precision measurements can only be conducted with the voltage taps of the primary resistor chain.

12 V) with regard to the high voltage to be measured. Since the voltages to be measured at CRYRING@ESR vary for every experiment, five scale factors were realized to be able to probe the full range of 35 kV. Here, a down-scaling of voltages up to 20 kV was favored because 20 kV corresponds to the maximum voltage that can be applied to the CRYRING@ESR electron cooler (see section 3.3.2). The third aspect focuses on

DVM safety. Each scale factor features at least two resistors connected in parallel to protect the attached DVM in case of resistor failure. If only one resistor per scale factor was used, the whole voltage would drop completely over the DVM in case of resistor failure. The exact configuration of the *low voltage taps* based on these considerations is shown in figure 6.7 on the right. Table 6.2 lists the corresponding input voltage specification for each G35 scale factor.

Table 6.2.: Voltage ranges for the G35 scale factors when using the most precise voltage range of the Keysight 3458A DVM. In its most precise range, the Keysight can measure voltages up to 12 V.

scale factor	voltage range (V)
M ₁₀₀	0 - 1205
<i>M</i> ₁₆₃	0 - 1952
<i>M</i> ₂₆₉	0 - 3227
M_{775}	0 - 9305
M ₃₄₅₂	0 - 35000

Secondary divider chain The secondary chain (also called control divider chain) consists of high voltage resistors and parallel high voltage capacitors. It is, therefore, an ohmic-capacitive divider. The resistors ensure a voltage drop between the planes, which approximately corresponds to the primary chain's voltage drop. This allows shaping the electrical field between the planes according to the high voltage applied to the corresponding precision high voltage resistor of the primary chain. This leads to a reduction of leakage currents and serves as protection against corona discharges (for details see section 6.4). Therefore, all four planes of the secondary high-voltage chain feature two $50\,M\Omega$ resistors of type Caddock $MX450\text{-}50M^2$ that are connected in parallel ($R_{\rm CD} = 25 \,\mathrm{M}\Omega$). The low voltage part of the secondary chain features two resistors of type Caddock MS260-10K and two resistors of type Caddock MS260-50K resulting in a total resistance of $R_{CD,LV} = 25 \text{ k}\Omega$. This leads to a down-scaling of the applied voltage with a ratio of 4001:1, which can also be measured via a dedicated secondary tap. However, due to the lower precision and stability of the secondary chain resistors, this tap is only used to monitor the applied input voltage via the divider's slow control system (see section 6.4.1). As for the primary divider chain, it was taken care that a minimum of two resistors are integrated into each plane to protect the attached measurement devices in case of resistor failure.

² Voltage proof up to 16 kV.



Figure 6.8: LTSpice simulation for protection circuit in case of fast transients. Shown is the transient simulation of the voltage drop across the primary chain's first resistor for a total transient voltage of 35 kV with a slope of 1 ns. The red curve shows the load over time if no protection circuit is in place. Blue shows the load for the actual G35 setup using the secondary ohmic-capacitive chain as protection circuit. While the red curve exceeds the maximum allowed voltage load for a single resistor (dashed black line) by a factor of 3, the blue line stays below the maximum allowed load.

The capacitive part of the secondary chain acts as a protection circuit of the primary chain to sudden transients, e.g., if an applied voltage of 35 kV instantly drops to zero due to power failure. For that purpose, each plane features additional capacitors in parallel to the corresponding Caddock resistors. Each high-voltage plane has three high-voltage Fischer und Tausche film capacitors³ with a combined capacity of $C_{\rm CD} =$ 7 nF connected in parallel (3 x 2.33 nF⁴). The tap plane has only one $C_{\rm CD, LV} = 2.33$ nF of the same type connected in parallel to the low voltage resistors. A similar setup has already been realized in both KATRIN dividers K35 and K65 (the capacitors used for the G35 are left-over parts from the K65 construction).

The presented secondary chain design was determined by dedicated electronic circuit simulations using LTSpice. The main goal of these simulations was to find a configuration that prevents voltage drops exceeding the specification voltage of 600 V for a single-precision chain resistor. In appendix A.4, the circuit diagram used for the simulation is presented. Besides the electronic parts described in this section, it features additional stray capacitances assumed to impact the divider response to a transient sig-

³ Voltage proof up to 20 kV.

 $^{^4}$ 2.33 nF corresponds to the measured value, the nominal value is 2.5 nF

nal. The positions and values of the stray capacities in the equivalent circuit diagram for the LTspice simulations are motivated by previous simulations and measurements performed by F. Hochschulz and S. Bauer for both KATRIN dividers [128, 130, 131].

Figure 6.8 shows the results for a simulation of the divider response to a 35 kV transient with a steepness of 1 ns. Here, the voltage drop over a single $1.84 \text{ M}\Omega$ Vishay resistor of the primary precision chain was investigated for two scenarios. The first scenario refers to a divider setup without the secondary divider capacitors (red line). In this case, the maximum allowed voltage of 600 V (dashed black line) is exceeded by more than a factor of 3, risking damage to the resistor. Adding the secondary chain resistors corresponding to figure 6.7 avoids resistor overload at all times (blue line), thus guaranteeing resistor integrity even in case of power failure.

In conclusion, an overview of the electrical characteristics for the final divider configuration is given in table 6.3.

Table 6.3.: Electrical characteristics of G35 divider. The combined resistance of the precision divider chain and the control divider is stated as total resistance. The current given in the table refers to the current flow through the precision chain only.

parameter	value	unit
max. voltage	35	kV
total resistance	55	MΩ
precision divider chain:		
resistance of precision chain	121	MΩ
nr. of resistors in high voltage section	65	
resistance of single resistor in high voltage section	1.84	MΩ
load per resistor at 35 kV	533	V
current at 35 kV	290	μΑ
power dissipation per resistor	154	mW
secondary divider chain:		
resistance of secondary chain	100	MΩ
resistance of single resistor in high voltage section	25	MΩ
capacitance of single capacitor	2.33	nF

6.4. Mechanical setup

The mechanical setup of the G35 precision HV divider is presented in figure 6.9, separated into the exterior (left) and interior part (right) of the G35.

All components used for the electrical setup introduced in the previous G35 exterior section are housed in a stainless steel vessel with a diameter of 600 mm and a height of 900 mm, which is pressed onto an o-ring on an aluminum base plate using a coupling ring. The vessel serves two main purposes. On the one hand, it serves as a Faraday cage and thus protects against influences of external disturbances like RF signals. On the other hand, it forms a sealed volume, which is critical for realizing the necessary temperature stabilization for the precision chain resistors and can optionally be filled with dry nitrogen gas with an over-pressure of approximately 100 mbar. The steel vessel is also thermally insulated with 32 mm thick ArmaFlex[®] elastomeric foam to decouple the interior from fluctuations in the ambient temperature and reduce thermal energy loss. On top of the divider is the high-voltage input realized with a Heinzinger HVB65 socket⁵ mounted into a flange that additionally features a gas outlet and a low-pressure gauge⁶. The complete divider setup is integrated into a transport frame consisting of Kanya aluminum profiles featuring four rolls to ensure mobility. This transport frame further accommodates the Keysight 3458A precision DVM and the divider's whole slow control system. The complete exterior setup of the divider is shown in the left picture of figure 6.9. An overview of the outer G35 dimensions is given in appendix A.5.

G35 interior In the right picture of figure 6.9, the divider interior is shown. It is separated into five planes that contain both divider chains. Each plane is enclosed by copper electrodes with a diameter of 434 mm that are supported by electrically insulating polyoxymethylene (POM) rods effectively forming a driven guard. The copper electrodes thus form a homogeneous electrical field in the vicinity of the precision chain resistors and shield the resistors against the vessel wall's ground potential. The high voltage transmission from the Heinzinger HV socket to the top copper electrode is realized via a gold-plated stainless steel hemisphere, which is pressed into a corresponding counterpart located on the top copper electrode. A spiral spring attached to the high-voltage socket holds the hemisphere, thus establishing the electrical con-

⁵ Voltage proof up to 65 kV

⁶ Swagelok PGI-63L-FG250-LAOX



Figure 6.9: Mechanical setup of G35 HV divider. **Left:** Setup of the G35 HV divider at CRYRING@ESR. The bottom part features the divider periphery with all components for the divider control and the precision DVM. The upper part features the divider vessel, which houses the precision divider chain. A thermal shielding encloses the vessel to reduce the loss of thermal energy from the temperature-stabilized interior. **Right:** Inside view of the G35 divider. Each of the five sections (four for the high voltage precision resistors and one for the tap resistors) is separated by copper electrodes that form a driven guard. The 65 precision high voltage resistors are arranged in a helix structure to minimize induction currents. Also shown is secondary the divider chain featuring resistors as well as capacitors. Further shown are parts of the heat exchange system (two of the four temperature sensors, the sensor flange, and the water pipes leading to a radiator below the bottom copper electrode).

tact. The bottom electrode is connected to ground so that the electrical potential of the electrodes in-between is determined by the secondary divider chain, which electrically connects to the copper electrodes (see figure 6.9, right). Simulations concerning the design and shape of the electrodes are presented in section 6.4.2.

The resistors of the primary divider chain are connected via custom-made rounded nickel-plated brass connectors that feature through holes for the resistor wires. The

resistors are fixed to the connectors through headless screws that are screwed into drillholes on the side of the connectors. Each connector through-hole holds an additional silver tube to avoid damage to the resistor wires when they are fixed to the connectors. Such silver tubes were first successfully used for the resistor connections of the K65 KATRIN divider after a repair by O. Rest [125]. Therefore, they were also used for the connections of the G35 divider. The connectors themselves are mounted to the inner POM rods. Starting from the top electrode, the high-voltage part of the primary divider chain is arranged in a helix around the central axis of the construction. The direction of the helix changes at each level to minimize the inductance of the resistor chain. Located in the lowest plane (called *tap plane*), are the five taps for the different scale factors of the G35. Each tap is connected to a DN 63 ISO-F flange, which solely features the five low voltage feed-throughs⁷ (see figure 6.10, left).



Figure 6.10: Tap flange (**left**) and Heinzinger high voltage input (**right**). The tap flange features the connections for all five scale factors of the G35 and the G35 ground connection. Each tap can be used by connecting the Keysight 3458A to the desired socket of the tap flange.

Not shown is the inner part of the heat exchange system (HES). Located below the bottom-most electrode on the central axis is a radiator with an attached fan to regulate the temperature inside the divider (for details see section 6.4.1). Heat exchange mainly occurs at the radiator, which is connected to a liquid circulation system located outside the steel vessel via the pipes shown in figure 6.9. Both pipes are welded into a DN 63 ISO-F flange (HES flange), which also features an extra pipe for the gas inlet. The fan distributes the thermalized gas from the radiator via a tunnel tube located on the central axis between the copper electrodes. The tunnel tube features a hole pattern to optimize the airflow inside the divider, which was determined through computational fluid dynamic simulations (see section 6.4.3). Additionally, four PT100 temperature

⁷ realized by Lemo low voltage connectors

sensors situated between the second and third divider planes respectively (counting from the bottom), are used to monitor the divider temperature. A humidity sensor inside the divider is used to detect possible leaks in the water circuit since the relative humidity will significantly increase if leakage occurs within the vessel. A dedicated sensor flange (DN 100 ISO-F flange), used for the supply and signal lines of the fan and described sensors, is located in the center of the aluminum plate. Also, the secondary voltage tap's connection to the divider slow control is realized via this flange. A detailed description of the G35 slow control and the heat exchange system is given in the next section.

