Determination of charmed decay constants via Lattice QCD

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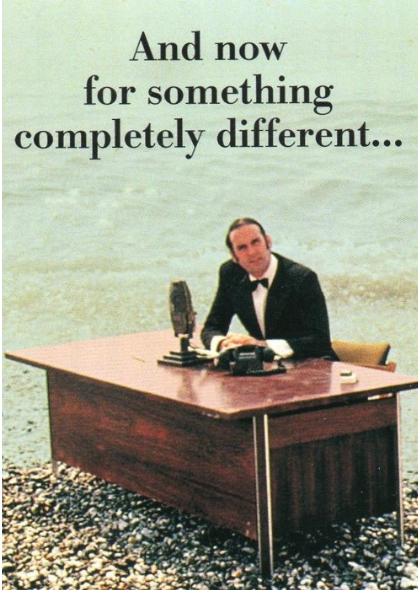
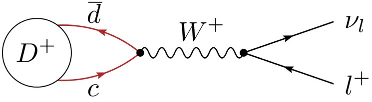

And now
for something
completely different...

Outline

- Motivation
- Lattice basics and utilized ensembles
- Analysis techniques
- Preliminary results
- Summary



Branching ratio for the leptonic decay of $D_{(s)}$:

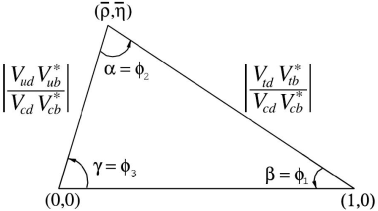
$$\mathcal{B}(D_{(s)} \to l\nu) = \frac{G_F^2 \, |V_{cq}|^2}{8\pi} f_{D_{(s)}}^2 m_l^2 m_{D_{(s)}} \left(1 - \frac{m_l^2}{m_{D_{(s)}}^2}\right)^2$$

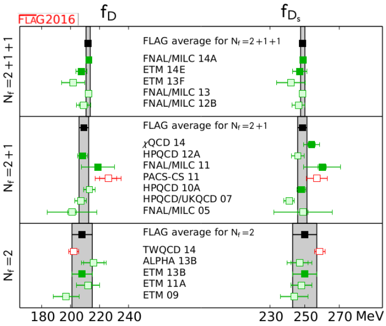
- experimentally accessible: product of $|V_{cq}| \times f_{D_{(s)}}$
- theoretical input on $f_{D_{(s)}}$ allows determination of V_{cq}

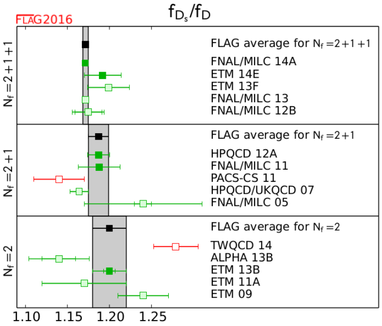
→ constrain unitarity triangle, test standard model, search for BSM physics

Goal of PhD project:

Precision computation of $f_{D_{(s)}}$ on the lattice.







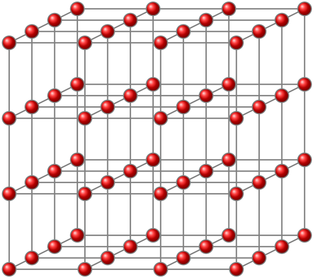
recent results from FLAG Working Group [arXiv:1607.00299]

→ errors now approaching the percent level

→ crucial to contrast with independent results

→ results so far show 2 σ deviation of lattice averages from experimental averages given by Particle Data Group [K.A. Olive et al (Particle Data Group). Chin. Phys. C, 2014, 38(9): 090001]

Ensembles were generated within Coordinated Lattice Simulations (CLS) effort. Work groups are currently active at:
CERN, DESY/NIC, Dublin, Berlin, Mainz, Madrid, Milan, Münster, Odense, Regensburg, Roma-La Sapienza, Roma-Tor Vergata, Valencia, Wuppertal



Properties of ensembles:

- $N_f = 2 + 1$ non-perturbatively $\mathcal{O}(a)$ improved Wilson-Sheikholeslami-Wohlert fermions

