Ward identities in $\mathcal{N}$=1 SU(3) SUSY Yang-Mills theory on the lattice

S. Ali[1], G. Bergner[2], H. Gerber[1], P. Giudice[1], I. Montvay[3], G. Münster[1], S. Piemonte[4]

[1]Institut für Theoretische Physik, Universität Münster, Germany; [2]Institut für Theoretische Physik, Universität Bern, Schweiz;
[3]Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany; [4]Institut für Theoretische Physik, Universität Regensburg, Germany

The Model

$\mathcal{N}=1$ SUSY Yang-Mills Theory

The action:

$$S_{\rm SYM} = {\rm Re}\int d^4x\, d^2\theta\, {\rm Tr}[W^\alpha W_\alpha] = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2}\bar\lambda^a \gamma_\mu \mathcal{D}_\mu \lambda^a \right\}$$

- Field strength tensor:
  $F_{\mu\nu} = -{\rm i}g F^a_{\mu\nu} T^a = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$
- Covariant derivative in adjoint representation:
  $\mathcal{D}_\mu \lambda^a = \partial_\mu \lambda^a + g\, f_{abc} A^b_\mu \lambda^c, \qquad a = 1, \ldots, N_c^2 - 1\,,$
- Vector supermultiplet:
  1) Gauge field $A^a_\mu(x)$, "Gluon"
  2) Majorana-spinor field $\lambda^a(x)$, $\quad \overline{\lambda} = \lambda^T C\,,$ "Gluino"
- SUSY transformations (on-shell):

$$\delta A^a_\mu = -2g\overline{\lambda}^a \gamma_\mu \varepsilon\,,$$
$$\delta \lambda^a = -\frac{\rm i}{g}\sigma_{\mu\nu} F^a_{\mu\nu}\varepsilon$$

- In contrast to QCD:
  1) λ is Majorana spinor field, "$N_f = \frac{1}{2}$"
  2) adjoint representation of SU(N_c)
- Gluino mass term $m_{\tilde g}\, \overline{\lambda}^a \lambda^a$ breaks SUSY softly

Motivation

- SUSY gives natural solution to hierarchy problem in the Standard Model
- Three couplings meet at one point at 2×10^{16} GeV
- SYM: simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended Standard Model
- Possible connection to ordinary QCD
- Similar to QCD:
  1) Asymptotic freedom
  2) Confinement
  3) Numerical lattice simulation of bound states

Solution of non-perturbative Problems:

- Spontaneous breaking of chiral symmetry $\quad Z_{2N_c} \to Z_2$
  $\longleftrightarrow$ Gluino condensate $\quad < \lambda\lambda > \neq 0$
- Spectrum of bound states $\to$ Supermultiplets
- Confinement of static quarks
- Spontaneous breaking of SUSY?
- SUSY restoration on the lattice
- Check predictions from effective Lagrangeans (Veneziano, Yankielowicz, ...)

Spontaneous breaking of chiral symmetry

U(1)$_\lambda$: $\lambda' = {\rm e}^{-{\rm i}\varphi\gamma_5}\lambda\,, \overline{\lambda}' = \overline{\lambda}\,{\rm e}^{-{\rm i}\varphi\gamma_5} \leftrightarrow$ R-symmetry, $J_\mu = \overline{\lambda}\gamma_\mu\gamma_5\lambda$

Anomaly: $\partial^\mu J_\mu = \frac{N_c g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$ $\quad$ breaks $\quad {\rm U}(1)_\lambda \to Z_{2N_c}$

Spontaneous breaking $\quad Z_{2N_c} \to Z_2$
by Gluino condensate $< \lambda\lambda > \neq 0$
$\leftrightarrow$ first order phase transition at $m_{\tilde g} = 0$

$N_c = 2: \quad < \lambda\lambda > = \pm C \Lambda^3$

Spectrum of bound states

Expect colour neutral bound states of gluons and gluinos
$\to$ Supermultiplets

Predictions from effective Lagrangeans:
Chiral supermultiplet (Veneziano, Yankielowicz)

