

## Ward identities in $\mathcal{N} = 1$ supersymmetric SU(3) Yang-Mills theory on the lattice

Sajid Ali

#### GRK 2149 Annual Retreat, 25-28.09.2017



 $\mathcal{N} = 1$  SYM theory Motivation

Bound states Supermultiplets

Ward identities

SUSY Ward identities on the lattice Renormalization Numerical results

Global method

Renormalized gluino mass

Generalized  $\chi^2$  method

Renormalized gluino mass

Adjoint pion Extrapolation to chiral limit

Summary





◆□ > ◆□ > ◆ □ > ◆ □ > □ = のへで





#### $\mathcal{N} = 1$ supersymmetric Yang-Mills theory

・ロト ・ 雪 ト ・ ヨ ト

э.



#### Supersymmetric Yang-Mills theory

$$S = \int d^4x \left\{ -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \bar{\lambda}^a \gamma_\mu (\mathscr{D}_\mu \lambda)^a - \frac{m_g}{2} \bar{\lambda}^a \lambda^a \right\}$$

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへで

#### Supersymmetric Yang-Mills theory

$$S = \int d^4x \left\{ -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \bar{\lambda}^a \gamma_\mu (\mathscr{D}_\mu \lambda)^a - \frac{m_g}{2} \bar{\lambda}^a \lambda^a \right\}$$

• Field strength tensor

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - \operatorname{i}gf^{a}_{bc}[A^{b}_{\mu}, A^{c}_{\nu}]$$

• Covariant derivative in adjoint representation

$$(\mathscr{D}_{\mu}\lambda)^{a} = \partial_{\mu}\lambda^{a} + g f^{a}_{bc}A^{b}_{\mu}\lambda^{c}, \qquad a = 1, \dots, N^{2}_{c} - 1$$

 $\begin{array}{c|c} \text{Outline} & \mathscr{N} = 1 \text{ SYM theory} \\ \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ \\ \end{array} \\ \begin{array}{c|c} \text{Bound states} & \text{Ward identities} & \text{Global method} & \text{Generalized } \chi^2 \text{ method} & \text{Adjoint pion} & \text{Summary} \\ \circ & \circ & \circ & \circ \\ \end{array} \\ \begin{array}{c|c} \text{Summary} & \text{Summary} & \text{Summary} \\ \text{Summary} & \text{Summary} & \text{Summary} \\ \text{Summary} & \text{Summary} & \text{Summary} \\ \end{array} \\ \end{array}$ 

#### Supersymmetric Yang-Mills theory

$$S = \int d^4x \left\{ -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \bar{\lambda}^a \gamma_\mu (\mathscr{D}_\mu \lambda)^a - \frac{m_g}{2} \bar{\lambda}^a \lambda^a \right\}$$

• Field strength tensor

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - \operatorname{i}gf^{a}_{bc}[A^{b}_{\mu}, A^{c}_{\nu}]$$

• Covariant derivative in adjoint representation

$$(\mathscr{D}_{\mu}\lambda)^{a} = \partial_{\mu}\lambda^{a} + g f^{a}_{bc}A^{b}_{\mu}\lambda^{c}, \qquad a = 1, \dots, N^{2}_{c} - 1$$

- Vector supermultiplet
  - 1) Gauge field(gluon)  $A_{\mu}^{a}$
  - 2) Majorana-spinor field (gluino)  $\lambda^a$ ,  $\bar{\lambda}^a = \lambda^{aT} C$

#### Supersymmetric Yang-Mills theory

• SUSY transformations (on-shell):

$$\delta A^a_\mu = -2g ar\lambda^a \gamma_\mu arepsilon \, , \ \delta \lambda^a = -rac{\mathrm{i}}{g} \sigma_{\mu 
u} F^{\ a}_{\mu 
u} arepsilon$$

