Multiple stability and pattern control in a photorefractive feedback system

Michael Schwab, Cornelia Denz
Institute of Applied Physics, Darmstadt University of Technology
Hochschulstr. 6, D-64289 Darmstadt, Germany
Phone: xx-49-6151/163482, Fax: xx-49-6151/164123
Michael.Schwab@physik.tu-darmstadt.de

Mark Saffman
Optics and Fluid Dynamics Department, Risø National Laboratory,
Postbox 49, DK-4000 Roskilde, Denmark

Abstract: We investigate the formation of square, rectangular or squeezed hexagonal patterns in a photorefractive single-feedback experiment. Although in general, hexagonal patterns are predominantly excited in these systems, non-hexagonal patterns may be stable in a certain range of the propagation length. Moreover, a coexistence of two hexagons with two different transverse scales is found and explained by the results of a linear stability analysis. To achieve control of these patterns, we introduce a two-arm feedback system with a Fourier filter in one arm and present results on the stabilization of roll and square pattern in a parameter region where the hexagonal pattern is the only output without control.
OCIS codes: (190.5330) Photorefractive Nonlinear Optics, (190.3100) Instabilities and Chaos, (999.9999) Pattern formation, (999.9999) Pattern Control

Introduction

Nonlinear optical systems are well-known to lead to the spontaneous formation of periodic spatial patterns, e.g. atomic vapours [1], liquid crystals (Kerr slices) [2, 3], organic films [4] or photorefractives [5], where squares and squeezed hexagons were first observed in experiment [6]. Photorefractive materials are well-suited for pattern observation since their intrinsically slow dynamics offers the opportunity to perform real-time measurements and observations. Moreover, low cw powers in the range of milliwatts are required and in the case of a diffusion-dominated crystal such as $KNbO_3$, no external voltage has to be supplied providing an all-optical pattern formation system. In all these systems, a single-feedback configuration creating two counterpropagating beams in the nonlinear optical medium gives rise to transverse modulational instabilities above a certain threshold. These instabilities generally lead to the formation of hexagonal patterns, which were first reported for a photorefractive system by Honda [5]. Following this pioneering work, various other publications offered improved insight into the stages of pattern formation in these photorefractive materials [7, 8]. A first approach to a nonlinear stability analysis [9] and close studies of pattern dynamics due to angular misalignment and competition behaviour were published recently [10]. Our focus of interest is to investigate more complex patterns that may arise in the same configuration for a certain range of the propagation length without changing the basic interaction geometry. Although some of these patterns have been observed earlier [6], the appropriate region of instability has not yet been investigated. We also explain the coexistence of two hexagons on different transverse scales by the results of the linear stability analysis.

Since we are able to observe a large number of different pattern states, it is our prefered aim to get controlled access to all these patterns. From the point of view of usefulness of spontaneous pattern formation in the growing field of optical information processing [11], it is of high interest to access a maximum number of different pattern states, or to manipulate the system in order to stabilize a desired solution. Spatio-temporal control is currently of high interest [12]. A common feature of all these control methods is that they are basically nonintrusive. When the desired state is reached, the control signal, i.e. the energy removed from, or added to the system tends to a very small level. The method of manipulation in the Fourier plane is known to provide all these basic properties of pattern control and is extremely well-known as a powerful tool in modern optics. The potential of this method was proposed theoretically [13] and soon proved experimentally [14, 15] for a photorefractive nonlinear system. Here, we present a novel control configuration using a two-arm feedback. The control is provided by a second control-arm without changing the basic interaction geometry. We will show that squares and rolls can be stabilized in parameter regions where hexagonal patterns are the only stable solution of the system.

Linear stability analysis

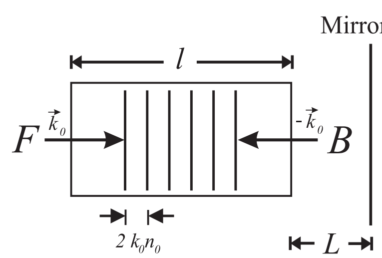

Fig. 1. Basic interaction geometry.