6.4.1. G35 slow control and heat exchange system

In this section, the individual parts of the G35 slow control and heat exchange system (HES) are described. Figure 6.11 gives a full schematic overview of all components referred to in the following paragraphs.

Temperature regulation To maximize the G35 precision, a constant temperature of 15 °C (alternatively 20 °C) inside the steel vessel is critical. A dedicated temperature-regulation system was, therefore, implemented in the G35 divider. The general concept is based on the regulation system designs of the KATRIN K35 and K65 dividers. The G35 temperature regulation control is realized via a LabVIEW 2014 based PID-control running on a NI cRIO 9063 real-time controller. To heat/cool the divider, the PID-control sets the input voltage of a Peltier element based liquid to air thermoelectric assembly⁸. Switching between heating and cooling is realized by a relay that switches the polarity of the supply voltage from the EA power supply⁹. The relay itself is control voltage from an NI 9263 analog output module to the relay. The thermoelectric assembly is integrated into a fluid circuit and heats/cools the liquid (a mix of 80 % water and 20 % Glysantin[®]) according to the control software's set temperature. The liquid is pumped into the HV divider via the HES flange by a rotary pump¹⁰ located in the periphery of the divider. Heat exchange in the divider occurs at a radiator/fan installation

⁸ Laird LA-160-24-02-00-00

⁹ EA-PS 2042-20B

¹⁰ EHEIM 1250



Figure 6.11: Slow control/HES (Heat Exchange **S**ystem) setup of the G35 divider. A detailed description of the individual parts is given in the text. For a better overview, all 230 V connections are omitted in this schematic. The primary and secondary chain resistors are also omitted for better visibility.

below the tunnel tube. The fan¹¹ blows the heated/cooled air from the radiator¹² into the tunnel tube. From the tunnel tube, the air gets distributed to each resistor of the

¹¹ Papst 8414 NHU

¹² Alphacool NexXxoS XT45

precision chain in order to homogeneously heat/cool all precision resistors. Additionally, a water reservoir¹³ is installed in the fluid circuit to account for the loss of fluid over time. The achieved regulation precision is approximately ± 0.1 K. To measure the temperature inside the steel vessel, the divider features four PT100 sensors in divider planes 2 and 3, as depicted in figure 6.11. All temperature sensors are supplied and read out via an NI 9216 RTD module. The average of the four temperature readings serves as the temperature the PID-control regulates. An additional temperature sensor is located outside the G35 to monitor the ambient temperature.

Humidity monitoring To maintain the divider's dielectric strength, the humidity inside the steel vessel is being monitored by a dedicated humidity sensor¹⁴. The sensor is located inside the vessel directly on top of the sensor flange. The supply voltage for this sensor is provided by the NI 9263 module attached to the cRIO system. Readout is conducted via an NI 9221 voltage input module.

High voltage measurements For the precision voltage measurements, the voltage signal is transferred from one of five voltage taps to the measurement device (Keysight 3458A) via the tap flange. The Keysight DVM is controlled and read out via its GPIB interface. The GPIB signal is converted to ethernet via a PROLOGIX GPIB-Ethernet controller connected to the Keysight DVM. To prevent electromagnetic interference in the measuring chain, the communication with the DVM is galvanically decoupled from all other devices via two Ethernet-optical converters. The secondary scale factor is fixed to approximately 4001:1, and the signal is transferred through the central sensor flange. The readout is conducted by the NI 9221 module and is fed into the cRIO data stream.

Data transfer The data transfer to the CRYRING@ESR slow control is conducted via an Ethernet switch located in the periphery of the divider. Two data streams are fed into the switch. The first data stream consists of the measured voltage from the Keysight 3458A DVM, and the second data stream consists of the cRIO data, including temperature regulation data, the measured humidity, and secondary tap voltage measurement data.

¹³ Aquatube G1/4 Delrin

¹⁴ Honeywell HIH-4000

Control software The G35 control software is a LabVIEW 2014 program that runs autonomically on the cRIO real-time controller. However, to verify the HV divider's functionality and status, it is also possible to monitor the software via a browser. Several control parameters, as well as sensor readouts, can be displayed via the browser interface. If required, the set temperature of the divider can be changed by assuming control over the software. Furthermore, all sensor data taken by the cRIO controller is recorded and stored on the cRIO system.

Power supply The 230 V supply voltage connections for nine devices in the periphery are realized by two multi sockets located at two sides of the KANYA frame. A constant 24 V supply voltage for the fan inside the divider tank, the liquid-to-air thermoelectric assembly fan, and the cRIO system is provided by an EA-PS 3032-05B power supply and distributed via a terminal. The dynamic 0 - 24 V, regulated via the PID-control for the Peltier element of the thermoelectric assembly, is provided by an EA-PS 2042-20B power supply.

6.4.2. Electrical field configuration

The shape and positions of the copper electrodes installed in the G35 were determined by electrical field simulations using Comsol Multiphysics[®]. Optimizations of the copper electrodes were investigated to achieve an electrical potential in the vicinity of the resistors that corresponds to the applied potential of each resistor. Two parameters were used to optimize the copper electrode geometry. The first parameter was the homogeneity of the electric field along the resistor positions. The second parameter was the maximum electrical field strength inside the setup, which has to be minimized to avoid corona- and spark discharges that can lead to measurement errors.

During previous investigations by F. Hochschulz [131], three different edge geometries were simulated to optimize both parameters. The electrode shape for the G35 electrodes was chosen based on these investigations. It features an edge curvature of 12 mm diameter while having a total diameter of 434 mm and a thickness of 2 mm. To further decrease the likelihood of discharges, all copper electrodes were electropolished. It has to be noted, that the chosen edge geometry was not the favored geometry for the previous K35 and K65 dividers since a more complex geometry with a double curvature provided superior field properties [129, 131]. However, differences between double and single curvature were found to be negligible with regard to raising the



Figure 6.12: Homogeneity of electric potential at approximate resistor positions. **Left:** Equipotential lines inside the simplified divider geometry (half-section view). A potential of 35 kV is applied to the top electrode. The voltage of each additional electrode corresponds to the scaling of the secondary resistor chain (approximately -8.75 kV per high voltage plane). Between the electrodes, the formation of a homogeneous electrical field at the resistor positions along a radius of r = 0.107 m (see labeled black lines) can be observed. **Right:** Evaluation of the electric potential at the resistor radius (at r = 0.107 m). The curve shows a steady slope, further substantiating the observed homogeneity of the electric potential at the indicated positions in the left figure.

risk of discharges. Therefore, the *single curvature* geometry was chosen as a trade-off between complexity of construction and optimal properties. Further design considerations investigated in the simulations were the distances between the electrodes and the distance between electrodes and the vessel wall. In the following, only the simulation results for the chosen copper electrode geometry are presented.

The simulation geometry featured a simplified 3D geometry that included the copper electrodes and the grounded steel vessel only. Comsol provides predefined material databases that were utilized to implement the corresponding material properties of each component. Simulations were conducted for the G35 maximum input voltage of 35 kV. The potential applied to each electrode was calculated according to the secondary divider chain's electrical setup, which determines the voltage drops between the copper electrodes (approximately -8.75 kV voltage drop between the electrodes of the high voltage planes). The bottom electrode was asserted with a ground potential also according to the real setup.

Figure 6.12 shows the results for the investigations of the equipotential field lines in-
side the simplified G35 divider geometry. The left figure shows the central plane of the geometry with the simulated equipotential field lines. Since the simplified Comsol geometry was rotationally symmetric, this view represents the general field distribution. All electrodes, except for the top electrode, feature a central hole with an 80 mm diameter for the tunnel tube used for airflow optimization inside the G35. The top electrode has a smaller hole for the mounting of the high voltage connection. With regard to the result, a special interest was taken in the electrical field's homogeneity at the precision chain resistor positions. The radius, on which all resistors are placed concentrically around the central divider axis, is marked by the black labeled lines at r = 0.107 m. The right plot of figure 6.12 shows the electric potential along the marked black lines of the simulation result on the left. The course of the potential shows a linear behavior indicating a homogeneous electrical field at the resistor radius. Consequently, a successful shielding against the vessel wall's ground potential is achieved with this copper electrode configuration.

The second part of the Comsol simulation was conducted to determine the maximum field strength inside the G35 geometry. Since high electrical field strengths facilitate discharges, they have to be kept as low as possible. In figure 6.13, the simulated field strength distribution inside the divider is presented. The largest potential gradient during divider operation is always located between the top electrode and the steel vessel wall because the input voltage is applied to the top electrode. This is emphasized by the enlarged image section showing the edge of the top electrode. The maximum field strength found amounts to 1.84 kV/mm, verifying the result of 1.86 kV/mm from F. Hochschulz for a similar electrode geometry [131]. Considering the similar maximum field strength of 1.72 kV/mm found for the KATRIN K65 electrode geometry, the simpler electrode geometry for the G35 was chosen in favor of reducing the electrode manufacturing process.

6.4.3. Thermal design

One of the key components of the G35 mechanical setup is the heat exchange system (HES). Most importantly, it guarantees stable and reproducible ambient temperature conditions for the Vishay precision resistors. As shown in section 6.2.3, the temperature dependency, and thus the voltage dependency of the resistors can be significantly reduced when operating the divider at a temperature of 15 °C. Since the resistors are situated at different positions inside the divider vessel, it is essential to ensure a homo-



Figure 6.13: Electric field strength inside the simplified divider geometry for an input voltage of 35 kV. **Left:** Half-section view for the full divider setup. The field strengths show maxima at the rounded edges of the copper electrode. The highest gradients are indicated at the top electrode, where the full 35 kV are applied. **Right:** Zoom of the maximum field strength region located at the edge of the top electrode. The maximum field strength in this geometry is 1.84 kV/mm. This result is consistent with previous simulations by F. Hochschulz [131] and represents a compromise between more complex geometries with lower field strengths and lower construction complexity.

geneous thermal distribution throughout the divider.

To investigate the thermal distribution in the interior, a simulation of the thermal divider characteristics was performed using the software Autodesk[®] CFD 2018. This software works according to the *finite element method* and is specialized in computational fluid dynamic simulations. It offers support for various fans and heat sources, has a large material database, and further allows direct imports of the CAD model of the G35 divider from the Autodesk[®] Inventor software. The goal of the simulation was to optimize the air/nitrogen flow inside the divider to maximize the heat exchange with the precision chain resistors. Since the HES designs of the KATRIN dividers proved to deliver a temperature stability of ±0.1 K over several years, the HES concepts were adopted for the G35 design. Considering that the G35 has to be operated at lower temperatures ($T_{G35} = 15$ °C) compared to the KATRIN dividers ($T_{K35/K65} = 25$ °C), higher differences between divider and ambient temperature are to be expected, especially since the CRYRING@ESR hall features no temperature regulation. Therefore, the

¹⁵ A schematic overview of the working principle and a description of all devices are given in section 6.4.1.



Figure 6.14: Visualization of Autodesk CFD fluid dynamic simulation used to optimize the thermal design of the high voltage divider. With this setup, a tunnel tube was simulated to optimize the heat transfer from the precision resistors. **Left:** Quarter-cut of CAD-based CFD setup. The setup is a simplified version of the CAD drawing used for the construction of the HV divider. Each material is asserted with the respective thermal properties needed for the fluid dynamic simulation. **Right:** Temperature distribution for a setup with the optimized tunnel tube installed in the high voltage divider.

thermoelectric cooling unit (max. cooling power 160 W), the fan (max. delivery rate $79 \text{ m}^3/\text{h}$) as well as the pump (max. delivery rate 1200 L/h) all feature larger power capacities than their counterparts in the KATRIN dividers¹⁵. Since these design considerations predetermined most of the components of the HES, the simulations focused on optimizing the tunnel tube used for the gas distribution inside the G35 divider.