  → Wilson Dirac operator for fermions, with Sheikholeslami-Wohlert term

$$D_w(m_{0,f}) = \frac{1}{2} \sum_{\mu=0}^{3} \left(\gamma_\mu(\nabla_\mu^* + \nabla_\mu) - a\, \nabla_\mu^* \nabla_\mu\right)$$

$$+ a c_{sw} \sum_{\mu,\nu=0}^{3} \frac{i}{4} \sigma_{\mu\nu} \hat{F}_{\mu\nu} + m_{0,f}$$

  → naive action already $O(a^2)$, but exibits fermion-doublers due to finite volume effects

  → $a\, \nabla_\mu^* \nabla_\mu$ removes fermion-doublers from theory, but introduces $O(a)$ effects → removed by third term

- tree-level $\mathcal{O}(a)$ improved Lüscher-Weisz gauge action → Lüscher-Weisz gauge action

$$S_g[U] = \frac{\beta}{6}\left(c_0 \sum_p \mathrm{tr}(1 - U(p)) + c_1 \sum_r \mathrm{tr}(1 - U(r))\right)$$

  with $\beta = \frac{6}{g_0^2}$ (g_0 being the bare gauge coupling), $c_0 = \frac{5}{3}$, $c_1 = -\frac{1}{12}$ being tree level coefficients

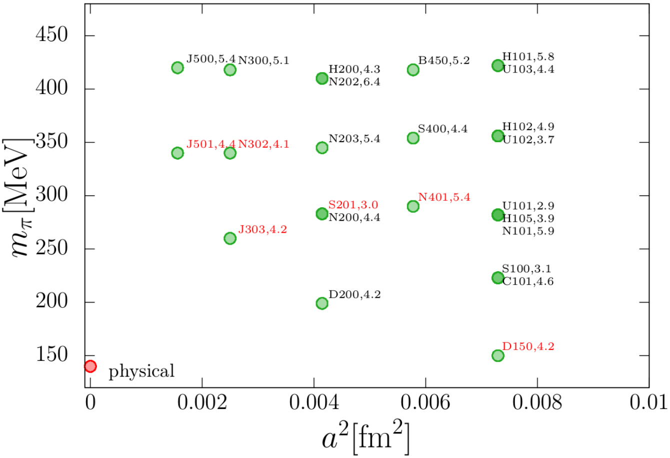
  → naive action already $O(a^2)$ improved at tree level, second term includes higher order corrections

- lattice spacings from $a \approx 0.085$fm to $a \approx 0.039$fm (corresponding to $\beta = 3.4$ to $\beta = 3.85$)

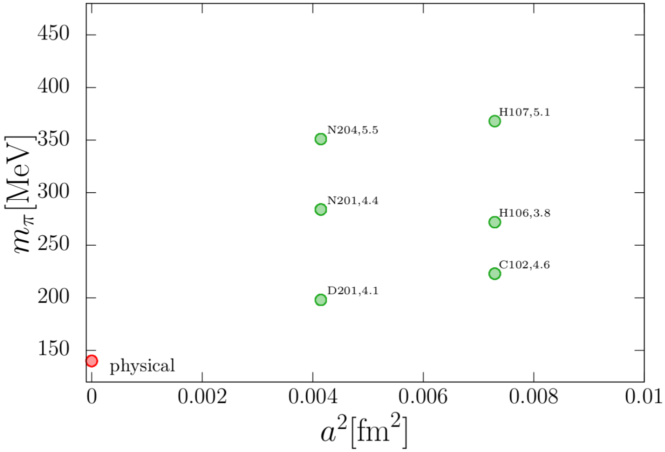
- open boundary conditions in temporal direction to avoid topological freezing

- pion masses varied from 422MeV to 223MeV

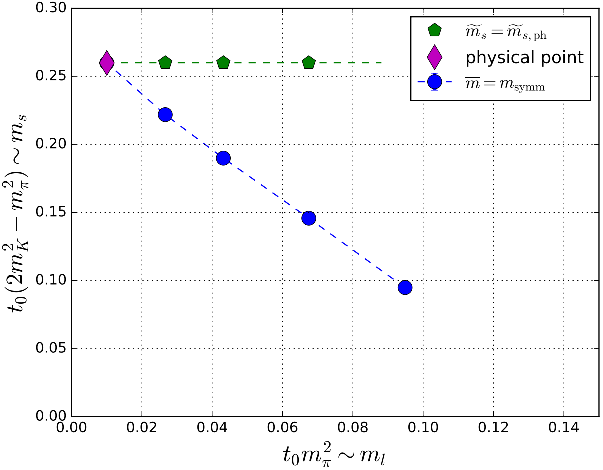
Ensembles for the $\overline{m} = m_{\mathbf{sym}}$ line