The basic interaction geometry is depicted in Fig. 1. A plane wave of complex amplitude F is incident on a thick photorefractive medium with length l. The backward beam B is produced by reflection at a mirror at a certain position L behind the medium. Our analysis is not restricted to positive propagation lengths since the 4f-4f-configuration enables us experimentally to produce negative propagation lengths which are essential for observing a multiple pattern stability. The principle function of this propagation length L is to introduce a phase lag relative to the central beam. A diffusion-dominated medium such as $KNbO_3$ offers beam coupling properties which are essential for pattern formation in this configuration. In this case, a dynamic photorefractive grating with spacing $2k_0n_0$ is written.

The linear stability analysis presented here is based on the treatment by Honda and Banerjee given in [8]. It is derived from Kukhtarev's equations for photorefractive two-beam coupling [16] and based on the assumption that reflection gratings are dominant in this configuration, which has been shown to be unstable against periodic disturbations. The usual equations for contradirectional two-beam coupling in a diffusion-dominated medium can be written as [8]

$$\frac{\partial F}{\partial z} - \frac{i}{2k_0 n_0}\nabla_\perp^2 F = i\gamma \frac{|B|^2}{|F|^2 + |B|^2} F \quad (1)$$

$$\frac{\partial B}{\partial z} + \frac{i}{2k_0 n_0}\nabla_\perp^2 B = -i\gamma^* \frac{|F|^2}{|F|^2 + |B|^2} B\,, \quad (2)$$

Performing a linear stability analysis for this system of partial differential equations and including the boundary conditions (phase lag by feedback), one can obtain a threshold condition $f(\theta, \gamma l, L) = 0$ for the modulational instability [8]. Given a certain mirror position L, a threshold curve can be plotted (see Fig. 2 a), where the absolute minimum provides information about the unstable sideband angle θ at a certain mirror position. If the relative minima of this threshold curve are numbered consecutively, the values of these minima depending on the mirror position for the first to fourth instability "balloon" are plotted in Fig. 2 b).

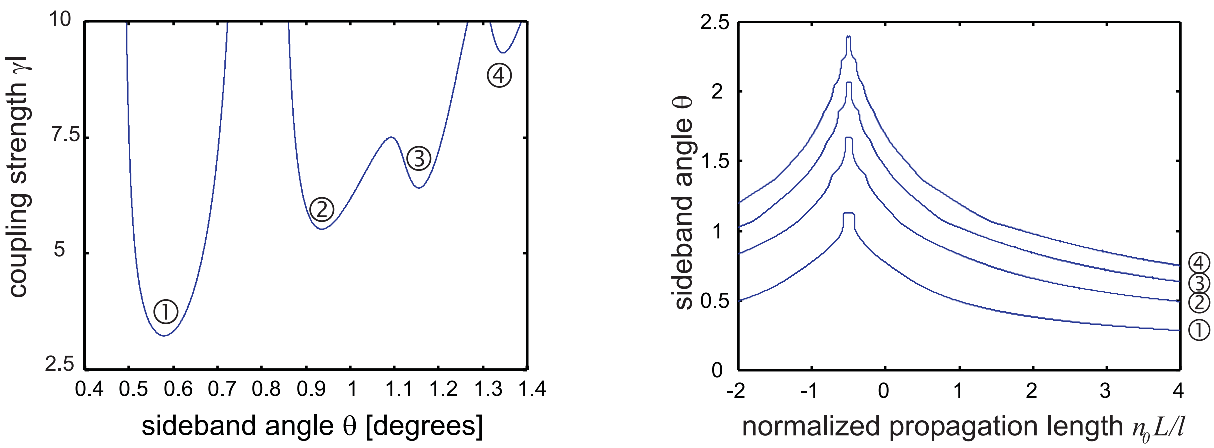

Fig. 2. Results from the linear stability analysis: a) Threshold curve for n_0L/l=-1.6, b) theoretical values for the minima of the threshold curves for varying values of L