For the presented simulations, the voltage divider's geometry was simplified to the geometry shown in figure 6.14, left. Parts like the resistor connectors or wires were neglected. The heat exchange system was also simplified to only feature the relevant components inside the divider, consisting of a radiator, fan, and tunnel tube. This had the advantage that the liquid flow throughout the pipes and all external heat exchanging processes could be neglected to significantly speed-up the simulation process. Instead, the radiator inside the G35 was fixed to a constant temperature of 15 °C, thus acting as an infinite thermal reservoir. The divider's surroundings were represented by an infinite thermal reservoir at 25 °C, which is an upper limit of the CRYRING@ESR



Figure 6.15: Overview of thermal simulation results for two different tunnel tube geometries. Shown are the average resistor temperatures in dependence of the position inside the divider. The areas between the black dashed lines indicate the different divider planes, where left is the lowest plane (tap plane), and right is the top plane. The turquoise and the purple points show the average resistor temperatures for two different geometries if an input voltage of 35 kV is applied to the G35. The slit geometry (turquoise) used for the final tube design shows a more homogeneous temperature distribution and a lower overall temperature of the resistors than for the *simple hole* geometry. The resistor temperature distribution smoothes out significantly for an input voltage of 20 kV (red), which is the maximum allowed voltage at the CRYRING@ESR electron cooler.

hall temperature¹⁶ (see figure 6.14, right). In order to simulate the cooling power of the system with regard to every precision chain resistor, each resistor in the divider was attributed with the corresponding Joule heating expected when applying an input voltage of 35 kV, e.g., 154 mW for a high-voltage precision chain resistor according to equation (6.2). The heat transport from the resistors is mainly determined by the fan in combination with the tunnel tube. In this setup, the fan's airflow¹⁷ is directed upwards through the radiator into the tunnel tube that runs from the bottom electrode to the top electrode. The tube features a hole for each precision chain resistor to individually direct the airflow towards the corresponding resistor. In order to have an airflow through the whole divider, four holes were bored into the top electrode as additional gas outlets. The goal of the simulations was to optimize the tunnel tube's hole pattern and, therefore, maximize the heat exchange at the precision chain resistors.

 $^{^{16}}$ The hall temperature measured with the outer G35 temperature sensor during the second half of 2018 never exceeded 23 $^\circ\mathrm{C}$

¹⁷ For the simulations, the gas inside the divider was nitrogen.

For that purpose, several tunnel tube geometries with different hole sizes, shapes, and patterns were investigated. For every simulation, the airflow vector from each outlet hole towards the corresponding resistor was analyzed and iteratively optimized for the current tunnel tube geometry. Each simulation was conducted until thermal equilibrium inside the divider was reached. As the main quality measure of the current tube design, each precision chain resistor's average temperature was regarded. Figure 6.15 shows a selection of three simulation results. Since over a hundred different designs were simulated, only this selection is presented for a better overview. In the figure, two different designs are compared, where one design was simulated for two different input voltages. The first approach taken during the simulations consisted of a simple hole geometry, where every precision chain resistor was asserted with a corresponding tunnel tube hole of 6 mm diameter positioned on the same height as the resistor (purple data points).

The resulting temperature distribution inside a single plane shows an inhomogeneous pattern with temperature differences of up to 5 °C within a plane and a maximum temperature difference of approximately 6 °C between the hottest and the coolest resistor of the whole divider (not counting the tap plane). Averaging the temperature of all highvoltage resistors yields an average of $T_{\rm HV,av}$ = 19.4 °C compared to $T_{\rm HV,av}$ = 18.0 °C for the optimized G35 slit design (turquoise data points). This optimized design features rounded slits with a width of 20 mm and a height of 8 mm for every high voltage resistor (see figure 6.16), thus allowing a higher airflow towards each resistor. It also features individual vertical positions, where the slits are typically below the corresponding resistors taking the preferred direction of the airflow towards the top of the divider into account. These positions were determined iteratively via numerous simulations. The optimized slit geometry thus leads to a more homogeneous temperature distribution inside a single plane with a maximum difference of 2 °C within a single plane or every resistor even. The maximum temperature difference of 4 °C between the hottest and the coolest high-voltage resistor indicates room for further optimizations by utilizing more complex designs, such as individual slit sizes for every plane. However, a simulation with an input voltage of 20 kV (red data points) shows a much smoother distribution of average resistor temperatures and significantly lower total temperatures (maximum at 17 $^{\circ}$ C). Since 20 kV corresponds to the maximum allowed input voltage of the CRYRING@ESR electron cooler, no further optimizations were made in favor of a less complex tube construction. The technical drawings of the final design are presented in appendix A.6 where the exact tunnel tube dimension, including all slit/hole positions and sizes, are given.



Figure 6.16: Final design of the tunnel tube of the heat exchange system for the G35 divider. The bottom holes are used for the precision resistors in the tap plane. Each slit in the upper part is dedicated to an individual resistor in the high-voltage part of the precision chain.

In conclusion, the simulated temperature distributions can only be regarded as qualitative due to the numerous assumptions made. It is, for example, likely that the radiator yields lower temperatures than the fixed 15 °C since the temperature inside the divider is regulated to the average temperature of the four implemented PT100 sensors. Further factors, such as imperfect thermal shielding or power dissipation by the fan, amongst others, were also not regarded in the simulation process. However, the trend by comparing the different tunnel tube geometries based on the same approximations should still yield an optimized design. It is further noted that the absolute divider temperature is not a critical quantity since the novel absolute calibration method described in section 6.6 provides the means to calibrate the scale factors depending on the divider temperature and the input voltage. The reproducibility of the temperature settings as used for the absolute calibration (15 °C and 20 °C), on the other hand, is crucial since the linearity is only reproducible for the respective temperature setting (as discussed in section 6.6.3). Therefore, the G35 temperature regulation reproducibility is directly reflected in the reproducibility of the scale factor calibration measurements, which are monitored via the G35 calibration histories.

6.5. Low voltage calibration

In this section, the *low voltage calibration* method is described, which has already been used to monitor the stability of the K35 and K65 dividers over the last years [125]. It has the advantage of a simple setup and full traceability by using commercial equipment only. The main disadvantage is that it is limited to 1 kV and, therefore, cannot be used to measure the linearity of the G35 scale factors. Despite this disadvantage, a calibration history was started to monitor the stability of each scale factor realized in the G35 divider. In the following, the method will first be described with a subsequent presentation of the measurement results.

The following description of the low voltage calibration method is taken verbatim from [13]:

"In order to calibrate the scale factor M_A of an HV divider, the general procedure is to apply a calibration input voltage U_{HV} and measure the output voltage U_1 with a precision DVM¹⁸. The input voltage has to be determined with a reference HV divider with well-known scale factor M_B and a second precision DVM measuring its output voltage U_2 :

$$U_{\rm HV} = M_{\rm B} \cdot U_2. \tag{6.5}$$

This set-up is shown schematically in figure 6.17. Following equations (6.1) and (6.5),



Figure 6.17: Connection scheme for the calibration of a HV divider with a HV supply ($U_{\rm HV}$) and a precision DVM to measure the output voltage U_1 of the scale factor $M_{\rm A}$. A reference HV divider with well-known scale factor $M_{\rm B}$ is connected to the same HV source. In combination with a second precision DVM (U_2) it is used to determine the input voltage $U_{\rm HV}$. With commercial equipment, this procedure is limited to 1 kV. Figure and caption taken from [13].

¹⁸ In the ideal case the input resistance of a DVM is infinitely high. In reality, the input resistance of the DVM $R_{in,DVM}$ (in the 100 GΩ to 1 TΩ range for high-end DVM) has to be more than a million times larger than R_{LV} to determine the scale factor with ppm-precision. Otherwise, the scale factor has to be corrected for $R'_{LV} = R_{LV} ||R_{in,DVM}$.



Figure 6.18: Connection scheme for the calibration of a HV divider with the two scale factors M_A and $M_{A'} > M_A$. Here the voltage created by a HV supply is not connected to the input of the unit under test, but to the scale factor M_A output connection. The scaled voltage U_1 is measured with a precision DVM at $M_{A'}$. A reference HV divider with scale factor M_B and a second DVM (U_2) are used to determine the input voltage. Figure and caption taken from [13].

the scale factor of the unit under test can be calculated to be

$$M_{\rm A} = \frac{U_2 \cdot M_{\rm B}}{U_1}.\tag{6.6}$$

Since commercial reference dividers with ppm-precision are only available for voltages up to 1 kV, the calibration with these devices in such a configuration is limited to 1 kV not probing the full range of $M_A(U_{HV})$. Secondly, this arrangement prefers scale factors of 100:1 or smaller to avoid that the output voltage U_1 gets far below the desired 10 V. To scale down high voltages well beyond 1 kV into the suitable range of high-precision DVM, much higher scale factors are necessary.

For example, for a scale factor of 2000:1, the output voltage measured with a DVM would be 0.5 V. Measuring such a small voltage would mean losing one digit of resolution of the most precise range of the DVM and is therefore not directly traceable to a 10 V reference source used to calibrate the DVM.

A standard procedure to avoid this problem is a step-up technique with 1 kV (low voltage) equipment. A prerequisite to apply this method is that the HV divider under test has multiple scale factors, one of them ideally scaling $M_A \approx 100$:1. In the first step, M_A has to be calibrated with the direct method mentioned above with 1 kV. In a second step the higher scale factor $M_{A'}$ is calibrated by applying U_{HV} not to the regular divider input, but to the M_A output connection. In this arrangement the voltage drop over the low voltage resistors R_{LV} at a calibration voltage $U_{HV} \leq 1$ kV is comparable to the voltage drop over the resistors at an input HV of $U_{HV} \cdot M_A$. The connection scheme for this calibration method is shown in figure 6.18. The set-up for the determination of the input voltage is similar to the previous method. For the calculation of $M_{A'}$ one

has to multiply the determined input voltage with M_A :

$$M_{\mathrm{A}'} = \frac{U_2 \cdot M_{\mathrm{B}}}{U_1} \cdot M_{\mathrm{A}}.$$
(6.7)

One disadvantage of this method is that the upper part of the divider with the resistors R_i is not loaded with the correct voltage $M_A \cdot U_{HV}$. This means that the voltage dependency of the scale factor M_A is not determined and included in the analysis properly. For a completely traceable calibration of an HV divider, the voltage dependency of the scale factors has to be taken into account correctly¹⁹." (End of verbatim copy of [13].)

6.5.1. Low voltage calibration results

According to the above described low voltage calibration method, a calibration history was started for every scale factor of the G35 divider. Due to the lack of temperature regulation in the CRYRING@ESR hall, it is possible that ambient conditions sometimes will not allow the G35 temperature regulation to reach the desired 15 °C divider temperature (e.g., during hot summers). For this purpose, all calibration measurements were conducted for divider temperatures of 15 °C and 20 °C. Figure 6.19 shows the calibration history of the M_{100} scale factor at 15 °C. All determined scale factors stay within a ±1 ppm band relative to the overall average (green line), thus indicating no drifts and a relative scale factor stability in the order of 1 ppm/year. This result is reinforced by all other measurements for the higher scale factors and the 20 °C calibrations presented in appendix A.7. To ensure a long-lasting divider stability, these results have to be constantly validated by performing regular calibrations at GSI. In conclusion of the low voltage calibration measurements, table 6.4 shows the latest results for all G35 scale factors determined with this method for a divider temperature of 15 °C.

scale factor	positive polarity	negative polarity
M ₁₀₀	100.514 86(3)	100.514 84(3)
M_{163}	162.714 20(16)	162.71444(16)
M_{269}	268.989 13(19)	268.98943(19)
M_{775}	775.484 56(38)	775.484 95(38)
M ₃₄₅₂	3451.6858(23)	3451.6866(23)

Table 6.4.: Latest 1 kV calibration values for G35 divider at 15 °C for positive and negative polarity. All values were taken between July and November 2018.

