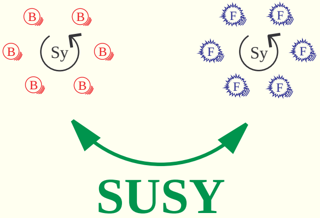
Numerical Simulation of Supersymmetric Yang-Mills Theory

S. Ali, G. Bergner, H. Gerber, P. Giudice,
I. Montvay, G. Münster, S. Piemonte

WWU Münster, Uni Bern, DESY Hamburg, Uni Regensburg

GRK 2149 Annual Retreat, November 24 – 26, 2015



Supersymmetry

Unsatisfactory properties of the Standard Model:

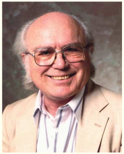
- 18 free parameters
- mass hierarchy
- neutrino masses
- dark matter

Is there anything beyond the Standard Model?

Supersymmetry

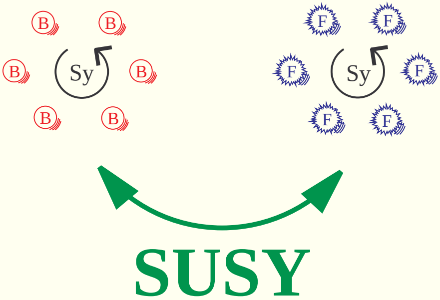
Supersymmetry (Golfand, Likhtman, Volkov, Akulov, Wess, Zumino)

J. Wess



B. Zumino

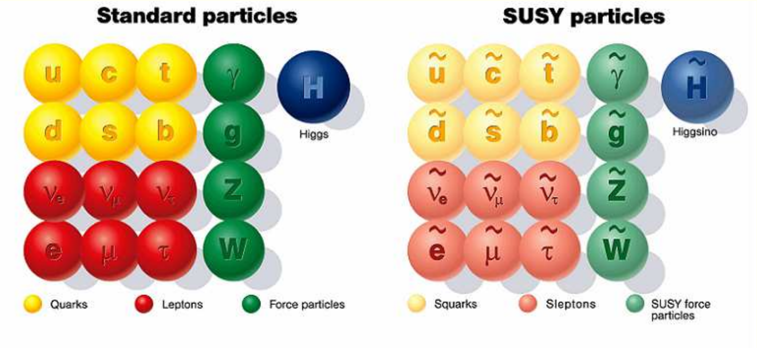
SUSY relates bosons to fermions:



$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Supersymmetry

Superpartners of the Standard Model

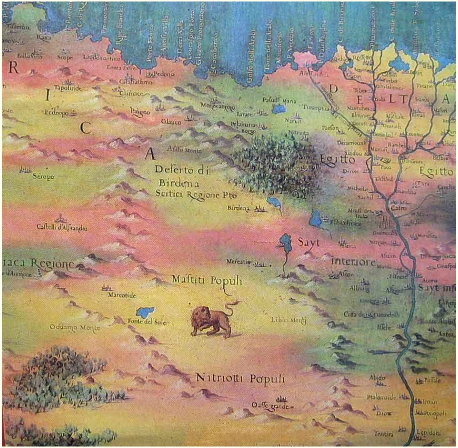


Supersymmetry

What are the properties of supersymmetric models?

Supersymmetry

What are the properties of supersymmetric models?



Hic sunt leones!

Methods: mainly perturbation theory and semiclassical methods

Non-perturbative investigations $\longrightarrow$ lattice discretisation

$\mathcal{N} = 1$ SUSY Yang-Mills Theory

- Simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended standard model

Vector supermultiplet:

- Gauge field $A_\mu^a(x)$, "Gluon"
  Gauge group SU(N_c)

- Majorana-spinor field $\lambda^a(x)$, "Gluino"

Lagrangean

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{\ a} F_{\mu\nu}^{\ a} + \frac{\mathrm{i}}{2} \overline{\lambda}^a \gamma_\mu (\mathcal{D}_\mu \lambda)^a$$

$\mathcal{N} = 1$ SUSY Yang-Mills Theory

- Similar to QCD

  Differences: λ : 1.) Majorana, "$N_f = \frac{1}{2}$"
  2.) adjoint representation of $SU(N_c)$

- Gluino mass term

  $$m_{\tilde{g}}\ \overline{\lambda}^a \lambda^a$$

  breaks SUSY softly.