Ensembles for the $m_s = $ const. line



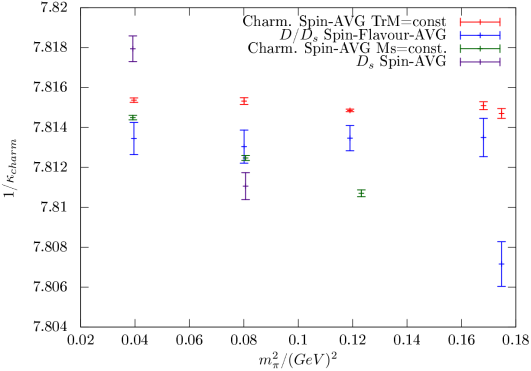
We follow two RG trajectories in the quark mass plane (here for $\beta = 3.4$):



- $\overline{m} = (2m_l + m_s)/3 = m_{sym}$ (sum of ren. quark masses const. up to $\mathcal{O}(a)$)
- renormalized strange quark mass $\tilde{m}_s = \tilde{m}_s^{phys} = const.$ (const. up to $\mathcal{O}(a)$)

In order to set κ_{charm} we simulate at two different values and interpolate

- most analyses so far based on estimation via $\overline{M}(1S) = \left(m_{\eta_c} + 3m_{J/\Psi}\right)/4$
- issues: neglected disconnected diagrams and possible flavour mixing introduce uncertainty; alternative: setting via mass combinations along RG trajectories
- Spin-flavour average $M_X = \left(6m_{D^*} + 2m_D + 3m_{D_s^*} + m_{D_s}\right)/12$ along $\overline{m} = const.$ line
- Spin-flavour average $M_X = \left(3m_{D_s^*} + m_{D_s}\right)/4$ along $\hat{m}_s = const.$ line



Leptonic decay constants defined via

$$\langle 0|A_\mu^{\mathrm{lc}}|\mathrm{D}(p)\rangle = \mathrm{i} f_{\mathrm{D}} p_\mu, \qquad \left\langle 0 \left| A_\mu^{\mathrm{sc}} \right| \mathrm{D_s}(p) \right\rangle = \mathrm{i} f_{\mathrm{D_s}} p_\mu$$

with the axial vector current $A_\mu^{\mathrm{lc}} = \overline{q}\gamma_\mu\gamma_5 c$ (with $q = l, s$). At zero spatial momentum this becomes:

$$\left\langle 0 \left| A_0^{\mathrm{qc}} \right| \mathrm{D}_{(s)} \right\rangle = \mathrm{i} f_{D_{(s)}} m_{D_{(s)}}$$

Remove $\mathcal{O}(a)$ discretization artifacts using the pseudoscalar current $P^{qc} = \overline{q}\gamma_5 c$

$$A_\mu^{\mathrm{qc,I}} = A_\mu^{\mathrm{qc}} + a c_{\mathrm{A}} \frac{1}{2} \left(\partial_\mu + \partial_\mu^*\right) P^{\mathrm{qc}}$$

Renormalization via

$$\left(A_\mu^{qc}\right)^{\mathrm{R}} = Z_{\mathrm{A}} \left[1 + a \left(b_{\mathrm{A}} m_{\mathrm{qc}} + 3\tilde{b}_{\mathrm{A}} \overline{m}\right)\right] A_\mu^{\mathrm{qc,I}} + \mathcal{O}(a^2)$$

c_A, Z_A and b_{A} have been determined non-perturbatively by Bulava, Della Morte, Heitger, Wittemeier in [1502.04999], [1604.05287] and Korcyl, Bali in [1607.07090];
$\tilde{b}_{\mathrm{A}} = 0$ in one-loop perturbative calculation

How to calculate expectation values on the lattice?