Fig. 2 b) extends previous results of the linear stability analysis [9], taking into account negative propagation lengths. One can see clearly the strange behaviour of the sideband angle-curve in a region near n_0L/l=-0.5. It is probable that the special shape of the curve may give rise to unexpected patterns in this parameter region. However, a nonlinear stability analysis is required for explaining the occurence of different pattern types.

Multiple pattern stability

The experimental setup is depicted in Fig. 3. Light obtained from a frequency-doubled Nd:YAG laser operating at 532 nm is focused by a lens L1 of focal length of f=600 mm onto the exit face of an Iron-doped $KNbO_3$ crystal (l=5 mm), producing a spot with a Gaussian diameter of 320 μm. The crystal was slightly inclined (about 4 degrees) in order to avoid undesired back-reflections from the crystal surfaces.

By means of a 4f-2L-4f-System with f=100 mm, the incoming beam is back reflected, thus providing the counterpropagating beam. Considering ABCD-matrix formalism, this configuration can be shown to be completely equivalent to a simple single mirror feedback configuration. Thus, a virtual mirror with a distance of L from the photorefractive medium is obtained. The basic

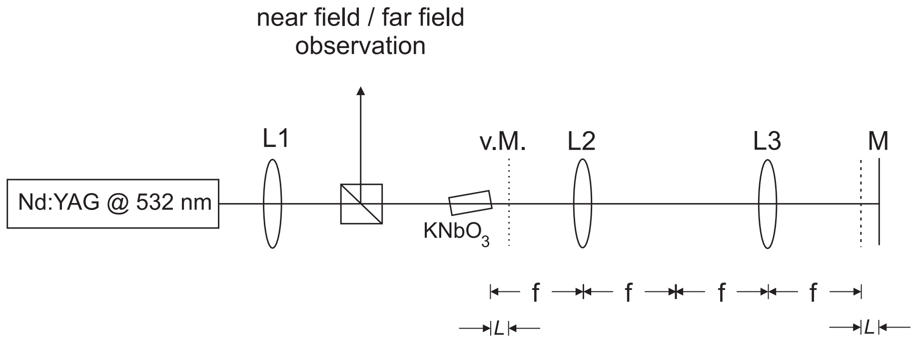

Fig. 3. Experimental setup. M=mirror, v.m.=virtual mirror, L=propagation length

advantage of this system is that negative propagation lengths can be achieved, which allows to access a broader range of stationary patterns, including squares and rectangles. The laser beam is linearly polarized along the crystal a-axis to exploit the large r_{13} component of the electrooptic tensor in this direction, resulting in a minimum input power for pattern observation of just 0.5 mW, which is extremly low compared to other pattern forming systems. A beam splitter between the focusing lens and the photorefractive medium enables to observe the far field, and by means of a lens and a microscope system, the near field, respectively. The direction of the crystal c-axis is arranged to give rise to depletion of the incoming and amplification of the backward reflected beam, a configuration that is necessary for the observation of transverse structures in this material. The reflectivity of the feedback system including all elements was measured to be $R = 83\%$. The only stable solution for positive propagation lengths is a hexagonal one, as depicted in Fig. 4 a).

