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Stabilization, manipulation and control of transverse optical patterns in a photorefractive feedback system

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Abstract. We present an experimental realization of an almost non-invasive stabilization and manipulation method of coexisting and unstable states of pattern forming systems. In a photorefractive single feedback system, a control path is used to realize amplitude and phase-sensitive Fourier-plane filtering, utilizing only a few per cent of the system's intensity. By that means, we were able to stabilize desired but not predominantly excited patterns in parameter space regions where several patterns are present as coexisting or underlying solutions. Changing the phase of the control signal allows one to switch between different pattern states.

Keywords: Pattern formation, control, nonlinear optics, photorefractive

1. Introduction

Transverse modulational instabilities of counterpropagating beams [1] are well known to lead to the spontaneous formation of a variety of complex spatial structures, among them conical rings, pairs of spots (rolls), or arrays of spots arranged in hexagonal or square symmetry. Many systems have revealed similar phenomena, as e.g. atomic vapours [2], liquid crystals [3], organic films [4] or, recently, photorefractives [5].

All systems have a large number of unstable pattern states in common, even in the presence of a single stable output. This is related to the breaking of the rotational and translational symmetry of the system. From an application point of view of pattern formation in these systems, it is of interest to access the whole range of solutions, and to control the structures inherent in the system. Controlling such a system by suppressing predominantly excited solutions, or by encouraging underlying solutions to become stable, therefore offers the opportunity to stabilize, select and manipulate these patterns in a well-defined way for a wide range of technological applications, e.g. in optical information processing.

A photorefractive single mirror feedback system allows one to realize and observe a rich variety of spontaneously formed patterns. In contrast to the classical single-feedback system for thin Kerr-media as introduced by D'Allesandro and Firth [6], we use a thick photorefractive medium providing a saturable nonlinearity. A recent realization of such a system led to the first observation of square patterns and squeezed hexagons in an optical pattern formation experiment [7]. Pattern formation in this system occurs through modulational instabilities that arise due to the formation of reflection gratings [8]. Above a certain threshold for the photorefractive coupling strength γl, satellite beams are generated with a particular angle Θ relative to the central beam. Because the nonlinearity of these materials is proportional to the intensity ratio of the interacting beams, this configuration allows for pattern formation with moderate laser power. Moreover, photorefractives are well-suited for experimental pattern control, since their intrinsically slow dynamics simplifies time-resolved measurements.

In experimental optical systems, propagational effects associated with pattern formation are easily observed in the far field, which is nothing more than the representation of the power spectrum of the pattern—its Fourier transform. The big advantage of optics is that the Fourier transform is easily obtained by use of a simple lens. Manipulation in the Fourier domain (spatial filtering), is one of the most important concepts in modern optics. The combination of feedback control with manipulation of the Fourier space therefore allows us to control the influence of certain spectral components on the nonlinear spatio-temporal pattern formation.

The easiest way to realize the method of Fourier filtering in a photorefractive feedback experiment is to insert a spatial filter in the original feedback path. This is in fact a simple, but

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very powerful means to access the whole range of underlying solutions. The method is strongly invasive and changes the feedback system as a whole. Therefore, it is desirable to control the system without changing it, i.e. to present a non-invasive control method.

A common approach to the non-invasive control of spatially extended systems is to provide a control signal that is strong enough to push the system to one member of a possibly infinite family of solutions that are inherent in the system. The technological aim is to produce a desirable behaviour by carefully applying the control signal that directs the system to the target state and keeps it there, while at the same time not changing or influencing the system dramatically. The intervention into the system should be as small as possible. Moreover, it is desirable to design the control signal in such a way that its magnitude decreases as the system approaches the desired state, and—in the absence of noise—vanishes when the system is locked to a certain solution. It is also our aim to control a state that is only approximately a true stable state of the system, e.g. if a real experimental system under stress produces states that are distorted compared with the solutions of the infinite, idealized system. In that case, one might expect the feedback to become small, but not to vanish completely. These features have led Martin *et al* [9] to suggest the possibility of using control techniques which operate in the spatial Fourier domain of the control arm to stabilize unstable patterns and to choose between alternative stable states. This technique has been applied theoretically to a variety of nonlinear optical systems, e.g. a feedback system including a liquid crystal light valve as the nonlinear element. Here, we describe an extension of a first experimental implementation of this technique [10].