- 0^- gluinoball $\quad$ a - $\eta' \quad \sim \quad \overline{\lambda}\gamma_5\lambda$
- 0^+ gluinoball $\quad$ a - $f_0 \quad \sim \quad \overline{\lambda}\lambda$
- spin $\frac{1}{2}$ gluino-glueball $\quad \sim \quad \sigma_{\mu\nu}\,{\rm Tr}\,(F_{\mu\nu}\lambda)$

Generalization (Farrar, Gabadadze, Schwetz):
additional chiral supermultiplet

- 0^- glueball
- 0^+ glueball
- gluino-glueball

possible mixing

Simulations

SUSY on the lattice

Lattice breaks SUSY. Restoration in the continuum limit?
Curci, Veneziano: use Wilson action, search for continuum limit with SUSY

$$S = -\frac{\beta}{N_c}\sum_p {\rm Re\,Tr}\; U_p$$

$$+\frac{1}{2}\sum_x \left\{ \overline{\lambda}^a_x \lambda^a_x - \kappa \sum_{\mu=1}^{4} \left[\overline{\lambda}^a_{x+\hat\mu} V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x + \overline{\lambda}^a_x V^t_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat\mu} \right] \right\}$$

$$\beta = \frac{2N_c}{g^2}, \quad \kappa = \frac{1}{2m_0+8} \quad \text{hopping parameter}, \quad m_0 : \text{bare gluino mass}$$

$$V_{ab,x\mu} = 2\,{\rm Tr}\,(U^\dagger_{x\mu} T_a U_{x\mu} T_b), \quad \text{adjoint link variables}$$

We study gauge group SU(3).

Fermion integration

Fermionic action

$$S_f = \frac{1}{2}\overline{\lambda}Q\lambda = \frac{1}{2}\lambda M \lambda\,, \qquad M \equiv CQ$$

Pfaffian

$$\int [d\lambda]\; {\rm e}^{-S_f} = {\rm Pf}(M) = \pm\sqrt{\det Q}$$

Effective gauge field action

$$S_{\rm eff} = -\frac{\beta}{N_c}\sum_p {\rm Re\,Tr}\; U_p - \frac{1}{2}\log\det Q[U]$$

Include sign Pf(M) in the observables.

Monte Carlo algorithm

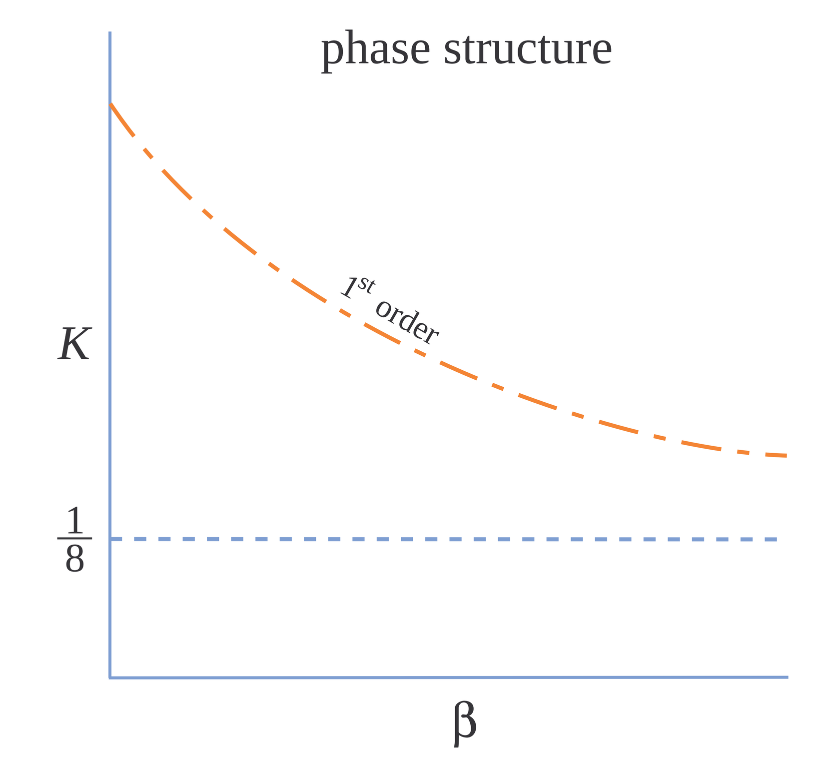
- Monte Carlo Simulation has been used to generate configurations.
- Each configuration contains numerical values of link variables (U).
- These configurations are used to compute correlation functions.

