# Charmed meson physics from three-flavour lattice QCD


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## Theoretical background

The branching ratio for the leptonic decay of a $D_{(s)}$ meson can be written as

$$\mathcal{B}(D_{(s)} \to l\nu) = \frac{G_F^2 \, |V_{cq}|^2}{8\pi} f_{D_{(s)}}^2 m_l^2 m_{D_{(s)}} \left(1 - \frac{m_l^2}{m_{D_{(s)}}^2}\right)^2 ,$$

with $q = \mathrm{d,s}$. By measuring this ratio the product $|V_{cq}| \, f_{D_{(s)}}$ can be determined experimentally. Combined with a precise theoretical prediction for the leptonic decay constants $f_{D_{(s)}}$, the CKM-matrix elements $|V_{cd}|$ and $|V_{cs}|$ can be determined, enabling the **unitarity of the CKM-matrix to be tested** in detail.

- In QCD: $f_{\mathrm{D}}$ and $f_{\mathrm{D_s}}$ given by non-perturbative matrix elements

$$\langle 0|\overline{q}\gamma_\mu\gamma_5 c|\mathrm{D}_{(s)}(p)\rangle = \mathrm{i} f_{\mathrm{D}(s)} \, p_\mu \, .$$

- On the lattice: computation of decay constants via stochastic Monte Carlo simulations; derived from two-point correlation functions of the *pseudoscalar* density $P^{rs} = \overline{\psi}_r\gamma_5\psi_s$ and the time component of the *axial* vector current $A_0^{rs} = \overline{\psi}_r\gamma_0\gamma_5\psi_s$, which are constructed from two mass non-degenerate valence quarks $r$ and $s$ as

$$f_{\mathrm{PP}}^{rs}(x_0) = -a^3 \sum_{\vec{x}} \langle P^{rs}(x) P^{sr}(0)\rangle \ , \ f_{\mathrm{AP}}^{rs}(x_0) = -a^3 \sum_{\vec{x}} \langle A_0^{rs}(x) P^{sr}(0)\rangle$$

$\rightarrow$ determined by Dirac structure at source and sink position and propagators (Green's functions of the massive lattice Dirac operator). Propagators $S_i$ defined by:

$$\sum_y \left(D[U](x,y) + m_i\right) S_i(y,z) = \mathbb{1}\delta_{x,z} \, .$$

## Motivation for distance preconditioning

Check numerically if condition

$$\left| \sum_y (D[U](x,y) + m_0) S^n(y) - \eta_t(x) \right| < r_{gl}$$

is satisfied, with $D[U]$: discretized lattice Dirac operator, $m_0$: quark mass in lattice units, $S^n(y)$: approximate solution of propagator at the n-th iteration of the solver procedure, $\eta_t(x)$: stochastic noise source located on single time-slice $t$, $r_{gl}$: *global* numerical accuracy one likes to achieve.

- Problem: time-slices $y_0$ far away from source at $x_0$ exponentially suppressed by factor $\propto \exp(-m_0 y_0)$
- Contributions to norm negligible for heavy quarks
- Solutions for large time extents $|x_0 - y_0|$ increasingly inaccurate

▶ Proposed improvement: implement Distance Preconditioning [1] via diagonal preconditioning matrix $P$:

$$P = \begin{pmatrix} p_1 & 0 & \cdots & \cdots & 0 \\ 0 & p_2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & p_T \end{pmatrix} \quad \text{with} \quad p_i = \exp\left(\alpha_0 \cdot |x_0 - y_{0_i}|\right)$$

$P$ is unity in spin, color and spatial coordinates $\rightarrow$ time-slices receive different exponential weight ($\alpha_0$ acts as control parameter; optimal: $\alpha_0 \approx \frac{m_{eff}}{2}$)

- Instead of original system consider preconditioned system:

$$AS = \eta \quad \text{with} \quad A = (D[U] + m_0) \longrightarrow A'S' = \eta' \Leftrightarrow \left(PAP^{-1}\right)(PS) = (P\eta)$$

$\Rightarrow$ solve for $PS$ and scale with $P^{-1}$ to obtain original solution $S$

## Computational details & techniques

Numerical tests were performed on several Coordinated Lattice Simulations ensembles (`https://twiki.cern.ch/twiki/bin/view/CLS/WebHome`) with tree level improved Lüscher-Weisz gauge action [2] & Sea of $N_{\mathrm{f}} = 2+1$ (2 light mass degenerate + strange) non-perturbatively $\mathrm{O}(a)$ improved Wilson quarks:

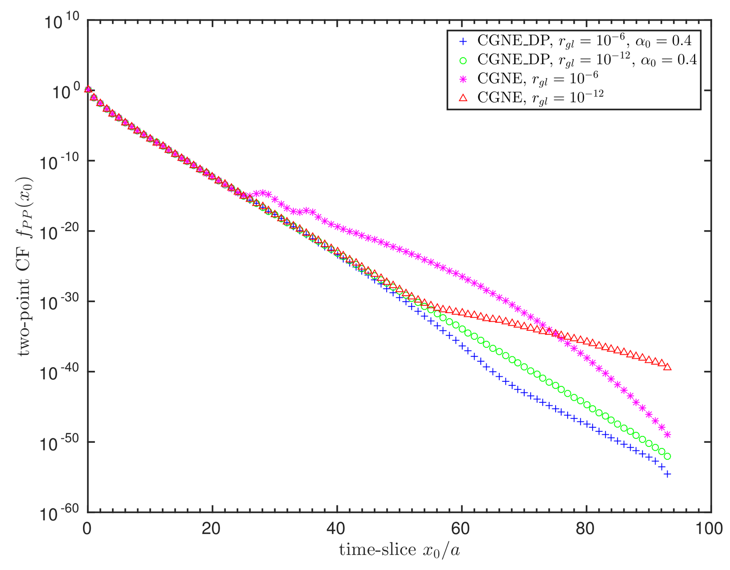
- Simulations performed using openQCD code [3], with overall computational setup described in detail in [4]
- Using 16 $U(1)$ noise sources $\eta_t(x) = \delta_{t,x_0}\exp(i\phi(\vec{x}))$ located on randomly chosen time slices $t$ [5] so that solving the Dirac equation once for each noise vector $\zeta_t^r = Q^{-1}(m_{0,r})\eta_t = a^{-1}(D + m_{0,r})^{-1}\gamma_5\eta_t$ suffices to estimate the two-point functions projected onto zero momentum

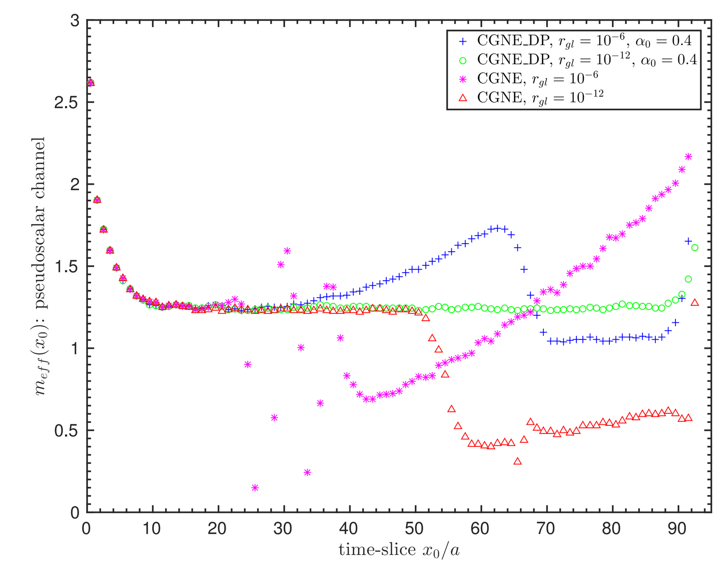
$$a^3 f_{\mathrm{XP}}^{rs}(x_0) = \sum_{\vec{x}} \langle [\zeta_t^r(x_0+t,\vec{x})]^\dagger \Gamma \zeta_t^s(x_0+t,\vec{x})\rangle \ , \ \Gamma = \mathbf{1}/\gamma_0 \text{ for } \mathrm{X} = \mathrm{P/A}$$

- Utilizing so called $\Gamma$ method developed within ALPHA Collaboration [6]
  $\rightarrow$ error estimation by explicit determination of autocorrelation functions and times (more certain error estimates than binning techniques, but requires continuous Monte Carlo series)