This paper is organized in the following way. In section 2, we give a description of the experimental set-up. We give a survey on the stationary patterns appearing in this photorefractive feedback system and discuss the main properties of the system. In section 3, an invasive method for stabilizing patterns using Fourier filtering techniques is realized. Section 4, in contrast, deals with a non-invasive control method. For this purpose, the control signal is supplemented by a Fabry–Perot-type feedback. The control signal contains only a small percentage of the total energy of the system. We present evidence that enhancement of a desired pattern is possible by positive (in-phase) control, and show that the suppression of a predominant pattern, in order to allow other patterns to stabilize, is possible by negative (out-of-phase) control. This is followed by a brief discussion and conclusions in section 5.

2. Stationary patterns

Our principle experimental set-up is shown in figure 1. A laser beam with a power of 23 mW, obtained from a frequency-doubled Nd:YAG laser, is focused onto the exit face of an iron-doped $KNbO_3$ crystal measuring 5.6 mm along its c-axis. The beam diameter was measured to be 350 μm. In order to avoid undesired back-reflections from the surfaces, the crystal was inclined about 6° relative to the direction of propagation. In this geometry, it is well known that energy coupling takes place. The crystal c-axis lies in the direction of the input beam, leading to depletion of the incoming beam. The direction of polarization is parallel to the crystal a-axis, thus allowing us to exploit the largest electro-optic coefficient r_{13} of $KNbO_3$. The counterpropagating, backward pump beam is generated by a dielectric mirror, variable in reflection by lateral movement. Thus, reflectivities that guarantee spontaneous roll, hexagon, or square pattern formation [11, 12] can be adjusted, as well as regions of instability that allow for observation of competition between different pattern types. The mirror is positioned at the end of a confocal (2f–2f) feedback system with a $f = 120$ mm focal lens in its middle. This specific configuration allows us to adjust positive as well as negative propagation lengths. Moreover, it allows more exact positioning compared with conventional feedback configurations. Beam splitter 1 enables observation of the feedback beam, carrying the transverse structures. Beam splitter 2 separates this beam for observation of the far field and, by means of a Fourier transforming lens, the near-field pattern via CCD 1 and CCD 2, respectively.

The predominant pattern the system selects is a hexagonal one, as depicted in the inset of figure 1. Second- and third-order sidebands are incorporated in the structure; the corresponding near field represents a honeycomb structure. This picture was taken for a maximum feedback reflectivity of $r = 66\%$ (including losses by Fresnel reflections at the crystal surfaces).

A linear stability analysis shows that above a certain threshold, a modulational instability of counterpropagating waves appears giving rise to spatial sidebands in the far field. To analyse the threshold condition for the instability of wavevectors in the transverse plane, we start with the usual photorefractive two-wave mixing equations, taking into account the beam profile in the transverse direction. With the assumption that reflection gratings are dominant in this configuration, these coupling equations can be written as [8]

$$\frac{\partial F}{\partial z} - \frac{\mathrm{i}}{2k_0n_0}\nabla_\perp^2 F = \mathrm{i}\gamma\frac{|B|^2}{|F|^2+|B|^2}F \tag{1}$$

$$\frac{\partial B}{\partial z} + \frac{\mathrm{i}}{2k_0n_0}\nabla_\perp^2 B = -\mathrm{i}\gamma^*\frac{|F|^2}{|F|^2+|B|^2}B, \tag{2}$$

where F and B are the amplitudes of the forward and backward wave (pump and feedback), z is the direction of propagation, k_0 the wavenumber in vacuum, n_0 denotes the linear refractive index of the nonlinear medium, γ is the complex photorefractive coupling coefficient and $_\perp$ denotes the transverse Laplacian. The amplitudes of the forward and backward wave with weak modulations of the wavevector in the transverse direction can be written as

$$F(\boldsymbol{r}) = F_0(z)[1 + F_{+1}(z)\exp(\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp) + F_{-1}(z)\exp(-\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp)] \tag{3}$$

$$B(\boldsymbol{r}) = B_0(z)[1 + B_{+1}(z)\exp(\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp) + B_{-1}(z)\exp(-\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp)], \tag{4}$$

where $F_{\pm1}$ and $F_{\pm1}$ are the relative amplitudes of the spatial sidebands. Assuming a feedback reflectivity of unity and no absorption in the medium (see [8] for details), the threshold condition for transverse instability can be