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへで

#### Supersymmetric Yang-Mills theory

• SUSY transformations (on-shell):

$$\delta A^{a}_{\mu} = -2g\bar{\lambda}^{a}\gamma_{\mu}\varepsilon,$$
  
 $\delta\lambda^{a} = -rac{\mathrm{i}}{g}\sigma_{\mu\nu}F^{a}_{\mu\nu}\varepsilon$ 

- In contrast to QCD:
  - 1)  $\lambda^a$  is Majorana spinor field, " $N_f = \frac{1}{2}$ "
  - 2) adjoint representation of  $SU(N_c)$
- Gluino mass term  $\frac{m_g}{2}\bar{\lambda}^a\lambda^a$  breaks SUSY softly



• SYM: simplest model with SUSY and local gauge invariance

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

- Part of the supersymmetrically extended Standard Model
- Possible connection to ordinary QCD

# Outline $\mathscr{N} = 1$ SYM theory of the states of the states

- SYM: simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended Standard Model
- Possible connection to ordinary QCD
- Similar to QCD:
  - 1) Asymptotic freedom
  - 2) Confinement
  - 3) Numerical lattice simulation of bound states

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <



#### Motivation

Solution of non-perturbative problems:

• Spontaneous breaking of chiral symmetry  $Z_{2N_c} \rightarrow Z_2$  $\longleftrightarrow$  Gluino condensate:  $\langle \lambda \lambda \rangle \neq 0$ 

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Spectrum of bound states → Supermultiplets



#### Motivation

Solution of non-perturbative problems:

• Spontaneous breaking of chiral symmetry  $Z_{2N_c} \rightarrow Z_2$  $\longleftrightarrow$  Gluino condensate:  $\langle \lambda \lambda \rangle \neq 0$ 

- Spectrum of bound states  $\rightarrow$  Supermultiplets
- Spontaneous breaking of SUSY?
- SUSY restoration on the lattice



#### Motivation

Solution of non-perturbative problems:

• Spontaneous breaking of chiral symmetry  $Z_{2N_c} \rightarrow Z_2$  $\longleftrightarrow$  Gluino condensate:  $\langle \lambda \lambda \rangle \neq 0$ 

- Spectrum of bound states  $\rightarrow$  Supermultiplets
- Spontaneous breaking of SUSY?
- SUSY restoration on the lattice

#### Supersymmetric Yang-Mills theory on the lattice

(a)

э



#### Supersymmetric Yang-Mills theory on the lattice



Link:  $U_{x\mu} = e^{iagA_{\mu}(x)}$ 

Plaquette:  $U_{x\mu\nu} = e^{ia^2 F_{\mu\nu}(x)}$ 

▲ロト ▲帰ト ▲ヨト ▲ヨト 三日 - の々ぐ

 $\begin{array}{c|c} \text{Outline} & \mathscr{N} = 1 \text{ SYM theory} \\ \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ \\ \end{array} & \begin{array}{c|c} \text{Bound states} & \text{Ward identities} & \text{Global method} & \text{Generalized } \chi^2 \text{ method} & \text{Adjoint pion} & \text{Summary} \\ \circ & \circ & \circ & \circ \\ \end{array} \\ \end{array} \\ \begin{array}{c|c} \text{Summary} & \text{Summary} & \text{Summary} \\ \text{Summary} & \text{Summary} & \text{Summary} \\ \text{Summary} & \text{Summary} & \text{Summary} \\ \end{array} \\ \end{array}$ 

#### Supersymmetric Yang-Mills theory on the lattice

$$S = -\frac{\beta}{N_c} \sum_{p} \operatorname{Re} \operatorname{Tr} U_p$$

$$+\frac{1}{2}\sum_{x}\left\{\overline{\lambda}_{x}^{a}\lambda_{x}^{a}-\kappa\sum_{\mu=1}^{4}\left[\overline{\lambda}_{x+\hat{\mu}}^{a}V_{ab,x\mu}(1+\gamma_{\mu})\lambda_{x}^{b}+\overline{\lambda}_{x}^{a}V_{ab,x\mu}^{t}(1-\gamma_{\mu})\lambda_{x+\hat{\mu}}^{b}\right]\right\}$$