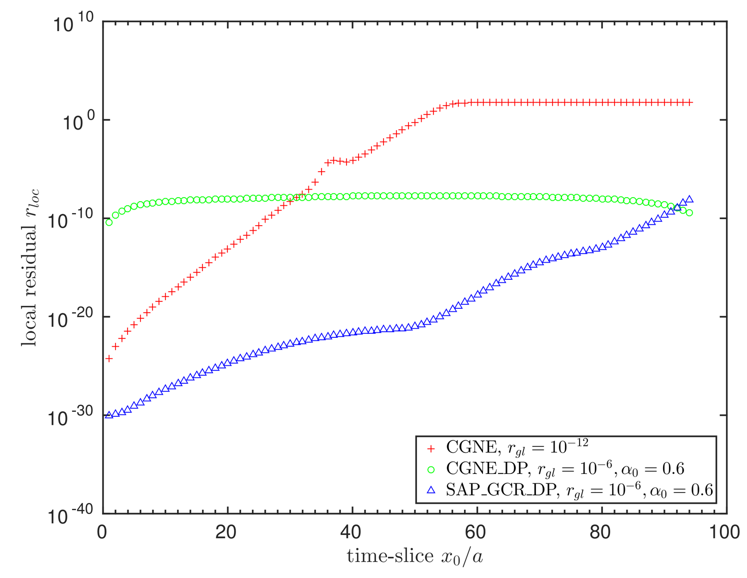
## Numerical tests of distance preconditioning

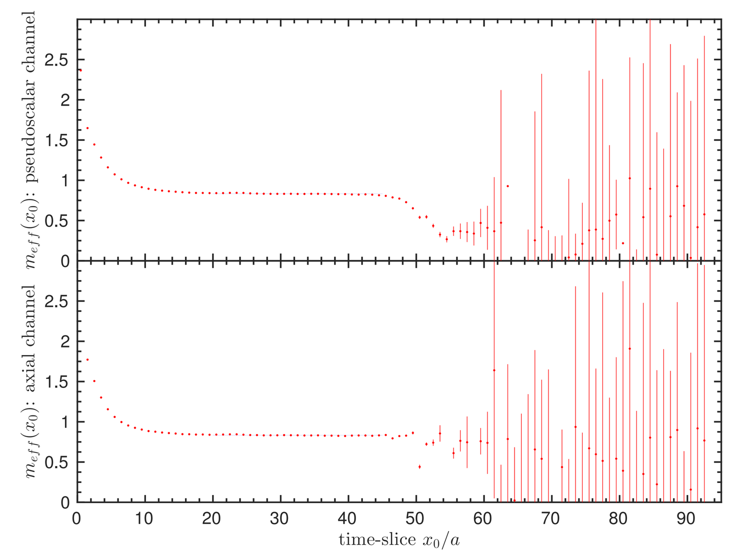
- Unmodified solver setup: locally deflated Schwarz preconditioned general conjugate residual solver (DFL_SAP_GCR) for light and strange quarks, conjugate gradient on the normal equations solver (CGNE) and DFL_SAP_GCR solver for heavy charm quarks
- Modified solver setup: DFL_SAP_GCR solver for l,s, distance preconditioned CGNE solver (CGNE_DP) & distance preconditioned SAP_GCR solver (SAP_GCR_DP) for h

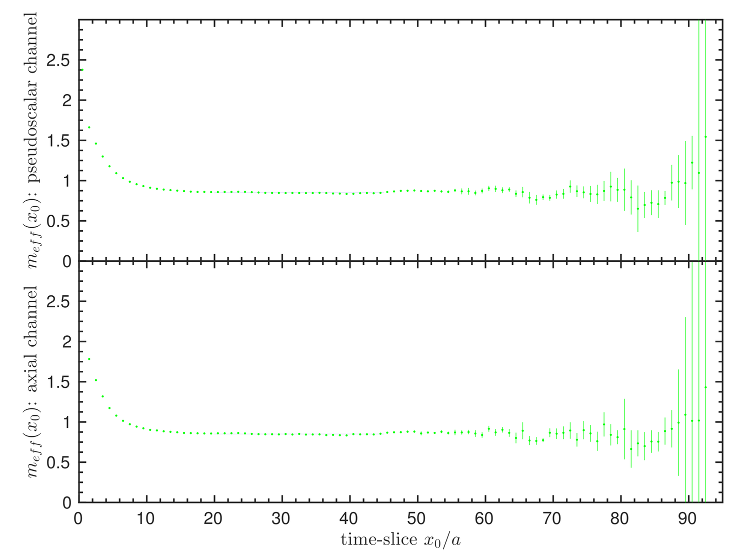




Numerical checks with modified CGNE_DP solver show increased accuracy for solution of heavy-heavy *correlator* on sample of CLS configurations (top left) and derived *effective* PS meson mass $m_{eff} = \ln\left(\frac{f_{PP}(x_0)}{f_{PP}(x_0+1)}\right)$ (top right). The behaviour of the *local* residual $r_{loc} = \frac{|AS - \eta_t|(x_0)}{|S(x_0)|}$ was tested for unmodified CGNE, modified CGNE_DP and modified SAP_GCR_DP solver (right; SAP_GCR_DP shown in blue)