- cannot efficiently use Grassmann variables on current machines
- use Wick's theorem to express N-point functions by correlators
- example: charged pion correlator $\pi^+(x) = \overline{d}^a_\alpha(x)(\gamma_5)_{\alpha\beta}u^a_\beta(x)$

$$\begin{aligned}\left\langle \pi^+(y)\overline{\pi}^+(x)\right\rangle &= \left\langle \overline{d}^a_\alpha(y)(\gamma_5)_{\alpha\beta}u^a_\beta(y)\overline{u}^b_\mu(x)(\gamma_5)_{\mu\nu}d^b_\nu(x)\right\rangle \\ &= -\left\langle [S_d]^{ba}_{\nu\alpha}(x,y)(\gamma)_{\alpha\beta}[S_u]^{ab}_{\beta\mu}(y,x)(\gamma_5)_{\beta\mu}\right\rangle \\ &= -\left\langle \mathrm{Tr}\left[S_d(x,y)\gamma_5 S_u(y,x)\gamma_5\right]\right\rangle\end{aligned}$$

- in general:

$$\left\langle \overline{q}_1(y)\Gamma q_2(y)\overline{q}_2(x)\overline{\Gamma}q_1(x)\right\rangle = -\left\langle \mathrm{Tr}\left[S_{q_1}(x,y)\Gamma S_{q_2}(y,x)\overline{\Gamma}\right]\right\rangle$$

Extract matrix elements from two-point functions

$$C_{\mathrm{A,I}}(x_0, y_0) = -\frac{a^6}{L^3} \sum_{\vec{x},\vec{y}} \langle A_0^{\mathrm{qc,I}}(x)\,(P^{\mathrm{qc}}(y))^\dagger \rangle,$$

$$C_{\mathrm{P}}(x_0, y_0) = -\frac{a^6}{L^3} \sum_{\vec{x},\vec{y}} \langle P^{\mathrm{qc}}(x)\,(P^{\mathrm{qc}}(y))^\dagger \rangle$$

Spectral decomposition for infinite T:

$$f_{\mathrm{PS}}(x_0) = \sum_{i=1}^{\infty} c_i \exp(-E_i x_0) \;\text{ with }\; E_1 = m_{\mathrm{PS}}\;,\; E_{i\geq 2} : \text{excited states}$$

$\rightarrow$ for large separations of source y_0 and sink x_0 these reduce to

$$C_{\mathrm{A,I}}(x_0, y_0) \approx \frac{\langle 0|A_0^{\mathrm{qc,I}}|\mathrm{D_q}\rangle\langle \mathrm{D_q}\,|P^{\mathrm{qc}}|\,0\rangle}{2m_{D_\mathrm{q}}}\,\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)} = \frac{f_{\mathrm{qc}}^{\mathrm{bare}}}{2}\,A(y_0)\,\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)}$$

$$\equiv A_{A,I}\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)},$$

$$C_{\mathrm{P}}(x_0, y_0) \approx \frac{|\langle 0\,|P^{\mathrm{qc}}|\,\mathrm{D_q}\rangle|^2}{2m_{D_\mathrm{q}}}\,\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)} = \frac{|A(y_0)|^2}{2m_{D_\mathrm{q}}}\,\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)}$$

$$\equiv A_P\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)}$$

Numerically one checks the condition

$$\left|\sum_x (D[U](y,x) + m_0)S^n(x) - \eta_t(y)\right| < r_{gl}$$

problem: time-slices x_0 far away from source at y_0 exponentially suppressed by factor $\propto \exp(-m_0|y_0 - x_0|)$,
solutions for large time extents $|y_0 - x_0|$ increasingly inaccurate
$\Rightarrow$ numerical instabilities

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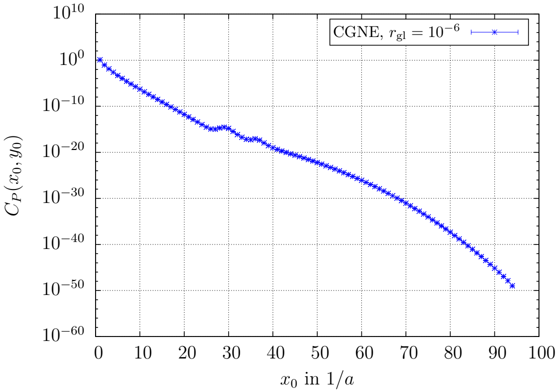