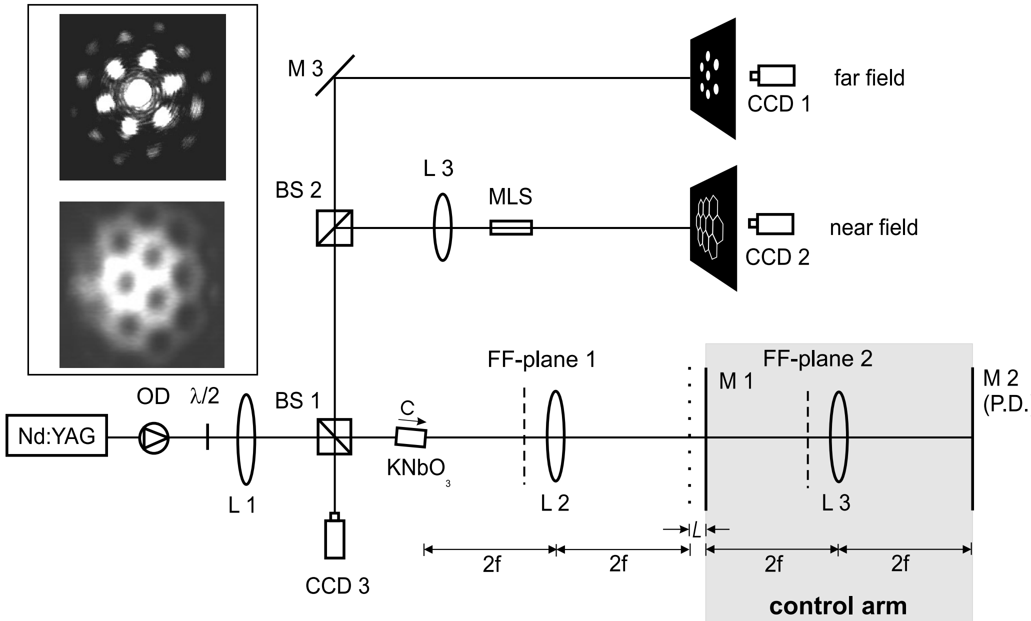

Figure 1. Experimental set-up. OD= optical diode, L= lenses, M= mirror, BS= beam splitter, MLS= microscope lens system, FF-plane= planes of Fourier filtering, dotted line: exact 2f–2f-position, L= propagation length. Shaded: Control arm for non-invasive control with a second linear feedback. Top of insert: typical hexagonal pattern with second- and third-order terms, bottom: corresponding near-field pattern.

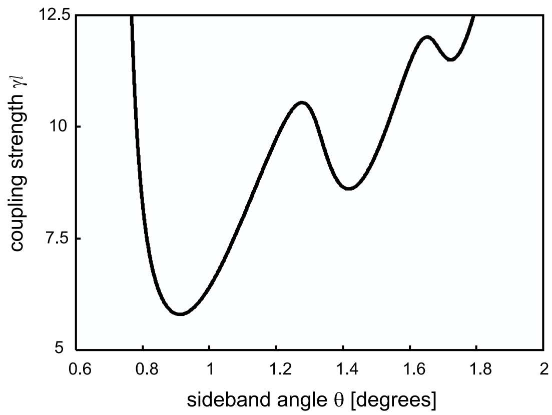

Figure 2. Plot of the threshold coupling strength γl for $n_0 L/l = -0.7$.

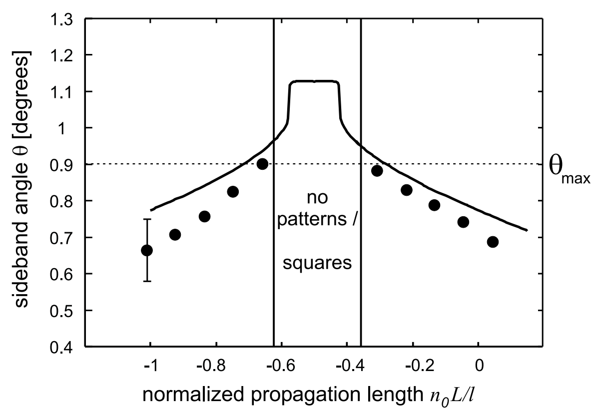

Figure 3. Sideband angle Θ as a function of the normalized propagation length; theoretical curve (full curve) and experimental results (dots).

obtained. In figure 2, an exemplary plot of the threshold coupling strength γl is shown for the normalized propagation length $n_0 L/l = -0.7$. For each value of the normalized propagation length $n_0 L/l$, the angle of sideband generation can be calculated, and a good agreement with the theoretical pattern size can be obtained experimentally (see figure 3). A maximum pattern size Θ_{max} appears in the experiment, which is due to a photorefractive coupling constant below threshold for the appearance of transverse structures in this parameter region [11].