Sign Problem:
monitoring of sign Pf(M)

- through spectral flow
- by calculation of real negative eigenvalues of Q with Arnoldi

$\to$ Negative Pfaffians occur in our simulations near κ_c, but rarely.

Phase transition for SU(3)



The dashed red line: $\kappa = \kappa_c(\beta)$, corresponds to the first order phase transition at zero renormalized gluino mass.

Ward identities

Noether Theorem in classical theory $\to$ Ward identities in quantum theory

Expression in the continuum: $\langle (\partial_\mu j^\mu(x))Q \rangle = -\left\langle \frac{\delta Q}{\delta\bar\varepsilon(x)} \right\rangle$

- $j^\mu(x)$ is the Noether current
- Q is an insertion operator.
- $\varepsilon(x)$ is the parameter of infinitesimal symmetry transformations.
- RHS of equation is contact term, which is zero if Q is localised at space-time points different from x.

Results

SUSY Ward identities on lattice

SUSY transformations on the lattice with P, T and gauge invariance:

$$\delta U_\mu(x) = -\frac{{\rm i}g_0 a}{2}\Big(\bar\epsilon(x)\gamma_\mu U_\mu(x)\lambda(x) + \bar\epsilon(x+\hat\mu)\gamma_\mu\lambda(x+\hat\mu)U_\mu(x) \Big)$$
$$\delta\lambda(x) = +\frac{1}{2}P^{(cl)}_{\mu\nu}(x)\sigma_{\mu\nu}\epsilon(x)$$

Above transformations result in following Ward identities:

$$\big\langle \big(\nabla_\mu S^{(sp)}_\mu(x)\big)Q(y)\big\rangle = m_0\big\langle\chi(x)Q(y)\big\rangle + \big\langle X^{ps}(x)Q(y)\big\rangle - \left\langle\frac{\delta Q(y)}{\delta\bar\epsilon(x)}\right\rangle$$

Terms containing $\chi(x)$ and $X^{(ps)}(x)$ break SUSY and come from non-zero bare gluino mass and lattice regularization.
After renormalization WI gets the following form:

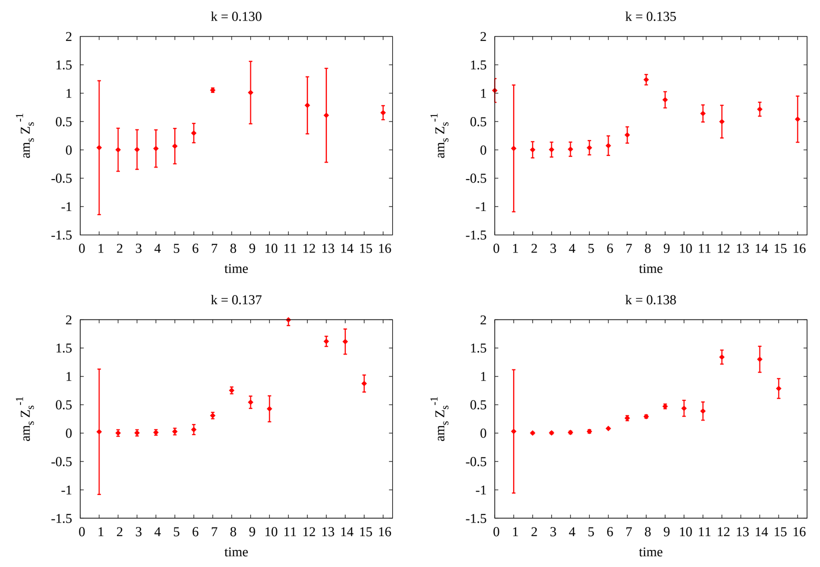
$$\big\langle\big(\nabla_\mu S_\mu(x)\big)Q(y)\big\rangle + \frac{Z_T}{Z_S}\big\langle\big(\nabla_\mu T_\mu(x)\big)Q(y)\big\rangle = \frac{m_S}{Z_S}\big\langle\chi(x)Q(y)\big\rangle + O(a)$$