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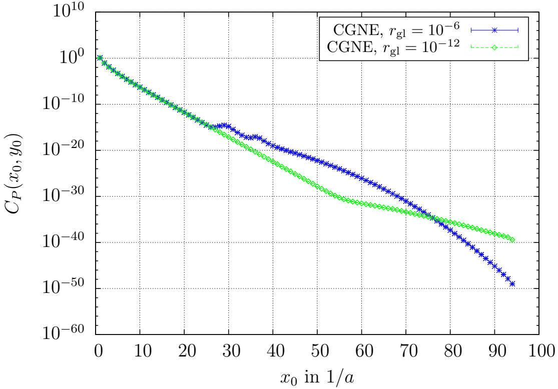

Instead of unconditioned system

$$AS = \eta \text{ with } A = \sum_x (D[U] + m_0)_{y,x}$$

$\Rightarrow$ compute solution $S' = PS$ within the preconditioned system:

$$A'S' = \eta' \Leftrightarrow \left(PAP^{-1}\right)(PS) = (P\eta)$$

To recover the original solution S scale back S' with P^{-1}.

Improvement by implementation of distance preconditioning [1]via diagonal preconditioning matrix P

$$P = \begin{pmatrix} p_1 & 0 & \cdots & \cdots & 0 \\ 0 & p_2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & p_T \end{pmatrix}$$

with $p_i = \exp\left(\alpha_0 \cdot |y_0 - x_{0_i}|\right)$. P is unity in spin, color and spatial coordinates $\rightarrow$ time-slices receive different exponential weight.

[1] G.M. de Divitiis, R. Petronzio, N. Tantalo, Phys.Lett. B 692 (2010) 157–160, arXiv:1006.4028.

heavy-heavy correlator; solution obtained with conjugate gradient solver without (CGNE) and with distance preconditioning (CGNE DP)

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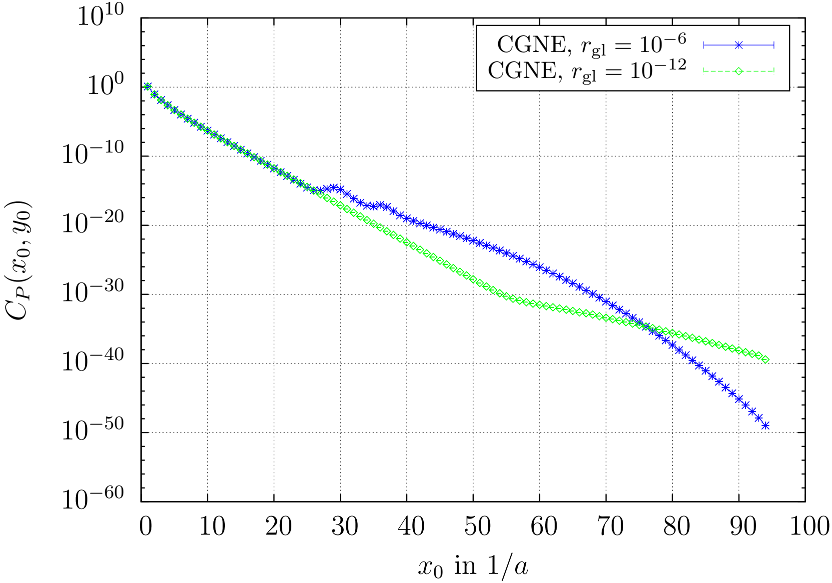

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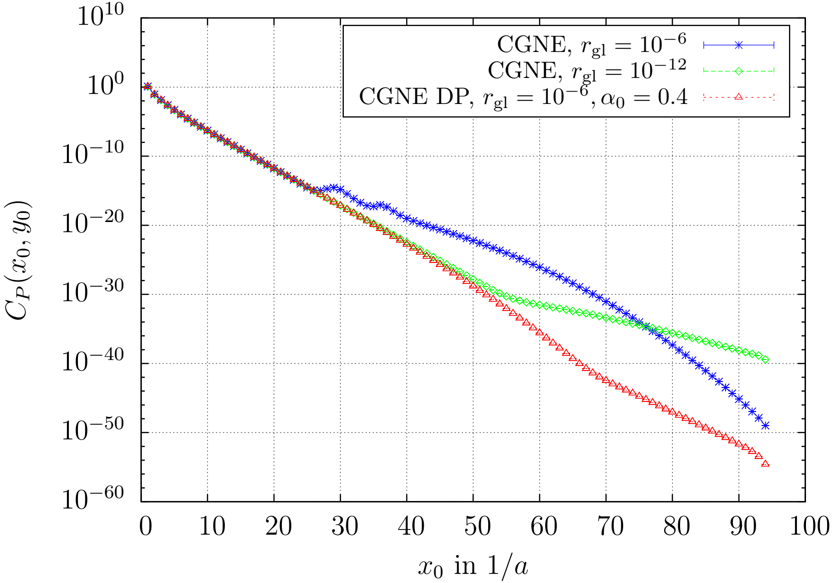