The systematic deviation from the theoretical curve may be explained by absorption and Fresnel losses changing the overall reflectivity of the feedback system ($r = B/F = 0.66$ due to Fresnel losses only), whereas theory uses a $r = B/F$ ratio of 1. The linear stability analysis gives the explanation for the appearance of spatial sidebands, i.e. the existence of a conical ring with a certain transverse wavevector, but is not able to predict the pattern the system selects. A first approach to a nonlinear stability analysis was published recently [13] and a set of Ginzburg–Landau equations was derived, but no regions of stability or instability of certain patterns were given. Nevertheless, the stability of hexagons was shown to be a consequence of wave processes between higher-order sidebands and the primary hexagon. This is in full agreement with our experimental results, as these second- and third-order sidebands were clearly visible (see inset of figure 1).

We have previously reported the observation of square patterns in this system for the same crystal with a coupling strength of $\gamma l = 11.5$, but for higher reflectivity of $r = 84\%$ achieved by using index-matching oil, and a virtual mirror position inside the crystal [7]. It should be noted that the pattern collapses in this region for smaller values of γl. When the virtual mirror was placed at a position between regions

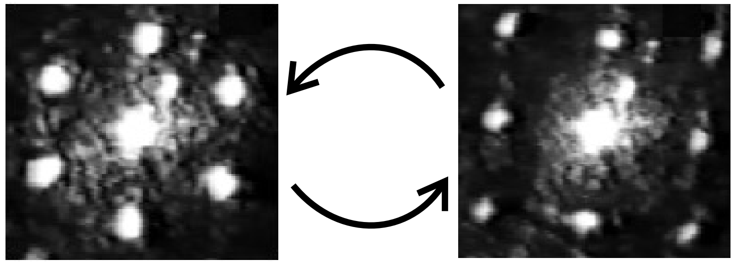

Figure 4. Hexagonal and square pattern alternating in time with a timescale of a few seconds. Virtual mirror position $n_0L/l = -0.25$.

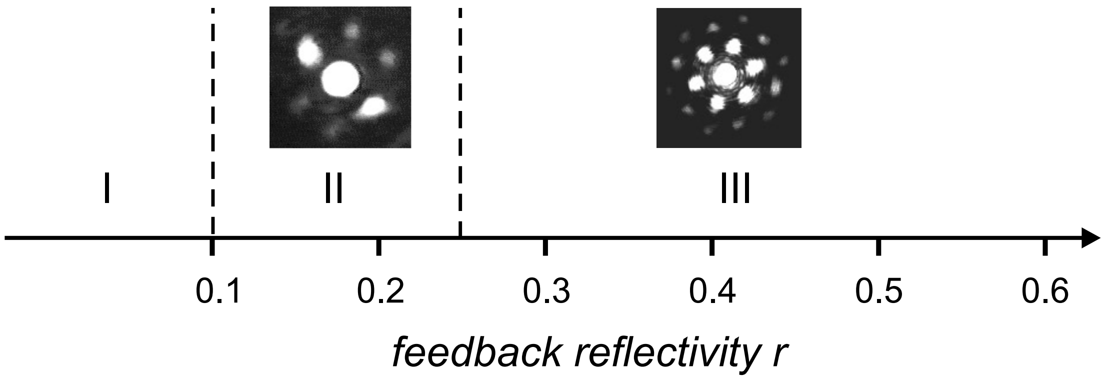

Figure 5. Dependence of pattern type on feedback reflectivity. Region I: no observable patterns, II: coexistence of roll and hexagonal pattern, transition to rolls, III: hexagonal pattern.

where stable hexagonal or square patterns were observed ($n_0L/l = -0.25$), hexagonal and square patterns (shown in figure 4) appeared alternately in time in a non-periodic manner.