Higher order harmonics of the hexagonal pattern are clearly seen in experiment. They saturate the explosive instability of the first order hexagon and are essential for the stability of a hexagonal pattern [9]. This hexagonal structure is well-known to be dominant for many different

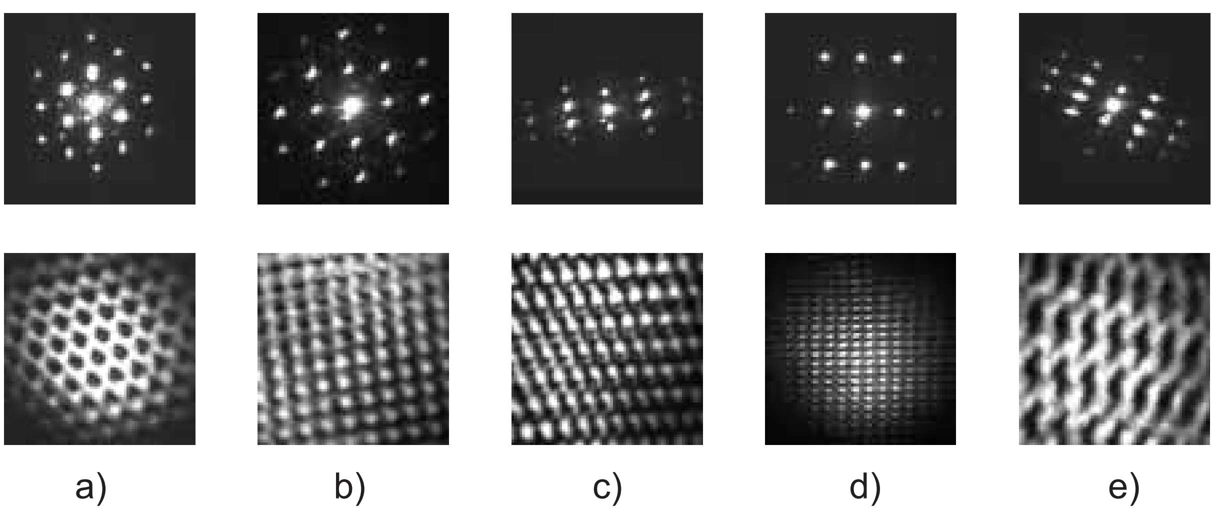

Fig. 4: Experimentally obtained patterns: a) predominant hexagonal structure, b-e) other pattern geometries for the multiple pattern region

nonlinear optical materials and is reported for a number of other non-optical pattern forming systems. However, when the virtual mirror is shifted into the crystal, i.e. when negative propagation lengths are achieved, a remarkable pattern transition occurs: In a small parameter region of the propagation length around $n_0L/l = -0.5$, different non-hexagonal structures may appear, being either stable or alternating in time due to pattern competition. Square patterns, squeezed hexagonal, rectangular or parallelogram-shaped strucures (see Fig. 4 b-e) can be found. Outside this multiple pattern region, no other patterns than hexagonal ones can be observed experimentally. The reason for the occurence of these patterns can only be found by a nonlinear analysis, since a linear stability analysis only accounts for the occurence of a special transverse wavevector to become unstable when excited beyond the instability threshold. A previously derived nonlinear stability analysis [9] only deals with positive propagation lengths and explains hexagons as the predominant stable solution.

However, detailed investigations of the results of the linear stability analysis taking into account higher order instability balloons may give useful information about the pattern type. Fig. 5 shows the measured values for the hexagon instability angle θ as a function of the normalized propagation length n_0L/l (virtual mirror is inside the crystal for values $-1 \leq n_0L/l \leq 0$) together with the theoretical results of the linear stability analysis (first and second instability balloon). The measured values agree well with the theoretical curves. As predicted in [10], a coexistence of two transverse $\vec{k}$-vectors appears for larger positive or negative propagation length as indicated in the figure.

