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Fourier control of pattern formation in an interferometric feedback configuration

M. Schwab, C. Denz, and M. Saffman[1]

We present a control method for a minimally invasive manipulation of pattern states occurring in feedback systems with an optical nonlinearity. An interferometric feedback configuration is used to control spontaneously formed patterns by the method of Fourier filtering. Here, we present experimental results for a photorefractive two-arm feedback system. A comparison with results of a numerical simulation based on a thin saturable Kerr-slice model is performed.

A large number of physical, chemical, or biological systems that are excited beyond an instability threshold are known to lead to the spontaneous formation of periodic spatial structures. Nonlinear optical systems are well-suited for the investigation of these structures due to the ease in accessing relevant control parameters. Moreover, pattern formation in optics offers a wide range of possible technological applications, for example in information processing [1]. A simple configuration using a nonlinear optical material and a feedback mirror for creating the counterpropagating beam is sufficient for observing transverse structures. Single feedback systems containing liquid crystals or liquid crystal light valves, atomic vapours, organic films, and photorefractive materials show similar behaviour when excited beyond the instability threshold (see [2] for references). The favored structure for the photorefractive single feedback system is a hexagonal one, but even squares, squeezed hexagonal or roll patterns [3, 2, 4] can be excited. For fixed parameters, the nonlinear optical system selects only one spatial pattern, whereas other patterns are suppressed and may not be accessible to the observer.

Optical pattern control represents a fast growing field of research and has attracted considerable interest [5]. Especially in the realm of technological applications, it is of interest to control the pattern the system selects, i.e. to access the maximum number of different existing patterns in a special parameter region. In this context, the stabilization and manipulation of otherwise unstable pattern states is currently of high interest. A control method based on spatial filtering in the Fourier spectrum of the pattern was proposed theoretically [6, 7], and soon afterwards demonstrated experimentally [8, 9, 10, 11]. Spectral control techniques are extremely attractive for optical systems since the Fourier space is accessible in real-time by using a single lens. The control method originally proposed in [6] has the important property that it is basically noninvasive. When the desired state is reached, the control signal declines to a very small level as in conventional electronic control schemes and otherwise unstable states of the underlying system are stabilized. Here, we present results for minimally-invasive control of a photorefractive feedback system by using a Michelson-interferometer feedback scheme.

As shown in fig. 1, we investigate a pattern forming system with a two-arm interferometric feedback. The pattern arm of the interferometer provides the majority of the energy in the feedback loop and serves as the unperturbed pattern forming system. The other arm, which we call the control arm, filters a small portion of energy in order to control and select the patterns in the pattern arm. This Michelson-like control method represents an extension and an improvement of our previously published ring control [9] or linear control scheme [10], since the pattern forming and the control arm are completely separated from each other. We distinguish between the terms positive control, where the control signal is fed back constructively (and thereby enhancing a desired pattern) and negative control. In the latter case, unwanted components are suppressed by destructive feedback of the control signal which nearly vanishes when achieving the desired state. In the case of the positive control, the control signal (i. e. the energy that is fed back into the system) tends to a certain steady-state level, whereas negative control implies that the control signal declines to a very small level in the desired pattern state. In this context, the term "minimally" means for the positive control that the control signal should be as small as possible at the very first stage of control and in the asymptotic steady state. By applying this positive control technique, we are able to excite rolls and squares in a parameter region where only a hexagonal pattern is experimentally avail-

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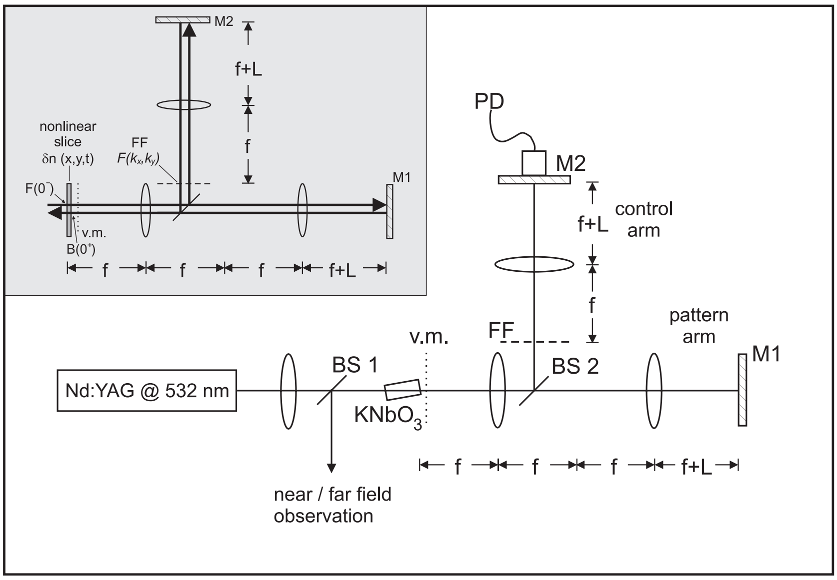