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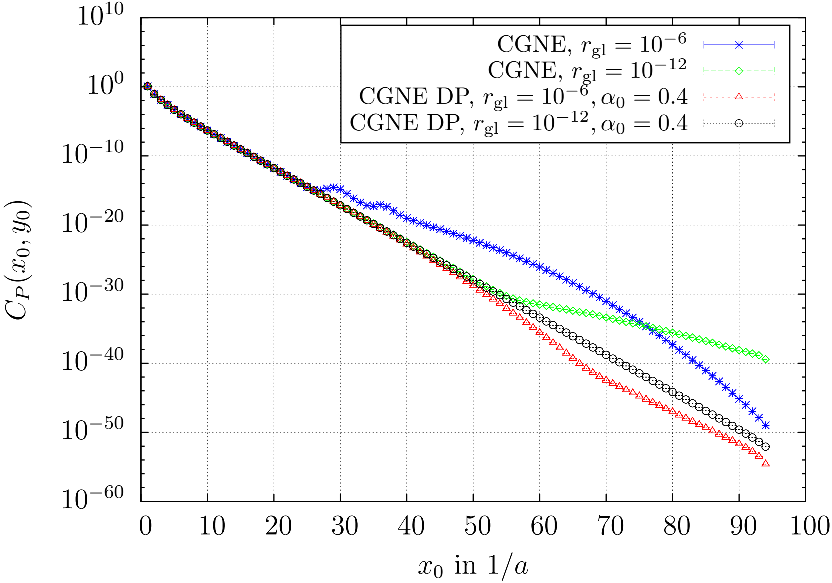
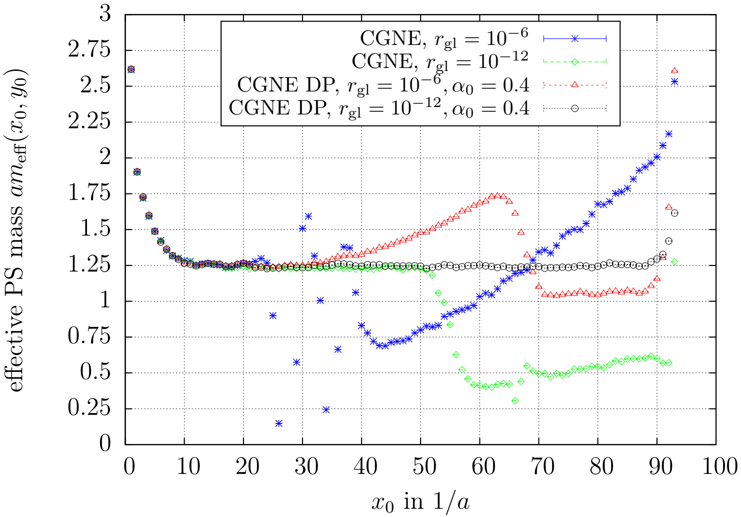
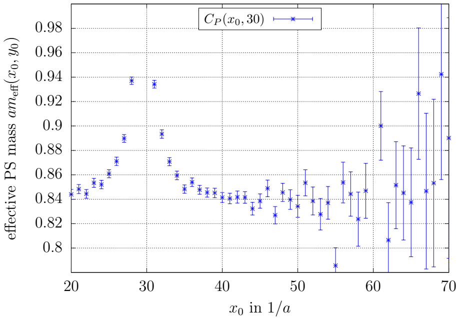


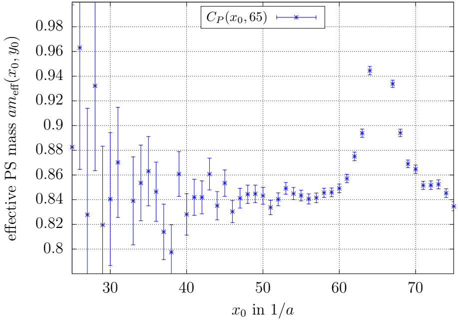
For large time separations $|y_0 - x_0| : m_{eff} = \ln\left(\frac{f_{PP}(x_0)}{f_{PP}(x_0+1)}\right)$