¹⁹ As described above, the traceability of the single resistors is possible.



Figure 6.19: Low voltage calibration history for M_{100} scale factor at 15 °C. Calibration measurements with positive input voltages are marked red, and for negative input voltage markers are blue. Further shown is the average of all measurements (green line) and a ±1 ppm band.

As previously stated, this calibration method is limited to 1 kV, so that the linearity of the G35 divider cannot be determined with this method. Since the resistor characterizations, presented in section 6.2, give clear indications of a non-negligible voltage dependency of the G35 scale factors, the low voltage calibration method is not sufficient to fully characterize the divider. For a precise determination of the voltage-dependent scale factors, the new absolute calibration method presented in the next section has to be applied.

6.6. Novel absolute high voltage calibration

From the investigated thermal characteristics of the Vishay precision resistors presented in section 6.2.3, it is to be expected that the G35 divider shows a substantial voltage dependency of the scale factors. This cannot be determined with the calibration limited to 1 kV presented in the previous section. With the novel absolute calibration method, this limitation was eliminated. The non-negligible voltage linearity of the G35 divider substantiates the importance of the newly developed calibration method, as only through the characterization of its linearity behavior can the G35 deliver voltage measurements with $1 \cdot 10^{-6}$ precision. In this chapter, the new method's principle and setup will be explained. Subsequently, the calibration results for the voltage-dependent scale factors of the G35 divider are presented.

As explained in the introduction of this chapter, a large part of this section are excerpts from the publications [13, 20].

6.6.1. Measurement principle

This subsection was taken verbatim from [13].

"The basic idea of the novel absolute calibration method is to determine the voltage dependency of the scale factors of an HV divider by measuring a differential scale factor directly at high voltages with commercially available equipment. This is especially important for scale factors up to 100:1 since they are used in a step-up technique to calibrate higher scale factors (see section 6.5). As defined in equation (6.1), the scale factor is the possibly voltage-dependent factor between the input- and output voltage of an HV divider. For a given input voltage, the corresponding output voltage can be approximated by a Taylor expansion around $U_{\rm HV} = 0$:

$$U_{\rm LV} = a \cdot U_{\rm HV} + b \cdot U_{\rm HV}^2 + c \cdot U_{\rm HV}^3 + d \cdot U_{\rm HV}^4 + \dots$$
(6.8)

with the coefficients *a*, *b*, *c* and *d* (neglecting higher orders²⁰). For the voltage-independent case the parameters *b*, *c* and *d* are zero and *a* is the inverse of the constant part of the scale factor M_0 :

$$a = \frac{1}{M_0}.\tag{6.9}$$

For the realistic case of a voltage-dependent scale factor, we can derive from equation

²⁰ The thermal heat scales with the electric power P, which scales with U^2 (see equation 6.2). The precision resistors of the HV dividers K65 and G35 we used were bulk metal foil resistors, which have a zero thermal coefficient at an optimal temperature T_0 . Therefore, the temperature dependence of each resistance near its optimal temperature is close to a quadratic curve. These two effects make a Taylor expansion to fourth power plausible. We expect the thermal control system of our HV dividers to give another reason for a non-linear behavior. In our measurements, the Taylor approximation of fourth order was sufficient for the G35 HV divider (see section 6.6.3). The higher orders necessary for the G35 HV divider reflect the fact that for technical reasons, it could not be operated at its optimal thermal point $T_0 = 8$ °C.



Figure 6.20: Illustration of scale factors as function of the input- and output voltages. Left: Output voltage as function of input voltage. Right: Scale factor as function of input voltage. A constant scale factor appears as a straight line. If the scale factor is dependent on the input voltage (see orange solid line), a deviation from the constant case is observed. For each input voltage U_{HV} , the differential scale factor is measured as a change of input- and output voltages. This is illustrated at the left at a certain input voltage $U_{\text{HV},0}$. The differential scale factor \tilde{M} appears as slope of the line through the two points $U_{\text{HV},0}$ and $U_{\text{HV},0} + \delta U_{\text{HV}}$ (blue dashed line). M_0 notifies the scale factor derived at $U_{\text{HV}} \approx 0$ (green dash-dotted line). Figure and caption taken from [13].

(6.1) and (6.8):

$$M = \frac{1}{a + b \cdot U_{\rm HV} + c \cdot U_{\rm HV}^2 + d \cdot U_{\rm HV}^3}.$$
(6.10)

We define a differential scale factor \widetilde{M} as the derivative of $U_{\rm HV}$ with respect to $U_{\rm LV}$ at $U_{\rm HV}$:

$$\widetilde{M} = \frac{\delta U_{\rm HV}}{\delta U_{\rm LV}} \Big|_{U_{\rm HV}} = \frac{1}{\frac{\partial U_{\rm LV}}{\partial U_{\rm HV}}} \Big|_{U_{\rm HV}}$$
$$= \frac{1}{a + 2 \cdot b \cdot U_{\rm HV} + 3 \cdot c \cdot U_{\rm HV}^2 + 4 \cdot d \cdot U_{\rm HV}^3}.$$
(6.11)

The measurement of \widetilde{M} at $U_{\rm HV}$ is done with the following procedure: at certain input voltages we increase $U_{\rm HV}$ by a small amount of $\delta U_{\rm HV}$ and measure the change of the output voltage $\delta U_{\rm LV}$. In the ideal case, the voltage increase $\delta U_{\rm HV}$ is infinitesimal small in order to determine the slope of the scale factor curve at $U_{\rm HV}$. However, due to technical limitations and because of the ambition to trace the voltage measurement



Figure 6.21: Connection scheme for the measurement of the ratio μ of the scale factors M_A and M_B . A HV U_{HV} is connected to both HV dividers and their output voltages are measured with two DVMs versus a counter voltage as a null volt measurement, which is monitored with a third DVM. The counter voltage labeled U_{HV}/M_A is adjusted such that $U_1 \approx 0$. Figure and caption taken from [13].

back to a 10 V reference, this is not possible. Hence, we increase the voltage by $\delta U_{\rm HV} =$ 1 kV, which can be measured with traceable equipment with ppm-precision. Therefore, we assume that the determined scale factor is valid for the input voltage $U_{\rm HV} + \delta U_{\rm HV}/2$. The two cases of the constant and voltage-dependent scale factor are sketched in figure 6.20. Additionally, \tilde{M} is illustrated for an exemplary input voltage $U_{\rm HV,0}$. By measuring the differential scale factor for different input voltages, the coefficients *a*, *b*, *c*, and *d* can be determined and used to calculate the scale factor *M* for any given input voltage.

The measurement of \widetilde{M} is split into two steps: figure 6.21 shows the experimental setup for the first step. A high voltage $U_{\rm HV}$ is connected to the HV divider whose scale factor $M_{\rm B}$ is to be calibrated. Its output voltage U_2 is measured with a precision DVM versus a very stable counter voltage²¹ $U_{\rm HV}/M_{\rm A}$ as a null volt measurement. By using a counter voltage instead of a measurement versus ground potential, it is ensured that the measured voltage is below 20 V, which can be traced back to a 10 V reference source. The counter voltage is either directly monitored with a third DVM²² (U_3) or converted via a reference divider²³ into the 0 to 20 V range. Additionally, a second HV divider ($M_{\rm A}$) is needed as reference for the unit under test, which is connected to the same HV source. The output voltage of the reference HV divider is also measured with a DVM

²¹ The ppm-stable counter voltage is provided by a Fluke Calibrator 5720A.

²² Since U_3 has to be very stable but does not need to be known such precisely we monitored this voltage with a 6.5 digit DVM of type Fluke 8846A.

²³ For this purpose we used a Fluke reference divider of type 752A.



Figure 6.22: Connection scheme for differential scale factor measurement. On top of a high-voltage potential, an additional calibration voltage is created, which is applied to the unit under test. The reference HV divider is unaffected by the calibration voltage. The devices in the blue shaded box are located in an HV cage and read out via an optical link. Figure and caption taken from [13].

 (U_1) versus the counter voltage. In this measurement, the ratio of the scale factors μ

$$\mu := \frac{M_{\rm A}}{M_{\rm B}} = \frac{U_2 + U_3}{U_1 + U_3} \approx 1 + \frac{U_2}{U_3} \tag{6.12}$$

can be determined applying Kirchhoff's circuit laws. The approximation on the right of equation (6.12) is only valid for $U_1 \approx 0$ and should only illustrate that μ does not require a precise determination of U_3 . This counter voltage is a key to achieve the ppm-precision for the novel absolute calibration method. The ratio μ can be measured with a short-term precision of the order of below 10^{-7} without knowing the single scale factors M_A and M_B since it only depends on the measured voltages $U_{1,2,3}$, which are determined with precision DVMs. Since both null volt measurements U_1 and U_2 are measured with the same counter voltage, both scale factors have to be of similar magnitude in order to not exceed the 20 V range of the DVM.

In the second step, the input voltage of the HV divider under test is increased by δU_{HV} , which is generated and measured on top of the HV potential U_{HV} (see figure 6.22).



Figure 6.23: Connection scheme for the corrected determination of $M_{A'}$. The input voltage U_{HV} is connected to the scale factor output M_A of the unit under test. The upper part of the HV divider with the resistors R_i is loaded with the voltage $U_{load} = U_{HV} \cdot M_A$ created by an additional HV supply, which is operated on the potential of U_{HV} in a HV cage. A second HV divider with the well known scale factor M_B is used to determine U_{HV} . Figure and caption taken from [13].

The input voltage of the reference HV divider stays constant as well as the counter voltage, any potential change would be detected by continuously measuring U_1 and U_3 . The DVM, which is used to measure the output voltage of the divider under test, will measure a voltage increase of $\delta U_{\rm HV}/\widetilde{M}_{\rm B}$. For this as well as for all other used DVMs, the measurement range has been kept fixed during the whole calibration procedure in order to avoid a change of input resistances and leakage currents.

According to Kirchhoff's circuit- and Ohm's laws, the differential scale factor is given by

$$\widetilde{M}_{\rm B} = \frac{U_1 \cdot M_{\rm A} + U_4 \cdot M_{\rm C}}{U_2 + (1 - \mu) \cdot U_3}.$$
(6.13)

As denoted in equation (6.13) the scale factor of the reference HV divider M_A is needed to calculate \widetilde{M}_B . However, the term $U_1 \cdot M_A$ is close to zero since U_1 is a null volt measurement against the stable counter voltage adjusted to $U_1 \approx 0$. Hence, the dominant factor of the numerator is $U_4 \cdot M_C$, which means that the absolute value of M_A needs to be stable but does not have to be known precisely in order to calibrate the unit under test to the ppm-level. The measurements, which are presented in the next section, showed that an uncertainty of up to $1 \cdot 10^{-4}$ can be allowed for M_A , without changing the calibration result for M_B on the $1 \cdot 10^{-7}$ level. Secondly, the uncertainty of U_3 is not important since the ratio of the scale factors μ is close to 1. Therefore, U_2 and its uncertainty are dominating the denominator for the determination of \widetilde{M}_B . For the calibration of scale factors $M_{A'} > 100:1$ the procedure similar to the one described in figure 6.18 can be used, but to load the resistors R_i correctly, the corresponding HV is additionally given to the input of the HV divider under calibration using an HV cage (see figure 6.23). The wanted scale factor $M_{A'}$ can be calculated according to equation (6.7). The critical scale factor $M_A \leq 100$ is determined with the novel absolute calibration method. Thus, the issues regarding traceability and the previously neglected voltage dependencies of M_A and M'_A vanish." (End of verbatim copy of [13].)