Fig. 1: Experimental setup. M=mirror, v.m.=virtual mirror, PD=piezo driver, FF=Fourier filter, L=propagation length, BS=beam splitter. Grey-shaded inset: Theoretical model with nonlinear slice and forward (**F**) and backward beam (**B**)

able without any control. For the negative control method, the minimally-invasive property also implies the minimization for the initial stage of the control process as well as the fact that the control signal declines to a very small but non-zero level. This negative control method is sufficient to rotate the spatial orientation of the hexagonal pattern. It is worth mentioning here that we do not restrict ourselves to a "noninvasive" stabilization of otherwise unstable states as proposed in [6, 7]. We perform a minimally-invasive manipulation and control of patterns, and demonstrate the selection of different pattern states. Our aim is to manipulate the system in a way that other patterns become stable in parameter space regions, where, without any control, the hexagonal pattern is the only experimentally obtainable solution.

The experimental setup is depicted in fig. 1. The beam derived from a frequency-doubled Nd:YAG-laser operating at a wavelength of $\lambda = 532$ nm was focused onto the exit face of a photorefractive $KNbO_3$:Fe-crystal measuring $l = 5.2$ mm along the c-axis. Polarization was chosen to be along the crystal's a-axis to exploit the large r_{13}-component of the electrooptic tensor. The propagation direction of the light was nearly parallel to the c-axis of the crystal which was oriented such that the incoming beam was depleted when passing the crystal. The power incident on the crystal was $P = 10$ mW and the Gaussian diameter of the beam was measured to be $d = 320 \mu$m. Feedback is provided by an interferometric setup of Michelson-type, separating the pattern from the control arm. Both feedback arms are basically identical 4f-4f imaging systems, containing lenses of focal lengths $f = 100$ mm and mirrors M1 and M2, respectively. A beam splitter right after the first lens in the feedback arm provides access to the control arm, where manipulation in Fourier space is performed. Amplitude masks of different shapes can be inserted in the Fourier plane that is created between the two lenses.

Let us now assume that only the pattern arm operates, i. e. the control arm is closed. The 4f-4f imaging system with a diffraction length L creates a virtual mirror at a distance L behind the crystal [12], incorporating the fundamental advantage that negative propagation lengths can be achieved and that the Fourier spectrum of the pattern is automatically created between the two lenses of the feedback arm. The lenses were adjusted to give a virtual mirror (v.m.) position of $L = 1$ mm behind the crystal in order to create a hexagonal pattern with a large aspect ratio.

Transverse instability in this geometry is known to be due to reflection gratings [2, 13] with a wave number of $k_g = 2k_0n_0$, where k_0 is the wave number of the incoming beam, and n_0 the linear refractive index of the crystal. $KNbO_3$ is known to be a purely diffusion-dominated material exhibiting pure energy-coupling between the counterpropagating beams. It is noteworthy that in certain parameter space regions, especially for situations when the virtual mirror is inside the crystal, a number of different pattern states exist, as square, squeezed hexagonal [3] or even rectangular or parallelogram-like patterns [14]. Nevertheless, the structure the system selects for the given virtual mirror position of L= 1 mm is purely hexagonal, as depicted in fig. 2, top left picture.

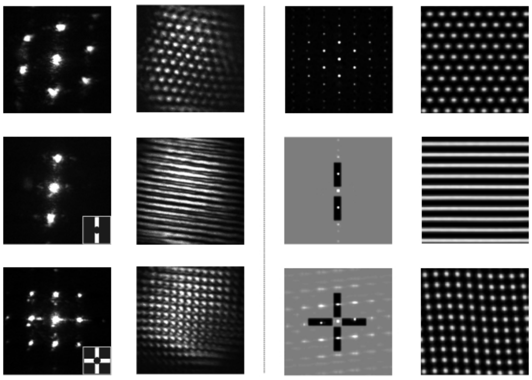

Fig. 2: Experimental (left) and theoretical results derived from a numerical simulation (right) for in-phase control. Top row: Hexagonal pattern appearing without control in the far and near field, middle and lower row: Results for the preparation of a roll and a square pattern with filters as indicated in the insets, grey-shaded in the simulations.