 $\beta = \frac{2N_c}{g^2}, \quad \kappa = \frac{1}{2m_0 + 8} \text{(hopping parameter)}, \quad m_0 \text{: bare gluino mass}$  $V_{ab.\times\mu} = 2 \operatorname{Tr} (U_{\times\mu}^{\dagger} T_a U_{\times\mu} T_b), \quad \text{adjoint link variables}$ 

## $\begin{array}{c} \begin{array}{c} \text{Outline} \\ \text{o} \end{array} \\ \begin{array}{c} \mathcal{N} = 1 \text{ SYM theory} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Bound states} \\ \bullet \end{array} \\ \begin{array}{c} \text{Ward identities} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Global method} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Generalized } \chi^2 \text{ method} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Adjoint pion} \\ \text{oo} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{oo} \end{array} \\ \begin{array}{c} \text{Outline} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{oo} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{oo} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{oo} \end{array} \\ \end{array}$

#### Supermultiplets

Colour neutral bound states of gluons and gluinos Predictions from effective Lagrangeans: Chiral supermultiplet (Veneziano, Yankielowicz)

- 0 $^-$  gluinoball a  $\eta^\prime~\sim~\overline{\lambda}\gamma_5\lambda$
- 0<sup>+</sup> gluinoball a  $f_0 ~\sim~ \overline{\lambda}\lambda$
- spin  $rac{1}{2}$  gluino-glueball  $\sim \sigma_{\mu
  u} {
  m Tr}(F_{\mu
  u}\lambda)$

## $\begin{array}{c|c} \text{Outline} & \mathscr{N} = 1 \text{ SYM theor} \\ \text{o} & \text{o$

#### Supermultiplets

Colour neutral bound states of gluons and gluinos Predictions from effective Lagrangeans: Chiral supermultiplet (Veneziano, Yankielowicz)

- 0 $^-$  gluinoball a  $\eta^\prime~\sim~\overline{\lambda}\gamma_5\lambda$
- 0<sup>+</sup> gluinoball a  $f_0~\sim~\overline{\lambda}\lambda$
- spin  $\frac{1}{2}$  gluino-glueball  $\sim \sigma_{\mu\nu} \operatorname{Tr}(F_{\mu\nu}\lambda)$

Generalization (Farrar, Gabadadze, Schwetz): Additional chiral supermultiplet

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

- 0<sup>-</sup> glueball
- 0<sup>+</sup> glueball

(Henning Gerber)

## $\begin{array}{c} \text{Outline} \quad \mathcal{N} = 1 \text{ SYM theory} \\ \text{o} \quad \begin{array}{c} \text{Bound states} \\ \bullet \end{array} \quad \begin{array}{c} \text{Ward identifies} \\ \text{o} \quad \begin{array}{c} \text{Global method} \\ \text{o} \end{array} \quad \begin{array}{c} \text{Generalized } \chi^2 \text{ method} \\ \text{o} \end{array} \quad \begin{array}{c} \text{Adjoint pion} \\ \text{oo} \end{array} \quad \begin{array}{c} \text{Summary} \\ \text{oo} \end{array}$

### Supermultiplets

Colour neutral bound states of gluons and gluinos Predictions from effective Lagrangeans: Chiral supermultiplet (Veneziano, Yankielowicz)

- 0 $^-$  gluinoball a  $\eta^\prime~\sim~\overline{\lambda}\gamma_5\lambda$
- 0<sup>+</sup> gluinoball a  $f_0~\sim~\overline{\lambda}\lambda$
- spin  $rac{1}{2}$  gluino-glueball  $\sim \sigma_{\mu\nu} \operatorname{Tr}(F_{\mu\nu}\lambda)$

Generalization (Farrar, Gabadadze, Schwetz): Additional chiral supermultiplet

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

- 0<sup>-</sup> glueball
- 0<sup>+</sup> glueball

(Henning Gerber) possible mixing and Baryons



#### Expression in the continuum

Noether Theorem in classical theory  $\rightarrow$  WIs in quantum theory

$$\left\langle \left(\partial_{\mu} j^{\mu}(x)\right) Q(y) \right\rangle = -\left\langle \frac{\delta Q(y)}{\delta \overline{\varepsilon}(x)} \right\rangle$$

RHS of equation is contact term, which is zero if Q is localized at space-time points different from x.