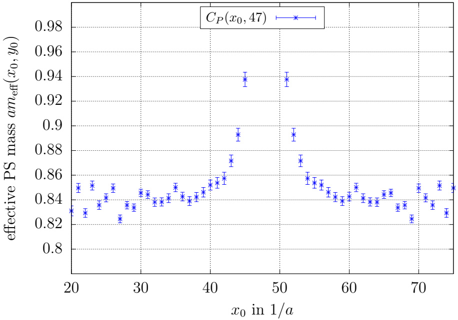
Several approaches so far, e.g. stochastic noise sources. Favored setup at the moment: Point-to-all propagators with smeared sources, placed at three different time-slices.

→ Example with H105 ensemble ($T = 96$ lattice points in temporal direction) with source positions of $x_0/a = 30, 47, 65$: Average over forward and backward propagating parts of correlators (30+65)





Average over forward and backward propagating parts of correlator with source at $y_0 = 47$



Repeat same procedure for axial correlator, in total four correlators. Pseudoscalar and axial correlators are separately fitted to the functional form

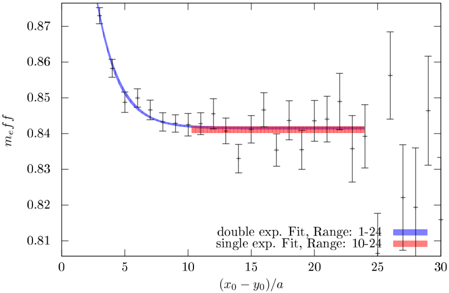
$$Ae^{-m(x_0-y_0)} + Be^{-M(x_0-y_0)}$$

with the second term representing the first exited state.

Find the range where exited state contributions are negligibly small via

$$\frac{|B|^2\, \mathrm{e}^{-M(x_0^{\mathrm{min}}-y_0)}}{2M} < \frac{1}{4}\, \Delta C_{\mathrm{P}}(x_0^{\mathrm{min}}, y_0)$$

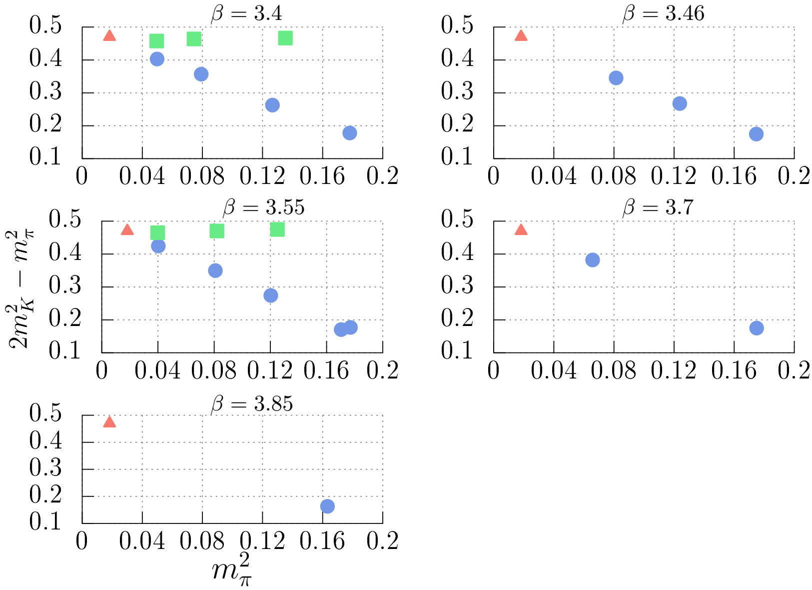
(Example on the right for D-meson on H105 with $y_0 = 30$)