The propagation lengths of both arms of the Michelson interferometer can be adjusted independently and the relative phase of the two arms can be monitored easily at the appropriate exit of the beam splitter. A piezo-driven mirror is used for controlled adjustment of the relative phase of both arms, i.e. for achieving negative (relative phase $\varphi = \pi$) or positive (relative phase $\varphi = 0$) control. This scheme allows us (by means of an attenuator in the control arm) to adjust continuously the control strength in order to minimize it. The reflectivity ratio of both feedback arms was $R_1/R_2 = 1.5$ with $R_1 = 15.8\%$ and $R_2 = 10.5\%$. R_1 and R_2 are defined as the intensity reflectivities (including both passes through the beam splitter, lens system and mirror reflectivities) of the pattern arm and the control arm, respectively. The propagation lengths of both arms were adjusted to be $L = 1$ mm leading to identical transverse scales of the hexagonal patterns created by both arms individually. Control is performed in the Fourier plane of the control arm, where Fourier filters of appropriate shapes can be inserted. The Fourier filters we use are binary amplitude masks with different shapes: A slit filter for positive control of rolls, a cross filter for positive control of squares and a three-fold slit filter for negative control of hexagons. Examples for the experimental positive control of a roll and a square pattern are depicted on the left of fig. 2. The central spot was always blocked in our experiments in order to avoid undesired effects from two-beam interference which may lead to different complicated pattern formation effects [15]. The system adjusts to the presented symmetry of the Fourier filter (as shown in the insets) and selects the corresponding solution. Clear and rapid switching behaviour on time scales comparable to the time constant of the crystal of less than 1 s were observed. When the control path was closed, the predominant hexagonal structure always reappeared. Note that in the case of positive control, the control signal does not vanish. Therefore, it is not a stabilization of unstable states, but an improvement compared to previous approaches that used a single feedback arm [8]. The control signal (defined as the control intensity with respect to the intensity in the original pattern forming arm without control) was just 4.7 %, but it was sufficient to control the behaviour of the whole system. This value was the best obtainable in our experiment, a further minimization may be possible by optimizing the experimental setup (wavefront alignment, relative phase of both arms etc.). The control strength in the notation given in [6, 7] can be calculated to be $s = 0.3$ for roll and square pattern control. In both cases, the power absorbed by the filter declines to a small level.

As an example of negative control, we manipulated the orientation of the hexagonal pattern by using a three-fold slit mask, where the slits were tilted by 60° relative to each other (see. fig. 3). Here, the control signal (the power reinjected into the system) tends to a very small level as suggested and calculated in [6] with the difference that, in this case, our aim is not to stabilize unstable states as the "negative control" defined in [6] requires. Here, we manipulate a given stable pattern regarding its orientation. Note that the system remained in the chosen position when the control arm was closed. For comparison with theory, we performed a numerical simulation using the same two-dimensional feedback (see inset of fig. 1). A numerical analysis of the photorefractive two-beam coupling equations in the presence of reflection gratings and diffraction is computationally expensive. Therefore we use a simplified generic model based on a transmission grating mediated interaction in a thin slice of a sluggish medium with cubic nonlinearity (see [16] for details). The numerical simulation was performed using the same values as given in the experiment (L=1 mm, λ=532 nm, and R_1/R_2=1.5). The simulation started from an initial hexagonal pattern (as depicted on the top right of fig. 2), which is the solution for this system without using a Fourier mask. When a spatial filter was inserted

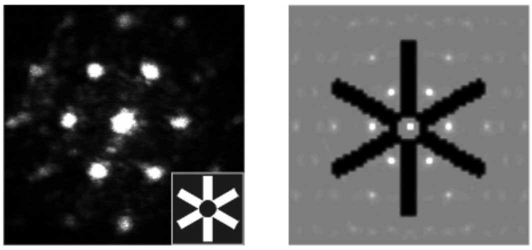

Fig. 3: Rotation of the hexagon position by out-of-phase control. Experimentally obtained pattern (left, with filter in the inset) and result from a numerical simulation, filter grey-shaded, on the right.

(grey-shaded region) with a certain geometry, the system adjusted to the situation given, and a roll or a square pattern could be excited. Comparing the experimental on the left and the numerical results on the right side of fig. 2 yields a complete qualitative agreement, despite the differences in the nonlinearities used for experiment and numerical simulation. We were also able to show pattern manipulation using negative control.

For manipulation of a hexagonal pattern, we used the same three-fold slit mask as in experiment (fig. 3). Starting with a hexagonal pattern with

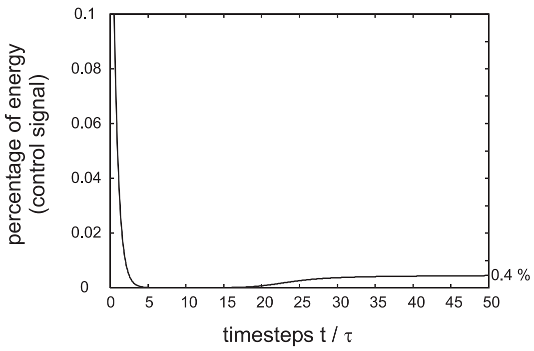

Fig. 4: Control signal fed back into the system as percentage of the energy with respect to the total energy in the pattern arm. Results of the numerical simulation leading to Figure 3 right.

two spots arranged on the vertical axis, we were able to turn the hexagon position by feeding back destructively unwanted components. As a result, a rotated hexagonal pattern with two spots on the horizontal axis can be obtained. Here, as an important property of this control method, the control signal nearly vanishes when the desired state is selected. The control signal, i. e. the fraction of energy re-injected into the system relative to the total power in the pattern arm, is depicted in fig. 4. The signal declines from the initial to a very small value of 1.35×10^{-5} at $t/\tau \approx 9$ and then approaches an asymptotic value of about 0.4 %.

In conclusion, we showed the efficiency of this method to control, manipulate and select spontaneously formed patterns. Only a small control signal was applied to the system proving the effectiveness and minimally-invasive character of this method.

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