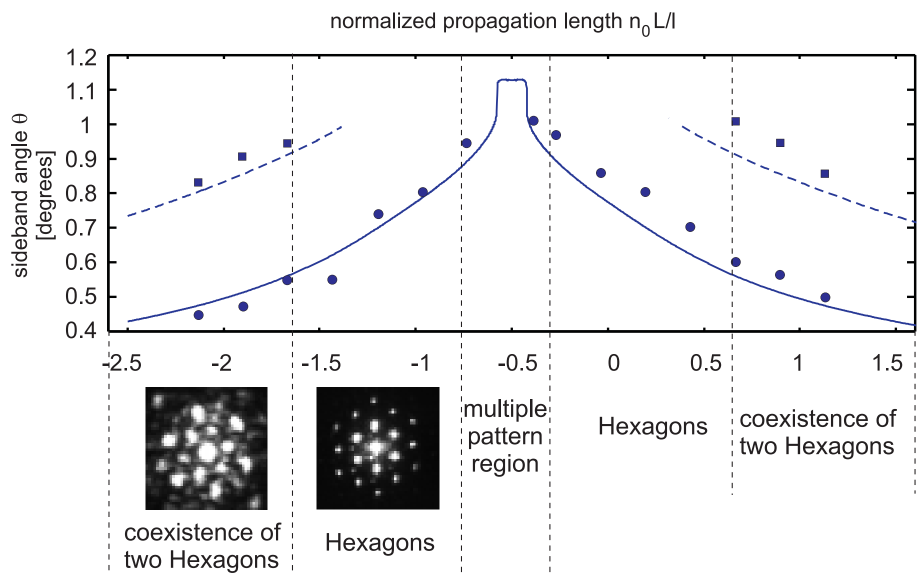

Fig. 5. Sideband angle θ as function of the normalized propagation length n_0L/l. Theoretical curve is displayed together with experimental values for the transverse scale (circles and squares)

Here, a second instability balloon (see Fig. 3 in [10]) takes the absolute minimum of the instability curve thus leading to a degeneration of the transverse wave-vector. This leads to a coexistence of two hexagons on two transverse scales tilted by 30 degrees relative to each other. The multiple pattern region described earlier is also indicated in the figure. Since a large number of various transverse $\vec{k}$-vectors occurs for $-1 \leq n_0L/l \leq 0$, no further experimental values are displayed here.

Previously, we reported a remarkable pattern collapse for the region where now the multiple patterns were found using a different crystal. We explained it with a photorefractive coupling strength being too low for the observation of transverse structures. Since the coupling gain was higher in the experiments we report here, this in turn proves our assumption in [10].

A transition to a roll pattern can be observed for all values of the propagation length, but low reflectivities ($R = 4\%$). The reflectivity border for pattern formation was just $R = 2\%$, which is remarkably low.

Pattern control by two-arm feedback

The preceding chapters show clearly that pattern forming systems offer a great variety of possible solutions. However, in most parameter regions, only one special pattern, predominantly a hexagonal one, is excited. Other patterns such as square or squeezed hexagonal patterns may be underlying as stable or unstable solutions.

These pattern forming systems are potential candidates for applications in the large growing field of optical information processing, e.g. for pattern recognition or encryption [11]. For this reason, one has to gain immediate access to a full set of possible solutions, either stable or unstable for a maximum number of basic functions. For this purpose, mechanisms need to be developed in order to access these different patterns. Since pattern formation is always accompanied by the stabilization of the appropriate $\vec{k}$-vectors, spatial filtering in the Fourier plane is an adequate means of pattern control and manipulation. An important aspect of a suitable control method is that the control signal should tend to a very small level when the desired state is reached. Consequently, Fourier-space techniques have been shown to be a proper choice for the stabilization and selection of unstable patterns [13]. Positive control enhances a desired solution by allowing only the correct Fourier components to pass the filter. On the other hand, negative control annihilates the undesired solution thus favouring a different, desired pattern. In the case of negative control, the feedback control signal tends to zero when the desired state is reached.

Here, we present an extension of previous control experiments, implementing a two-arm, Michelson-like feedback configuration [3] which avoids the problems of bidirectional signal propagation in ring control configuration in [15]. The setup for pattern control is depicted in Fig. 6.