Our analysis is restricted to the case of collinear pump and feedback beams, whereas a slight asymmetry leads to various types of dynamics, including competition between different patterns and periodic pattern movements [11, 12]. However, a spontaneous temporal alternation of patterns without an induced asymmetry can be observed in this system. When the reflectivity of the feedback mirror is lowered, a remarkable pattern transition occurs: for $10\% \leqslant r \leqslant 25\%$, a coexistence of rolls and hexagons was apparent, and for $r \leqslant 10\%$, no patterns were observed at all (see figure 5).

3. Invasive method for stabilizing desired patterns

In the previous section we reported that this system contains a variety of different solutions in certain parameter regions. Here we present a powerful tool to select and stabilize an otherwise unstable or underlying stable solution—the method of direct Fourier filtering, a strongly invasive method. It was first shown experimentally by Mamaev *et al* [14] for a similar system, also using a photorefractive nonlinearity.

Near the position of lens L2 in the feedback path, the far field of the pattern is clearly visible, thus indicating the possibility to perform Fourier filtering in this plane (Fourier plane 1 in figure 1). By inserting specially shaped filters, stable system solutions (e.g. hexagons) can be suppressed, thereby selecting a novel and desired solution. When a variable slit was inserted and the slit size was narrowed, a remarkable transition from the hexagonal over the squeezed hexagonal to the final roll solution could be observed, as depicted in figures 6(*a*)–(c). A square solution can also be obtained (figure 6(*d*)) by use of an appropriate Fourier filter shown in the inset. The system always adjusts to the presented symmetry. Rotating the filter enables to select every desired orientation of a pattern. For the case of roll and square patterns, it was possible to rotate the patterns continuously by 360° around the central spot by slowly rotating the Fourier filter.

Roll patterns have an interesting property that the system control by the filter (the power absorbed) declines to zero in the equilibrium state. In contrast, in the case of hexagonal or square patterns, higher-order spots are emitted during readout of the grating in the photorefractive medium by the forward beam, and are observable at the filter. These higher-order spots are blocked by the Fourier filter, leading to small but non-zero power losses in the system for the latter case, even in the equilibrium state.

Note that the complete Fourier filter is not necessary to achieve a desired pattern. For the case of the square pattern, the symmetry of the system requires only one fourth of the full filter. No other patterns than the one inherent to the system could be achieved, though a variety of differently shaped filters were used. Furthermore, it was not possible to achieve any pattern in parameter regions where, without a Fourier filter, no pattern was observed.

4. Non-invasive control of pattern formation

The method of direct Fourier filtering, as described above, is an easy and extremely flexible tool to stabilize and select

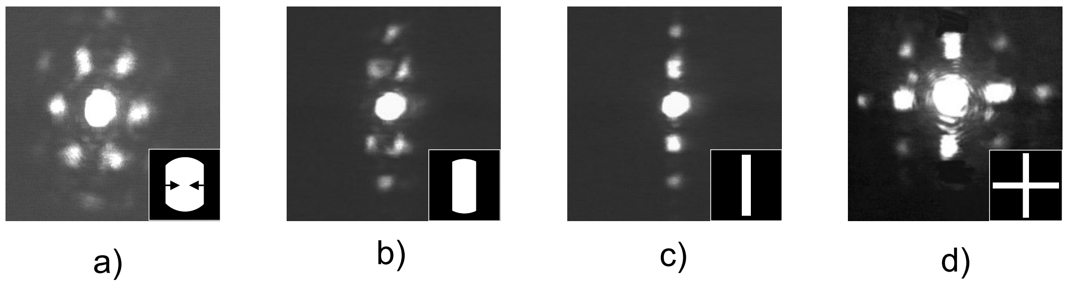

Figure 6. Invasive method to manipulate patterns. Preparation of (*a*) roll pattern, (*b*) squeezed hexagonal pattern, (*c*) square pattern in a parameter region where hexagonal patterns are predominantly excited. The inset shows the corresponding Fourier filter.