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

**SUSY** transformations on the lattice with P, T, Majorana nature and gauge invariance:

00000

Outline  $\mathscr{N}=1$  SYM theory Bound states Ward identities Global method Generalized  $\chi^2$  method Adjoint pion Summary

$$\begin{split} \delta U_{\mu}(x) &= -\frac{\mathrm{i}ga}{2} \bigg( \bar{\varepsilon}(x) \gamma_{\mu} U_{\mu}(x) \lambda(x) + \bar{\varepsilon}(x+\hat{\mu}) \gamma_{\mu} \lambda(x+\hat{\mu}) U_{\mu}(x) \bigg) \\ \delta \lambda(x) &= +\frac{1}{2} P_{\mu\nu}^{(cl)}(x) \sigma_{\mu\nu} \varepsilon(x) \end{split}$$

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへで

SUSY transformations on the lattice with P, T, Majorana nature and gauge invariance:

00000

Outline  $\mathscr{N}=1$  SYM theory Bound states Ward identities Global method Generalized  $\chi^2$  method Adjoint pion Summary

$$\delta U_{\mu}(x) = -\frac{\mathrm{i}ga}{2} \left( \bar{\varepsilon}(x) \gamma_{\mu} U_{\mu}(x) \lambda(x) + \bar{\varepsilon}(x+\hat{\mu}) \gamma_{\mu} \lambda(x+\hat{\mu}) U_{\mu}(x) \right)$$
$$\delta \lambda(x) = +\frac{1}{2} P_{\mu\nu}^{(cl)}(x) \sigma_{\mu\nu} \varepsilon(x)$$

Above transformations result in following Ward identities:

$$\langle (\nabla_{\mu} S_{\mu}(x)) Q(y) \rangle = m_0 \langle D_S(x) Q(y) \rangle + \langle X(x) Q(y) \rangle - \left\langle \frac{\delta Q(y)}{\delta \overline{\varepsilon}(x)} \right\rangle$$
$$S_{\mu}(x) = -\sum_{\rho \nu} \sigma_{\rho \nu} \gamma_{\mu} \operatorname{Tr} \left( P_{\rho \nu}^{(cl)}(x) \lambda(x) \right)$$

(日) (日) (日) (日) (日) (日) (日) (日)

SUSY transformations on the lattice with P, T, Majorana nature and gauge invariance:

Outline  $\mathscr{N} = 1$  SYM theory Bound states Ward identities Global method Generalized  $\chi^2$  method Adjoint pion Summary

$$\begin{split} \delta U_{\mu}(x) &= -\frac{\mathrm{i}ga}{2} \bigg( \bar{\varepsilon}(x) \gamma_{\mu} U_{\mu}(x) \lambda(x) + \bar{\varepsilon}(x+\hat{\mu}) \gamma_{\mu} \lambda(x+\hat{\mu}) U_{\mu}(x) \bigg) \\ \delta \lambda(x) &= +\frac{1}{2} P_{\mu\nu}^{(cl)}(x) \sigma_{\mu\nu} \varepsilon(x) \end{split}$$

Above transformations result in following Ward identities:

$$\langle (\nabla_{\mu} S_{\mu}(x)) Q(y) \rangle = m_0 \langle D_S(x) Q(y) \rangle + \langle X(x) Q(y) \rangle - \left\langle \frac{\delta Q(y)}{\delta \overline{\varepsilon}(x)} \right\rangle$$

$$S_{\mu}(x) = -\sum_{\rho \nu} \sigma_{\rho \nu} \gamma_{\mu} \operatorname{Tr} \left( P_{\rho \nu}^{(cl)}(x) \lambda(x) \right)$$