6.6.2. Measurement setup

This section gives a brief overview of the measurement devices used for the absolute calibration measurements. Further details of the technical realization are given in [13, 20, 125]. The setup schematics for the three relevant measurement setups are given in the previous section (figures 6.21 to 6.23).

For the calibration measurements presented in section 6.6.3, the divider to be calibrated $M_{\rm B}$ was the G35 divider, while KATRIN divider K65 acted as the reference unit $M_{\rm A}$. Using the K65 as reference had the advantage that the K65 could also be used to simultaneously crosscheck the results since it has a negligible linearity below $1 \cdot 10^{-6}$ over a voltage range of 35 kV [13]. To measure the voltage $\delta U_{\rm HV}$ (provided by an MCP 14-1250 voltage supply), a reference divider of type Fluke 752A ($M_{\rm C}$) was used in combination with a Keysight 3458A precision DVM (U_4). Further voltage measurements of U_1 , U_2 , and U_3 were conducted by precision multimeters of type Fluke 8508A, Agilent 3458A, and Fluke 8846A. The high voltage $U_{\rm HV}$ was provided by an FuG HCP 70M-35000 voltage supply, and the ppm-stable counter voltage $U_{\rm HV}/M_{\rm A}$ was supplied by a Fluke Calibrator 5720A.

A special prerequisite of the absolute calibration method is that multiple devices of the precision equipment have to be operated on a high voltage. For this purpose, a dedicated high-voltage cage was constructed by C. Huhmann and O. Rest from the University of Münster. The cage is shown in figure 6.24. It features a dedicated high voltage divider that can also be used for the absolute calibration measurements as reference unit.



Figure 6.24: Custom-made high voltage cage for absolute calibration, constructed by C. Huhmann and O. Rest from the University of Münster. **Left:** Picture of the complete custom-made cage with the used measurement devices. The cage features an inner cage, which can be operated on HV potential up to 35 kV. The electrical devices within the inner cage are supplied by an insulating transformer below the inner cage. **Right:** Custom-made high voltage divider installed at the back-side of the high-voltage cage. This divider was not used for the measurements presented in section 6.6.3. Pictures taken from [13, 125].

6.6.3. Absolute calibration results

The calibrations presented in this section mainly result from a measurement campaign in early 2018 with the K65 and G35 dividers. Besides the calibrations of the scale factors, the main goal was to check the long-term stability and the reproducibility of the novel absolute calibration method, as well as prove its capability to determine the voltage dependency of the scale factors. A full overview of the results from the measurement campaign and detailed descriptions of the individual measurement steps are given in [13, 125]. Here, only the results with regard to the G35 are presented concluding with a paragraph describing the determination of all G35 scale factors in dependence of the input voltage and a paragraph describing a method to update the absolute calibration by utilizing the much less complex *low voltage calibration* method.

Figure 6.25 shows the result for the calibration of the G35 100.46:1 scale factor at 15 °C. The red points represent the differential scale factors \widetilde{M} measured at different input voltages over the full range of the allowed G35 input voltages (up to ±35 kV). In order to determine the coefficients for the reconstruction of the scale factor $M_{\rm B}$, a third-order polynomial fit (red line) was applied to the differential scale factors \widetilde{M} and the low voltage scale factor $M_{1 \text{ kV}}$ (blue point), which was previously determined with the low voltage calibration method presented in section 6.5. The determined coefficients *a* through *d* are given in the legend. These parameters were then applied to equation (6.10) to cal-



Figure 6.25: Voltage dependency of the G35 100.46:1 scale factor at 15 °C determined with the newly developed absolute calibration method. The differential scale factors \tilde{M} measured at different voltages (red points), and the low voltage scale factor $M_{1 \text{ kV}}$ (blue point) are fitted with a polynomial of third order (red line). The error-bars include the statistical and systematic uncertainties. The obtained coefficients are used to calculate the real scale factor M for a voltage range from 0 to 35 kV (blue line). In order to verify the result for the G35, the two months earlier calibrated K65 was used to crosscheck the voltage dependency (green points). Note that all green data points are shifted by $-2 \cdot 10^{-7}$ in y-direction (see text). Figure and caption taken from [13]. The figure has been modified to fit the format of this thesis.

culate the real scale factor M depicted by the blue line. The derived scale factor $M_{\rm B}$ shows relative deviations of up to $3.3 \cdot 10^{-6}$ at $-35 \,\rm kV$ compared to the low voltage scale factor $M_{1\,\rm kV}$. This matches the expected behavior from the resistor characterization shown in section 6.2, as only resistors with negative TCR were available for the construction of the divider.

As stated in section 6.6.2, the K65 was used as reference divider for the calibration measurements. Since the scale factor of the K65 is known to the $< 1 \cdot 10^{-6}$ -level and shows a negligible linearity also below $1 \cdot 10^{-6}$, it could also be used to crosscheck the scale factor determined with the new calibration method. The green points depict the scale factors determined with the K65. These points were shifted by a constant (relative) offset of $-2 \cdot 10^{-7}$ over the full range of -35 kV to get an excellent agreement with the voltage dependence of the scale factor determined with the new calibration method. Despite this tiny shift exceeding the combined short-term uncertainties, this shift appears reasonable, as the K65 calibration, which the green data points rely on, was conducted two months prior to the presented measurement. Therefore, an additional relative uncertainty of $\pm 5 \cdot 10^{-7}$ for the K65 scale factor can be assumed since

Table 6.5.: Estimated uncertainty budget for systematic uncertainty of the G35 differential scale factor with most important contributions (shown for an exemplary measurement at approximately -14.5 kV). For all parameter values p, a Gaussian distribution (1σ) of the uncertainty Δp was considered (see section 6.6.2 for details about the used devices). The contribution of each parameter is the product of the sensitivity coefficient $\partial \tilde{M}/\partial p$ and Δp . The relative importance of each contribution is derived by $(\partial \tilde{M}/\partial p \cdot \Delta p)^2 / (\Delta M_{tot})^2$. Additionally, all values have been rounded to fit the table format.

parameter	value <i>p</i>	abs. unc. Δp	unit	sensitivity coeff.	contribution	rel. imp. (%)
U_4 DVM (cal. with 10 V)	-10.0002262	0.000 002 0	V	10.0462	0.0000202	21.94
U_4 DVM (\widetilde{M} , see figure 6.22)	-10.0026608	0.0000020	V	-10.0438	$-0.000\ 020\ 2$	21.94
M_C HV divider (see figure 6.22)	100.000 000	0.000017		1.0046	0.0000167	15.08
U_2 DVM (\widetilde{M} , see figure 6.22)	-10.0258629	0.0000012	V	10.0906	0.000 011 9	7.65
U_1 DVM (cal. with 10 V)	-10.0000854	0.0000012	V	-10.0464	$-0.000\ 011\ 8$	7.56
U_1 DVM (\widetilde{M} , see figure 6.22)	-0.0000230	0.0000011	V	-10.0957	$-0.000\ 011\ 5$	7.13
U_1 DVM (μ , see figure 6.21)	0.0000109	$0.000\ 001\ 1$	V	10.0956	$-0.000\ 011\ 5$	7.13
other uncertainties					0.0000147	11.57
total uncertainty	100.464 628	0.000 043				100.00

all previous low and high-voltage calibrations showed this level of uncertainty when repeated later on a time scale of weeks/months. For the same reason, this additional uncertainty also has to be assumed for the results of the G35 scale factor calibrations presented in this thesis.

Table 6.5 shows the estimated uncertainty budget for an exemplary differential scale factor measurement (measured at approximately -14.5 kV). The total (relative) uncertainty of about $4.3 \cdot 10^{-7}$ (derived by $\Delta p/p$) is mainly dominated by the two devices with a combined contribution to the uncertainty of almost 60 %, which are operated on the HV potential: the 1 kV reference divider $M_{\rm C}$ and the corresponding DVM U_4 with its associated calibration. Alongside the devices and their calibrations used for the measurements, the resistances of the used cables become relevant for the uncertainty determination at this level of precision. The resulting uncertainty contributions are included within *other uncertainties* shown in the table. A detailed description of these contributions is further given in [13].

Determination of voltage dependent scale factors $M_{A'}$ The above results only describe the determination of the M_{100} scale factor linearity. For the full characterization of the G35 divider, all higher scale factors were determined with the measurement technique presented in figure 6.23. Table 6.6 shows all parameters determined with the absolute calibration measurements for a divider temperature of 15 °C. From the

presented coefficients, each scale factor $M_{A'}$ can be calculated for a given voltage by combining equations (6.7) and (6.10)

$$M_{A'} = \underbrace{\frac{1}{a + b \cdot U_{in}[kV] + c \cdot (U_{in}[kV])^2 + d \cdot (U_{in}[kV])^3}_{M_{100}}} M_{A'_{factor}}$$
(6.14)

where $M_{A'}$ is the scale factor to be determined (M_{100} , M_{163} , M_{269} , M_{775} or M_{3452}), and U_{in} is the input voltage in kV. Here the fraction $U_2 \cdot M_B / U_1$ of equation (6.7) is defined as the sub-scale factor $M_{A'}$ _factor. It has to be noted that the parameters are only valid for negative polarity, as the absolute calibration was only conducted with negative input voltages.

The 1σ uncertainties for the respective scale factors can be derived via uncertainty propagation using the following equation while taking the given covariances into account

Table 6.6.: Coefficients *a*, *b*, *c*, *d*, their respective covariances and sub-scale factors $M_{A'_{factor}}$ to determine the voltage dependent G35 scale factors for a divider temperature of $T_{G35} = 15$ °C and negative polarity. To calculate the voltage-dependent scale factors, the values in this table are applied to equation (6.14). Here, the updated value for the reciprocal M_{100} scale factor *a* and the original value a_{orig} (from the measurement of figure 6.25) are both presented. Applying *a* to equation (6.14) yields the blue curve in figure 6.26 and applying a_{orig} yields the gray curve. The uncertainties are determined by applying the covariances according to equation (6.15).

coefficient	value	uncertainty	measurement date
<i>a</i> *	$9.95374461 \cdot 10^{-3}$	$2.42 \cdot 10^{-9}$	10/10/2018
a _{orig}	$9.95374155 \cdot 10^{-3}$	$2.02 \cdot 10^{-9}$	07/16/2018
b	$3.42 \cdot 10^{-10}$	$4.70\cdot10^{10}$	07/16/2018
с	$-7.6 \cdot 10^{-12}$	$2.45\cdot10^{\text{-}11}$	07/16/2018
d	$7.20 \cdot 10^{-13}$	$3.61 \cdot 10^{-13}$	07/16/2018
M _{100 factor}	1		
M_{163} factor	1.6196176	$1.5 \cdot 10^{-6}$	07/14/2018
$M_{269 \text{ factor}}$	2.6774509	$1.6 \cdot 10^{-6}$	07/14/2018
$M_{775 \text{ factor}}$	7.7189804	$2.7\cdot 10^{-6}$	07/14/2018
M_{3452} factor	34.357179	$2.0 \cdot 10^{-5}$	07/14/2018
$\operatorname{Cov}(\overline{a,b})$	-5.96· 10 ⁻¹⁹		
$\operatorname{Cov}(a,c)$	$2.54 \cdot 10^{-20}$		
$\operatorname{Cov}(a,d)$	$-3.33 \cdot 10^{-22}$		
Cov(b, c)	$-1.12 \cdot 10^{-20}$		
$\operatorname{Cov}(b,d)$	$1.57 \cdot 10^{-22}$		
$\operatorname{Cov}(c,d)$	$-8.72 \cdot 10^{-24}$		

*Measured with 1 kV calibration at GSI.

$$u_M^2 = J^T(\mathbf{x}) \cdot V_{\mathbf{x}} \cdot J(\mathbf{x}).$$
(6.15)

Here, u_M^2 is the variance associated with the uncertainty of the scale factor u_M , $J(\mathbf{x}) = \partial M / \partial x_i(\mathbf{x})$ is the Jacobian matrix, $V_{\mathbf{x}}$ is the covariance matrix, and \mathbf{x} is the parameter vector. As described above, an additional uncertainty of $5 \cdot 10^{-7}$ for each derived scale factor value has to be assumed.

