Outlook 00

# Supermultiplets in $\mathcal{N}=1\,$ SU(2) SUSY Yang-Mills Theory

### Henning Gerber

Münster-DESY(-Regensburg-Jena) collaboration

#### GRK Retreat 2017 - Marienheide

Westfällsche Wilhelms-Universität Münster



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• 
$$\mathcal{L} = \frac{1}{2} \operatorname{Tr} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\mathrm{i}}{2} \overline{\tilde{g}} D \widetilde{g} - \frac{m_g}{2} \overline{\tilde{g}} \widetilde{g} \right)$$

•  $A^a_\mu(x)$  : gauge fields

• g(x): gluino fields, Majorana fermions in adjoint representation

• 
$$(D_{\mu}\tilde{g})^{a} = \partial_{\mu}\tilde{g}^{a} + gf_{abc}A^{b}_{\mu}\tilde{g}^{c}$$
, here:  $f_{abc} = \epsilon_{abc}$ 

•  $\frac{m_g}{2}\overline{\tilde{g}}\widetilde{g}$ : soft SUSY-breaking term

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G. R. Farrar, G. Gabadadze, M. Schwetz, ", Phys. Rev. D60 (1999) 035002, arXiv:hep-th/9806204.

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SYM So Expectation values in lattice field theory Results 0000 Outlook 00

• Interested in 
$$\langle O \rangle = \frac{\int \mathcal{D} \left[\Phi, \Psi\right] e^{-S_{\mathcal{E}}\left[\Phi\right]} O\left[\Phi, \Psi\right]}{\int \mathcal{D}[\Phi, \Psi] e^{-S_{\mathcal{E}}\left[\Phi, \Psi\right]}}$$

• Integrate out the fermions  $\Rightarrow Z = \int \mathcal{D} [\Phi, \Psi] e^{-S_G[\Phi]} e^{-S_F[\Phi, \Psi]} = \int \mathcal{D} [\Phi] e^{-S_G[\Phi]} \det Q$ 

• det  $Q=\int \mathcal{D}\left[\Psi
ight]e^{-ar{\Psi}Q\Psi}$  : Fermion determinant

- For different lattice spacings *a* and (bare) gluino-masses  $m_g$  generate ensembles of lattice gauge configurations with probability distribution  $e^{-S_E[\Phi]} \det Q$ 
  - Hybrid Monte Carlo (PHMC)
- Calculate  $\langle O 
  angle$  on the ensembles
- for each *a* perform chiral extrapolation  $m_{\pi}^2 
  ightarrow 0$
- continuum extrapolation a 
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SYM Sympositive 

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$$\langle O \rangle = \frac{\int \mathcal{D}[\Phi, \Psi] e^{-S_{E}[\Phi]} O[\Phi, \Psi]}{\int \mathcal{D}[\Phi, \Psi] e^{-S_{E}[\Phi, \Psi]}}$$

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Techniques Expectation values in lattice field theory

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• continuum extrapolation  $a \rightarrow 0$ 



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SYM So Extracting masses from Correlation Function

• Correlation function  $ilde{C}(x,y) = \left< O(x) O^{\dagger}(y) \right>$ 

- Operator O is interpolating field,
   e.g. O(x) = \$\tilde{g}(x)\$\tilde{g}(x)\$
- Translation invariance + 0-momentum projection leads to  $C(\Delta t) = N \sum_{\vec{x}} \langle O(\Delta t) O^{\dagger}(0) \rangle$

$$^{ ext{a-}}\eta'$$
- glueball @ $eta=1.9,\ \kappa=0.14435$ 

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• Spectral decomposition:

$$C(\Delta t) = a_0^2 + \sum_{n=1}^{\infty} a_n^2 e^{-E_n \Delta t} \pm e^{-E_n(T - \Delta t)}$$

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Results

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a- $\eta'$ - glueball @ $eta=1.9,\ \kappa=0.14435$ 



Glueballs:

•  $O(x) = \sum_{i < j} P_{ij}(x)$ 



• Gluino-glue:  $O^{\alpha}(x) = \sum_{i < i} \sigma_{ij}^{\alpha\beta} Tr_c \left[ P_{ij}(x) \tilde{g}^{\beta}(x) \right]$ 

• Mesons:  $O(x) = \overline{\tilde{g}}(x)\Gamma \tilde{g}(x)$ 

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- Improve signal-to-noise ratio by applying smearing techniques:
- For gluonic operators: APE Smearing

$$U_{\mu}'(x) = U_{\mu} + \epsilon_{\mathsf{APE}} \sum_{
u=\pm 1, 
u 
eq \mu}^{\pm 3} U_{
u}^{\dagger}(x+\mu)U_{\mu}(x+\hat{
u})U_{
u}(x)$$

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- For mesonic operators: Jacobi Smearing  $\tilde{g}'(\vec{x},t) = \sum_{\vec{y}} F(\vec{x},\vec{y}) \tilde{g}(\vec{y},t)$
- F(x, y) = Iterative solution of 3d Klein-Gordon equation for source and sinks :

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### SYM Smearing

Jacobi Smearing

 $R_J =$ 

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Radius [a]

Smearing Radius:

 $\kappa_J = 0.1 \\ \kappa_J = 0.15$ 

 $\kappa_J = 0.2$   $\kappa_J = 0.225$  $\kappa_I = 0.25$ 

#### Techniques ○○○○○●○○○

 $\frac{\sum_{\vec{x}} |\vec{x}|^2 |F(\vec{x},0)|^2}{\sum_{\vec{x}} |F(\vec{x},0)|^2}$ 



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#### APE smearing:

- similar analysis as for Jacobi smearing
- $\epsilon_{\rm APE} \sim 0.4$

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- smearing levels up to 95 for  $32^3 \times 64$  lattice
- i.e. {5,15,...,95} for gluino-glue

70 80 90 100

40 50 60 Smearing Level

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### SYM Smearing

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## Techniques

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Results

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# Smearing

Jacobi Smearing

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Smearing Radius:

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#### Techniques 0000000000

 $\frac{\sum_{\vec{x}} |\vec{x}|^2 |F(\vec{x},0)|^2}{\sum_{\vec{x}} |F(\vec{x},0)|^2}$ 

Results

### APE smearing:

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Smearing Level  $\Rightarrow$ Use  $\kappa_{\text{Jacobi}} = 0.2$  $\Rightarrow$ Use smearing levels up to 80  $\Rightarrow$ Optimizing the signal lead to choosing smearing levels 0, 40 and 80 Supermultiplets in SYM Henning Gerber

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sym oo Variational method Techniques

Results

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- Correlation Matrix  $C_{ij}(\Delta t) = \left\langle O_i(\Delta t) O_j^{\dagger}(0) 
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  angle$
- Generalized Eigenvalue Problem (GEVP):  $C(t)\vec{v}^{(n)} = \lambda^{(n)}(t, t_0)C(t_0)\vec{v}^{(n)}$
- $\lim_{t\to\infty} \lambda^{(n)}(t,t_0) \propto e^{-m_n(t-t_0)} \left(1 + \mathcal{O}\left(e^{-\Delta m_n(t-t_0)}\right)\right)$ 
  - $\Delta m_n = \min_{l \neq n} |m_l m_n|$

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Outlook 00

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Techniques

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Outlook 00

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sym oo Variational method

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Results

### SYM Solution Supersymmetry in GEVP

Results

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- Mixing of meson and glueball states allows using a larger operator basis for determining the masses in the  $0^{++}{\rm and}~0^{-+}{\rm channel}$ 
  - e.g.  $a-f_0$  and  $0^{++}$ -glueball
- Build correlation matrix C from mesonic and gluonic operators:

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- Configurations produced at Jülich Supercomputing Centre
- Measurements performed on
  - local HPC clusters in Münster
    - PALMA, NWZPHI (Xeon-Phi)
  - Jülich Supercomputing Centre