 $\left. \begin{array}{l} D_S(x) : \text{ due to bare gluino mass } (m_0) \\ X(x) : \text{ due to lattice regularization} \end{array} \right\} Break \ SUSY$ 



After renormalization WI gets the following form:

$$\langle (\nabla_{\mu}S_{\mu}(x))Q(y)\rangle + \frac{Z_{T}}{Z_{S}}\langle (\nabla_{\mu}T_{\mu}(x))Q(y)\rangle = \frac{m_{S}}{Z_{S}}\langle D_{S}(x)Q(y)\rangle + O(a)$$

 $T_{\mu}(x)$  is mixing current.



After renormalization WI gets the following form:

$$\langle (\nabla_{\mu}S_{\mu}(x))Q(y)\rangle + \frac{Z_{T}}{Z_{S}}\langle (\nabla_{\mu}T_{\mu}(x))Q(y)\rangle = \frac{m_{S}}{Z_{S}}\langle D_{S}(x)Q(y)\rangle + O(a)$$

 $T_{\mu}(x)$  is mixing current.

Zero spatial momentum WI and expansion in a basis of 16 Dirac matrices; the surviving contributions form a set of two non-trivial independent equations:

$$1x_{b,t} + Ay_{b,t} - Bz_{b,t} = 0, \qquad b = 1,2$$

▲ロト ▲帰ト ▲ヨト ▲ヨト 三日 - の々ぐ

After renormalization WI gets the following form:

$$\langle (\nabla_{\mu}S_{\mu}(x))Q(y)\rangle + \frac{Z_{T}}{Z_{S}}\langle (\nabla_{\mu}T_{\mu}(x))Q(y)\rangle = \frac{m_{S}}{Z_{S}}\langle D_{S}(x)Q(y)\rangle + O(a)$$

 $T_{\mu}(x)$  is mixing current.

Zero spatial momentum WI and expansion in a basis of 16 Dirac matrices; the surviving contributions form a set of two non-trivial independent equations:

$$1x_{b,t} + Ay_{b,t} - Bz_{b,t} = 0, \qquad b = 1,2$$

$$\sum_{\alpha} A_{\alpha} x_{i\alpha} = 0, \quad i = (b, t), \alpha = 1, 2, 3$$

Outline	$\mathcal{N} = 1$ SYM theory	Bound states	Ward identities	Global method	Generalized $\chi^2$ method	Adjoint pion	Summary
0	0000000	0	00000	0	0	00	00

#### Supercomputer



 $\begin{array}{c|c} Outline & \mathscr{N} = 1 \text{ SYM theor} \\ \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ \\ \end{array} & \begin{array}{c|c} \mathsf{Global method} & \mathsf{Generalized} \ \chi^2 \ \mathsf{method} & \mathsf{Adjoint pion} \\ \circ & \circ & \circ \\ \circ & \circ \\ \end{array} & \begin{array}{c|c} \mathsf{Summary} & \mathsf{Summary} \\ \mathsf{Summary} \\ \mathsf{Summary} \\ \mathsf{Summary} & \mathsf{Summary} \\ \mathsf{Summary$ 

#### Numerical results

Figure: 
$$V = 16^3 \cdot 32$$
,  $\beta = 5.5$ ,  $\kappa = 0.1673$ 





#### Global method

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへで

Calculate A and B by minimizing the quantity:

$$\sum_{b=1}^{2} \sum_{t=t_{min}}^{t_{max}} \frac{(x_{b,t} + Ay_{b,t} - Bz_{b,t})^2}{\sigma^2}$$