Update of sub-scale factors $M_{A'_{factor}}$ From the given uncertainty evaluation, it can be concluded that the voltage-dependent scale factor can be determined with a systematic uncertainty in the order of less than $1 \cdot 10^{-6}$. However, the measurements are complex to perform and also time-consuming if the set-up is not already in place. Therefore, it can be suitable to update the voltage-dependent scale factors with measurement results obtained from the low voltage calibration presented in section 6.5. This update can be conducted under three assumptions:

First, it has to be assumed that the general shape of the curve from figure 6.25 does not change on timescales in the order of the time difference of an absolute calibration and the low voltage calibration to be used as an update. The shape of the curve is directly related to the warm-up behavior of the resistors, which could generally change due to aging effects. However, the Vishay resistors did not show long-term drifts over the past years in both KATRIN dividers K35 and K65, which is why it is reasonable to assume negligible aging effects for the G35 resistors on such timescales.

The second assumption is related to updating the coefficient *a* of equation (6.14). As described in section 6.6.1, this factor is the inverse of the constant part of the scale factor M_0 . By assuming a negligible warm-up of the precision chain resistors for an input voltage of 1 kV, the factor *a* can be updated with the inverse of the $M_{1 \text{ kV}}$ scale factor shown in figure 6.25, which is obtained from the low voltage calibration. Since the maximum power consumed by a single resistor for that input voltage is 0.13 mW, which is even lower than the power for the reference resistor R_{ref} used for the resistor characterizations in section 6.2.2, this assumption is also reasonable.

The last assumption refers to the equivalence of the calibration methods for scale factors $M_{A'} > 100$: 1 between the low voltage calibration and the absolute high voltage calibration method. The only difference between both methods is that during the absolute calibration of $M_{A'}$, a load is applied to the high voltage part of the precision divider chain, which is not the case during the low voltage calibration. Table 6.7 shows that the difference between both methods is, in this case, almost negligible, as the sub-scale factors $M_{A'}$ factor determined with both methods do not deviate by more than 0.5 ppm.

Table 6.7.: Relative difference between low voltage calibration and absolute high voltage calibration for higher scale factor determination $M_{A'} > 100 : 1$ at a divider temperature of $T_{G35} = 15$ °C. The relative differences between both methods stay considerably below 1 ppm, indicating a negligible warm-up behavior of the tap plane resistors. All measurements compared in this table were conducted on the same day.

sub-scale factor $M_{A'_{factor}}$	low voltage calibration	absolute calibration	rel. difference (ppm)
M ₁₆₃ factor	1.6196175	1.619 617 6	-0.06
M_{269} factor	2.677 451 2	2.677 450 9	0.11
M_{775} factor	7.718 976 6	7.718 980 4	-0.49
	34.357 195	34.357 179	0.47

Since all assumptions discussed above appear reasonable, an update of the absolute calibration measurements can be performed with more recent values for $a \approx 1/M_{1 \text{ kV}}$ and $M_{\text{A'}_{\text{factor}}}$ obtained with the low voltage calibration. The most recent value for a is already shown in table 6.6, which was measured approximately three months after the latest absolute calibration measurement (see) figure 6.25). The expunged value a_{orig} refers to the value obtained from the original absolute high voltage calibration. A comparison between the original and the updated M_{100} scale factor is presented in figure 6.26. Here, the gray line refers to the original curve (identical with the blue curve in figure 6.25) and the blue line to the updated curve.

Furthermore, same as for the low voltage calibration, all measurements were performed for two divider temperatures (15 °C and 20 °C). An overview of all results for 20 °C and the graphs for the 15 °C results are presented in appendix A.8. As expected from the results of the thermal investigations regarding the resistors in section 6.2.3, the scale factor for 20 °C is shifted, indicating a change in total resistance of the Vishay resistors depending on the absolute temperature. Additionally, the relative deviation between the low voltage scale factor $M_{1 \text{ kV}}$ and the scale factor at -35 kV becomes larger when operating the G35 at 20 °C ($3.3 \cdot 10^{-6}$ at 15 °C vs. $5.6 \cdot 10^{-6}$ at 20 °C). This is also expected, because the optimum operating temperature for the Vishay resistors of the precision chain has been determined to range between 5 °C and 13 °C.

In conclusion, a novel absolute calibration method was developed and successfully applied to calibrate the G35 high-voltage divider for the electron cooler of the CRY-RING@ESR storage ring. Applying the new method allowed to determine the voltage-



Figure 6.26: Updated M_{100} scale factor for a divider temperature of $T_{G35} = 15$ °C. The blue curve shows the updated scale factor over the whole divider range of 35 kV. The gray line represents the result from the original absolute calibration measurements for comparison.

dependent M_{100} scale factor of the new divider with a relative systematic uncertainty in the order of less than $1 \cdot 10^{-6}$ and the higher scale factors in the order of $1 \cdot 10^{-6}$. The calibration history up to now shows no measurable drifts of the scale factors, indicating a long-term stability on the (sub)ppm level per year similar to the KATRIN divider K35 and K65. However, this level of stability has to be verified by regular calibration measurements over the coming years. Finally, the possibility of updating the voltage-dependent scale factors by combining the results from the absolute calibration method with results from the easier to apply low voltage calibration was discussed.

Chapter 7

Conclusion and outlook

To advance the development of modern measurement techniques for the investigation of (heavy) highly-charged ions at the future research center FAIR, two precision measuring instruments were developed or improved in this work.

The first instrument is an XUV detection system for in-vacuum detection of extreme ultraviolet fluorescence photons in the scope of laser spectroscopy measurements at the Experimental Storage Ring (ESR). Its main field of application is the detection of photons in the 10 nm-range, as this corresponds to the expected de-excitation photon wavelength (in the laboratory system) of an experiment proposed Winters et al. for the measurement of the $(1s^22s2p)^3P_0^{-3}P_1$ level splitting in beryllium-like krypton $(^{84}\text{Kr}^{32+})$ [6]. The detection system is a refined revision of the system constructed by J. Vollbrecht as part of his doctoral thesis [19]. Improvements leading to the described setup were carried out in close cooperation with C. Egelkamp, as described in detail in his master thesis [77]. The detection principle is based on the conversion of fluorescence photons into secondary electrons, which are then magnetically guided onto an MCP detector. Since ions in the ESR are stored at relativistic velocities, the emission of fluorescence photons is forward peaked due to the Lorentz boost. Therefore, the detector utilizes a movable cathode plate with a central slit that can be positioned around the ion beam axis, collecting mainly forward emitted photons. Compared to the first design, the cathode geometry was optimized, and a CsI coating was added to the stainless steel surface to optimize fluorescence photon collection and conversion efficiency. A second detector enhancement was achieved through a new magnet coil system featuring counteracting magnetic fields to optimize the detector flux tube, thus imaging the whole cathode area onto the MCP surface.

In consequence of the detector improvements developed for this thesis, the detection system was successfully commissioned during a beamtime with lithium-like carbon ions (¹²C³⁺) at the ESR. During that beamtime, the (1s²2s) ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ and (1s²2s) ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splittings of the carbon ions were measured and the corresponding wavelengths determined to be

$$(1s^22s)^2 S_{1/2} - {}^2P_{1/2} : 155.0779(12)_{sys}(1)_{stat} \,\mathrm{nm}\,,$$
 (7.1)

and

$$(1s^22s)^2 S_{1/2} - {}^2P_{3/2} : 155.8211(12)_{sys}(2)_{stat} \text{ nm},$$
 (7.2)

in the rest frame of the ions. With this result, measurements performed with fundamentally different measurement methods (interferometry and plasma spectroscopy) could be confirmed as both results agree within the uncertainty intervals of the results obtained [87]. Furthermore, the uncertainty of the beamtime results could be improved by a factor of more than three compared to a similar laser spectroscopy measurement at the ESR in 2006 [83]. On the one hand, this improvement can be attributed to an improved voltage measurement of the ESR electron cooler voltage and, on the other hand, to the new XUV detection system. In particular, it should be emphasized that the detector's contribution to the systematic uncertainty of the result is negligible ($\leq 0.05\%$ of the total systematic uncertainty), which underlines the high sensitivity of the system to XUV photons. Based on the uncertainty analysis of the presented measurements, a further improvement in accuracy by a factor of two can potentially be achieved for similar experiments if the three main (electron cooler voltage related) contributions of the calculated uncertainty budget are reduced as discussed in section 5.3.6 of this thesis.

However, the measurements also revealed that the detector still needs to be improved with respect to ion- and laser-induced background. Due to the high background rate, the detector could not be operated as intended in the direct vicinity of the ion beam but had to be positioned on the side of the beam pipe during the presented beamtime. At this position, the focusing effect of the Lorentz boost is less pronounced, leading to some loss in the measured signal rates. Therefore, further detector improvements concerning background suppression are being developed and implemented as part of the doctoral thesis by A. Buß [96]. This development will prove crucial for experiments with a much lower signal-to-noise ratio (SNR) than the SNR of the ${}^{12}C^{3+}$ measurements presented in this thesis.

The second part of the thesis dealt with the construction of a precision high voltage

divider (G35) for the CRYRING@ESR electron cooler of the future FAIR facility. The construction of a dedicated divider was mainly motivated by the fact that limitations in the precision of some laser spectroscopy experiments at GSI were caused by the imprecise knowledge of the electron cooler acceleration voltages [8, 9]. For measurements at ESR and CRYRING@ESR, the electron cooler determines the ion velocity and momentum spread of the ions by superimposing the ion beam with a mono-energetic electron beam, which is why it is such a critical quantity for the accuracy of the experiments. The new divider presented in this thesis provides the possibility to measure high voltages up to $35 \,\text{kV}$ with an accuracy of $< 2 \,\text{ppm}$. Its design is based on the world's most precise high-voltage dividers MT100 [14] of PTB and K35/K65 of the KATRIN experiment [11, 12]. For the construction of the new divider, resistors were available that were leftover from the construction of the KATRIN dividers. Since the left-over resistors showed only negative temperature coefficients (TCR), it was not possible to match resistors with positive and negative TCR to minimize the divider's' voltage dependency, as was done for the KATRIN dividers. The G35 divider, therefore, shows a non-negligible voltage dependency over the whole input voltage range, which has been measured with ppm-precision by utilizing a newly developed absolute calibration method.