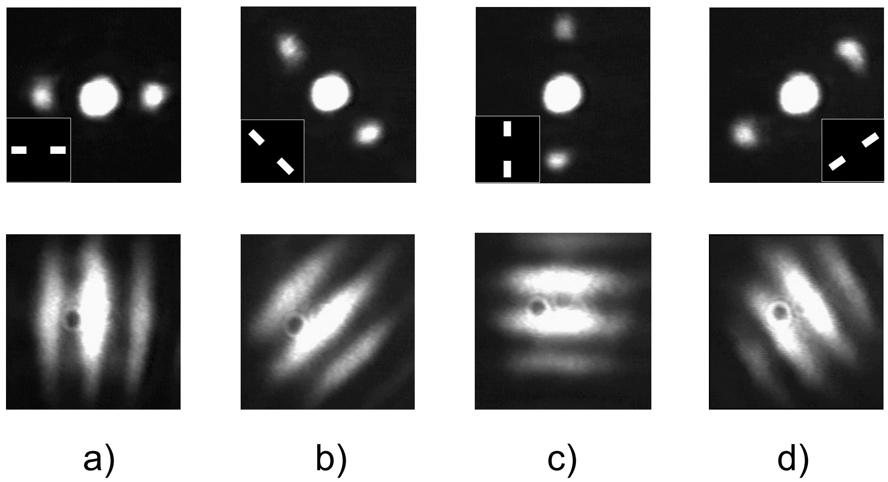

Figure 7. Examples for non-invasive pattern manipulation. Top: far-field pattern, bottom: near-field pattern. (*a*)–(*d*) Roll patterns in different directions with non-invasive control using the filters shown in the insets.

certain stable or unstable patterns, but has the obvious disadvantage that the removal of certain spectral components from the feedback constitutes a strong invasion into the system. However, in order to access the whole range of solutions without changing the system dramatically, we are looking for a realization of a minimum invasive method to control pattern formation. There are several possible experimental realizations for pattern control with minimal invasion, as e.g. a ring feedback [10]. Here, we focus on a second, linear feedback behind the original feedback system which is depicted in the shaded region of figure 1. From an experimental point of view, this system offers an easier access to pattern manipulation and control avoiding the problem of bidirectional oscillations for the ring feedback geometry in [10]. This new system offers improved results compared with the previous one, but the qualitative results are basically identical.

The small transmission signal of mirror M1 is fed back by another 2f–2f-system which consists of lens L3 with a focal length of $f = 120$ mm (the same focal length as lens L2) and a piezo-driven mirror M2 as the feedback mirror for accurate phase-adjustment. The plane of Fourier filtering is again near the lens (Fourier plane 2), where the far field of the pattern is clearly observable. In order to avoid Fabry–Perot resonances, the central spot is again blocked throughout all measurements. The intensity that is fed back into the system was always in the range of 1–2% of the original intensity in the feedback path corresponding to 10–14% of the field amplitude.

We applied the Fourier filtering technique in different parameter regions of the feedback reflectivity r, which represents an important control parameter of the system. We distinguish between a region of pure and stable hexagonal patterns (r large) and an intermediate region where roll and hexagonal patterns are coexisting (see again figure 5).

The stationary pattern that is obtained for the intermediate reflectivity region (around $r = 25\%$) represents a mixed hexagonal-roll state (figure 5 or figure 2 in [10]). Using a slit filter in the Fourier plane, it was possible to stabilize the nearest solution of the system, which is a roll pattern in the filter direction, as shown in figures 7(*a*)–(d). By changing the orientation of the filter, rolls can be excited in any desired orientation by rotating the slit Fourier filter in the control arm. If only a single spot is allowed to pass the Fourier filter and is reinjected into the system, the corresponding roll solution in the appropriate direction is again excited. Note that all these results were obtained using in-phase feedback. This feedback method suppresses undesired directions and offers the system a preferred direction, by feeding back only the ‘correct’ Fourier components.

It was not possible to stabilize squares by this non-invasive control method in any region of reflectivity. For the intermediate reflectivity region where rolls and hexagons are coexisting, a square filter was used to switch between two orthogonal roll directions. A roll solution in one of the two directions could easily be achieved. A subsequent π-phase

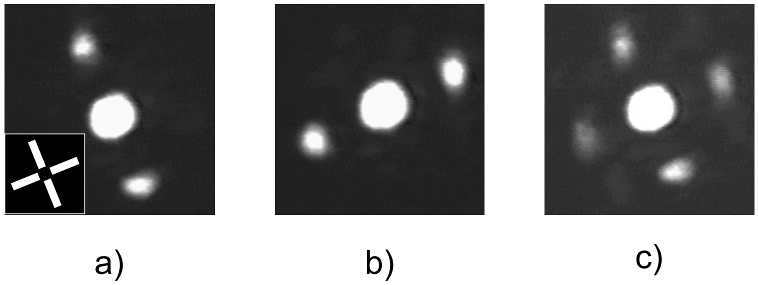

Figure 8. Switch between two orthogonal roll orientations. (*a*) Roll pattern in the direction of one slit (Fourier filter depicted in the inset), (*b*) roll pattern excited by a π phase shift with orthogonal orientation, (*c*) snapshot of an unstable square pattern for an intermediate phase.