#### Global method



(日)、

э



#### Global method



## Generalized $\chi^2$ method

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへで

$$\sum_{\alpha}A_{\alpha}x_{i\alpha}=0$$

We employ the method of maximum likelihood

$$L = \frac{1}{2} \sum_{i,\alpha,j,\beta} (A_{\alpha} \bar{x}_{i\alpha}) (D^{-1})_{ij} (A_{\beta} \bar{x}_{j\beta})$$

 $\begin{array}{c} \text{Outline} \\ \text{o} \\ \text{$ 

## Generalized $\chi^2$ method

$$\sum_{\alpha} A_{\alpha} x_{i\alpha} = 0$$

We employ the method of maximum likelihood

$$L = \frac{1}{2} \sum_{i,\alpha,j,\beta} (A_{\alpha} \bar{x}_{i\alpha}) (D^{-1})_{ij} (A_{\beta} \bar{x}_{j\beta})$$

where

$$D_{ij} = \sum_{\alpha,\beta} A_{\alpha} A_{\beta} (\overline{x_{i\alpha} x_{j\beta}} - \overline{x}_{i\alpha} \overline{x}_{j\beta})$$

find  $A_{\alpha}$  such that L is minimum

Outline  $\mathscr{N} = 1$  SYM theory Bound states Ward identities Global method Generalized  $\chi^2$  method Adjoint pion Summary

Generalized  $\chi^2$  method

$$\sum_{\alpha} A_{\alpha} x_{i\alpha} = 0$$

We employ the method of maximum likelihood



$$L = \frac{1}{2} \sum_{i,\alpha,j,\beta} (A_{\alpha} \bar{x}_{i\alpha}) (D^{-1})_{ij} (A_{\beta} \bar{x}_{j\beta})$$

where

$$D_{ij} = \sum_{\alpha,\beta} A_{\alpha} A_{\beta} (\overline{x_{i\alpha} x_{j\beta}} - \overline{x}_{i\alpha} \overline{x}_{j\beta})$$

find  $A_{\alpha}$  such that L is minimum

Outline  $\mathscr{N} = 1$  SYM theory Bound states Ward identities Global method Generalized  $\chi^2$  method Adjoint pion Summary

## Generalized $\chi^2$ method

$$\sum_{\alpha} A_{\alpha} x_{i\alpha} = 0$$

We employ the method of maximum likelihood

$$L = \frac{1}{2} \sum_{i,\alpha,j,\beta} (A_{\alpha} \bar{x}_{i\alpha}) (D^{-1})_{ij} (A_{\beta} \bar{x}_{j\beta})$$

where

$$D_{ij} = \sum_{\alpha,\beta} A_{\alpha} A_{\beta} (\overline{x_{i\alpha} x_{j\beta}} - \bar{x}_{i\alpha} \bar{x}_{j\beta})$$

find  $A_{\alpha}$  such that L is minimum







## Adjoint Pion

- The m<sub>a-π</sub> is obtained numerically in simulations of *N* = 1 SYM theory
- It is being used for the tuning of critical point

## $\begin{array}{c} \text{Outline} \\ \text{o} \end{array} \\ \begin{array}{c} \mathcal{N} = 1 \text{ SYM theory} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Bound states} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Ward identifies} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Global method} \\ \text{Global method} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Generalized } \chi^2 \text{ method} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Adjoint pion} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{o} \end{array} \\ \begin{array}{c} \text{o} \end{array} \\ \begin{array}{c} \text{o} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array}$ \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \\ \begin{array}{c} \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array}

### Adjoint Pion

- The m<sub>a-π</sub> is obtained numerically in simulations of *N* = 1 SYM theory
- It is being used for the tuning of critical point



Figure: Extrapolation to the critical point /chiral limit using  $m_{a-\pi}^2$ 

(日)、

## $\begin{array}{c} \text{Outline} \\ \text{o} \end{array} \\ \begin{array}{c} \mathcal{N} = 1 \text{ SYM theory} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Bound states} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Ward identities} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Global method} \\ \text{Global method} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Generalized } \chi^2 \text{ method} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Adjoint pion} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{o} \end{array} \\ \begin{array}{c} \text{o} \end{array} \\ \begin{array}{c} \text{o} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array}$ \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \text{Summary} \end{array} \\ \end{array}