The calibration method is based on an idea of C. Weinheimer to determine the voltagedependent scale factor M by measuring a differential scale factor \widetilde{M} . The development was a joint effort by O. Rest, D. Winzen, and C. Weinheimer (published in [13, 20, 125]). In conclusion, the new method constitutes the most precise, fully traceable calibration to determine precision high-voltage divider scale factors for absolute input voltages greater than 1 kV. Applying this method brings the G35's scale factor precision into the same 1 ppm-regime as the KATRIN and PTB dividers since all five available scale factors could be characterized over the whole voltage range. The first months of calibration measurements indicate a measurement reproducibility in the sub-ppm regime over a month and a stability of the scale factors in an ± 1 ppm interval over a year. After construction and full characterization, the divider was delivered to GSI, where it is in use for electron cooler voltage measurements at CRYRING@ESR. To guarantee the divider's longterm-stability in the future, further monitoring has to be conducted via calibration measurements on a regular basis over the coming months and years. Consequently, operating the G35 with up-to-date calibrations from the new method opens up new possibilities for high precision measurements at CRYRING@ESR in the coming years.

In addition to the static DC-measurements of the electron cooler voltage at CRYRING-@ESR, voltage measurements in time intervals of approximately 10 ms are planned within the framework of dielectric recombination measurements [135] with a desired accuracy of approximately 10 ppm. Studies in the context of the work of T. Dirkes [136] and D. Roth [137] have shown that the G35 does not meet the precision requirements for these fast voltage measurements in the required time interval. Therefore, as an extension to the existing divider, a frequency-compensated high voltage divider add-on specially adapted to the recombination measurements has been developed that is able to deliver voltage measurements in the required time interval with the desired precision. The results of this development were published in the master thesis by T. Dirkes [138] end of 2020. Appendix A



A.1. XUV detector alignment



Figure A.1: Scheme of the scrapers which are used to point the position of the ion beam in vertical and horizontal direction. They can be driven to certain positions, and the cross-hair of the telescope can be adjusted to the exact position. The detector will then be orientated according to the cross-hair. Figure and caption taken from [77].

A.2. Complete fluorescence scans



Figure A.2: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 248 \text{ mA}$



Figure A.3: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 238 \text{ mA}$



Figure A.4: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 228 \text{ mA}$



Figure A.6: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 208 \text{ mA}$



Figure A.5: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 218 \text{ mA}$



Figure A.7: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 198 \text{ mA}$



Figure A.8: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 188 \text{ mA}$



Figure A.9: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 179 \text{ mA}$



Figure A.10: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 169 \text{ mA}$



Figure A.12: Fluorescence scan analysis for ${}^{2}S_{1/2} - {}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 149 \text{ mA}$



Figure A.11: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 159 \text{ mA}$



Figure A.13: Fluorescence scan analysis for ${}^{2}S_{1/2} {}^{-2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 139 \text{ mA}$



Figure A.14: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 129 \text{ mA}$



Figure A.15: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 119 \text{ mA}$



Figure A.16: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 109 \text{ mA}$



Figure A.18: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 99 \text{ mA}$



Figure A.17: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 109 \text{ mA}$



Figure A.19: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 89 \text{ mA}$



Figure A.20: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 79 \text{ mA}$



Figure A.22: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 60 \text{ mA}$



Figure A.24: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 40 \text{ mA}$



Figure A.21: Fluorescence scan analysis for ${}^{2}S_{1/2} - {}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 69 \text{ mA}$



Figure A.23: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 50 \text{ mA}$



Figure A.25: Fluorescence scan analysis for ${}^{2}S_{1/2} {}^{-2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 30 \text{ mA}$


Figure A.26: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 19 \text{ mA}$



Figure A.27: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 11 \text{ mA}$



Figure A.28: Fluorescence scan analysis for ${}^{2}S_{1/2} {}^{-2}P_{1/2}$ level splitting at cooler current $I_{ecool} \approx 248 \text{ mA}$



Figure A.30: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 248 \text{ mA}$



Figure A.29: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 248 \text{ mA}$



Figure A.31: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 238 \text{ mA}$



Figure A.32: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 228 \text{ mA}$



Figure A.34: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 199 \text{ mA}$



Figure A.36: Fluorescence scan analysis for ${}^{2}S_{1/2} - {}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 159 \text{ mA}$



Figure A.33: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 218 \text{ mA}$



Figure A.35: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 179 \text{ mA}$



Figure A.37: Fluorescence scan analysis for ${}^{2}S_{1/2} {}^{-2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 139 \text{ mA}$



Figure A.38: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 119 \text{ mA}$



Figure A.39: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 99 \text{ mA}$



Figure A.40: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 79 \text{ mA}$



Figure A.42: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 50 \text{ mA}$



Figure A.41: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 60 \text{ mA}$



Figure A.43: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 40 \text{ mA}$



Figure A.44: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 19 \text{ mA}$



Figure A.45: Fluorescence scan analysis for ${}^{2}S_{1/2}$ - ${}^{2}P_{3/2}$ level splitting at cooler current $I_{ecool} \approx 11 \text{ mA}$

A.3. Uncertainty budget for (1s²2s)²S_{1/2}-²P_{3/2} level splitting

Table A.1.: Estimated uncertainty budget for ${}^{2}S_{1/2} - {}^{2}P_{3/2}$ transition for all relevant contributions. The uncertainties are ordered by the size of their contribution. For all parameters p given in the table, a Gaussian distribution (1σ) for the respective uncertainty Δp was considered. The contribution of each parameter is the product of the sensitivity coefficient $\partial \lambda / \partial p$ and Δp . The relative importance of each contribution is derived by $(\partial \lambda / \partial p \cdot \Delta p)^{2} / (\Delta \lambda)^{2}$. All values have been rounded to fit the table format.

parameter	value p	abs. unc. Δp	unit	sensitivity coeff.	contribution (nm)	rel. imp. (%)
work function	2.84	1.42	eV	$5.72\cdot 10^{-4}$	0.000 81	48.65
scale factor	9999.698	0.155		$-3.85\cdot10^{-3}$	-0.00060	26.33
DVM gain	$1.000\ 000\ 6$	0.0000131		-38.46	-0.00050	18.67
e-cooler current	var.	2	mА	n/a	0.000 26	5.04
DVM offset	0.00000	0.00002	V	-5.72	-0.00011	0.96
ion space charge	var.	max.	V	n/a	0.00004	0.14
laser frequency	291.433 40	0.00007	THz	$-5.31 \cdot 10^{-13}$	-0.00004	0.10
MCP gain	var	max.		n/a	0.000 03	0.06
angle laser - ion	0.000	+0.036	0	n/a	0.000 03	0.05
total uncertainty	154.82107	0.001 16	nm			100.00

A.4. G35 LTSpice simulation



Figure A.46: Equivalent circuit diagram of the G35 high voltage divider used for the LTspice simulations. From left to right: Stray capacities between copper electrodes and tank wall C_tank, secondary divider chain (protection chain) consisting of resistors and capacitors further subdivided into the secondary HV chain (green) and the secondary tap (magenta), stray capacities between copper electrodes and primary precision chain (C_feedthr., C_meas.), primary precision chain subdivided into primary HV chain (red) and primary taps (blue), voltage source (brown) and measurement equipment (orange) consisting of the cable capacitance and the DVM input capacitance and resistance.



Figure A.47: Small signal analysis of G35 primary divider chain with M_{3452} tap attached. Shown is the reciprocal scale factor $1/M_{3452}$ in dependence of the input signal frequency (red, left y-axis) and the corresponding phase shift (blue, right y-axis).



Figure A.48: Small signal analysis of G35 secondary divider chain. Shown is the reciprocal scale factor $1/M_{\text{secondary}}$ in dependence of the input signal frequency (red, left y-axis) and the corresponding phase shift (blue, right y-axis).

A.5. G35 outer dimensions



Figure A.49: Outer dimensions of G35 high-voltage divider.

A.6. G35 tunnel tube technical drawings



Figure A.50: Tunnel tube dimensions for 0° view.



Figure A.51: Tunnel tube dimensions for 45° view.



Figure A.52: Tunnel tube dimensions for 90° view.



Figure A.53: Tunnel tube dimensions for 135° view.



Figure A.54: Tunnel tube dimensions for 180° view.



Figure A.55: Tunnel tube dimensions for 225° view.



Figure A.56: Tunnel tube dimensions for 270° view.



Figure A.57: Tunnel tube dimensions for 315° view.

A.7. G35 low voltage calibration results



Figure A.58: Low voltage calibration history for G35 M_{100} scale factor at 15 °C.



Figure A.59: Low voltage calibration history for G35 M_{163} scale factor at 15 °C.



Figure A.60: Low voltage calibration history for G35 M_{269} scale factor at 15 °C.



Figure A.61: Low voltage calibration history for G35 M₇₇₅ scale factor at 15 °C.



Figure A.62: Low voltage calibration history for G35 M_{3452} scale factor at 15 °C.



Figure A.63: Low voltage calibration history for G35 M_{100} scale factor at 20 °C.



Figure A.64: Low voltage calibration history for G35 M_{163} scale factor at 20 °C.



Figure A.65: Low voltage calibration history for G35 M₂₆₉ scale factor at 20 °C.



Figure A.66: Low voltage calibration history for G35 M_{775} scale factor at 20 °C.



Figure A.67: Low voltage calibration history for G35 M₃₄₅₂ scale factor at 20 °C.

A.8. G35 absolute calibration results



Figure A.68: Voltage dependency of the G35 100.46:1 scale factor at 20 °C determined with the newly developed absolute calibration method. From the measurement of the differential scale factor (red points), the real scale factor (blue line) can be determined by fitting the differential scale factor and obtaining the fit parameters *a*, *b*, *c*, and *d*. The result has been cross-checked with a second precision HV divider (K65) (green points). Measured at 20 °C. Figure and caption taken from [20]. The figure has been modified to fit the format of this thesis.

Table A.2.: Coefficients *a*, *b*, *c*, *d* and their respective covariances to determine the G35 scale factors for a divider temperature of 20 °C and negative polarity. To calculate the voltage dependent scale factors the values in this table are applied to equation (6.14). The uncertainties are determined by applying the covariances according to equation (6.15). In contrast to the 15 °C measurements, the values have not been updated since July 2018.

coefficient	value	uncertainty
a	$9.95377088 \cdot 10^{-3}$	$2.56 \cdot 10^{-9}$
b	$6.74 \cdot 10^{-10}$	$5.02 \cdot 10^{-10}$
с	$1.5 \cdot 10^{-12}$	$2.54 \cdot 10^{-11}$
d	$7.11 \cdot 10^{-13}$	$3.71 \cdot 10^{-13}$
$M_{100 \text{ factor}}$	1	
M_{163} factor	1.6196137	$1.5 \cdot 10^{-6}$
$M_{269 \text{ factor}}$	2.6774450	$1.6 \cdot 10^{-6}$
M_{775} factor	7.7189633	$2.7 \cdot 10^{-6}$
M_{3452} factor	34.357172	$2.0\cdot 10^{-5}$
Cov(a, b)	$-8.71 \cdot 10^{-19}$	
Cov(a, c)	$3.60 \cdot 10^{-20}$	
Cov(a, d)	$-4.64 \cdot 10^{-22}$	
Cov(b, c)	$-1.24 \cdot 10^{-20}$	
Cov(b, d)	$1.72 \cdot 10^{-22}$	
$\operatorname{Cov}(c,d)$	$-9.29 \cdot 10^{-24}$	



Figure A.69: Voltage dependency of the G35 100.46:1 scale factor at 15 °C determined with the newly developed absolute calibration method.



Figure A.70: Voltage dependency of the G35 100.46:1 scale factor at 20 °C determined with the newly developed absolute calibration method.