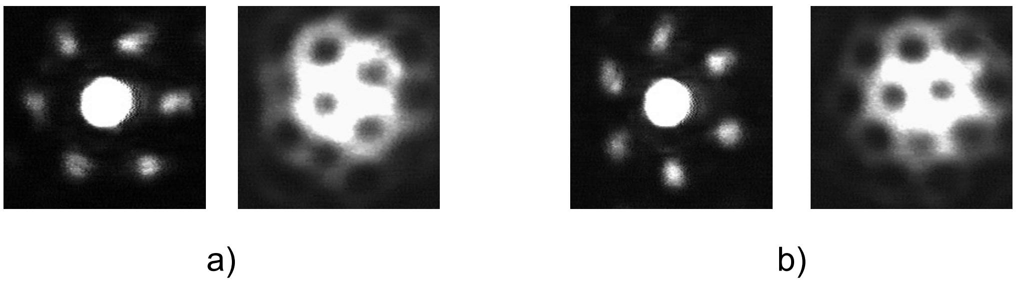

Figure 9. Positive and negative filtering in the case of a hexagonal pattern. The control signal is a roll pattern oriented in the horizontal plane. (*a*) Positive control emphasizing the pattern by constructive interference, (*b*) negative control suppressing the horizontal direction by destructive interference, thus favouring the orthogonal direction.

shift then enabled us to switch to the orthogonal roll direction. For an intermediate phase, a square appeared as a transition pattern, but could actually not be stabilized in this parameter region (see figure 8).

In a region where the reflectivity is chosen to be high enough to guarantee a stable hexagonal solution ($r > 40\%$), our control method enables us to choose between different hexagon orientations by in-phase and out-of-phase feedback. Using positive feedback, the direction of the hexagon adjusts to the position of the filter with the hexagon containing these two spots (figure 9(*a*)). Negative feedback leads to a different hexagon position, avoiding the two spots that are fed back as a control signal (figure 9(*b*)). Switching between positive and negative feedback, therefore, allows one to choose between the two spatially orthogonal hexagon orientations. For the case of the negative feedback, the feedback signal vanishes when the desired state is achieved.

By rotating the slit filter, a continous rotation of the hexagon was possible. After switching off the control signal, the hexagon remained unchanged in its position. This indicates clearly that the orientation of a hexagonal pattern (without any Fourier control) depends only on small variations in the starting conditions or on a slight symmetry breaking of the system in the early phase of its development.

For higher reflectivities, $r > 40\%$, the hexagonal solution was dominant and no other patterns could be excited in these parameter regions using both non-invasive control methods, though the invasive control method indicated that other solutions were underlying and could be excited. Here, the control signal was obviously too weak ($<0.5\%$) to induce any changes to the hexagonal pattern dominating in the system.

5. Discussion, conclusion and outlook

A photorefractive single feedback system offers a great variety of different solutions. Although the predominantly excited pattern is a hexagonal one, rolls and squeezed hexagonal or square patterns can also be observed. The method of Fourier plane filtering allowed us to stabilize, select and manipulate different patterns that can be formed in such a feedback system, realized with a photorefractive nonlinearity. An invasive feedback control with a spatial filter shows that the system adapts to the geometry of the filter, but only solutions inherent to the system are selected. We could achieve squeezed hexagonal, roll, and square patterns, and were able to choose the position of these patterns in space.

We have adapted this method to a non-invasive control system, where only a small amount of the system energy is coupled out, Fourier-filtered, and reinjected into the system. With only a small percentage of the system energy, we could achieve a selection of patterns in regions where two solutions were coexisting or where other stable solutions were underlying. Accurate wavefront alignment was not necessary in all our experiments.