Non-perturbative Problems

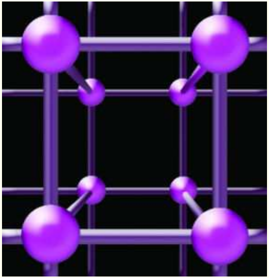
- Spontaneous breaking of chiral symmetry $Z_{2N_c} \quad \rightarrow \quad Z_2$

  $\updownarrow$

  Gluino condensate $< \lambda\lambda > \neq 0$
- Confinement of static quarks.
- Spontaneous breaking of SUSY?
- SUSY restauration on the lattice
- Spectrum of bound states
  $\rightarrow$ Supermultiplets
- Check predictions from
  effective Lagrangeans
  (Veneziano, Yankielowicz, ...)

Lattice field theory

Continuous space-time $\longrightarrow$ lattice



Lattice spacing a provides momentum cut-off

Lattice field theory

K. Wilson's approach: Euclidean lattice field theory

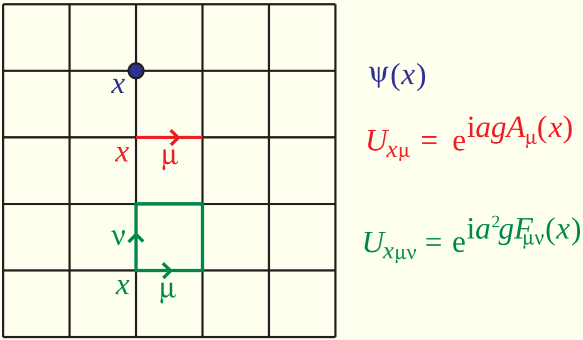
Basic principles

- 4-dimensional space-time lattice
- Imaginary time $t = -\mathrm{i}\tau$
- Functional integral approach à la Feynman

allows Monte Carlo simulations

Lattice field theory

Fields on the lattice



link variable $U_{x\mu} \in \mathrm{SU}(N)$

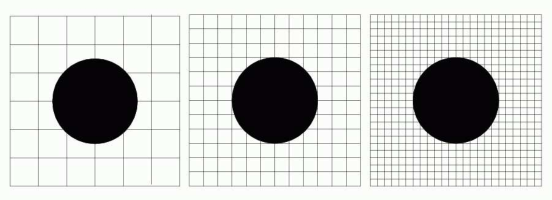
SUSY on the lattice

Lattice breaks SUSY.

$$\{Q, Q^{+}\} \sim P_\mu$$

Restauration in the continuum limit?

Search for continuum limit $a \to 0$ with restored SUSY



SYM project

Calculate

Functional integral

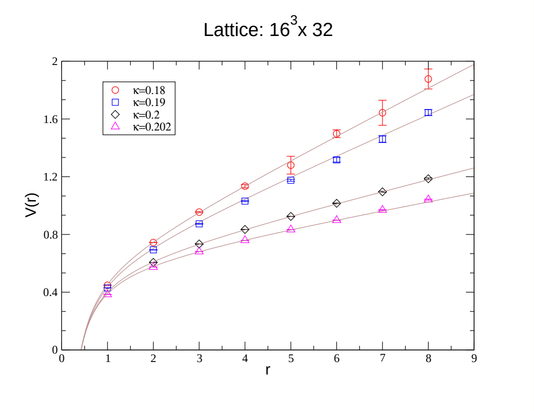
$$Z = \int [DU][D\lambda]\ \mathrm{e}^{-(S_g + S_f)}$$

by means of numerical simulations with the Monte Carlo method.