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collaboration arXiv 1512 07014

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Linear chiral extrapolation at  $\beta = 1.9$ 

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Linear chiral extrapolation at  $\beta = 1.9$ 

Quadratic chiral extrapolation at  $\beta = 1.9$ 

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#### Techniques

Results ○○●○ Outlook 00

• Eigenvalues need to be ordered correctly



Generalized eigenvalues of 0<sup>++</sup>-channel at eta= 1.9,  $\kappa=$  0.1433 sorted by value

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SYM	Techniques	Results	Outlook
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Challenges			





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SYM	Techniques	Results	Outlook
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Challenges			

• Eigenvalues need to be ordered correctly



Generalized eigenvalues of 0<sup>++</sup>-channel at  $\beta=$  1.9,  $\kappa=$  0.1433 sorted by new method

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Supermultiplets in SYM

SYM	Techniques	Results	Outlook
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Supermultiplets in SYM

SYM	Techniques	Results	Outlook
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Challenges			

- Eigenvalues need to be ordered correctly
- Eigenvalues don't always behave as expected
  - large autocorrelation



 $0^{-+}$  glueball  $@\beta = 1.9, \ \kappa = 0.14435$ 

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SYM	Techniques	Results	Outlook
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Challenges			

- Eigenvalues need to be ordered correctly
- Eigenvalues don't always behave as expected
  - large autocorrelation
  - use "Derivative trick":  $\Rightarrow$  Use  $\tilde{C}(t) =$ C(t) - C(t+1)''



 $0^{-+}$  glueball  $@\beta = 1.9, \ \kappa = 0.14435$ 

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- Second order polynomial fit fits the data too well
  - errorbars overestimated?
- Within errors there seems so be no mixing in the 0<sup>-+</sup>-channel
  - off-diagonal entries of the correlation matrix are zero within errors
- 0<sup>-+</sup>-glueball and  $a \eta'$  masses differ

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SYM	Techniques	Results	
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Summary & Outlook			

- Agreement with the predicted supermultiplet structure for the groundstate
   ⇒ SUSY is restored in the continuum limit
- 0<sup>-+</sup>-glueball groundstate heavier than groundstate multiplet ⇒might become lighter in the continuum limit ⇒maybe it belongs to the first excited multiplet
- Work to be done:
  - Continuum extrapolation of the excited multiplet
  - reanalysis of groundstate multiplet using the full correlation matrix
  - Determine mixing between glueball and mesonic states using the variational method:
    - learn about mixing in the SUSY-phase

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Outlook 0

## Thank you for your attention!!

Westfälische Wilhelms-Universität Münster

Graduiertenkolleg 2149 Research Training Group

Supermultiplets in SYM

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(groundstate)

Extrapolation to the chiral limit (first excited state)

S. Kuberski, Masterthesis, Uni Münster

Supermultiplets in SYM

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- 日本 (四本) (日本 (日本)

- Calculate the lowest eigenvalues of *Q* and corresponding eigenvectors
  - using Arnoldi (ARPACK)
  - Chebyshev Polynomials of order 11
  - Even/Odd-Preconditioning
- Stochastic estimator technique for space orthogonal to the previously calculated eigenvectors:

• 
$$\frac{1}{N_{S}}\sum_{i}^{N_{S}}\left|\eta^{i}\right\rangle\left\langle\eta^{i}\right|=\mathbb{1}+\mathcal{O}\left(\sqrt{N_{S}}\right)$$

use ℤ₄-noise

• 
$$Q |s^{i}\rangle = |\eta^{i}\rangle$$
  
•  $Q^{-1} = \frac{1}{N_{s}} \sum_{i}^{N_{s}} |s^{i}\rangle \langle \eta^{i}\rangle$ 

- Conjugate gradient
- $N_S = 40$  for  $\beta = 1.9$ ,  $32^3 \times 64$

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