However, not all patterns could be excited. Even if we were able to stabilize a certain pattern with the invasive manipulation method, the non-invasive control method did not necessarily work for this special pattern. This may be due to a too weak control signal fed back into the system. In a reflectivity region where hexagonal and roll patterns are coexisting, we were able to select a certain pattern by using only 1–2% of the system energy as the feedback signal. For example, inserting a slit filter into the control path induced a cleary visible roll solution. Additionally, we were able to choose the position of the pattern by rotating the filter. Using a square filter, we could switch between two orthogonal roll solutions, but no square pattern could be achieved, though the invasive manipulation indicated that this solution was underlying. The square pattern was visible for a short period of time as a transition state, but could not actually be stabilized.

For reflectivities in the region of stable hexagons ($r = 35\%$), we could adjust different hexagon orientations by positive and negative feedback with only 1% of the system energy as a feedback signal. In the latter case, the system control by the filter (the pattern reinjected into the system) declines to zero when the desired pattern state is reached. In addition, the hexagon could be tracked to one position even if the control was turned off. Although usually negative control is defined only for the stabilization and tracking of unstable states [9], this manipulation of an obviously stable hexagonal pattern proves the usefulness of negative feedback, in full agreement with the theoretical predictions. For a feedback signal $< 0.5\%$ of the system energy, pattern control was not possible in a region where the hexagonal pattern appeared to be the predominant solution of the system.

Since a complete nonlinear stability analysis is still missing for this photorefractive feedback system, we do not know anything about the regions of stability for the different pattern types. Our aim is to explore these regions experimentally by use of the Fourier filtering control technique.

In all our experiments, the filters were inserted and removed manually. Automatizing this procedure, e.g. by using an amplitude modulator, would result in a more detailed analysis concerning the reaction of the system to a new presented Fourier filter. Without disturbances, it may be possible to lock the system to a solution after ‘switching off’ the filter in the case of the invasive control method. This in turn proves the stability or instability of a certain solution.

In conclusion, the Fourier filtering control technique is a powerful and flexible method to control pattern formation without influencing the system dramatically. Otherwise, underlying stable or unstable patterns can be excited and explored by use of this method. In the case of the negative control, the control intensity vanishes when the desired state is reached, which has a remarkable similarity to conservative electric control units. We state that this stabilization method is not restricted to photorefractive media and is applicable to a variety of pattern forming nonlinear optical systems.

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References

[1] Vlasov S N and Talanov V I 1979 *Optical Phase Conjugation in Nonlinear Media* ed V I Bespalov (Gorki: Institute of Applied Physics, USSR Academy of Sciences)
Grynberg G 1988 *Opt. Commun.* **66** 321
Firth W J and Par C 1988 *Opt. Lett.* **13** 1096
[2] Grynberg G, LeBihan E, Verkerk P, Simoneau P, Leite J R R, Bloch D, LeBioteux S and Ducloy M 1988 *Opt. Commun.* **67** 363
Pender J and Hesselink L 1990 *J. Opt. Soc. Am.* B **7** 1361
[3] Macdonald R and Eichler H J 1992 *Opt. Commun.* **89** 289
Thüring B, Neubecker R and Tschudi T 1993 *Opt. Commun.* **102** 111
[4] Glückstad J and Saffman M 1995 *Opt. Lett.* **20** 551
[5] Honda T 1993 *Opt. Lett.* **18** 598
[6] D’Allesandro G and Firth W J 1991 *Phys. Rev. Lett.* **66** 2597
[7] Honda T, Matsumoto H, Sedlatschek M, Denz C and Tschudi T 1997 *Opt. Commun.* **133** 293
[8] Honda T and Banerjee P P 1996 *Opt. Lett.* **21** 779
[9] Martin R, Scroggie A J, Oppo G-L and Firth W J 1996 *Phys. Rev. Lett.* **77** 4007
[10] Juul Jensen S, Schwab M and Denz C 1998 *Phys. Rev. Lett.* **81** 1614
[11] Denz C, Schwab M, Sedlatschek M, Tschudi T and Honda T 1998 *J. Opt. Soc. Am.* B **15** 2057
[12] Schwab M, Sedlatschek M, Thüring B, Denz C and Tschudi T 1999 *Chaos, Solitons Fractals* accepted
[13] Lushnikov P M 1998 *J. Exp. Theor. Phys.* **86** 614
[14] Mamaev A V and Saffman M 1998 *Phys. Rev. Lett.* **80** 3499