Z_S , Z_T and Z_χ are renormalization coefficients. Zero spatial momentum and expansion in a basis of 16 Dirac matrices; the surviving contributions form a set of two non-trivial independent equations:

$$C^{(S,\mathcal{O})}_{\mathbb{1}}(t) + (Z_T Z_S^{-1})C^{(T,\mathcal{O})}_{\mathbb{1}}(t) = (a m_S Z_S^{-1})C^{(\chi,\mathcal{O})}_{\mathbb{1}}(t)$$
$$C^{(S,\mathcal{O})}_{\gamma_0}(t) + (Z_T Z_S^{-1})C^{(T,\mathcal{O})}_{\gamma_0}(t) = (a m_S Z_S^{-1})C^{(\chi,\mathcal{O})}_{\gamma_0}(t)$$

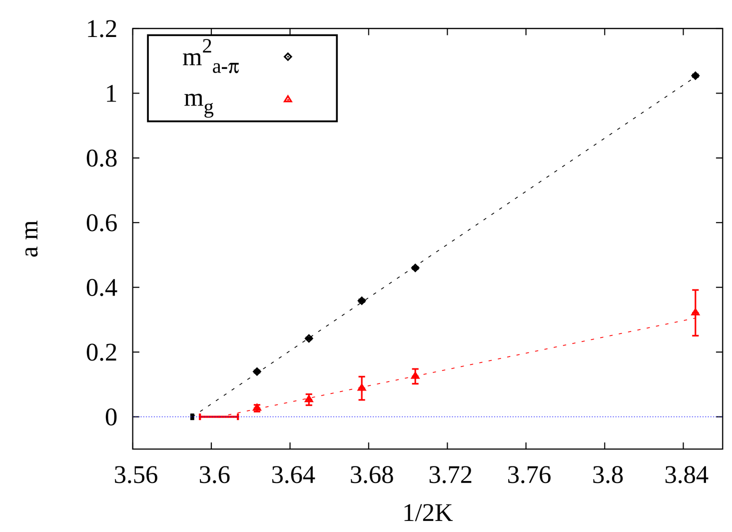
Numerical results for SU(3)

Gluino masses ($a m_S Z_S^{-1}$) as a result of simulations of WIs on a lattice:
Lattice volume $= 16^3 \cdot 32$, $\quad$ gauge coupling $\beta = 4.0$
Final values of masses are taken at $t = 4, 5$ for each κ



Extrapolations to the chiral limit

- Masses are fitted and extrapolated to zero to get κ_c. It is the point in parameter space where theory is characterized by massless gluino (Chiral limit).
- Consistent with restoration of SUSY up to lattice artefacts



Summary

- First order phase transition at $m_{\tilde g} = 0$
- SUSY WIs on the lattice
- Determination of $m_{\tilde g}$ using WIs
- Extrapolations towards vanishing gluino mass using WIs
- Extrapolations towards vanishing gluino mass using $m^2_{a-\pi}$
- Consistency between κ_c from WIs and from $m_{a-\pi^2}$
- Consistency with restoration of SUSY

References

References

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