Gauge group SU(2) in our previous work.

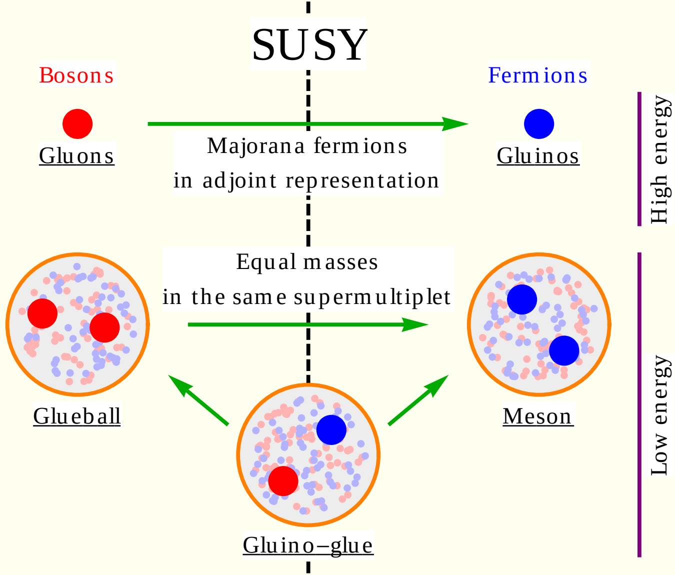
Static potential

Static quark potential $V(R)$ $(\beta = 1.6)$



Linear rise $\leftrightarrow$ Confinement

Bound states

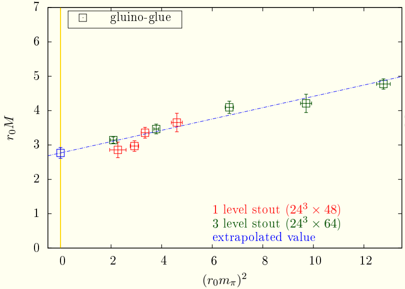


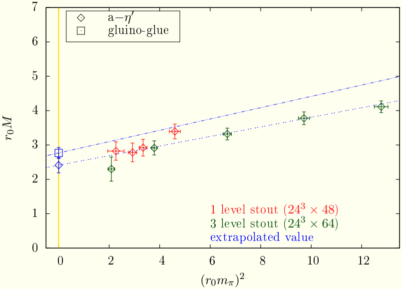
Bound states

Lattices $16^3 \cdot 32, \quad 24^3 \cdot 48, \quad 32^3 \cdot 64,$

$a \sim 0.086$ fm, 0.053 fm, 0.036 fm in QCD units

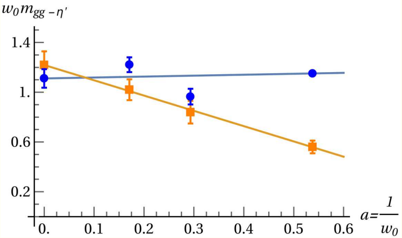
Extrapolations to $m_{\tilde{g}} = 0$

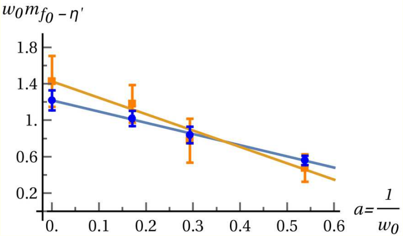




Bound states

Extrapolations to the continuum





Status

Status:

- Gauge group SU(2)
- Consistency with SUSY Ward identities
- Quantitative results about the low-energy spectrum
- Results are consistent with the formation of degenerate supermultiplets

Project

Gauge group SU(3): "real" gluons + gluinos

- different phase structure, spontaneous CP violation
- new types of bound states
- finite temperature studies
- . . .