#### Adjoint Pion

- The m<sub>a-π</sub> is obtained numerically in simulations of *N* = 1 SYM theory
- It is being used for the tuning of critical point

$$(am_{a-\pi})^{2} \simeq A(\frac{1}{\kappa} - \frac{1}{\kappa_{c}})$$
$$am_{S}Z_{S}^{-1} = \frac{1}{2}(\frac{1}{\kappa} - \frac{1}{\kappa_{c}})$$
$$(am_{a-\pi})^{2} \propto am_{S}Z_{S}^{-1}$$



Figure: Extrapolation to the critical point /chiral limit using  $m_{a-\pi}^2$ 

(日)、



Figure: Extrapolation to Chiral limit using  $m_{a-\pi}^2$  and  $m_g$ 



◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで



#### Summary

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

- Introduction of  $\mathcal{N} = 1$  supersymetric Yang-Mills theory
- SYM theory on the lattice
- Spectrum of bound states



### Summary

- Introduction of  $\mathscr{N}=1$  supersymetric Yang-Mills theory
- SYM theory on the lattice
- Spectrum of bound states
- SUSY Ward identities
- Determination of  $am_S Z_S^{-1}$  using WIs by Global method
- Determination of  $am_S Z_S^{-1}$  using WIs by GCS method

- ロ ト - 4 回 ト - 4 □ - 4

## $\begin{array}{c} \text{Outline} \quad & \mathcal{N} = 1 \text{ SYM theory} \\ \text{o} & \text{ocococo} & \text{o} \end{array} \end{array} \\ \begin{array}{c} \text{Bound states} \\ \text{o} & \text{ocococo} \end{array} \\ \begin{array}{c} \text{Ward identities} \\ \text{o} & \text{ocococ} \end{array} \\ \begin{array}{c} \text{Global method} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Generalized } \chi^2 \text{ method} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Adjoint pion} \\ \text{o} \end{array} \\ \begin{array}{c} \text{Summary} \\ \text{o} \end{array} \\ \begin{array}{c} \text{o} \end{array} \\ \end{array}$

#### Summary

- Introduction of  $\mathscr{N}=1$  supersymetric Yang-Mills theory
- SYM theory on the lattice
- Spectrum of bound states
- SUSY Ward identities
- Determination of  $am_S Z_S^{-1}$  using WIs by Global method
- Determination of  $am_S Z_S^{-1}$  using WIs by GCS method
- Extrapolations towards vanishing gluino mass using WIs
- Extrapolations towards vanishing gluino mass using  $m_{a-\pi}^2$

## $\begin{array}{c|c} \mbox{Outline} & \ensuremath{\mathscr{N}} = 1 \mbox{ SYM theory} & \mbox{Bound states} & \mbox{Ward identities} & \mbox{Global method} & \mbox{Generalized } \chi^2 \mbox{ method} & \mbox{Adjoint pion} & \mbox{Adjoint pion} & \mbox{Outline} & \m$

#### Summary

- Introduction of  $\mathscr{N}=1$  supersymetric Yang-Mills theory
- SYM theory on the lattice
- Spectrum of bound states
- SUSY Ward identities
- Determination of  $am_S Z_S^{-1}$  using WIs by Global method
- Determination of  $am_S Z_S^{-1}$  using WIs by GCS method
- Extrapolations towards vanishing gluino mass using WIs
- Extrapolations towards vanishing gluino mass using  $m_{a-\pi}^2$

- Consistency between  $\kappa_c$  from WIs and from  $m_{a-\pi}^2$
- Consistency with restoration of SUSY

# Thank You!



Figure: Schlossgebäude: Westfälische Wilhelms-Universität Münster.

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