Figure A.71: Voltage dependency of the G35 162.71:1 scale factor at 15 °C determined with the newly developed absolute calibration method.



Figure A.72: Voltage dependency of the G35 162.71:1 scale factor at 20 °C determined with the newly developed absolute calibration method.



Figure A.73: Voltage dependency of the G35 268.99:1 scale factor at 15 °C determined with the newly developed absolute calibration method.



Figure A.74: Voltage dependency of the G35 268.99:1 scale factor at 20 °C determined with the newly developed absolute calibration method.



Figure A.75: Voltage dependency of the G35 775.48:1 scale factor at 15 °C determined with the newly developed absolute calibration method.



Figure A.76: Voltage dependency of the G35 775.48:1 scale factor at 20 °C determined with the newly developed absolute calibration method.



Figure A.77: Voltage dependency of the G35 3451.69:1 scale factor at 15 °C determined with the newly developed absolute calibration method.



Figure A.78: Voltage dependency of the G35 3451.69:1 scale factor at 20 °C determined with the newly developed absolute calibration method.

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List of Abbreviations

ASF	atomic state function	DVM	digital voltmeter
BBO	beta-barium borate	ECDL	external cavity diode laser
BKG	background	ECR	electron cyclotron resonance
BPS	beam position stabilization	ECRIS	electron cyclotron resonance
BW	Bohr-Weisskopf		ion source
CAD	computer-aided design	ESR	Experimental Storage Ring
CC	coupled-cluster	FAIR	Facility for Antiproton and
CFD	computational fluid dynamics		Ion Research
CFD	constant fraction	fns	finite nuclear size
	discriminator	FPGA	field programmable gate
CR	Collector Ring		array
CSF	configuration-state function	FSCC	Fock space coupled-cluster
CsI	cesium iodide	FWHM	full width at half maximum
CW	continuous wave	GSI	Gesellschaft für
DAC	digital-to-analog converter		Schwerionenforschung
DAQ	data acquisition	HCI	highly charged ions
DC	direct current	HES	heat exchange system
DHF	Dirac-Hartree-Fock	HESR	High-Energy Storage Ring

HITRAP	Highly charged Ion Trapping	MP	Möller-Plesset
	facility	NEG	non-evaporable getter
HKR	Hauptkontrollraum (main control room)	NI	National Instruments
Hl	hydrogen line	PID	proportional-integral-
HLI	Hochladungsinjektor (high charge state injector)	РОМ	polyoxymethylene
HV	high voltage	PPLN	periodically poled lithium
HVC	high-voltage calibration		niobate
HVDC	high voltage direct current	ppm	parts per million
HWHM	half width at half maximum	РТВ	Physikalisch-Technische Bundesanstalt
HZDR	Helmholtz-Zentrum Dresden–Rossendorf	QED	quantum electrodynamics
IR	infrared	RC	resistor-capacitor
JRL	Julie Research Lab	RCI	relativistic configuration
KATRIN	Karlsruhe Tritium Neutrino		interaction
	Experiment	RF	radio frequency
KIT	Karlsruhe Institute of Technology	RFQ	radio frequency quadrupole
LiBELLE	Lithium like Bismuth	RMBPT	relativistic many-body perturbation theory
	at the ESR	RTD	resistance temperature
LVC	low-voltage calibration		detector
MBS	Multi Branch System	SCF	self-consistent field method
МС	multiconfiguration	SE	self-energy
MCDHF	multiconfiguration Dirac-Hartree-Fock	SIS	Schwerionensynchrotron (Heavy-Ion-Synchrotron)
МСР	microchannel plate detector	SNR	signal-to-noise ratio

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SPARC	Stored Particles Atomic	UNILAC	Universal Linear Accelerator
	Physics Research	ΙW	ultraviolet
	Collaboration	U V	uttraviolet
TCR	temperature coefficient of	VP	vacuum polarization
	resistance	VUPROM	VME Universal Processing
TDC	time-to-digital-converter		Module
TU	Technische Universität	WLM	wavelength meter
	(technical university)	XUV	extreme ultraviolet
UHV	ultra-high vacuum		

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Eine entscheidende Rolle während dieser Arbeit haben aber nicht nur die AG Weinheimer und das IKP Münster gespielt. Da die Instrumente in dieser Arbeit für Experimente bei FAIR an der GSI eingesetzt werden, gab es natürlich immer eine sehr enge Kooperation mit den Kollegen der GSI und der TU Darmstadt. Besonders bedanken möchte ich mich bei Danyal und Sebastian. Trotz der vielen Arbeit habe ich die Zeit an der GSI immer genossen, was vor allen Dingen an den beiden Kollegen lag. Auch danke, dass ihr immer geduldig Fragen beantwortet habt, die besonders während des Schreibens aufgekommen sind. Generell waren an der GSI die Kollegen, sei es am ESR oder am CRYRING, immer sehr aufgeschlossen und hilfsbereit, was gerade das Experimentieren vor Ort erfolgreich gemacht hat.

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Persönliche Informationen Geburtsdatum 07.03.1984 Geburtsort Münster Familienstand verheiratet Berufserfahrung seit 08/2020 Softwareentwickler, SAFELOG GmbH, Markt Schwaben. • Softwareentwickung für fahrerlose Transportsysteme (FTS). • Softwareentwickung für intelligente Kommissionierwagen. • Prozessautomatisierung im Bereich Intralogistik. • Hardwareanbindung. • Datenbankanbindung. 05/2014 - 08/2020 Wissenschaftlicher Mitarbeiter, WWU Münster, Institut für Kernphysik. • Entwicklung und Konstruktion eines Präzisionshochspannungsteilers mit international führender Genauigkeit inkl. eines integrierten Regel- und Messaufbaus zum autonomen Betrieb an der Teilchenbeschleuniger-Anlage FAIR in Darmstadt. • Mitentwicklung und Anwendung der weltweit präzisesten Kalibrationsmethode für Präzisionshochspannungsteiler, u. a. in Kooperation mit der Physikalisch-Technischen Bundesanstalt (PTB). • Weiterentwicklung und Inbetriebnahme eines Fluoreszenzphotonendetektors am Ionenspeicherring ESR in Darmstadt. • Analyse und Visualisierung komplexer Experimentierdaten. • Betreuung von Bachelor- und Masterarbeiten sowie von Studierenden im Rahmen von experimentellen Praktika und Seminarvorträgen. 10/2010 – 04/2014 Studentische Hilfskraft, WWU Münster, Institut für Kernphysik. • Installation eines Drahtelektrodensystems für das KATRIN Experiment am Karlsruher Institut für Technologie (KIT). • Diverse Laborarbeiten im Institut für Kern- und Teilchenphysik der WWU Münster. 10/2003 – 10/2004 Freiwilliges Soziales Jahr, DRK-Kreisverband Münster e.V. • Tätigkeit als Erste-Hilfe-Ausbilder. Ausbildung

seit 10/2014 Promotionsstudium Physik, WWU Münster, Institut für Kernphysik. Thema: "Laser spectroscopy of lithium-like carbon with a novel XUV detection system at the ESR and construction of a precision high voltage divider for the CRYRING@ESR electron cooler", Abschluss: voraussichtlich August 2020.
11/2014 – 12/2016 Stipendium, HGS-HIRe for FAIR.

Stipendium der "Helmholtz Graduate School for Hadron and Ion Reasearch" (HGS-HIRe) für Forschungsarbeiten in Verbindung mit GSI- und FAIR-Projekten.

- 10/2004 04/2014 Studium Physik, WWU Münster, Institut für Kernphysik. Abschluss: Diplom, Note: 1,9. Abschlussarbeit: "Development of an angular selective electron gun for the KATRIN main spectrometer".
 - 1996 2003 **Abitur**, *Annette-von-Droste-Hülshoff-Gymnasium*, Münster. *Abschluss*: Allgemeine Hochschulreife, Note: 2,2.
 - 1994 1996 Gymnasium, Wilhelm-Hittorf-Gymnasium, Münster.

Publikationen (Auswahl)

- 2020 Laser spectroscopy of the ²S_{1/2}-²P_{1/2}, ²P_{1/2} transitions in stored and cooled relativistic C³⁺ ions, submitted to Scientific Reports, D. Winzen, V. Hannen *et al.*
- 2020 Absolute Calibration of a Ppm-Precise HV Divider for the Electron Cooler of the Ion Storage Ring CRYRING@ESR, LNEE 599 pp. 1500-1512 (2020), O. Rest, D. Winzen et al.
- 2019 A novel ppm-precise absolute calibration method for precision high-voltage dividers, *Metrologia* 56 045007 (2019), O. Rest, D. Winzen *et al.*
- 2017 A pulsed, mono-energetic and angular-selective UV photo-electron source for the commissioning of the KATRIN experiment, *Eur. Phys. J. C (2017) 77:* 410, J. Behrens *et al.*

Präsentationen (Auswahl)

- 2010 2019 **DPG Frühjahrstagungen (3 Vorträge und 6 Posterpräsentationen)**. Auszeichnung (2015): 1. Posterpreis von 119 Teilnehmern.
 - 11/2018 Atomphysik-Seminar an der GSI (eingeladener Vortrag), Darmstadt. "High-voltage measurement and calibration techniques for precision experiments with electron coolers at ion storage rings".
 - 09/2018 **19th International Conference on the Physics of Highly Charged Ions (Vortrag)**, Lissabon (Portugal). "Results from the commissioning of a detection system for forward emitted XUV photons at the ESR".

Fortbildungen/Seminare (Auswahl)

- 2016 2017 HGS-HIRe Soft Skills Training I-III, Eiterfeld.
 Fortbildungen mit den Themen Karriereentwicklung und Mitarbeiterführung, durchgeführt von Experten der britischen Firma "Coaching4Careers".
 - 08/2015 **International Summer School on Metrology 2015**, Drübeck. Themenschwerpunkte: Grundkonzepte der Messtechnik, Neudefinition der Einheiten im SI-Einheitensystem, Bestimmung von Messunsicherheiten

Kenntnisse und Fähigkeiten (Auswahl)

- Hardware Ansteuerung und Auslesen von Sensoren, Fahrerlose Transportsysteme, Messtechnik, Prozessautomatisierung, Hochspannungstechnik, Detektorentwicklung, Elektrotechnik, Vakuumtechnik.
- IT-Kenntnisse Fundierte Kenntnisse: Python, NI LabVIEW, Linux, MS-Office, La Katolice, La Katolice, Datenvisualisierung, C#, SQL; Grundkenntnisse: C++, LTspice
- Fremdsprachen Englisch (verhandlungssicher).

Ehrenamtliche Tätigkeiten

2016 - 2019	Mitorganisator "Astroseminar an der WWU", Münster.	
	Münsters größte jährliche Veranstaltung im Bereich Wissenschaftskommunikation (ca. 400	
	Teilnehmer)	
	Aufgaben: Verwaltung der Finanzen, Beantragung von Fördergeldern, Anwerben von Red-	
	nern, Design von Werbematerial, Tagungsorganisation.	
2015 - 2019	Vermittler "Netzwerk Teilchenwelt" (NTW).	
	Durchführung eintägiger Workshops zum Thema Astroteilchenphysik mit Oberstufenschü-	
	lern.	
2005 - 2010	Helfer im Verein "AusZeit im UKM", Münster.	
	Unterstützung krebskranker Kinder auf der kinderonkologischen Station des Universitäts-	
	klinikums Münster.	
2004 - 2005	DRK-Kreisverband Münster e.V., Münster.	

Tätigkeit als Erste-Hilfe-Ausbilder.

Münster, 2. Januar 2021

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