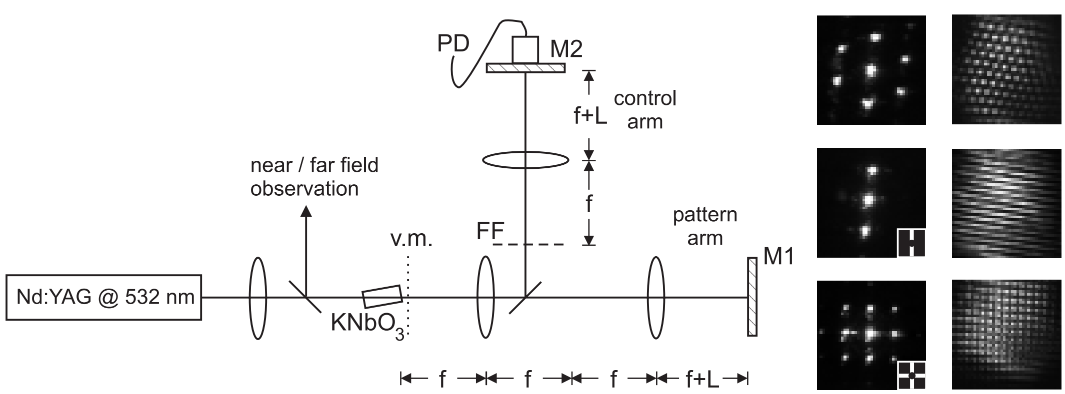

Fig. 6. Left: Experimental setup for control. FP=Fourier plane, PD=Piezo Driver. Right: Experimental results for pattern control, top: without control arm, middle and bottom: with control arm and Fourier filter shaped as indicated in the figure.

In contrast to Fig. 3, an additional beam splitter in the feedback arm allows for separation of the pattern arm and the control arm. In the second (control) arm, a Fourier mask is inserted with an appropriate shape to achieve pattern control. We also used a piezo-driven mirror for controlled adjustment of the relative phase of both arms. Only a part of the system's energy is coupled out and reinjected into the system compared to [14], where Fourier control was performed invasively in the feedback arm itself. Nevertheless, in both cases, the energy absorbed by the filter tends to a very small level.

The main physical parameters remained the same as in the appropriate experiment with only one feedback arm. The reflectivity ratio of the two arms is $R_1/R_2 = 1.5$, and the propagation length is adjusted to be L=1 mm for both arms. We also made sure that for both arms individually, hexagons with the same size (thus confirming the equality of both propagation lengths) appeared as the only stable solution. The experimental results are depicted in the inset of Fig. 6 together with the corresponding Fourier filter shape. The central spot was always blocked by the Fourier masks in our experiment in order to avoid undesired effects from two-beam interference which may lead to different pattern formation effects [3].

In a parameter space region where only hexagonal patterns are predominant, we managed to excite a roll or square pattern by positive control in every desired orientation. Thus, we were able to switch from hexagons (no control) to rolls or squares (controlled system) by using a slit or a cross-shaped mask. The control signal was in both cases very small, only 4.7 % of the energy in the original control arm. The time to form these new, controlled patterns was less than one second, and in the range of the photorefractive time constant for this crystal. By using an appropriate three-fold slit filter, we also succeeded in locking the hexagon position to a desired orientation by positive and negative control. In the latter case, the control signal nearly vanished when the desired hexagon orientation was chosen. We repeated this control scheme in the multiple pattern region (for $n_0L/l = -0.5$), which gave similar results. A more detailed analysis of this two-arm feedback control system is being published elsewhere, together with a theoretical model based on a simplified model with a saturable Kerr medium as the nonlinear element. This novel control scheme is a clear improvement to previous results, since we were able to separate the pattern forming arm and the control arm.