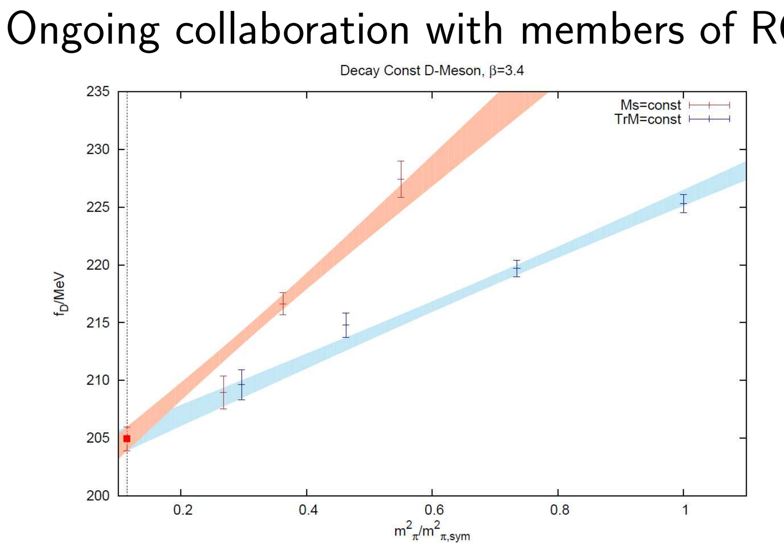


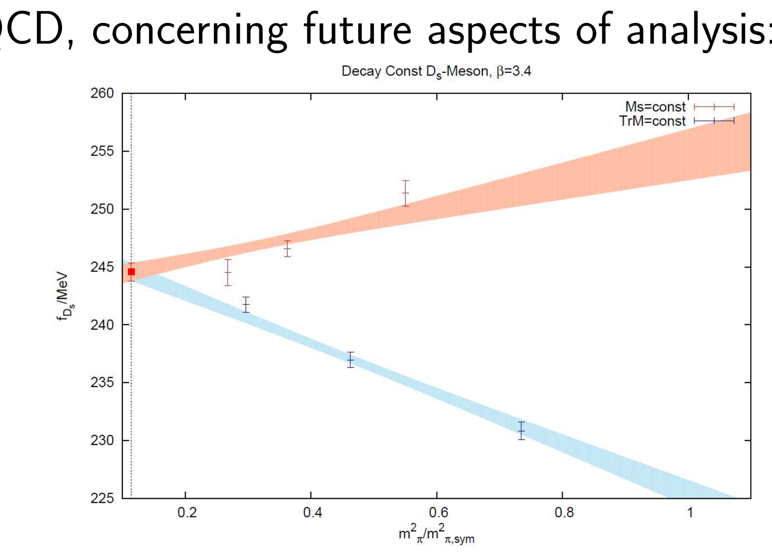


Considerable accuracy gain from unmodified solver setup (50 configurations of CLS H105r002 ensemble, top left) to modified setup for heavy-strange (with $r_{gl} = 10^{-4}$, $\alpha_0 = 0.7$, $r_{loc} < 10^{-10}$, top right).

## Further steps and outlook

Ongoing collaboration with members of RQCD, concerning future aspects of analysis:





- Chiral extrapolation to physical point, two proposed strategies: $(2m_l + m_s) = const.$ or $m_s = const.$ (lines of constant physics along renormalization group trajectories); examples for $f_D$ (top left) and $f_{D_{(s)}}$ (top right) borrowed from complementary poster by Stefan Hofmann (RQCD) presented at "Lattice 2016" and extracted from

$$C_A(x_0, y_0) = \frac{f_{PS}}{2} A(y_0) e^{-m_{PS}(x_0 - y_0)} \ , \ C_P(T - y_0, y_0) = \frac{|A(y_0)|^2}{2m_{PS}} e^{-m_{PS}(T - 2y_0)}$$

$$\text{with } A(y_0) = \langle 0|P|PS\rangle \ , \ f_{PS} \cdot m_{PS} = \langle 0|A_0|PS\rangle$$

- Continuum limit extrapolation (lattice spacing $a \rightarrow 0$), extra care has to be taken with regard to extremely localized charm quark
- Non-perturbative determination of renormalization factors (recently computed within ALPHA Collaboration)

Goal: **Precision measurement of the pseudoscalar decay constants $f_D$ and** $f_{D_{(s)}}$ to further explore already existing tensions between lattice calculations and experimental findings in the heavy flavour sector.

## References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