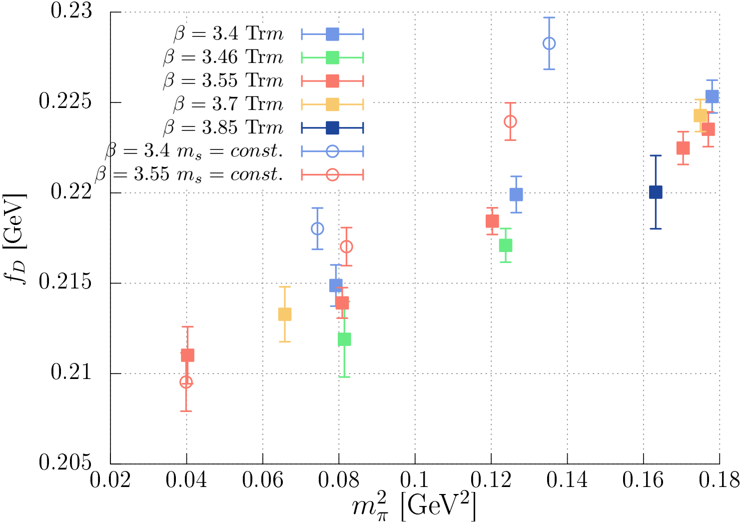
- x_0^{min} determines fit-range for combined one-state fit of all four correlators
- allow different amplitudes for axial and pseudoscalar correlators, but enforce same amplitude for different source positions
- extract decay constant from fitted amplitudes $A_{A,I}$ and A_P

$$\Rightarrow f_{D_{(s)}}^{bare} = \frac{A_{A,I}}{\sqrt{A_P}}\sqrt{\frac{2}{m_{D_\mathrm{q}}}}$$

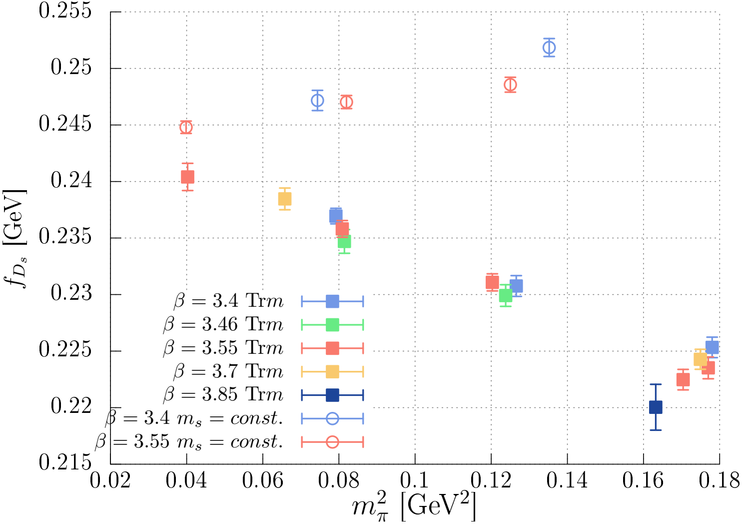
Ensembles analysed so far:

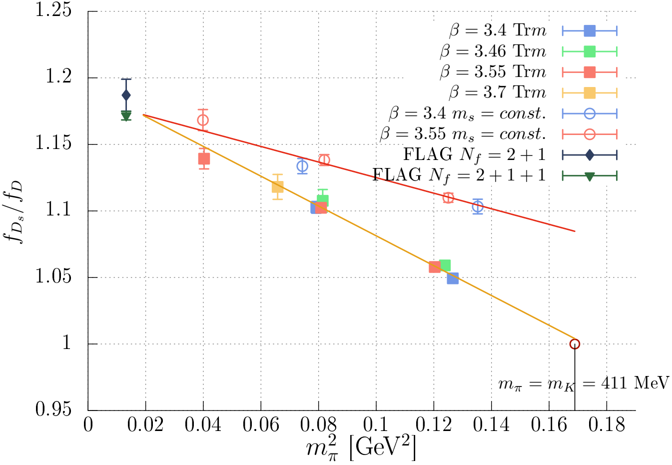


Results for the decay constant f_D on the ensembles analysed so far:



Results for f_{D_s}:





careful continuum/chiral extrapolation as soon as fitting and statistical error estimation are under full control

→ data consistent with FLAG results, no strong discretization effects, ratio compatible with one at symmetric point

Summary:

- multitude of ensembles at different β and quark masses already analysed
- combined two- and one-state-fits show good stability
- preliminary results of $f_{D_{(s)}}$ promising
- discretization artifacts and finite size effects small and well under control

Outlook:

- inclusion of further ensembles at small lattice spacings
- increase of statistics for several ensembles already included in analysis
- preparation of chiral/continuum extrapolation with full control over statistical and systematic uncertainties

Thank you for your attention!

Literature

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