Conclusion

We have shown that a photorefractive feedback system offers a variety of different spatial patterns. The occurence of nonhexagonal patterns is restricted to a small parameter region where the virtual mirror is placed inside the crystal, allowing for negative propagation lengths. This multiple pattern region coincides with a strange shape of the corresponding curves for pattern size vs. propagation length derived from a linear stability analysis. In the multiple pattern region, a temporal alternation of different patterns is possible. This is, to the best of our knowledge, the first observation of a multiple pattern parameter region. This observation may not be restricted to photorefractives and could be observed in other optical pattern forming systems.

We also discovered a coexistence of two hexagonal pattern which can be explained by the existence of two instability balloons competing for the absolute minimum of the threshold curve. We could clearly separate the parameter regions of these coexisting hexagons, pure hexagons and multiple stability.

In addition, we presented a control system in a two-arm feedback configuration allowing for the stabilization of these various patterns in a parameter region where only hexagonal patterns were dominant. Negative control was achieved for turning the hexagon position. This novel control method is an improvement to earlier achievements, since a clear separation between the pattern forming arm and the control arm is possible.

Acknowledgements

We would like to acknowledge support by Prof. Dr. T. Tschudi. M. Schwab acknowledges partial support by a DAAD scholarship.

References

1. G. Grynberg, E. LeBihan, P. Verkerk, P. Simoneau, J.R.R. Leite, D. Bloch, S. LeBioteux and M. Ducloy, Opt. Comm. **67**, 363 (1988); J. Pender, L. Hesselink, J.Opt.Soc.Am. B**7**, 1361 (1990).
2. R. Macdonald, H.J. Eichler, Opt. Comm. **89**, 289 (1992); B. Thüring, R. Neubecker, T. Tschudi, Opt. Comm. **102**, 111 (1993).
3. M.A. Vorontsov, A.Yu. Karpov, Opt. Lett. **20** No. 24, 2466 (1995); M.A. Vorontsov, A.Yu. Karpov, J.Opt.Soc.Am. B **14**, 34 (1997).
4. J. Glückstad, M. Saffman, Opt. Lett. **20**, 551 (1995).
5. T. Honda, Opt. Lett. **18**, 598 (1993);
6. T. Honda, H. Matsumoto, M. Sedlatschek, C. Denz, T. Tschudi, Opt. Comm. **133**, 293 (1997).
7. M. Saffman, A.A. Zozulya and D.Z. Anderson, J.Opt.Soc.Am.B **11**, 1409 (1994); T. Honda and H. Matsumoto, J.Opt.Soc.Am.B **11**, 1983 (1994); T. Honda, Opt. Lett. **20**, 851 (1995).
8. T. Honda, P.P. Banerjee, Opt. Lett. **21**, 779 (1996).
9. P.M. Lushnikov, JETP **86** No.3, 614 (1998).
10. C. Denz, M. Schwab, M. Sedlatschek, T. Tschudi, T. Honda, J.Opt.Soc.Am.B **15** No. 7, 2057 (1998).
11. M.A. Vorontsov and W.B. Miller (eds.), *Self-Organization in Optical Systems and Applications in Information Technology*, Springer, Berlin (1995).
12. W. Lu, D. Yu, and R.G. Harrison, Phys. Rev. Lett. **76**, 3316 (1996); R. Martin, A.J. Kent, G. D'Allesandro, G.L. Oppo, Opt. Comm. **127**,161 (1996)
13. R. Martin, A.J. Scroggie, G.-L. Oppo, W.J. Firth, Phys. Rev. Lett. **77**, 4007 (1996).
14. A.V. Mamaev and M. Saffman, Phys. Rev. Lett. **80** No. 16, 3499 (1998).
15. S. Juul Jensen, M. Schwab, C. Denz, Phys. Rev. Lett. **81** No. 8, 1614 (1998).
16. N.V. Kukhtarev, V.B. Markov, S.G. Odulov, M.S. Soskin, V.L. Vinetskii, Ferroelectrics **22**, 961 (1979).